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ANALYSIS OF A TRAVELING-WAVE BEAM ELECTRODE

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For
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Introduction

Several authors[1,2] have studied the response characteristics of wire-loop types of beam monitoring electrodes for use in particle accelerators. The purpose of this analysis is to obtain a rigorous solution for the frequency response characteristics of the traveling-wave type of beam pickup electrode shown schematically in Fig. 1. We consider the pickup wire and the vacuum tank wall as constituting a transmission line which can support transverse electromagnetic waves coupled from the $\overline{E}$ and $\overline{H}$ fields of the particle beam. We further want to consider matching the impedance seen by these waves at one port and to determine the resulting voltage appearing at the other (open-circuit) port. In the process of solving for the voltage, we derive explicit expressions for the electric field associated with a particle beam in a toroidal metal pipe of circular cross-section. The response of the pickup electrode to current waves on the inner conductor of a coaxial transmission line is also calculated and the results compared with those for a particle beam. MKS units are used throughout the discussion.

I. Method of Solution

Rumsey[3] has shown that the open-circuit voltage received by a metal antenna in an isotropic medium is given by

$$V = -\frac{1}{I_a}\int \overline{J}_a \cdot \overline{E}_b^{(+)}\, d\, J_a \qquad (1)$$

where V = the voltage appearing at the terminals of antenna $\underline{a}$ due to a source $\underline{b}$; $\overline{E}_b^{(+)}$ = the electric field of $\underline{b}$ which would exist in the absence of $\underline{a}$; $\overline{J}_a$ = the conduction current density which would flow on the surface of $\underline{a}$ if it were excited by a current source I_a with $\underline{b}$ turned off. ($\overline{J}_a$ will hereafter be referred to as the "test current".) These quantities are written in a-c complex form with $e^{-i\omega t}$ time dependence.

The distribution of the test current takes into account the interaction between the physical structures of a and b. For the beam monitoring problem, b becomes the charged particle beam and the metal pipe, and a becomes the length of line running along the pipe wall. We will assume that the pickup line can be made so as to present a constant impedance to waves traveling along its length and to completely absorb any energy incident upon whichever port we choose; the other port is left open and we calculate the voltage at the open end. To consider the voltage appearing at the downstream port, we imagine a current source of strength I_a connected between the tank wall and the pickup line at $z = \ell$, $r = a$. Neglecting the width of the wire, the test current is written as

$$\bar{J}_a = \begin{cases} I_a(-\hat{r}) \dfrac{\delta(\theta - \theta_b)\,\delta(z - \ell)}{r} e^{-i\beta(r - a)}, & a \leq r \leq b \\ I_a(-\hat{z}) \dfrac{\delta(\theta - \theta_b)\,\delta(r - b)}{r} e^{-i\beta(z - \ell + b - a)}, & 0 \leq z \leq \ell \\ I_a(\hat{r}) \dfrac{\delta(\theta - \theta_b)\,\delta(z)}{r} e^{+i\beta(r - 2b + \ell + a)}, & a \leq r \leq b \end{cases}$$

Here, $\beta = \omega/v_\phi$; v_ϕ = the characteristic phase velocity of the pickup line transmission system; $\hat{r}$ and $\hat{z}$ are unit vectors in cylindrical coordinates. (v_ϕ will be equal to c, the speed of light in free space, unless some dielectric material is purposely introduced into the system.)

II. Response to a Coaxial Line Source

The voltage appearing at each port when a sinusoidal current wave (or Fourier component of a current pulse) is traveling down a conductor placed at $r = 0$ is easily calculated from Eq. (1) if we assume that $(a - b) \ll b$. Under these circumstances, we can ignore any distortion of the test current due to the conductor at $r = 0$.

$E_b^{(+)}$ is now given by

$$\bar{E}_b^{(+)} = \frac{\eta I}{2\pi r} e^{i\gamma z} \hat{r}$$

where I = peak magnitude of current on inner conductor, $\eta = (\mu/\epsilon)^{1/2}$, and $\gamma = \omega(\mu\epsilon)^{1/2}$.

Performing the integration indicated in Eq. (1) yields the complex form of the voltage appearing at the downstream port when the upstream port is matched

$$V(\text{down}) = \frac{\eta I}{2\pi} \; e^{i\gamma(\ell+a)}[e^{-2ib\gamma} (\ln a/b + i\gamma\Delta) - \ln a/b + i\gamma\Delta] \qquad (3)$$

$$\Delta \equiv a - b$$

(We have put $\int_a^b \frac{e^{-i\gamma r}}{r} \, dr = \ln b/a + i\gamma\Delta$.)

The corresponding expression for the upstream voltage can be found by assuming an incident field with exponential dependence $e^{-i\gamma(z-\ell)}$. The result is

$$V(\text{up}) = \frac{\eta I}{2\pi} \; e^{i\gamma a} \; [e^{2i\gamma(\ell-b)} (\ln a/b + i\gamma\Delta) - \ln a/b + i\gamma\Delta]. \qquad (4)$$

Plots of the "transfer impedances" $\frac{V(\text{down})}{I}$ and $\frac{V(\text{up})}{I}$ as functions of the wave number γ of the incident field are shown in Figs. 2 and 3. The response shows directional properties similar to those predicted by Firestone and Knechtli.[4,5] We do not see "infinite directivity" in the sense of zero voltage appearing at the downstream port, however, because we have included the effects of the radially directed segments of the pickup line. The low frequency slope of the voltage curve corresponding to Fig. 2 is very close to that predicted by application of Faraday's law to the loop. Observation of the pulse response of a test model shows that the transfer function for the upstream port is given quite accurately by the simple expression

$$Z(f) = -2iAe^{+\pi i f \Lambda} \sin(\pi f \Lambda) \qquad (5)$$

where $\Lambda = \frac{2\ell}{v_\phi}$ = the time required for a signal to travel <u>twice</u> the length of the pickup line; A is a real constant which depends upon the lengths a and b. For a = 4.76 cm, b = 4.44 cm, the value of A is about two ohms.

III. Field Expressions for a Particle Beam

A. General Solution

We consider the $\overline{E}$ and $\overline{H}$ fields associated with a charged particle beam circulating in a perfectly conducting toroid of circular cross-section. The geometry is shown in Fig. 4. The method of solution of Maxwell's equations inside the tank is that of Dr. V. K. Neil and is described in detail in Ref. 6. It will be assumed for this analysis that $R \gg a$, and that the beam is of negligible cross-sectional area. The beam current density is written as

$$\overline{J} = \frac{\delta(r - r_o)\,\delta(\theta - \theta_o)}{r}\,\omega_o\, f\,(\phi - \omega_o t)\,\hat{\phi} \tag{6}$$

The corresponding charge density is

$$\rho = \frac{|\overline{J}|}{v_p} = \frac{\delta(r - r_o)\,\delta(\theta - \theta_o)}{r}\;\frac{f(\phi - \omega_o t)}{R} \tag{7}$$

Equation (6) defines a circulating current of magnitude $\omega_o f(\phi - \omega_o t)$ amperes located in the pipe cross-section at $r = r_o$, $\theta = \theta_o$; the charges are traveling with angular velocity $\omega_o = \frac{v_p}{R}$ radians/second and are distributed in azimuth according to the function $f(\phi)$.

Assuming a time dependence of the form $e^{-i\omega t}$, the $\overline{E}$ and $\overline{H}$ vectors in the tank can be expressed as a sum of the eigenfunctions characterized by potentials $\overline{A}_\lambda$ and ψ_σ which are solutions of the homogenous wave equations

$$\nabla^2 \overline{A}_\lambda + \omega_\lambda^2 \mu\epsilon\, \overline{A}_\lambda = 0$$

$$\nabla^2 \psi_\sigma + \omega_\sigma^2 \mu\epsilon\, \psi_\sigma = 0$$

The total vector magnetic potential $\overline{A}$ and scalar potential function ψ are expanded in terms of the $\overline{A}_\lambda$ and ψ_σ.

$$\overline{A}(r, \theta, z, t) = \sum_\lambda q_\lambda(t)\;\overline{A}_\lambda(r, \theta, Z) \tag{8}$$

$$\psi(r, \theta, z, t) = \sum_\sigma q_\sigma(t)\;\psi_\sigma(r, \theta, Z) \tag{9}$$

The subscripts λ and σ will be seen to imply several parameters over which one must sum to obtain the general solution. Once $\bar{A}$ and ψ are obtained, the electric field is

$$\bar{E} = -\nabla\psi - \frac{\partial\bar{A}}{\partial t} \tag{10}$$

$\bar{A}$ and ψ are related to the sources $\bar{J}$, ρ, by

$$\nabla^2\bar{A} - \mu\epsilon \frac{\partial}{\partial t}(\Delta\psi) - \mu\epsilon \frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J} \tag{11a}$$

$$\nabla^2\psi = -\rho\,\epsilon \tag{11b}$$

$$\nabla\cdot\bar{A} = 0 \tag{11c}$$

If the functions $\bar{A}_\lambda$ and ψ_σ are normalized such that

$$\int_V |\bar{A}_\lambda|^2\, dV = \int_V |\psi_\sigma|^2\, dV = 1$$

where V = volume of the toroid, then insertion of (8) and (9) into (11) yields the following relations for determining the sets $q_\lambda(t)$ and $q_\sigma(t)$:

$$\frac{d^2q_\lambda}{dt^2} + \omega_\lambda^2 q_\lambda = 1/\epsilon \int_V \bar{J}\cdot\bar{A}_\lambda^*\, dV \tag{12}$$

$$q_\sigma = \frac{1}{\omega_\sigma^2\,\mu\epsilon^2} \int_V \rho\,\psi_\sigma^*\, dV \tag{13}$$

(The asterisk denotes the complex conjugate.)

Since a large radius of curvature is assumed, the solutions of the homogeneous wave equations may be taken as those appropriate to a straight pipe, with the restriction that the distance $2\pi R$ must contain an integer number of wavelengths in the z direction.

The $\overline{A}$ and ψ_σ functions are therefore

$$\overline{A}_\lambda \cdot \hat{r} = - N_\lambda \frac{imk_\lambda}{R} J'_n (k_\lambda r) \; e^{i(n\theta + \frac{m}{R} z)}$$

$$\overline{A}_\lambda \cdot \hat{\theta} = N_\lambda \frac{nm}{rR} J_n(k_\lambda r) \; e^{i(n\theta + \frac{m}{R} z)}$$

$$\overline{A}_\lambda \cdot \hat{Z} = - N_\lambda k_\lambda^2 \; J_n(k_\lambda r) \; e^{i(n\theta + \frac{m}{R} z)}$$

$$\psi_\sigma = N_\sigma J_n (k_\sigma r) \; e^{i(n\theta + \frac{m}{R} z)}$$

(Primes denote differentiation with respect to the argument.)

The condition $J_n (k_i a) = 0$ insures the vanishing to the tangential $\overline{E}$ at the tank walls, and the eigenfrequencies ω_σ, ω_λ are found from

$$\omega_i^2 \mu\epsilon = k_i^2 + \frac{m^2}{R^2} ; \qquad i = \sigma \text{ or } \lambda; \; m = 1, 2, \ldots$$

The normalization constants N , N_λ are

$$N_\sigma = \frac{1}{2\pi} [R \int_0^a J_n^2 (k_\sigma r) \, r \, dr]^{-1/2}$$

$$N_\lambda = \frac{1}{2\pi} \{ \frac{m^2 k_\lambda^2}{R} \int_0^a [J'_n (k_\lambda r)]^2 \, r \, dr + n^2 m^2 \int_0^a J_n^2 (k_\lambda r) \, dr + R \, k_\lambda^4 \int_0^a J_n^2 (k_\lambda r) \, r \, dr\}^{-1/2}$$

The solutions to equations (12) and (13) become

$$q_\lambda(t) = \frac{-N'_\lambda N_m \omega_o k_\lambda^2 R J_n (k_\lambda r_o)}{\epsilon(\omega_\lambda^2 - m^2 \omega_o^2)} e^{-i(n\theta_o + m\omega_o t)}$$

$$q_\sigma(t) = \frac{N_\sigma N_m}{\omega_\sigma^2 \mu\epsilon^2} J_n(k_\sigma r_o) \; e^{-i(n\theta_o + m\omega_o t)}$$

The factor N_m is related to a Fourier decomposition of the beam current and has units of charge.

$$N_m \equiv \int_0^{2\pi} f(\phi)\, e^{-im\phi}\, d\phi$$

The summations for the electric field components can now be written

$$E_r = \frac{-c^2}{\epsilon} \sum_{n,m,\sigma} \frac{N_\sigma^2 N_m k_\sigma}{\omega_\sigma^2} J_n(k_\sigma r_o) J_n'(k_\sigma r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}$$

$$- \frac{\omega_o^2}{\epsilon} \sum \frac{N_\lambda^2 N_m m^2 k_\lambda^3}{(\omega_\lambda^2 - m^2\omega_o^2)} J_n(k_\lambda r_o) J_n'(k_\lambda r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}.$$

$$E_\theta = -\frac{ic^2}{r\epsilon} \sum_{n,m,\sigma} \frac{N_\sigma^2 N_m n}{\omega_\sigma^2} J_n(k_\sigma r_o) J_n(k_\lambda r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}$$

$$- \frac{i\omega_o^2}{r\epsilon} \sum_{n,m,\lambda} \frac{N_\lambda^2 N_m m^2 n k_\lambda^2}{(\omega_\lambda^2 - m^2\omega_o^2)} J_n(k_\lambda r_o) J_n(k_\lambda r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}.$$

$$E_z = -\frac{ic^2}{R\epsilon} \sum_{n,m,\sigma} \frac{N_\sigma^2 N_m m}{\omega_\sigma^2} J_n(k_\sigma r_o) J_n(k_\sigma r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}$$

$$+ \frac{iv_p^2}{R\epsilon} \sum_{n,m,\lambda} \frac{N_\lambda^2 k_\lambda^4 N_m m}{(\omega_\lambda^2 - m^2\omega_o^2)} J_n(k_\lambda r_o) J_n(k_\lambda r)\, e^{i(n\theta - n\theta_o + \frac{mz}{R} - m\omega_o t)}.$$

In these relations, v_p is the particle velocity and $c = (\mu\epsilon)^{-1/2}$ is the velocity of light in the tank.

The magnetic field components H_θ and H_r can be found from

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

The summations include integer values of $\underline{n}$ from $-\infty$ to ∞, integer values of $\underline{m}$ from 0 to ∞; the summations over σ and λ implying summing over all N_σ, N_λ, ω_σ, ω_λ k_σ and k_λ for each set of values n, m.

B. Electric Field of Centered Beam

The general solution for $\overline{E}$ simplifies considerably when $r_o = 0$. We further simplify the expressions by considering each frequency component of the field, $\overline{E}_m$, separately, and by deleting the time factor $e^{-im\omega_o t}$. From the centered beam then

$$E_{rm} = -\frac{N_m}{\epsilon} e^{i\frac{mz}{R}} \left[c^2 \sum_\sigma \frac{N_\sigma^2 k_\sigma}{\omega_\sigma^2} J_o'(k_\sigma r) + \right.$$

$$\left. \frac{v_p^2 m^2}{R^2} \sum \frac{N_\lambda^2 k_\lambda^3}{(\omega_\lambda^2 - m^2\omega_o^2)} J_o'(k_\lambda r) \right] \tag{14}$$

$$E_{\theta m} = 0$$

$$E_{zm} = \frac{im N_m}{\epsilon R} e^{i\frac{mz}{R}} \left[- c^2 \sum_\sigma \frac{N_\sigma^2}{\omega_\sigma^2} J_o(k_\sigma r) + \right.$$

$$\left. v_p^2 \sum_\lambda \frac{k_\lambda^4 N_\lambda^2}{(\omega_\lambda^2 - m^2\omega_o^2)} J_o(k_\lambda r) \right] \tag{15}$$

The corresponding expressions for N_σ and N_λ become

$$N_\sigma^2 = 2\pi^2 a^2 R J_1^2 (k_\sigma a) \tag{16}$$

$$N_\lambda^2 = 2\pi^2 a^2 k_\lambda^2 J_1^2 (k_\lambda a) \left(\frac{m^2}{R} + k_\lambda^2 R\right) \tag{17}$$

and as before

$$J_o(k_i a) = 0$$

$$\omega_i^2 \mu\epsilon = \frac{m^2}{R^2} + k_i^2 \qquad i = \sigma, \lambda$$

Equations (14) and (15) define a transverse magnetic, circularly symmetric electromagnetic field at frequency $\frac{m\omega_6}{2\pi}$. The wavelength in the z direction is $\lambda_z = \frac{2\pi R}{m}$. The axial component of the electric field is ninety degrees out of time phase with the radial component, and vanishes as $v_p \to c$.

For frequency components for which $\frac{2\pi a}{\lambda_z} \ll 1$, Eq. (14) and Eq. (15) can be further simplified to

$$E_{rm} \approx \frac{N_m e^{i\frac{mz}{R}}}{2\pi^2 R \epsilon} \sum_\sigma \frac{J_1(k_\sigma r)}{a^2 k_\sigma J_1^2(k_\sigma a)} .$$

$$E_{zm} \approx \frac{-im N_m e^{i\frac{mz}{R}}}{2\pi^2 R^2 \epsilon} (1 - v_p^2/c^2) \sum_\sigma \frac{J_o(k_\sigma r)}{a^2 k_\sigma^2 J_1^2(k_\sigma a)} .$$

Noticing that

$$\sum_\sigma \frac{J_1(k_\sigma r)}{a^2 k_\sigma J_1^2(k_\sigma a)} = \frac{1}{2r}, \text{ and}$$

$$\sum_\sigma \frac{J_o(k_\sigma r)}{a^2 k_\sigma^2 J_1^2(k_\sigma a)} = \frac{-\ln r/a}{2} ,$$

the "long wavelength" fields are written as :

$$E_{rm} = \frac{N_m e^{i\frac{mz}{R}}}{4\pi^2 \epsilon R r}$$

$$E_{zm} = \frac{im N_m (1 - v_p^2/c^2)}{4\pi^2 \epsilon R^2} (\ln r/a) e^{i\frac{mz}{R}} .$$

The radial component of field becomes of the same form as would result from current flowing in a coaxial line with a phase velocity equal to the particle velocity, v_p.

It should be noted at this point that expressions for the electric field when $r_o = 0$ can be obtained in closed form directly from Maxwell's equations. The expressions are complicated, however, and bear little practical advantage over the series formulations of Eq. (14) and Eq. (15).

IV. Response to a Beam Source

We can now perform the integration indicated in Eq. (1) for an arbitrary beam current. Since the frequency response of the pickup will depend upon the distance Δ only through a magnitude factor, we let $r_o = 0$. The beam current is assumed to be unaffected by the presence of the pickup wire.

Inserting Eq. (2), Eq. (14) and Eq. (15) into Eq. (1) gives the following expression for the mth frequency component of the voltage received at the downstream port.

$$V_m = \frac{N_m}{\epsilon} \Bigg\{ \Bigg\{ i\, m\, e^{i\beta(\ell+a-b)} \left[\frac{e^{i\ell\left(\frac{m}{R} - \beta\right)} - 1}{i(m/R - \beta)} \right] \left[\frac{-c^2}{R} \sum_\sigma \frac{N_\sigma^2}{\omega_\sigma^2} J_o(k_\sigma b) + \frac{v_p^2}{R} \sum_\lambda \frac{k_\lambda^4 N_\lambda^2}{(\omega_\lambda^2 - m^2\omega_o^2)} J_o(k_\lambda b) \right] + e^{i\beta a} \Bigg\{ e^{i\beta(\ell-2b)} \Bigg[-c^2 \sum_\sigma \frac{k_\sigma N_\sigma^2}{\omega_\sigma^2} f_\sigma - m^2 \omega_o^2 \sum_\lambda \frac{N_\lambda^2 k_\lambda^3}{(\omega_\lambda^2 - m^2\omega_o^2)} f_\lambda \Bigg] + e^{i\frac{m\ell}{R}} \Bigg[c^2 \sum_\sigma -\frac{k_\sigma N_\sigma^2}{\omega_\sigma^2} f_\sigma^* + m^2\omega_o^2 \sum_\lambda \frac{N_\lambda^2 k_\lambda^3}{(\omega_\lambda^2 - m^2\omega_o^2)} f_\lambda^* \Bigg] \Bigg\} \Bigg\} \Bigg\} \tag{18}$$

$$f_i \equiv \int_b^a e^{+i\beta r} J_1(k_i r)\, dr; \quad i = \sigma \text{ or } \lambda.$$

$\beta = \dfrac{m\omega_o}{v_\phi}$, and the asterisks again denote the complex conjugate.

The corresponding solution for the voltage received at the upstream port when the downstream port is terminated can be found from Eq. (18) by changing the sign of v_p and multiplying by $e^{-im\ell/R}$.

Since the magnitude and phase of each frequency component of the received voltage depends upon the particular distribution of the charge around ϕ, it is convenient to define a transfer impedance function which can be used independent of the charge distribution. We therefore define Z(f) at each port as the ratio of the voltage at frequency $\frac{m\omega_0}{2\pi}$ appearing at that port divided by the corresponding Fourier component of beam current. Once these functions are known, the output voltage waveform for any beam current can be calculated.

The functions Z(f) are found by assuming a particular form for $f(\phi)$, evaluating Eq. (18) for different values of m, and dividing by the corresponding Fourier current components. Numerical results using parameters applicable to a large proton synchrotron are given in Figs. 5 through 10. Note that for $v_p \approx c$, the upstream response becomes essentially that calculated for the coaxial line, but differs considerably for less relativistic velocities (see Fig. 7). For the case $v_p = v_\phi < c$, Fig. 6 shows an increase in the downstream output above that for unmatched velocities (Fig. 5). The fundamental voltage component at frequency $f_1 = \frac{M\omega_0}{2\pi}$, M being the number of charge bunches around the machine, is shown in Figs. 11 and 12 for varying v_p with $\ell = \frac{c}{2f_1} = \lambda_1/2$ when $v_p = c$. For a pickup line modified such that $v_\phi < c$, the fundamental voltage component corresponding to Figs. 11 and 12 becomes zero at the velocity $v_p = v_\phi$ and increases linearly with increasing velocity.

V. Discussion and Conclusions

The upstream transfer function given by Eq. (5) applies to a beam source when $v_p/c \approx 1$. (The constant factor A differs only slightly from that for a coaxial line source for a given distance Δ.) This function theoretically generates an accurate reproduction of the beam current pulses if the period of the pulses exceeds the "round trip time", Λ, plus the pulse width, τ, and if $\Lambda > \tau$. However, since the beam pulses in an actual machine will vary, one should design the electrode such that $\Lambda \leq T/2$. (It should also be noted that a practical pickup would have to be shielded from the effects of stray particles and secondary electron emission.)

A possible beam-pulse form is shown in Fig. 13. The resulting upstream output voltage for $\Delta = 0.32$ cm. and $v_p \approx c$ is shown in Figs. 14 through 18 for different input periods, pulse widths, and round trip times Λ. It is interesting to note that when $\Lambda = T$, the negative going excursions of the output pulses just cancel the positive excursions, and we see zero output at this port. (These curves assume the matching conditions at each port can be maintained at frequencies many times the fundamental frequency of the pulse train.)

We can conclude that the type of beam monitor studied here can be used effectively to observe the shape of current pulses in a high energy accelerator beam. The magnitude of the output voltage at the upstream port should be great enough to be amplified and displayed easily. For example, the voltage pulses pictured in Fig. 14 would have peaks of one millivolt or more for all machines for which $N \geq \frac{10^{-3}\tau}{2Ae}$, where N is the total number of particles circulating, e is the charge per particle, M is the number of charge bunches around the machine, and 2A is the peak amplitude of the upstream transfer function. A good approximation for A when $v_\varphi = v_p = c$ is $A \approx 30 \ln (a/b)$. If one desired a pickup with $v_\varphi < c$, an alternative to dielectric loading might be to spiral the line inside the beam pipe, forming a helix. For the helical line, we would have $v_\varphi/c = [1 + \pi^2 (a + b)^2 P^2]^{-\frac{1}{2}}$, P being the number of turns per meter, and we assume that the turns are spaced far enough apart to be isolated electrically.

The directional properties of the pickup might conceivably be used in an electron-positron storage ring or other machines having two beams of unlike charge circulating in opposite directions; each port would respond primarily to the current component for which it appeared as the "upstream" port. Four pickups placed around the beam pipe could serve as part of a beam-position monitoring system.

An experimental model was constructed at the Laboratory to check the accuracy of the calculations described here. The model used a large coaxial line as the current source. The line was terminated in its characteristic impedance to avoid reflections. The pickup electrode consisted of a half-inch strip of 75 cm. length with a spacing from the main line wall of 1/8 inch. The observed outputs matched the predicted forms for continuous and pulsed inputs. The predicted magnitude of voltage (Eq. 4) for a sine-wave input matched the measured value within three percent at 100 mc/sec. Figure 19 is a photograph of the test setup.

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7. Numerical calculations were done on the Laboratory's IBM 7094 computer using programs "LARGE 2" from downstream voltage and "LARGE 3" for upstream voltage. These programs calculated the transfer impedances for specified values of a, b, M, ℓ, v_p, v_ϕ and R.

LIST OF FIGURES

Fig. 1 Schematic of pickup electrode in beam pipe.

Fig. 2 Magnitude of the upstream transfer impedance vs. wave number for a coaxial line source with ℓ = 75 cm., Δ = 0.125 inch.

Fig. 3 Magnitude of the downstream transfer impedance vs. wave number for a coaxial line source with ℓ = 75 cm., Δ = 0.125 inch.

Fig. 4 (a) Top view of toroidal beam pipe. The particle orbits lie in the plane of the paper.
(b) Cross section showing cylindrical coordinates r, θ, z.

Fig. 5 Magnitude of downstream transfer impedance vs. frequency for a beam source with $f_1 = \frac{M\omega_o}{2\pi} \approx 100$ mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = c$.

Fig. 6 Magnitude of downstream transfer impedance vs. frequency for a beam source with f_1 = 100 mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = v_p = 0.8$ c.

Fig. 7 Magnitude of upstream transfer impedance vs. frequency for a beam source with f_1 = 100 mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = c$.

Fig. 8 Angle of upstream transfer impedance vs. frequency for a beam source with f_1 = 100 mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = c$.

Fig. 9 Magnitude of upstream transfer impedance vs. frequency for f_1 = 100 mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = v_p = 0.8c$.

Fig. 10 Angle of upstream transfer impedance vs. frequency for f_1 = 100 mcs; ℓ = 37.5 cm.; Δ = 0.5 inch; $v_\varphi = v_p = c$.

Fig. 11 Magnitude of upstream transfer impedance at the proton bunch repetition frequency vs. v_p with ℓ = 1.5 meters; Δ = 0.5 inch; $v_\varphi = c$.

Fig. 12 Magnitude of downstream transfer impedance at the proton bunch repetition frequency vs. v_p with ℓ = 1.5 meters; Δ = 0.5 inch; $v_\varphi = c$.

Fig. 13 A beam current pulse-train with T = 10 nsec.; τ = 1.18 nsec.

Fig. 14 Upstream output voltage pulse-train corresponding to Fig. 13 assuming $v_p = c$; Δ = 0.125 inch; Λ = 2.5 nsec.

Fig. 15 Beam current pulse-train with T = 10 nsec; τ = 3 nsec.

Fig. 16 Upstream output voltage pulse-train corresponding to Fig. 15 assuming $v_p = c$; Δ = 0.125 inch; Λ = 3 nsec.

Fig. 17 Beam current pulse-train with T = 10 nsec; τ = 6 nsec.

Fig. 18 Upstream output voltage pulse-train corresponding to Fig. 17 assuming $v_p = c$; $\Delta = 0.125$ inch; $\Lambda = 2.5$ nsec.

Fig. 19 Photograph of experimental model. Part of the outer conductor is removed to display the pickup line.

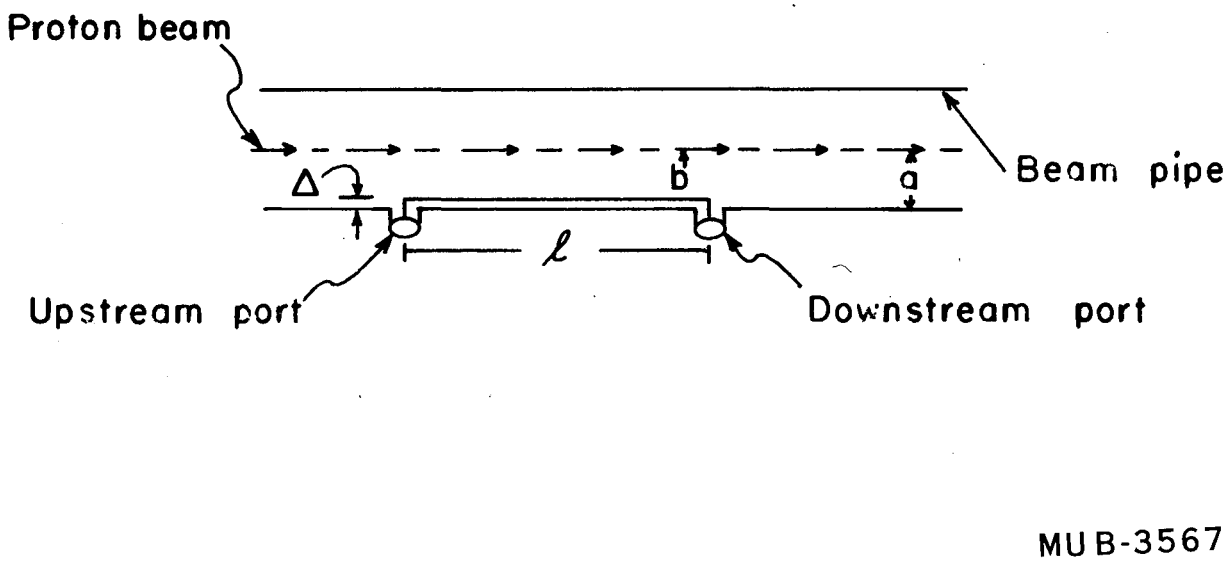
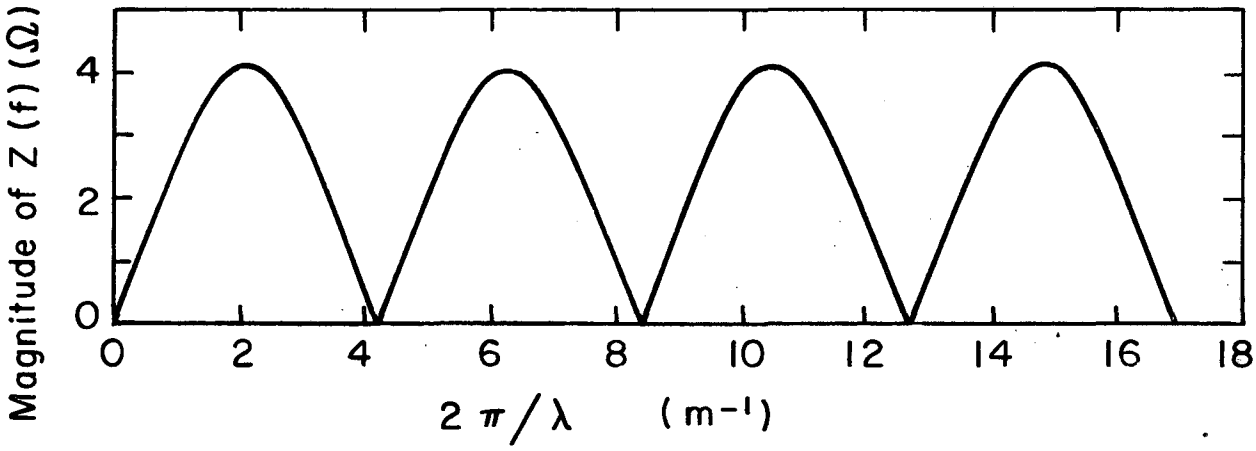
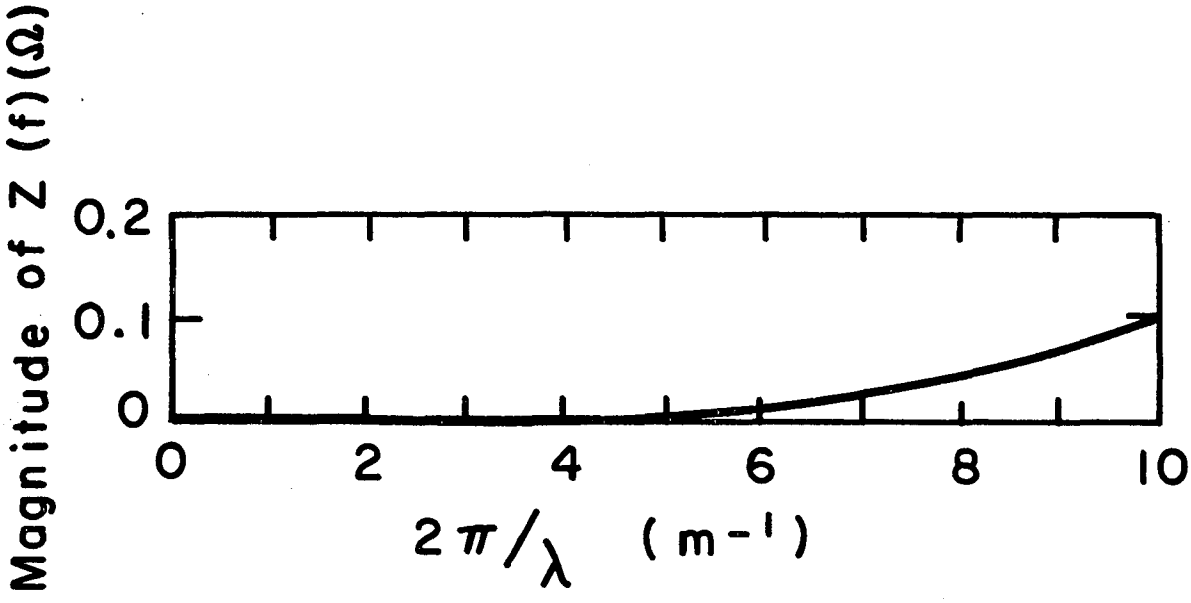

Proton beam
Beam pipe
Δ
b
a
ℓ
Upstream port
Downstream port
MUB-3567

Fig. 1



MUB-3568

Fig. 2



MUB-3569

Fig. 3

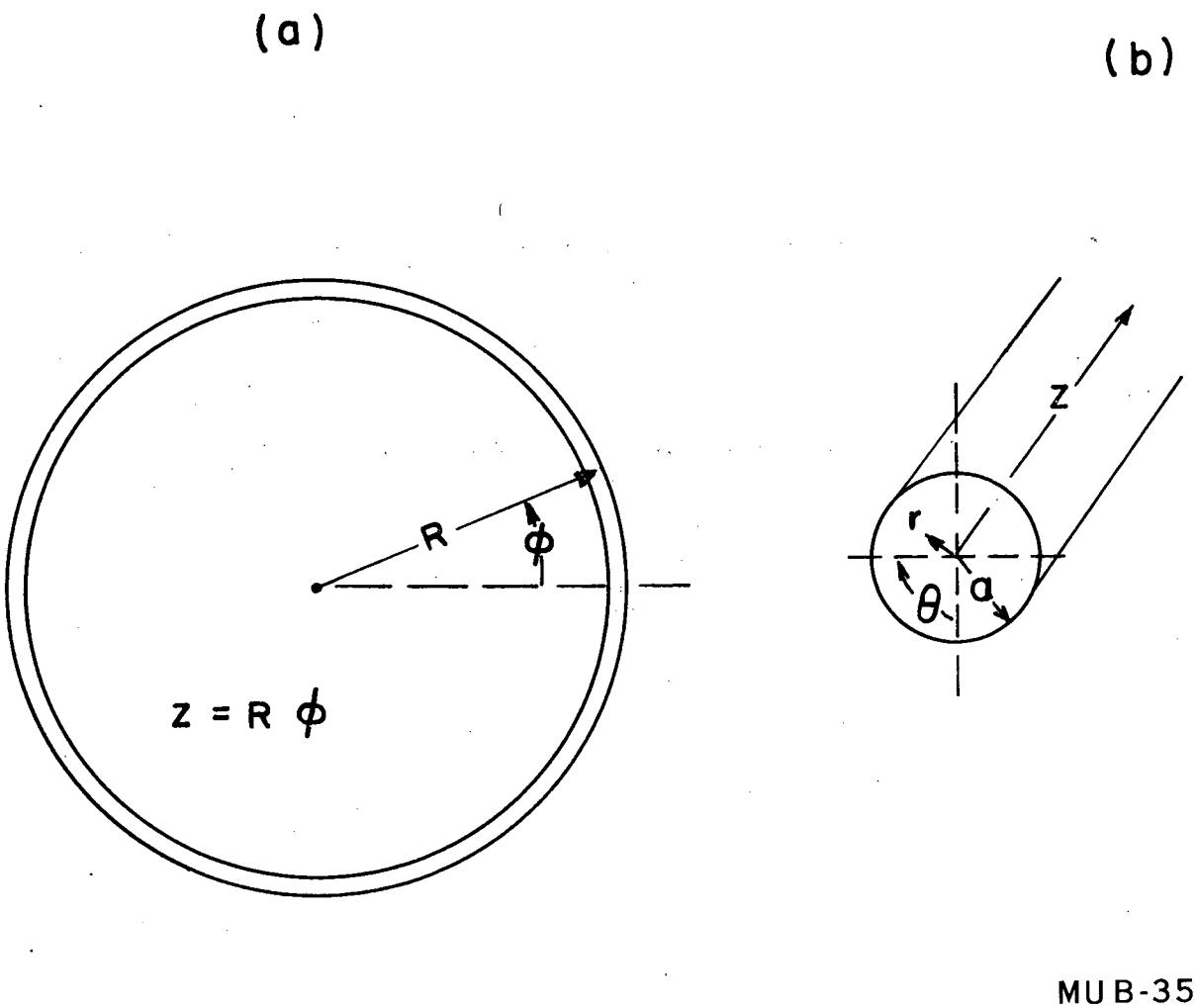
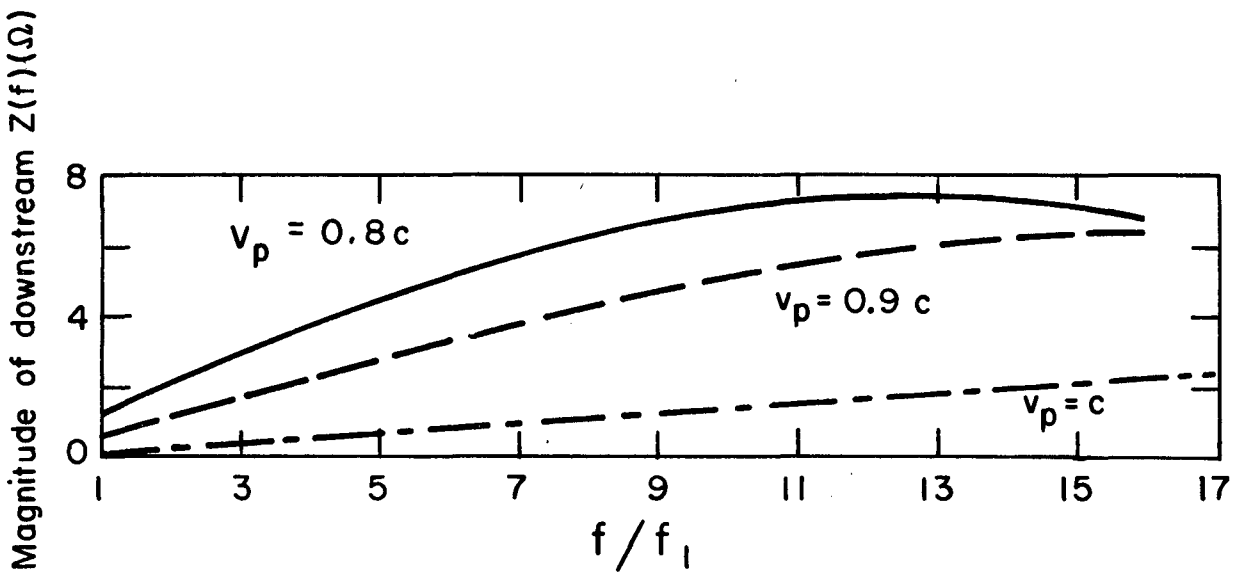


Fig. 4



MUB-3571

Fig. 5

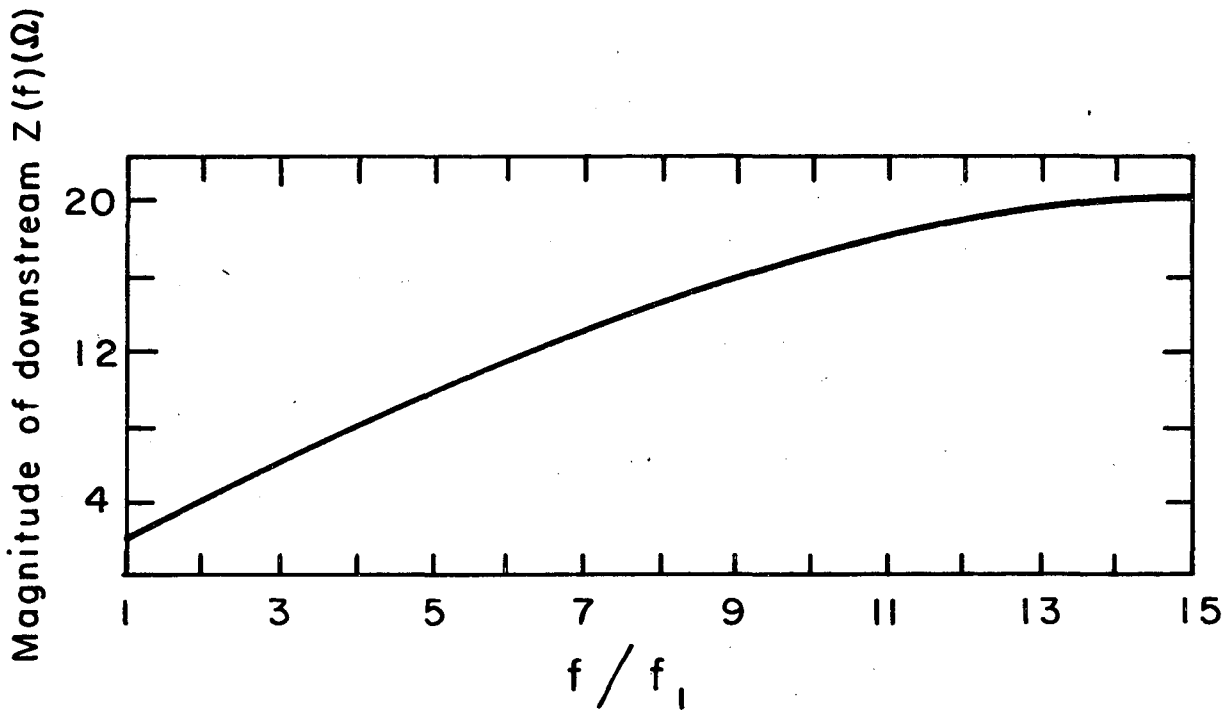


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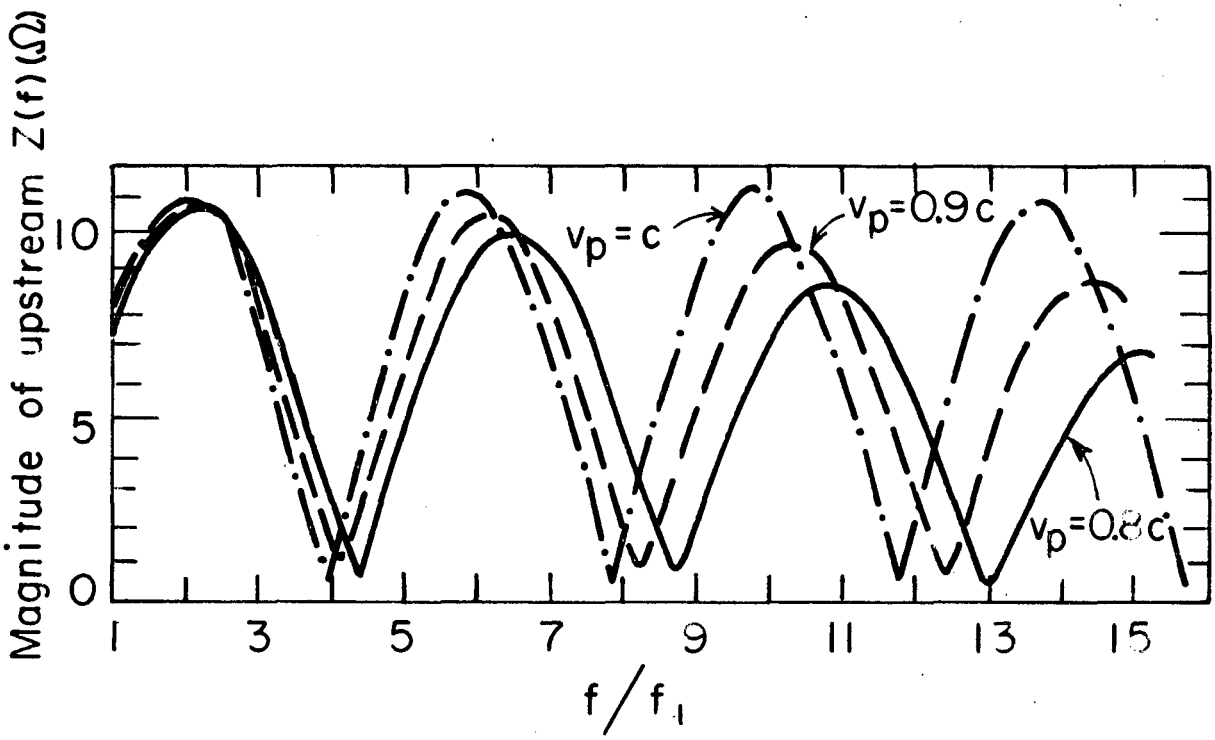
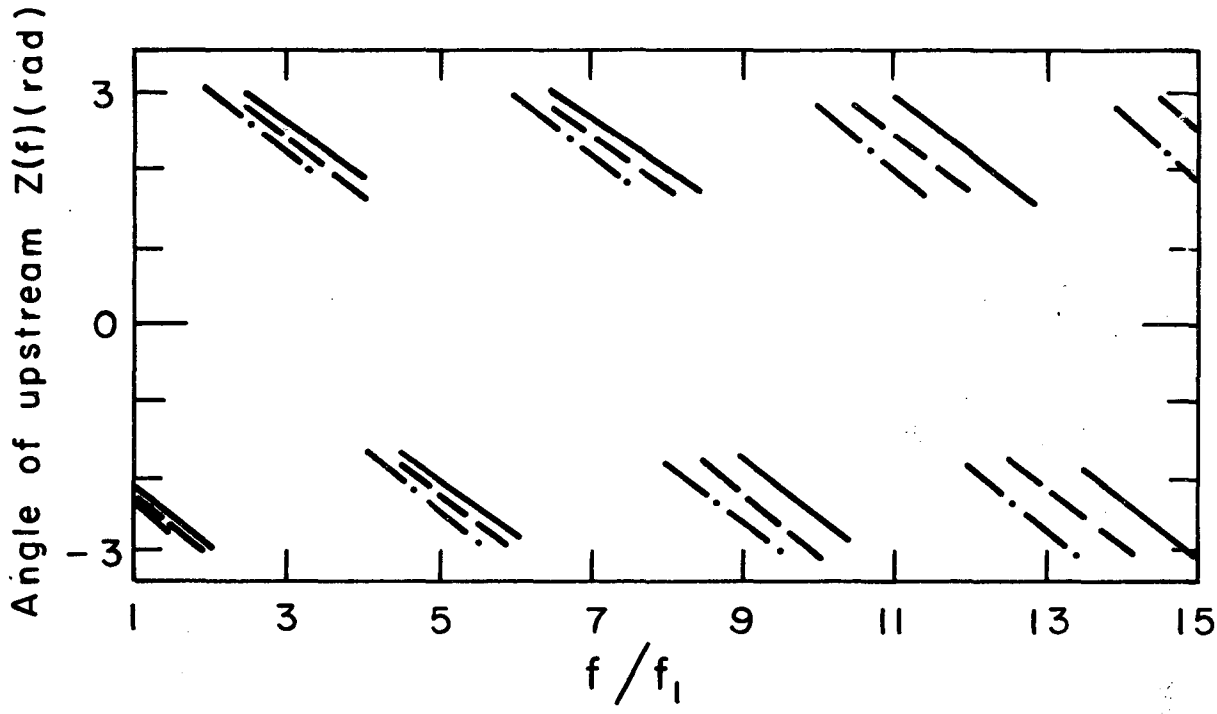
Fig. 6



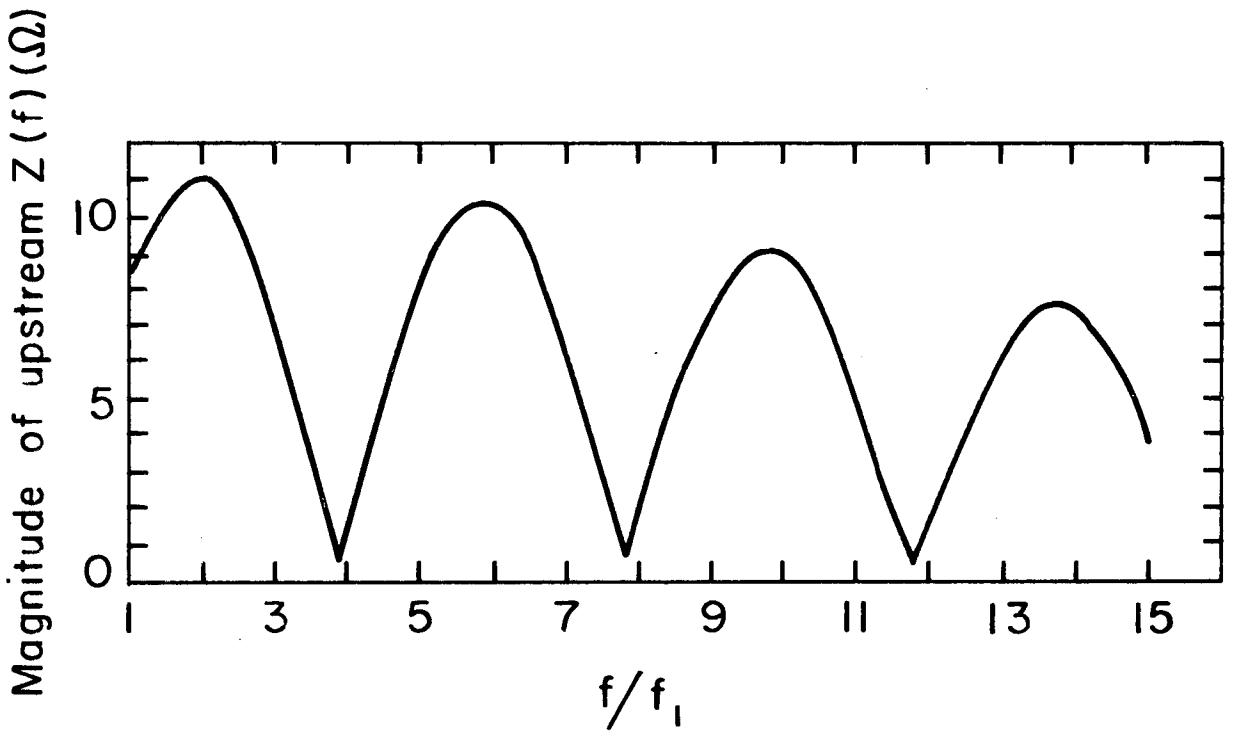
MUB-3573

Fig. 7



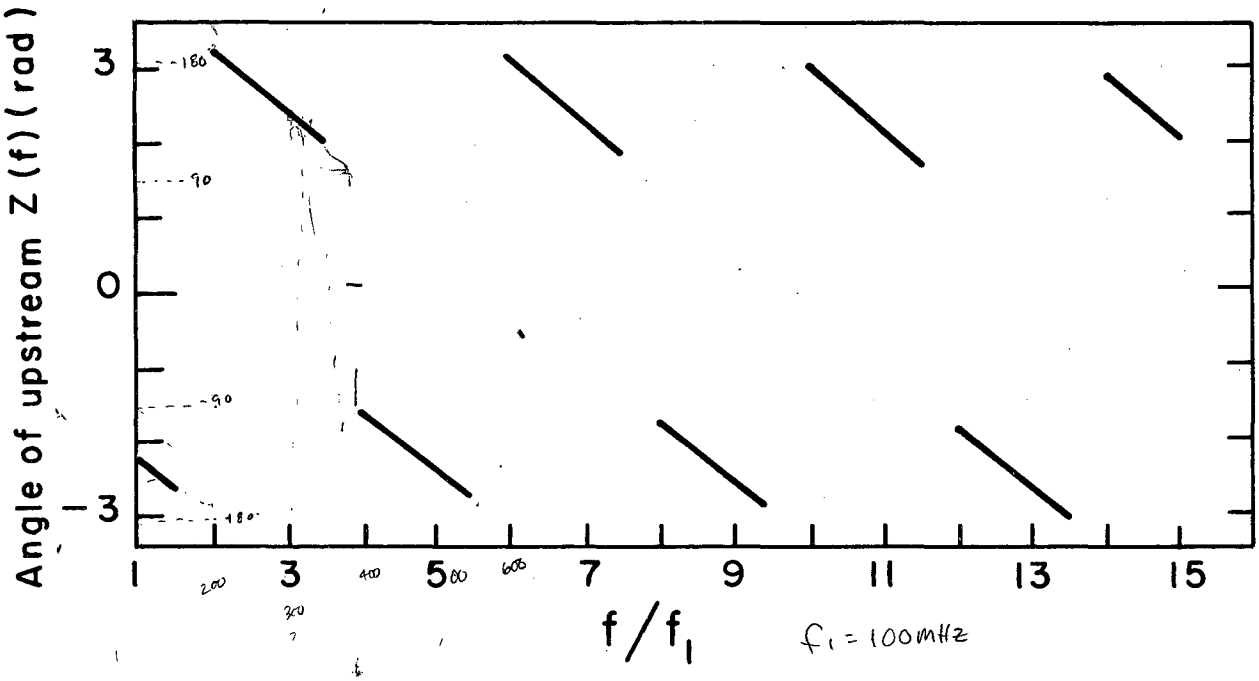
MUB-3574

Fig. 8



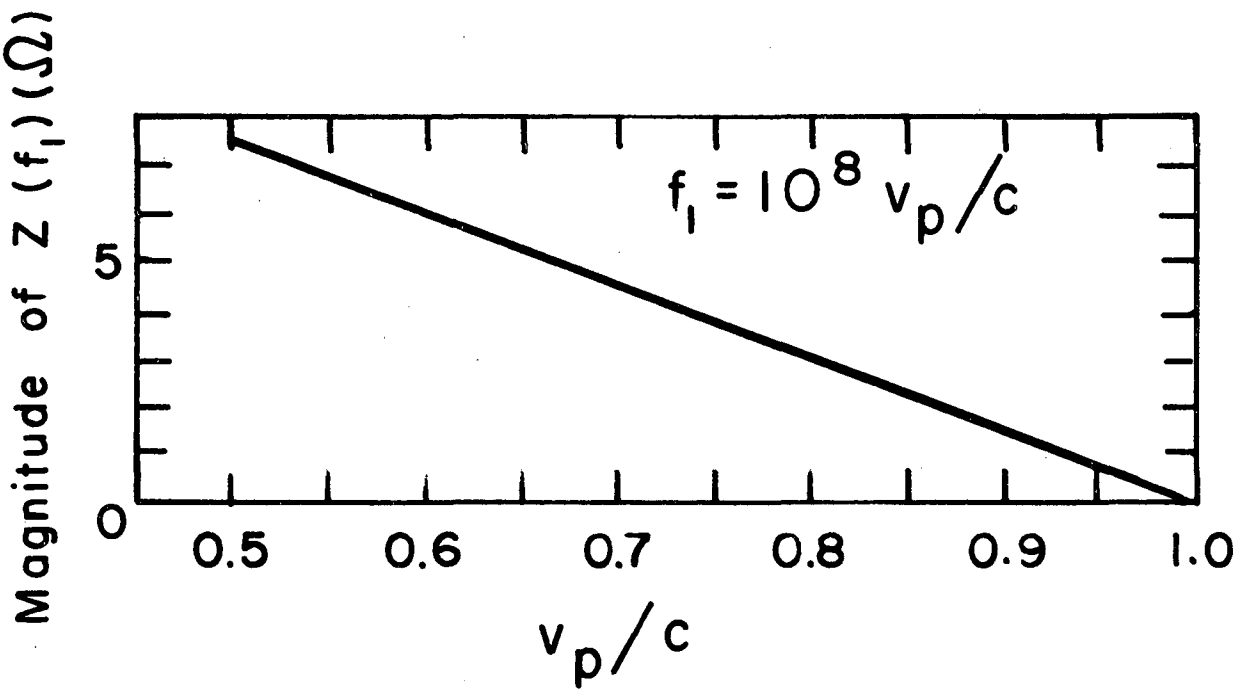
MUB-3575

Fig. 9



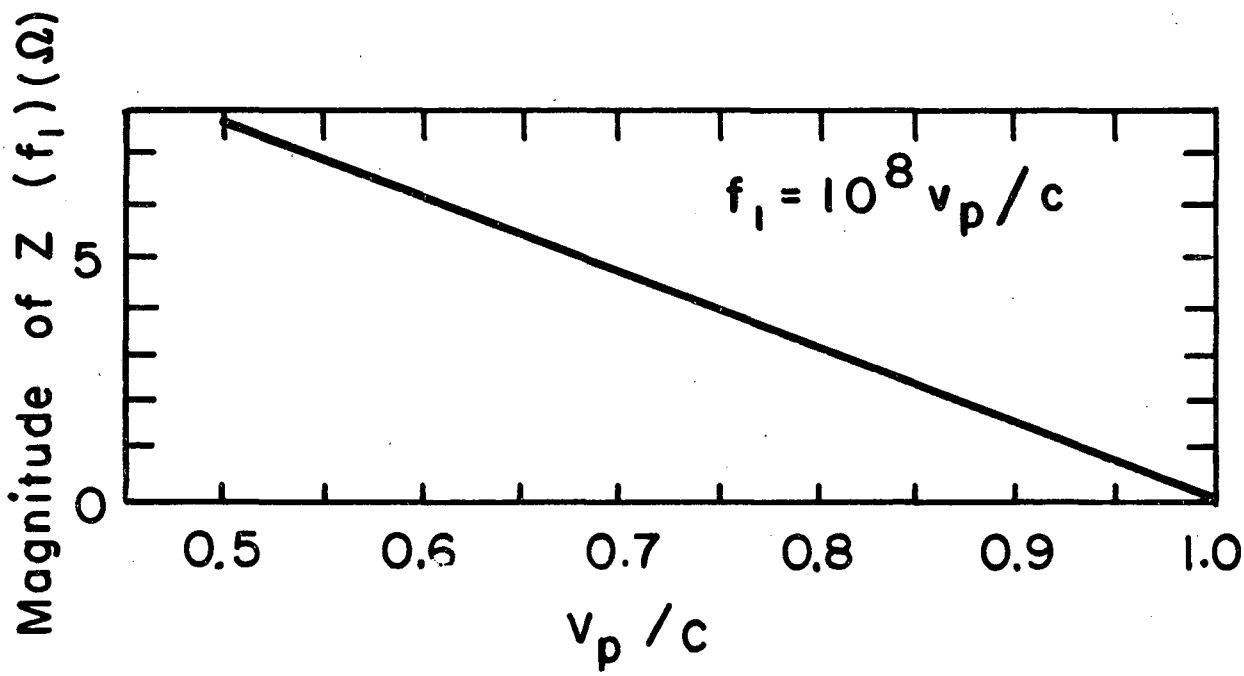
MUB-3576

Fig. 10



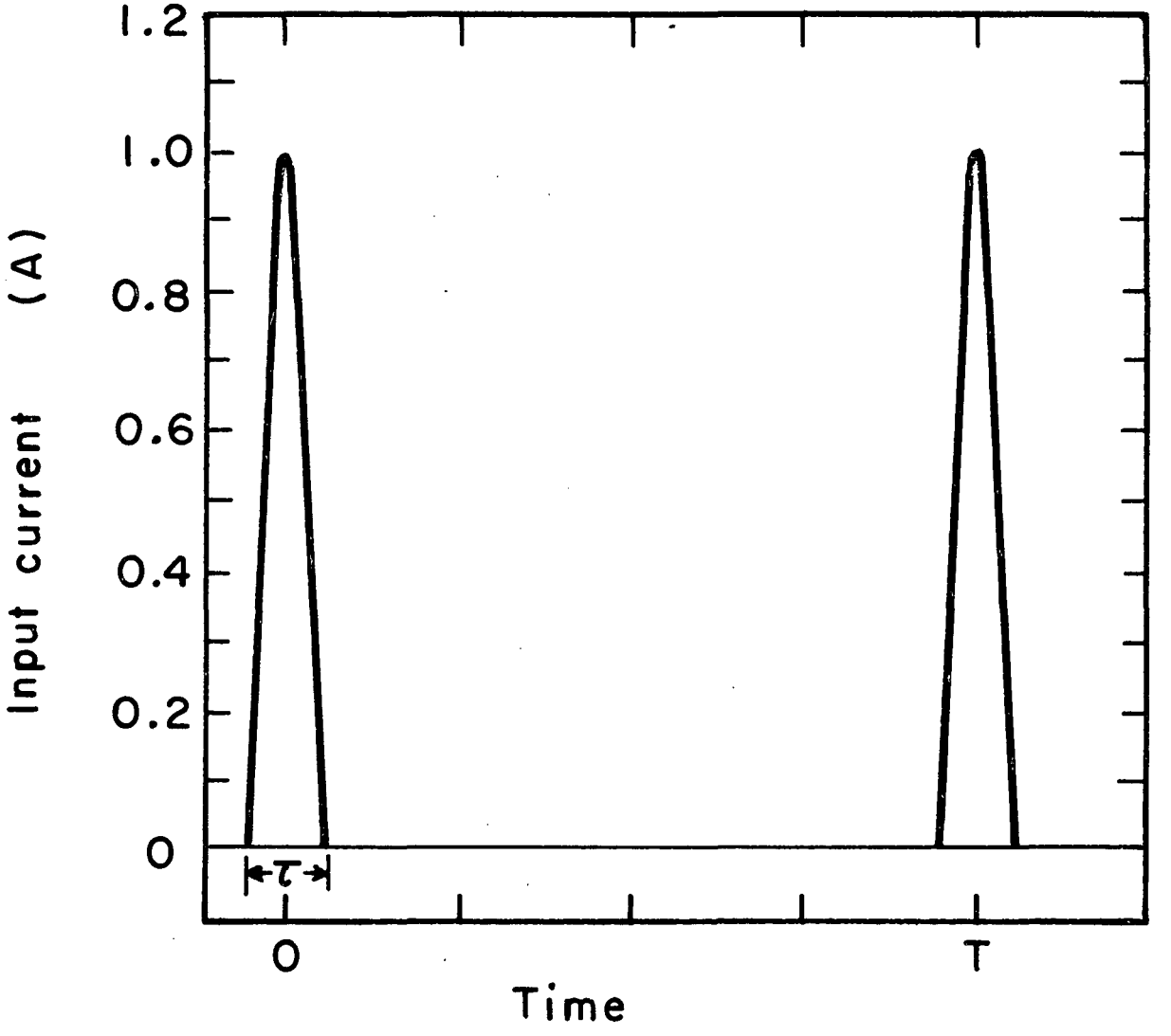
MUB-3577

Fig. 11



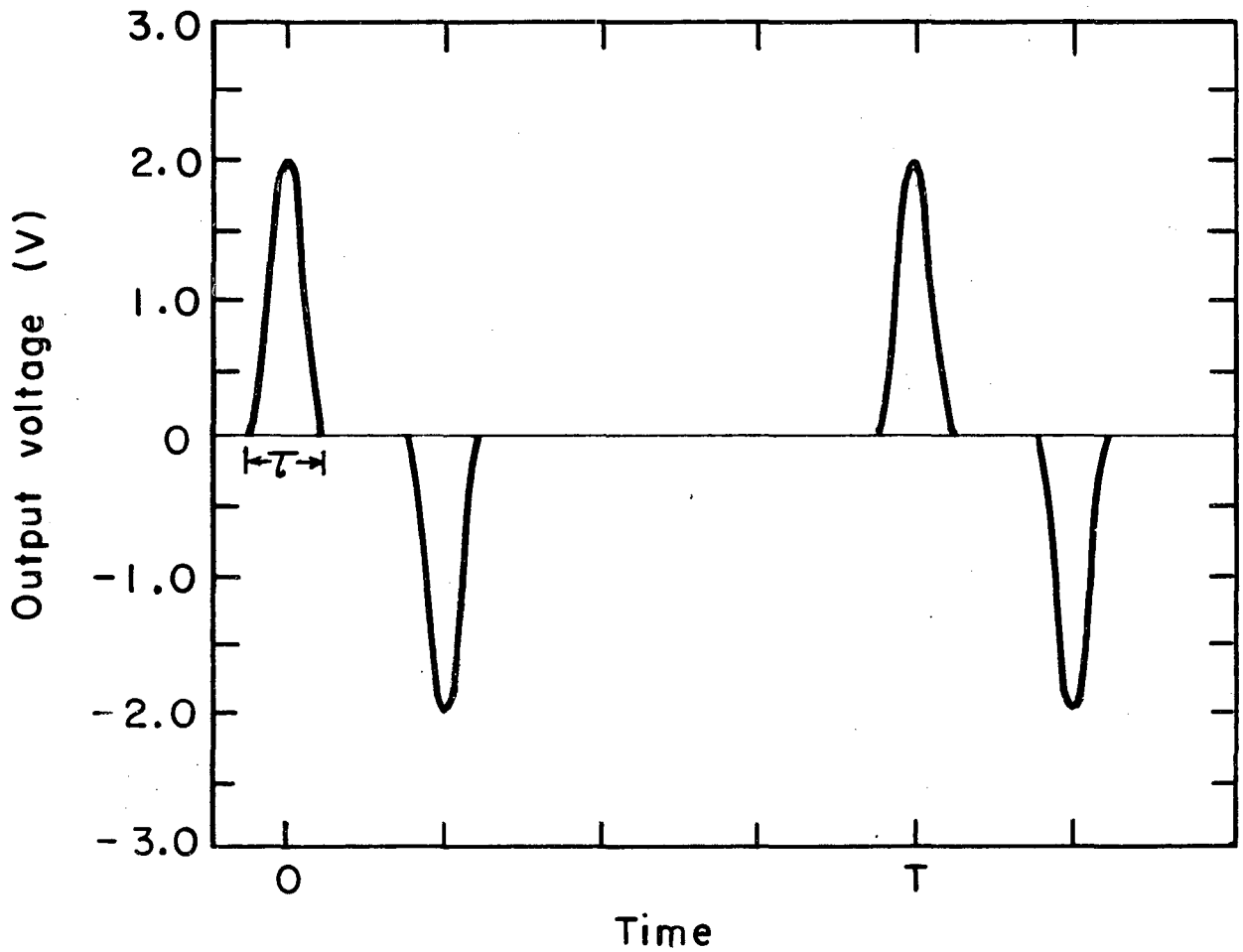
MUB-3578

Fig. 12



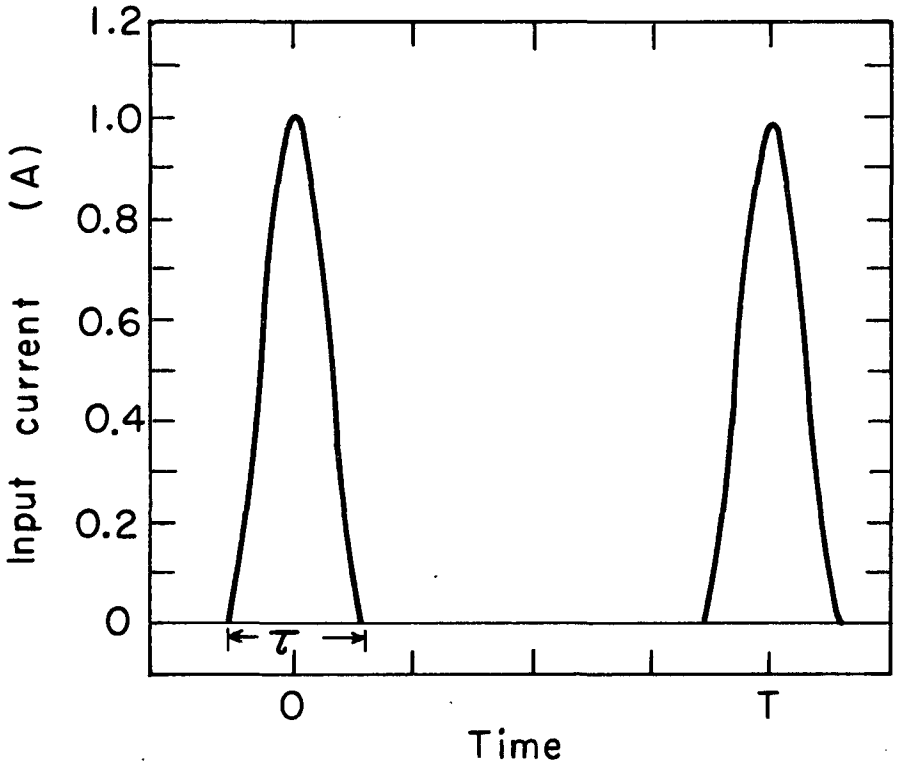
MUB-3579

Fig. 13



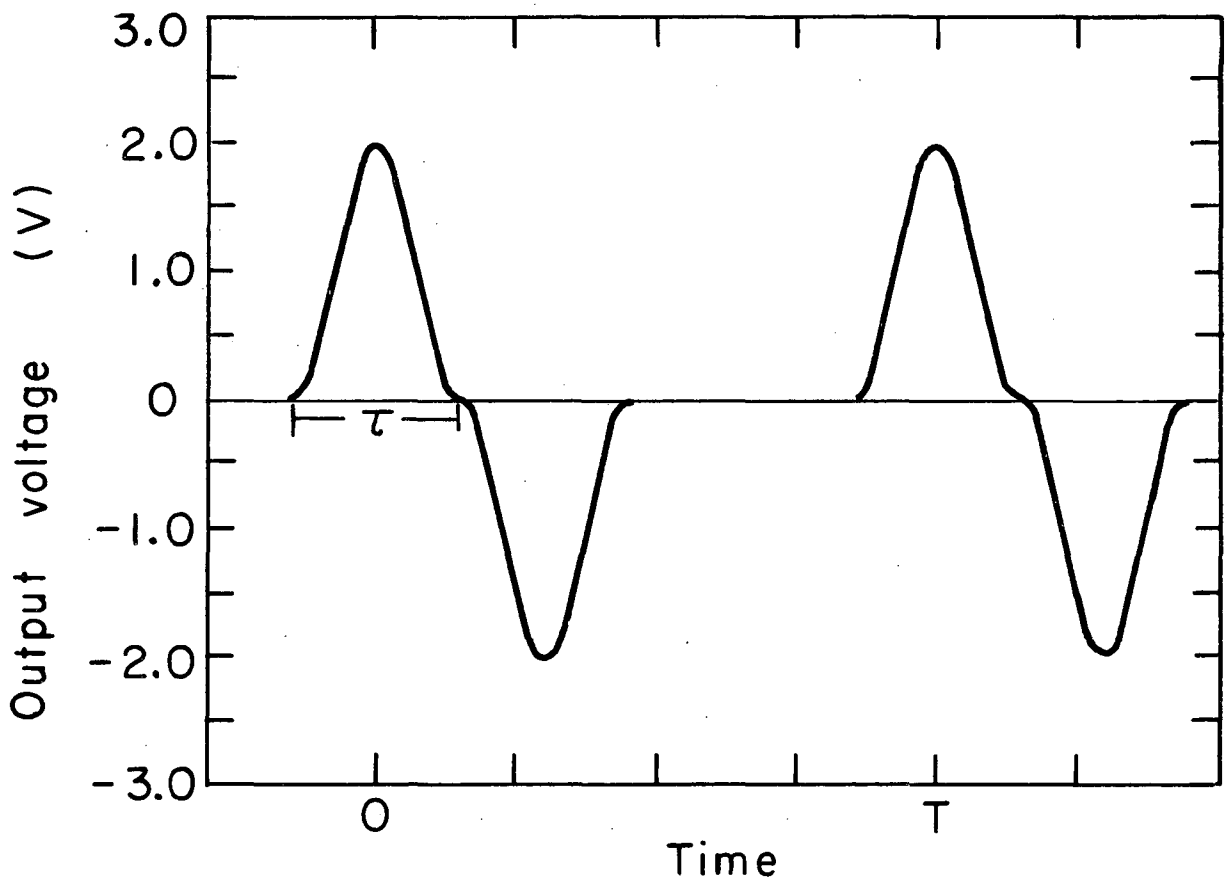
MUB-3580

Fig. 14



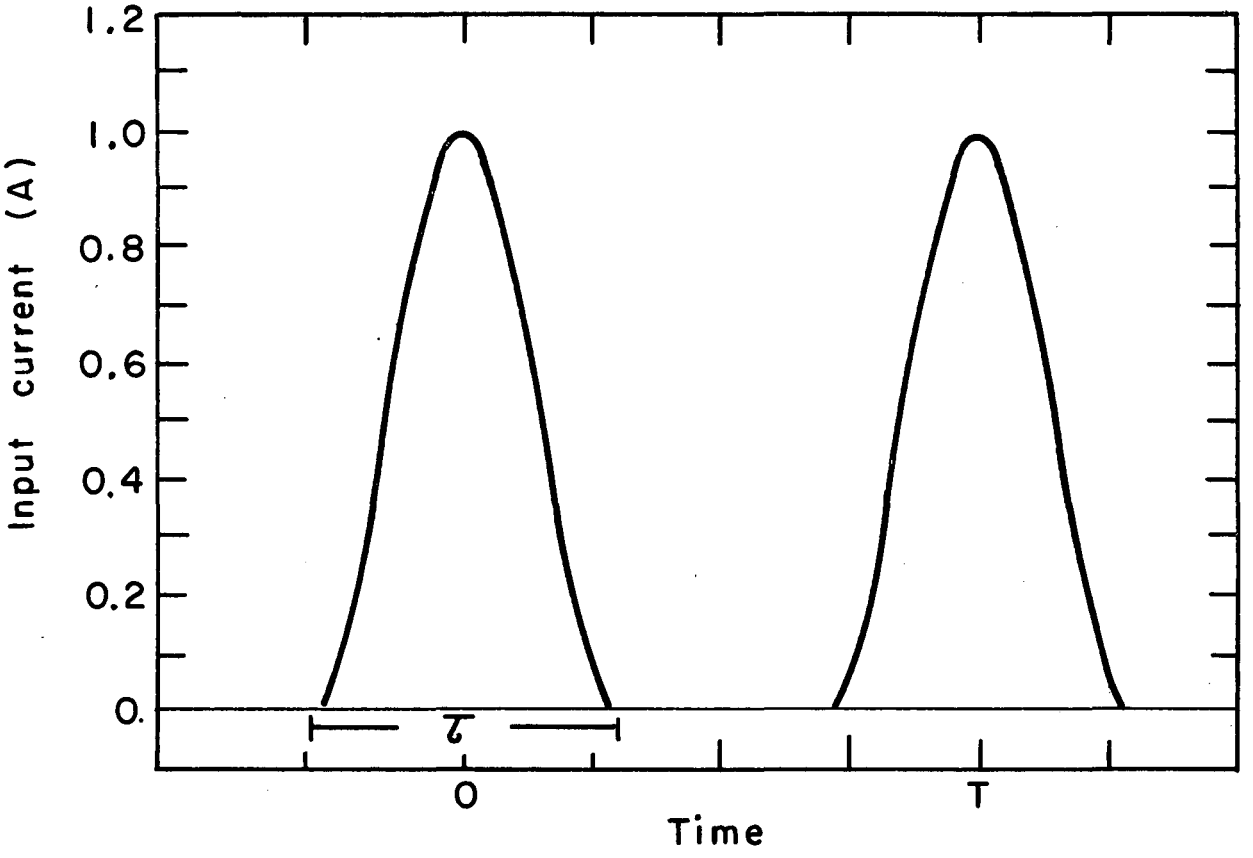
MUB-3581

Fig. 15



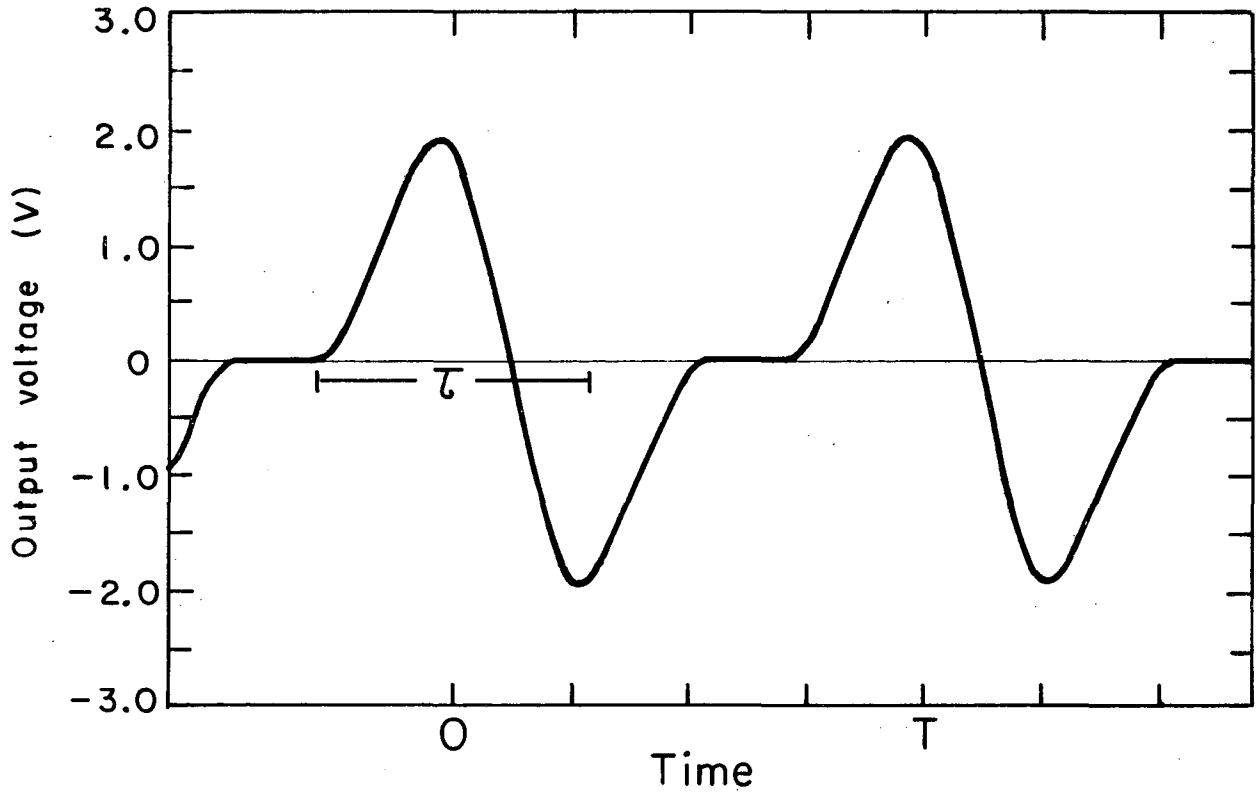
MUB-3582

Fig. 16



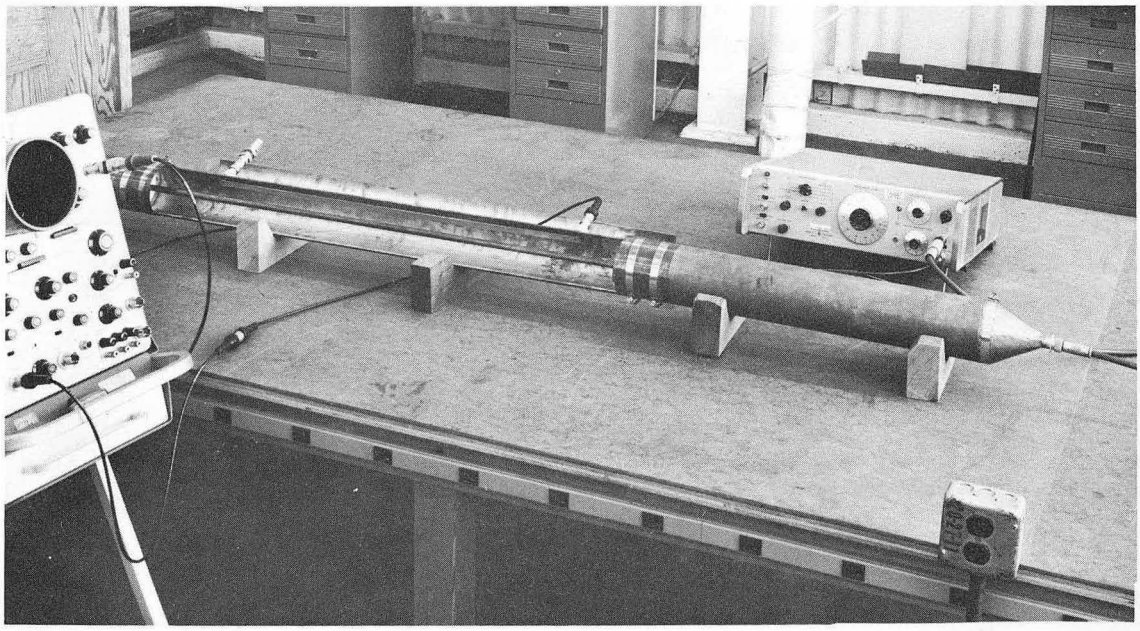
MUB-3583

Fig. 17



MUB-3584

Fig. 18



ZN-4348

Fig. 19

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