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HYPERFINE STRUCT

Berkeley, California

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Abstract

A theoretical analysis is made of the hyperfine structure of the twelve levels of EuI $4f^7(^8S)6s6p$, using intermediate-coupled eigenfunctions obtained from a least-squares fit of the energies of the levels. Relativistic effects for the 6p electron are calculated throughout by tensor-operator techniques. Good agreement is obtained with the observed A values, treating as parameters the polarization of the core (by the f electrons) and the hyperfine interaction constant of the 6s electron. The magnitude of the core polarization is related to data on EuI $4f^7(^8S)6s^2$, EuII $4f^7(^8S)6s$, and EuIII $4f^7(^8S)$. The hyperfine-structure anomalies also fall into a consistent pattern. The observed B values are related to quadrupole moments of ^{151}Eu and ^{153}Eu.

1. INTRODUCTION

The levels of the configuration EuI $4f^7(^8S)6s6p$ have long been of interest to spectroscopists. The lines corresponding to the transitions to the ground level, $4f^7(^8S)6s^2$, lie in the visible. By studying the hyperfine structures of these lines, Schüler & Schmidt (1935) established that the nuclear spin I for both isotopes, ^{151}Eu and ^{153}Eu, is 5/2. They also showed that the nuclei are not perfectly spherical, and their data were used in 1936 by Casimir (see Casimir 1963) to estimate quadrupole moments of approximately 1.5b and 3.2b (1b = 10^{-24} cm^2). More precise measurements of the hyperfine structures were later carried out by Brix (1952), and by Krebs & Winkler (1957, 1961). Recent advances in experimental technique have enabled Müller, Steudel & Walther (1965) to improve still further the accuracy with which the hyperfine structure constants for the levels are known. Their results stimulated us to attempt a fairly complete theoretical analysis of the configuration $4f^7(^8S)6s6p$, our aim being to correlate as far as possible the various experimental data. This problem is the more interesting because a considerable amount of information has accumulated for allied configurations; the hyperfine structure of the ground level $4f^76s^2\ ^8S_{7/2}$ has been very accurately measured by atomic beam methods (Sandars & Woodgate 1960, Pichanick, Woodgate & Sandars 1960), and data are also available for the ions EuII (Krebs & Winkler 1960a, b) and EuIII (Baker & Williams 1962). Such information is very useful for interpreting the core polarization and the hyperfine structure anomalies in EuI $4f^7(^8S)6s6p$. As a by-product, we obtain quite accurate values of a number of parameters. It is hoped that they will prove useful in interpreting the spectra involving configurations of the type $4f^N6s6p$ in other rare-earth ions.

2. LEVELS

The first step is to obtain accurately coupled eigenfunctions for the twelve levels of $4f^7(^8S)6s6p$. These will then be available for the hyperfine structure calculations. The energies of the levels, as found by Russell & King (1939), are given in table 1; for a general impression of their arrangement, see figure 1. Distinct multiplets ^6P, y^8P, z^8P, and ^{10}P can be easily picked out, and this suggests taking the states

$$|4f^7\ {}^8S,\ (6s6p)S_1P,\ S_2PJM_J)$$

as the basis for the calculation. The energies of the levels depend on the spin-orbit coupling for the 6p electron and on the Coulomb interactions between the nine electrons $4f^76s6p$. Spin-spin and spin-other-orbit interactions are expected to amount to only a few cm^{-1}, and are therefore neglected. Tensor-operator techniques are used to evaluate the matrix elements (see, for example, Edmonds 1960, particularly p. 111). For the spin-orbit interaction $\zeta_p \underset{\sim}{s}\cdot\underset{\sim}{\ell}$, we obtain

$$(4f^7\ {}^8S\ (6s6p)S_1P,S_2PJ|\zeta_p \underset{\sim}{s}\cdot\underset{\sim}{\ell}|4f^7\ {}^8S\ (6s6p)S_3P,S_4PJ')$$

$$= 3\zeta_p\delta(J,J')(-1)^{S_1+S_2+S_3+S_4+J-1/2}\{[S_1][S_2][S_3][S_4]\}^{1/2}$$

$$\begin{Bmatrix} S_2 & 1 & S_4 \\ 1 & J & 1 \end{Bmatrix} \begin{Bmatrix} S_2 & 1 & S_4 \\ S_3 & \frac{7}{2} & S_1 \end{Bmatrix} \begin{Bmatrix} S_1 & 1 & S_3 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix},$$

in which the traditional abbreviation $[x] \equiv 2x+1$ has been made. The 6-j symbols can be immediately found from the tables of Rotenberg, Bivins, Metropolis & Wooten (1959).

There are five matrix elements of the Coulomb interaction: the energies of the terms ^6P, ^{10}P, and of the two ^8P terms, and also the interaction energy between the two ^8P terms. As a first run-through, a least squares fit was obtained for the twelve levels on the assumption that these five quantities and also the spin-orbit coupling constant ζ_p could be regarded as adjustable parameters. A seventh parameter was also introduced to allow for small intermediate-coupling effects within the $4f^7$ shell. A RMS deviation of 27 cm^{-1}, or only 0·4% of the configuration, was obtained. However, when the Coulomb matrix elements were examined, it was found that they implied a ratio τ, defined in terms of the energies E of the terms of $4f^7(^8S)6p$ and $4f^7(^8S)6s$ by the equation

$$\tau = [E(^7P)-E(^9P)]/[E(^7S)-E(^9S)],$$

that was only 0·29; whereas for EuII, the experimental value is 0·65. [See Russell, Albertson & Davis 1941. To get the term difference from the levels, we use $E(^7P)-E(^9P) = E(^7P_2)-E(^9P_5)$.] This suggested that the parameters were absorbing extraneous effects, and it was soon found that this was primarily due to the insensitivity of the energies of the ^8P levels to the matrix element coupling them.

The least squares procedure was therefore repeated with $\tau = 0{\cdot}65$. Only four parameters instead of seven were used; in the notation of Condon & Shortley (1935), their final values in cm^{-1} are $\zeta_p = 1198$, $G^3(4f, 6s) = 1259$, and $G^1(6p, 6s) = 8172$. The fourth parameter corresponds to a displacement of the configuration as a whole, and is of no special interest. No allowance was made for intermediate coupling within the 4f shell, mainly because the parameter introduced in the first trial run seemed of doubtful significance. Although the number of parameters is only four instead of seven, the RMS deviation

increases merely to 50 cm^{-1}. Details are given under the heading "calculation α" in table 1. We immediately notice that the largest discrepancies between theory and experiment appear in just two levels of the multiplet y^8P. The lowest level of the configuration $4f^7 5d6p$ lies only 6759 cm^{-1} above $y^8P_{9/2}$, and we would therefore expect the theoretical fit to be poorest for this multiplet. The nine z levels show a RMS deviation of only 12 cm^{-1}, or 0·3% of that part of the configuration.

On the suggestion of Professor A. Steudel and Dr. H. Walther, we carried out a third least-squares fitting procedure. This time we rejected the two levels of y^8P that, in the previous calculation (calculation α), had shown the large discrepancies between experiment and theory. The parameter τ was also left free. The results are given in table 1 under the heading "calculation β". The values of the parameters in cm^{-1} are $\zeta_p = 1191$, $G^3(4f, 6s) = 1216$, $G^1(6p, 6s) = 8181$, while $\tau = 0{\cdot}71$. The RMS deviation for the ten levels included in this analysis is reduced to 9·8%.*

*After these calculations had been performed, Dr. B. G. Wybourne kindly gave us a preprint of some work that he and Dr. G. Smith had carried out on several configurations of EuI, including $4f^7(^8S)6s6p$. Their calculation is the same as our calculation α, except that τ was taken as a variable parameter. They find $\zeta_p = 1227$, $G^3(4f, 6s) = 1470$, $G^1(6p, 6s) = 8103$ and $\tau = 0{\cdot}4$. The RMS deviation is 39 cm^{-1}. The unreasonably low value of τ, coupled with the appreciable RMS deviation, suggests that their eigenfunctions should be less reliable than ours, at least for the nine z levels.

3. STATES

The least squares fitting procedure determines not only the energies of the levels but also the corresponding eigenfunctions. For the level labelled $^6P_{7/2}$, for example, we find that the actual state $|[^6P_{7/2}])$ is given for calculation α by

$$|[^6P_{7/2}]) = 0{\cdot}1177|^8S,\ ^3P,\ ^{10}P_{7/2}) + 0{\cdot}5676|^8S,\ ^3P,\ ^8P_{7/2})$$
$$+ 0{\cdot}1100|^8S,\ ^1P,\ ^8P_{7/2}) + 0{\cdot}8072|^8S,\ ^3P,\ ^6P_{7/2}).$$

By squaring the coefficient 0·8072, it can be seen that the state $^6P_{7/2}$ is only 65% pure. This demonstrates the importance of making allowance for deviations from perfect Russell-Saunders (RS) coupling. For calculation β, the four coefficients become 0·1154, 0·5699, 0·1040, and 0·8089 respectively. The changes are slight, but not insignificant.

The theory so far has been non-relativistic. However, the hyperfine interaction depends mainly on those parts of the electronic eigenfunctions lying close to the nucleus; and the corresponding velocities of the electrons are not negligible compared to c, the velocity of light. Every single-electron eigenfunction, though part of an elaborately coupled scheme, must obey Dirac's equation for a charged particle moving in a spherically symmetric central field. The changes that are necessary can be succinctly described by the substitution

$$(R/r)|s\ \ell\ jm) \rightarrow \begin{pmatrix} (F/r)|s\ \ell'\ jm) \\ (G/ir)|s\ \ell\ jm) \end{pmatrix}, \qquad (1)$$

where the quantity on the right is a column matrix in which $\ell' = \ell \pm 1$ according as $j = \ell \pm 1/2$. Instead of one radial function R, we now have two, F and G. They can be assumed to be real.

4. OPERATORS

The vector potential $\underset{\sim}{A}$ for a nucleus with a magnetic moment $\underset{\sim}{\mu}_N$ is given by

$$\underset{\sim}{A} = (\underset{\sim}{\mu}_N \wedge \underset{\sim}{r})/r^3.$$

It is straightforward to treat a Dirac electron moving in such a field. On writing $\underset{\sim}{\mu}_N = \beta_N \mu_N \underset{\sim}{I}/I$, where β_N ($=e\hbar/2Mc$) is the nuclear magneton, it is found that the effective Hamiltonian can be taken to be $\underset{\sim}{I}.\underset{\sim}{X}$, where $\underset{\sim}{X}$ is a vector whose amplitude, for a given ℓ and between two states j (with radial functions F and G) and j' (with radial functions F' and G') is defined by

$$(j'\|X\|j) = \frac{e\mu_N\beta_N}{I}(-1)^{\ell}\sqrt{\{2[j][j']\}}\begin{pmatrix} j' & 1 & j \\ -\frac{1}{2} & 1 & -\frac{1}{2}\end{pmatrix}\int_0^{\infty}\frac{FG'+F'G}{r^2}\,dr. \qquad (2)$$

This form is not convenient, since, to use it, we would have to transform all our states to jj-coupling. This difficulty has been elegantly circumvented by Sandars (private communication), who pointed out that $\underset{\sim}{X}$ can be replaced by any combination of single-particle vector operators, provided that Eq. (2) is satisfied by all four possibilities $j = \ell \pm 1/2$, $j' = \ell \pm 1/2$. Now the states defined by j and j' in Eq. (2) are relativistic, in as much as they stand for column matrices of the type on the right in (1); but, if we wish, we may evidently ask that the replacement for $\underset{\sim}{X}$ correctly reproduces the right-hand side of Eq. (2) if the simple non-relativistic states $|s\ \ell\ jm)$ are used, rather than the full relativistic forms.

For the non-relativistic operators, it is convenient to use the coupled double tensors $\underset{\sim}{w}^{(\kappa\ k)K}$, defined for a particular electron ℓ by

$$\underset{\sim}{w}^{(\kappa\ k)K} = (\underset{\sim}{t}^{(\kappa)}\underset{\sim}{v}^{(k)})^{(K)},$$

where $\underset{\sim}{t}^{(\kappa)}$ and $\underset{\sim}{v}^{(k)}$ act in the spin and orbital spaces respectively, and whose amplitudes are defined by

$$(s\|t^{(\kappa)}\|s) = [\kappa]^{1/2}, \qquad (\ell\|v^{(k)}\|\ell) = [k]^{1/2}.$$

In four cases the tensors $\underset{\sim}{w}^{(\kappa\ k)K}$ are simply related to familiar hyperfine operators. Thus, on comparing reduced matrix elements, we find

$$\begin{aligned}
\underset{\sim}{w}^{(01)1} &= \left[\frac{3}{2\ell(\ell+1)(2\ell+1)}\right]^{1/2} \underset{\sim}{\ell}\,, \\
\underset{\sim}{w}^{(10)1} &= \left[\frac{2}{2\ell+1}\right]^{1/2} \underset{\sim}{s}\,, \\
\underset{\sim}{w}^{(12)1} &= -\left[\frac{10(2\ell-1)(2\ell+3)}{\ell(\ell+1)(2\ell+1)}\right]^{1/2} (\underset{\sim}{s}^{(1)}\underset{\sim}{C}^{(2)})^{(1)}, \qquad (3) \\
\underset{\sim}{w}^{(02)2} &= -\left[\frac{5(2\ell-1)(2\ell+3)}{2\ell(\ell+1)(2\ell+1)}\right]^{1/2} \underset{\sim}{C}^{(2)}\,;
\end{aligned}$$

in the notation of Edmonds (1960).

A tedious but straightforward calculation indicates that the required replacement for X is given by

$$\underset{\sim}{X} \rightarrow a(10)\underset{\sim}{w}^{(10)1} + a(01)\underset{\sim}{w}^{(01)1} + a(12)\underset{\sim}{w}^{(12)1}, \qquad (4)$$

where

$$\begin{aligned}
a(10) &= -\frac{2\beta_N\mu_N e}{I}\left[\frac{8}{9(2\ell+1)^3}\right]^{1/2}[(\ell+1)^2F_{++} + \ell(\ell+1)F_{+-} + \ell^2F_{--}], \\
a(01) &= \frac{2\beta_N\mu_N e}{I}\left[\frac{2\ell(\ell+1)}{3(2\ell+1)^3}\right]^{1/2}[-2(\ell+1)F_{++} + F_{+-} + 2\ell F_{--}], \qquad (5)
\end{aligned}$$

$$a(12) = \frac{2\beta_N \mu_N e}{3I} \left[\frac{\ell(\ell+1)}{(2\ell+1)^3(2\ell+3)(2\ell-1)} \right]^{1/2}$$

$$\times \; [4(\ell+1)(2\ell-1)F_{++} - (2\ell-1)(2\ell+3)F_{+-} + 4\ell(2\ell+3)F_{--}].$$

In these expressions

$$F_{++} = \int_0^\infty \frac{F_+G_+}{r^2}\, dr,$$

$$F_{+-} = \int_0^\infty \frac{F_+G_- + F_-G_+}{r^2}\, dr, \qquad (6)$$

$$F_{--} = \int_0^\infty \frac{F_-G_-}{r^2}\, dr,$$

where F_+ and G_+ are the radial functions corresponding to $j = \ell + 1/2$; the functions F_- and G_- correspond to $j = \ell - 1/2$.

As a check on our formulae, we may let c tend to infinity, corresponding to the passage to the non-relativistic limit. Each term in (4) goes over into its non-relativistic counterpart: the first term corresponds to the contact interaction between an s electron and the nucleus; the second represents the interaction between $\underset{\sim}{\mu}_N$ and the magnetic field produced by the purely orbital motion of the electron; and the third represents the interaction between $\underset{\sim}{\mu}_N$ and the magnetic moment $-2\beta\underset{\sim}{s}$ of the electron, where β is the Bohr magneton.

5. MAGNETIC HYPERFINE STRUCTURE

In principle, replacements of the type (4) must be made for each of the three kinds of electrons (4f, 6s and 6p) that comprise the configuration $4f^7 6s6p$ of EuI. However, we see at once that all matrix elements of $\underset{\sim}{w}^{(\kappa\ k)1}$ vanish for an s electron when $k > 0$; and since we are studying only those states deriving from the ^8S term of $4f^7$, we can similarly drop all 4f-electron tensors $\underset{\sim}{w}^{(\kappa\ k)K}$ for which $k > 0$. Using the more familiar operators of Eqs. (3), we conclude that the most general relativistic expression for the magnetic hyperfine interaction can be taken to be

$$\underset{\sim}{Q}\cdot\underset{\sim}{I} = (\underset{\sim}{Q}_1 + \underset{\sim}{Q}_2 + \underset{\sim}{Q}_3)\cdot\underset{\sim}{I}\ , \tag{7}$$

where $\underset{\sim}{Q}_1 = a_f\underset{\sim}{S}$ for the 4f electrons; where $\underset{\sim}{Q}_2 = a_s\underset{\sim}{s}$ for the single 6s electron; and where

$$\underset{\sim}{Q}_3 = a_p\underset{\sim}{s} + b_p\underset{\sim}{\ell} - c_p(10)^{1/2}(\underset{\sim}{s}^{(1)}\underset{\sim}{C}^{(2)})^{(1)}$$

for the 6p electron.

We could now attempt to fit the observed hyperfine structure constants A for the various levels of $4f^7\ (^8S)6s6p$, using the states we have already calculated and taking the quantities a_f, a_s, a_p, b_p and c_p as five adjustable parameters. However, it seemed more worthwhile to reduce the degree of freedom, so that any agreement that we might subsequently obtain for the constants A would appear all the more striking. First, it was decided to calculate a_p, b_p and c_p, while leaving a_s and a_f free. The reason for this choice is that there exists a fairly reliable method for calculating the parameters for a p electron from the spin-orbit coupling constant, ζ_p; and this quantity has been

accurately determined from the diagonalization described in Sec. 2. This procedure for an electron $\ell(>0)$ runs as follows. An effective value Z_{eff} of the nuclear charge Z is decided upon. The equation

$$\langle 1/r^3 \rangle = 2m^2c^2 \zeta_\ell / Z_{eff} H \hbar^2 e^2$$

is used to calculate $\langle 1/r^3 \rangle$, where H is a relativistic correction factor (see Casimir 1963, p.51). The integrals $F_{\pm\pm}$ are now found by means of the formulae

$$F_{--} = \hbar(\ell+1)F'\langle 1/r^3 \rangle / 2mc,$$

$$F_{+-} = \hbar G \langle 1/r^3 \rangle / 2mc,$$

$$F_{++} = -\hbar \ell F'' \langle 1/r^3 \rangle / 2mc,$$

where F', G and F'' are relativistic correction factors. Suitable tables for finding H, F', G and F'' have been prepared by Kopfermann (1958). The constants $a(\kappa\ k)$ of Eqs. (5) can now be calculated in terms of the nuclear moment μ_N.

This method was adopted for the 6p electron. The starting point is $\zeta_p = 1198\ \mathrm{cm}^{-1}$ (corresponding to calculation α). For Z_{eff} we used $Z-3 = 60$. This is a compromise between the traditional formula $Z-4$ (see, for example, Kuhn 1962, p. 335) and $Z-2{\cdot}5$, which appears appropriate for a p electron in triply ionized rare-earth ions (Judd 1963). For ^{151}Eu, the value $\mu_N = 3{\cdot}42$ of Sandars & Woodgate (1960) was replaced by $\mu_N = 3{\cdot}463$ (Sandars & Woodgate, private communication). The final values are $a_p = -0{\cdot}099b_p$, $c_p = 1{\cdot}337b_p$, and $b_p = 17{\cdot}42\mathrm{mK}$. The non-relativistic equations would be $a_p = 0$, $c_p = b_p$.

The matrix elements of $\underset{\sim}{Q}$ were calculated by tensor-operator techniques for all the twelve levels, using the eigenfunctions of calculation α. There are two parameters, a_f and a_s. The A values of the seven levels $^{10}P_{11/2}$, $^{10}P_{9/2}$, $z^8P_{9/2}$, $y^8P_{9/2}$, $^{10}P_{7/2}$, $y^8P_{7/2}$, and $^6P_{5/2}$ have been measured by Müller et al. (1965), and, although the experimental errors vary considerably from one level to another, a simple least-squares fit was carried out. The justification for doing this is that in all cases these errors are much smaller than the differences between experiment and theory that emerge from the fitting procedure. The discrepancies are of the order of 2%. The parameters turn out to be a_s = 330·39mK and a_f = -2·35mK. These lead to values of A for $^6P_{7/2}$ and $y^8P_{5/2}$ that lie within the bounds set for them by Müller et al. (1965). Details are given in table 2 in the rows labelled α.

To get an idea of the sensitivity of the parameters to the conditions of the fitting procedure, the calculation was repeated with the eigenfunctions of calculation β. It was also decided to test the calculated value of b_p by leaving it as a variable parameter, the theoretical ratios b_p/a_p and b_p/c_p being preserved. The agreement with the A values is now slightly better, as can be seen from the rows labelled β of table 2. In particular, the large negative value of $A(^6P_{5/2})$ is accounted for. On the other hand, agreement with the A values that are known the most accurately experimentally has worsened somewhat. The parameters are given by a_s = 332·54mK, a_f = -2·52mK, and b_p = 18·3mK. As Müller et al. (1965) point out, the 5% increase in b_p can be accounted for if one uses $Z_{eff} = Z-6$ (a formula due to Barnes and Smith 1954) in place of $Z_{eff} = Z-3$. Our own feeling is that a small discrepancy of this kind could quite possibly arise from other sources, in particular, from the interaction with higher configurations.

6. CORE POLARIZATION

Throughout the preceding discussion, it has been assumed that the configuration $4f^7 6s6p$ is pure. However, the interactions between configurations can make themselves felt in various ways, and, for the hyperfine constants, A, the most important effect is produced by the polarization of the core of s electrons by the electrons in unfilled shells. From the point of view of perturbation theory, it may be said that the Coulomb interaction admixes states such as those of $4s4f^7 6s6p8s$ into the states of $4s^2 4f^7 6s6p$; and although the admixture is extremely small, the matrix elements of the hyperfine operator between 4s8s and $4s^2$ may be large. The total effect is the sum of all possible contributions, corresponding to the (virtual) excitation of every s electron to all possible vacant s-electron orbitals, including those in the continuum. When the normal contribution to the hyperfine structure from the valence electrons is small, the effect of core polarization is particularly striking. For example, for ^{151}EuI$4f^7 6s^2\ {}^8S_{7/2}$, Sandars & Woodgate (1960) find A = -0·668mK; but for ^{151}EuIII $4f^7\ {}^8S_{7/2}$, Baker & Williams (1962) give A = -3·430mK. The eigenfunctions of the 4f electrons are expected to be almost the same for the neutral atom as for the ion: the larger value of A for the ion must be ascribed to the absence of 6s-electron excitation together with the possibility of exciting inner s electrons into the now vacant 6s orbitals.

If all the shells of an atom save one, say $(n\ell)^N$, are closed, a fairly simple expression can be obtained for the contribution $A(n_1 n_2, n\ell)$ to the hyperfine structure constant A corresponding to the excitation of an electron $(n_1 s)$ to the shell $(n_2 s)$. Denoting the amplitude of the eigenfunction for an electron $(n's)$ at the nucleus by $\psi_0(n')$, we find by standard perturbation-theory techniques that

$$A(n_1 n_2, n\ell) = 2(g-1)(16\pi\beta\beta_N\mu_N/3I)$$
$$\psi_0(n_1)\psi_0(n_2)R^{\ell}(n\ell n_1 s, n_2 s\, n\ell)/(2\ell+1)\Delta(n_1 n_2),$$

where R^{ℓ} is a Coulomb integral (in the notation of Condon & Shortley 1935), where $\Delta(n_1 n_2)$ is the (positive) energy of excitation, and where g is the Landé g value for the level of $(n\ell)^N$ under examination. To obtain the total contribution, we must sum over all appropriate values of n_1 and n_2. We should also visualize n_2 extending beyond infinity, as it were, in order to label states of the continuum.

The situation is more complicated for EuI $4f^7(^8S)6s6p$; for, in addition to the unclosed 4f and 6p shells, the 6s shell is only half filled. However, a detailed calculation reveals that for all twelve levels of $4f^7(^8S)6s6p$, the various s-electron excitations can be exactly represented by contributions δa_f, δa_s, and δa_p to the three parameters a_f, a_s, and a_p. The complicated summations for these contributions are difficult to estimate. Fortunately, many of them recur in precisely the same algebraic form when analyses are performed for EuI $4f^7 6s^2$, EuII $4f^7 6s$, and EuIII $4f^7$. If the zero-order contributions to the hyperfine structure are represented by truncated Hamiltonians of the type $\underset{\sim}{Q}_1 \cdot \underset{\sim}{I}$ (for EuI $4f^7 6s^2$) or $(\underset{\sim}{Q}_1 + \underset{\sim}{Q}_2) \cdot \underset{\sim}{I}$ (for EuII $4f^7 6s$), it is found that the contributions from core polarization can again be represented by changes to a_f, and, in the case of EuII, by a simultaneous change of a_s. The details are shown in table 3, where identical algebraic summations share the same symbol A_j. Thus

$$A_1 = \sum_{n'>6} A(6n', 4f),$$

$$A_2 = \sum_{n'>6>n} A(nn', 4f),$$

$$A_3 = \sum_{n'>6} A(6n', 6p),$$

etc. Of course, we know that $R^3(4f^4s, 7s^4f)$, for example, is not the same for EuI as for EuIII: but nevertheless it is interesting to ignore differences of this sort and simply to suppose that all s-electron eigenfunctions, all 4f-electron eigenfunctions, and all excitation energies are the same for the four electronic structures included in table 3. For, if we make this simplifying assumption, the contributions to a_f from the 4f electrons themselves will be invariant; and from this fact and table 3 the following approximate equation is readily obtained:

$$a_f[\mathrm{EuI}4f^7(^8S)6s6p] = 1/2a_f[\mathrm{EuI}4f^7(^8S)6s^2] + 1/2a_f[\mathrm{EuIII}4f^7(^8S)] .$$

In other words, we would expect a_f for the twelve levels of $4f^7(^8S)6s6p$ to be roughly 1/2(-0·668) + 1/2(-3·430), namely -2·05mK. The values that we have actually obtained (-2·35 and -2·52 for calculations α and β) are not far from this number; indeed, the agreement is very striking.

This successful correlation of core polarization in EuI and EuIII can be pursued to include EuII. From table 3, we see

$$a_f[\mathrm{EuI}4f^7(^8S)6s6p] = a_f[\mathrm{EuII}4f^7(^8S)6s].$$

With a little manipulation, it can be shown that, for EuII,

$$a_f[\mathrm{EuII}4f^7(^8S)6s] = 1/2A(^9S_4) + 1/2A(^7S_3).$$

According to Krebs & Winkler (1960 a), $A(^9S_4)$ = 51·11 ± 0·15mK, and

$A(^7S_3) = -55{\cdot}85 \pm 0{\cdot}15$mK. These data lead to an anticipated value of a_f for EuI $4f^7(^8S)6s6p$ of $-2{\cdot}37 \pm 0{\cdot}15$mK, in excellent accord with our calculated values.

By fixing a_p in Sec. 5, we have implicitly set the total core polarization produced by the 6p electron, namely $1/2A_3 + A_4 + 1/2A_6$, equal to zero. The good agreement between experiment and theory shown in table 2 indicates that at present there is little need to modify this; indeed, our eigenfunctions are probably not known accurately enough to serve as a basis for an analysis of such subtle corrections. If we suppose further that the individual contributions A_3, A_4 and A_6 are themselves small, table 3 indicates that for EuI $4f^76s6p$,

$$\begin{aligned} \delta a_s &= 7/2A_1 - 7/2A_5 \\ &= 7/2a_f[\text{EuI } 4f^7(^8S)6s^2] - 7/2a_f[\text{EuIII } 4f^7(^8S)] \\ &= 9{\cdot}67 \text{ mK.} \end{aligned}$$

Hence the value of a_s that can be legitimately assigned to the 6s electron itself, say a_{6s}, is not 331 ± 2 mK, as implied by the analysis of Sec. 5, but rather

$$a_{6s} = 331 - 9{\cdot}67 = 321 \pm 2 \text{ mK.}$$

7. HYPERFINE STRUCTURE ANOMALIES

For the two isotopes ^{151}Eu and ^{153}Eu, the hyperfine structure anomaly Δ of a level with A values $A(151)$ and $A(153)$ is given by

$$\Delta = [A(151)\mu_N(153)/A(153)\mu_N(151)] - 1.$$

Sandars & Woodgate (private communication) have revised their earlier value for the ratio of the magnetic moments (Sandars & Woodgate 1960). The latest figure, namely

$$\mu_N(151)/\mu_N(153) = 2{\cdot}2650 \pm 0{\cdot}0010,$$

is now in agreement with the work of Baker & Williams (1962) on the ion EuIII, which yielded

$$\mu_N(151)/\mu_N(153) = 2{\cdot}2632 \pm 0{\cdot}0026.$$

The measurements of Müller et al. (1965) are sufficiently accurate for the values of Δ to be deduced for three levels of EuI $4f^7(^8S)6s6p$. These are given in the third column of table 4, assuming a value of 2·2650 for the ratio of the magnetic moments. The assigned limits of error are calculated entirely from uncertainties in the A values, and not from the uncertainty of 2·2650. This was done so that subsequent comparisons of hyperfine structure anomalies should be as meaningful as possible; for it is easily seen that a small change in the ratio $\mu_N(151)/\mu_N(153)$ produces identical displacements to all Δ values.

Hyperfine structure anomalies are produced principally by s electrons, which are the most sensitive to changes in the shape of the nucleus. If, for

a given level, $A_s(151)$ and $A_s(153)$ are the respective contributions to A(151) and A(153) coming from a particular s electron, theoretical considerations indicate that the quantity $\Delta(s)$, defined by

$$\Delta(s) = [A_s(151)\mu_N(153)/A_s(153)\mu_N(151)] - 1,$$

should be virtually independent of the type of s electron chosen (see Kopfermann 1958, p. 130). It follows that $\Delta(s)$ should be the same for EuI as for EuIII, since inner s shells are not appreciably altered if a few outer electrons are removed from a heavy atom. This property was used by Baker & Williams (1962) to deduce $\Delta(s) = -0{\cdot}78 \pm 0{\cdot}11\%$ from the resonance results on the ground levels of these two ions. With the new value of 2·2650 for the ratio of the magnetic moments, we recalculate $\Delta(s) = -0{\cdot}65\%$. The uncertainty in 2·2650, which, as explained above, we wish to keep distinct from other errors, corresponds to 0·10%.

Each level of EuI $4f^7(^8S)6s6p$ for which Δ has been found can be used to deduce a value of $\Delta(s)$, since the total contributions to the A values coming from s electrons are known (see table 2). In fact, Δ can be expressed simply by a factor times $\Delta(s)$. These factors are given in the second column of table 4. Three separate calculations for $\Delta(s)$ can now be performed by comparing theory with experiment for $^{10}P_{9/2}$, $z^8P_{9/2}$, and $^{10}P_{7/2}$. The results are given in table 4. All values of $\Delta(s)$ are in excellent agreement with the figure of -0·65% derived from the resonance results. We conclude that the hyperfine structure anomalies for EuI $4f^7(^8S)6s^2$, for EuIII $4f^7(^8S)$, and for those levels of EuI $4f^7(^8S)6s6p$ that have so far been investigated, fall into a consistent pattern.

8. QUADRUPOLE OPERATORS

We have now to consider the interaction between the electrons and the quadrupole moment Q of the nucleus. To treat this relativistically, without having to transform the states to jj coupling, we carry out an analysis similar to that for the magnetic operators (see Sec. 4). We find that the effective Hamiltonian

$$\left(\frac{3}{2}\right)^{1/2}(\underset{\sim}{Z}^{(2)} \cdot \{\underset{\sim}{I}^{(1)}\underset{\sim}{I}^{(1)}\}^{(2)})$$

is to be used, where, in complete analogy to Eq. (2),

$$(j'\|Z^{(2)}\|j) = -\frac{e^2Q}{I(2I-1)}(-1)^{j'+1/2}\sqrt{([j][j'])}\begin{pmatrix} j' & 2 & j \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix}\int_0^\infty \frac{FF'+GG'}{r^3}\,dr.$$

The substitution corresponding to (4) is

$$\underset{\sim}{Z}^{(2)} \rightarrow b(11)\underset{\sim}{W}^{(11)2} + b(13)\underset{\sim}{W}^{(13)2} + b(02)\underset{\sim}{W}^{(02)2},$$

where

$$b(11) = -\frac{e^2Q}{I(2I-1)}\left[\frac{4\ell(\ell+1)}{25(2\ell+1)^3}\right]^{1/2}[-(\ell+2)R_{++} + 3R_{+-} + (\ell-1)R_{--}],$$

$$b(13) = -\frac{e^2Q}{I(2I-1)}\left[\frac{6(\ell-1)\ell(\ell+1)(\ell+2)}{25(2\ell-1)(2\ell+1)^3(2\ell+3)}\right]^{1/2}$$

$$\times\ [(2\ell-1)R_{++} + 4R_{+-} - (2\ell+3)R_{--}],$$

$$b(02) = +\frac{e^2Q}{I(2I-1)}\left[\frac{2\ell(\ell+1)}{5(2\ell-1)(2\ell+1)^3(2\ell+3)}\right]^{1/2}$$

$$\times\ [(2\ell-1)(\ell+2)R_{++} + 6R_{+-} + (\ell-1)(2\ell+3)R_{--}].$$

In these expressions,

$$R_{++} = \int_0^\infty \frac{F_+^2 + G_+^2}{r^3}\, dr,$$

$$R_{+-} = \int_0^\infty \frac{F_+F_- + G_+G_-}{r^3}\, dr,$$

$$R_{--} = \int_0^\infty \frac{F_-^2 + G_-^2}{r^3}\, dr.$$

In the non-relativistic limit, these integrals all tend to $\langle 1/r^3 \rangle$. The coefficients b(11) and b(13) become zero, and

$$b(02)\underset{\sim}{w}^{(02)2} \rightarrow -\frac{e^2Q}{I(2I-1)}\, \underset{\sim}{C}^2 \langle 1/r^3 \rangle .$$

The quadrupole coupling constant for the ground level of EuI $4f^7(^8S)6s^2$ is, from an optical standpoint, extremely small. This indicates that we need only consider the electron 6p in the calculation of the B factors for the levels of EuI $4f^7(^8S)6s6p$. For such an electron, all matrix elements of $\underset{\sim}{w}^{(13)2}$ are zero, and we have only to calculate the matrix elements of $\underset{\sim}{w}^{(02)2}$ and $\underset{\sim}{w}^{(11)2}$. Tensor-operator techniques are used; for example,

$$(^8S(sp)S_1P,\ S_2PJ \| \underset{\sim}{w}^{(11)2} \| {}^8S(sp)S_3P,\ S_4PJ)$$

$$= 3\sqrt{5}[J]\{[S_1][S_2][S_3][S_4]\}^{1/2}(-1)^{S_1+S_2+S_3+1/2}$$

$$\times \begin{Bmatrix} S_2 & 1 & S_4 \\ S_3 & \frac{7}{2} & S_1 \end{Bmatrix} \begin{Bmatrix} S_1 & 1 & S_3 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} S_2 & S_4 & 1 \\ 1 & 1 & 1 \\ J & J & 2 \end{Bmatrix}.$$

The integrals $R_{\pm\pm}$ are found from $R_{+-} = S\langle 1/r^3\rangle$ and $R_{++} = R''\langle 1/r^3\rangle$, where S and R" are the correction factors of Casimir (1963). To present the results, it is convenient to tabulate the number Ω, defined for each level by the equation

$$B = e^2 Q\langle 1/r^3\rangle\Omega.$$

Values of Ω are set out in the second column of table 5. From the observations of Müller et al. (1965), values of $e^2Q\langle 1/r^3\rangle$ can be found that give moderately good agreement with the experimental data. In carrying out the fitting procedure, only the accurate data on $^{10}P_{9/2}$, $z^8P_{9/2}$ and $^{10}P_{7/2}$ were used. We find $e^2Q\langle 1/r^3\rangle = 29{\cdot}3$ and $73{\cdot}9$ mK for ^{151}Eu and ^{153}Eu respectively. Details are given in table 5.

9. QUADRUPOLE MOMENTS

To determine Q from the product $e^2Q\langle 1/r^3\rangle$, we need $\langle 1/r^3\rangle$. This can be obtained from the calculation of Sec. 5; but we also note that it led to a value of b_p that was 5% smaller than the one determined from experiment. As a compromise, Müller et al. (1965) take the value of $\langle 1/r^3\rangle$ that leads to the average value of b_p, namely 17·8 mK, and assign an error that includes both limits. In quoting values for Q, the error should be widened to allow for the discrepancies in table 5 between the experimental and theoretical values of $B(^{10}P_{9/2})$ and $B(^8P_{9/2})$. When this is done, one obtains

$$\begin{aligned} Q(151) &= 1{\cdot}16 \pm 0{\cdot}08 \text{ b}, \\ Q(153) &= 2{\cdot}92 \pm 0{\cdot}20 \text{ b}. \end{aligned} \tag{8}$$

These are the quadrupole moments given by Müller et al. (1965). No Sternheimer corrections have been applied.

There is a marked discrepancy between theory and experiment in table 5 for $B(^{151}Eu\ y\,^{8}P_{9/2})$, a datum not used in the fitting procedure. However, we notice that the experimental limits of error are comparatively large, and also that $y\,^{8}P_{9/2}$ is one of the two levels showing a large discrepancy between its observed and theoretical energy (see table 1). This suggests that is has been perturbed by higher levels, and that its eigenfunctions may not be as accurately known as those of the other levels. Nevertheless, we should not dismiss this difficulty too lightly, since the levels of $4f^{7}6s6p$ labelled by the symbol y correspond to the coupling $(6s6p)\,^{1}P$, and quadrupole moments derived from the higher term ^{1}P of the configuration 6s6p are systematically smaller than those derived from ^{3}P. As examples, we may cite Hg (Blaise & Chantrel 1957), Yb (Ross & Murakawa 1964) and Ba (Jackson & Duong Hong Tuan 1963, 1964; Putlitz 1963). The most obvious explanation for these discrepancies is that the 6p electron has a rather more extended orbit in the ^{1}P state than in the ^{3}P state, thereby decreasing $\langle 1/r^{3}\rangle_{6p}$ for ^{1}P and so giving a smaller quadrupole interaction (Jackson & Duong Hong Tuan 1964).

This idea can be expressed in terms of perturbation theory. Exchange interactions between 6s6p and 6snp possess opposite signs for ^{3}P and ^{1}P; and by admixing np into 6p through this mechanism, the shape of the 6p eigenfunction becomes different for the two terms ^{1}P and ^{3}P of 6s6p. However, this point of view is not of much practical value, since one would presumably have to go far into the continuum to get realistic results. It would probably be better to solve for the eigenfunction of the 6p electron in two central fields that differ, one from the other, in the effective exchange potential produced by the 6s electron. If this explanation for the discrepancies in the quadrupole moments is correct, there should be no need to alter the values of Q given in Eqs. (8). This is because we have tied the properties of the 6p electron to

the spin-orbit coupling constant ζ_p, and this has been taken directly from experiment. Since the levels that determine ζ_p correspond to the coupling $(6s6p)^3P$, the terms ^{10}P, z^8P, and ^6P of EuI $4f^7(^8S)6s6p$ should give the most reliable values of Q.

Of course, other types of configuration interaction may intervene. The coupling between $4f^N6s6p$ and $4f^{N-1}6s^25d$ could lead to errors in the calculation of Q; though, since the levels of EuI $4f^7(^8S)6s6p$ corresponding to the coupling $(6s6p)^3P$ are further from possible perturbing levels than those corresponding to $(6s6p)^1P$, we would again prefer the Q values of Eqs. (8) to any derived from the levels of y^8P.

Having expressed considerable confidence in Eqs. (8), it must now be stated that the quadrupole moments given by these equations differ significantly from the ones published by Krebs & Winkler (1960 b), namely

$$Q(153) = 2{\cdot}42 \pm 0{\cdot}2 \text{ b}, \qquad Q(151) = 0{\cdot}95 \pm 0{\cdot}1 \text{ b}. \tag{9}$$

These are derived from the B values of just one level, namely $^9P_5$ of EuII $4f^7(^8S)6p$. This value of Q(153) agrees with that of Elbek (1963), obtained from Coulomb excitation. We have repeated the calculations of Krebs & Winkler, using methods analogous to those employed here. A least squares fit was performed for the configuration $4f^7(^8S)6p$, and from the value of ζ_p so obtained (1707 cm^{-1}), the radial integrals were calculated. Casimir correction factors appropriate to $Z_{eff} = 60$ were used. Accepting the values of B for $^9P_5$, we obtained quadrupole moments essentially in agreement with those of Krebs & Winkler.

On the face of it, it seems difficult to reconcile Eqs. (8) and (9). The differential Sternheimer effect for a 6p electron in atom and ion should be very much smaller than the existing discrepancy of 20%. Perhaps the configuration $4f^7(^8S)6p$ is perturbed by neighbouring levels. Effects of this kind would show up if accurate B values could be found for other levels of ^9P and ^7P of EuII. In any case, it would be valuable if a more accurate measurement of $B(^9P_5)$ could be performed. The experiments on EuII now being started in Hannover by Dr. Walther and his collaborators should go far to clarify the situation, since at the moment it is difficult to discount completely the possibility that the source of the discrepancy is experimental.

We are most grateful to the group at Hannover for allowing us to use their hyperfine structure measurements, and particularly to Dr. H. Walther, who sent us data as soon as they were obtained. Thanks also go to Dr. P. G. H. Sandars, for informing us of his method for treating relativistic effects, and also for permission to quote the revised value of the ratio of the moments of the two europium isotopes. The later stages of the share of the work of one of the authors (B.R.J.) were done under the auspices of the United States Atomic Energy Commission.

REFERENCES

Baker, J. M. & Williams, F. I. B. 1962 Proc. Roy. Soc.A, 267, 283.

Barnes, R. & Smith, W. 1954 Phys. Rev. 93, 95.

Blaise, J. & Chantrel, H. 1957 J. Phys. Radium 18, 193.

Brix, P. 1952 Z. Phys. 132, 579.

Casimir, H. B. G. 1963 On the interaction between atomic nuclei and electrons. San Francisco: Freeman & Co.

Condon, E. U. & Shortley, G. H. 1935 The theory of atomic spectra. Cambridge University Press.

Edmonds, A. R. 1960 Angular momentum in quantum mechanics. Princeton University Press.

Elbek, B. 1963 Determination of nuclear transition probabilities by coulomb excitation. Copenhagen: Ejnar Munksgaards Forlag.

Jackson, D. A. & Duong Hong Tuan 1963 Proc. Roy. Soc. A,274, 145.

Jackson, D. A. & Duong Hong Tuan 1964 Proc. Roy. Soc. A,280, 323.

Judd, B. R. 1963 Proc. Phys. Soc. 82, 874.

Kopfermann, H. 1958 Nuclear moments. New York: Academic Press.

Krebs, K. & Winkler, R. 1957 Ann. Phys., Lpz., 20, 60.

Krebs, K. & Winkler, R. 1960a Z. Phys. 160, 320.

Krebs, K. & Winkler, R. 1960b Naturwiss. 47, 490.

Krebs, K. & Winkler, R. 1961 Ann. Phys., Lpz., 7, 77.

Kuhn, H. G. 1962 Atomic spectra. London: Longmans.

Müller, W., Steudel, A. & Walther, H. 1965 Z. Phys.

Pichanick, F. M., Sandars, P. G. H. & Woodgate, G. K. 1960 Proc. Roy. Soc. A, 257, 277.

Putlitz, G. zu 1963 Ann. Phys., Lpz., 11, 248.

Ross, J. S. & Murakawa, K. 1964 J. Phys. Soc. Japan 19, 249.

Rotenberg, M., Bivins, R., Metropolis, N. & Wooten, J. K. Jr. 1959 The 3-j and

6-j symbols. Cambridge, Massachusetts: Technology Press, Massachusetts Institute of Technology.

Russell, H. N., Albertson, W. & Davis, D. N. 1941 Phys. Rev. 60, 641

Russell, H. N. & King, A. S. 1939 Astrophys. J. 90, 155.

Sandars, P. G. H. & Woodgate, G. K. 1960 Proc. Roy. Soc. A, 257, 269.

Schüler, H. & Schmidt, Th. 1935 Z. Phys. 94, 457.

TABLE 1. ENERGIES OF THE LEVELS OF EuI $4f^7(^8S)6s6p$

Level	Experiment	Calculation α		Calculation β	
		Theory	Difference	Theory	Difference
$^{10}P_{11/2}$	15581·58	15588·5	-6·9	15581·6	0·0
$^{10}P_{9/2}$	14563·57	14554·5	9·1	14559·8	3·8
$z^8P_{9/2}$	16611·79	16595·0	16·8	16600·3	11.5
$y^8P_{9/2}$	21761·26	21643·0	118·3	-	-
$^{10}P_{7/2}$	14067·86	14079·3	-11·4	14083·7	-15·8
$z^8P_{7/2}$	15952·31	15953·1	-0·8	15949·9	2·4
$y^8P_{7/2}$	21605·17	21602·5	2·7	21605·5	-0·3
$^6P_{7/2}$	17340·65	17324·8	15·9	17329·3	11·3
$z^8P_{5/2}$	15890·53	15886·8	3·7	15879·5	11·0
$y^8P_{5/2}$	21444·58	21564·2	-119·6	-	-
$^6P_{5/2}$	17707·42	17716·3	-8·9	17716·1	-8·7
$^6P_{3/2}$	17945·49	17964·4	-18·9	17960·6	-15·1

All energies are in cm^{-1}

TABLE 2. VALUES OF A FOR ^{151}EuI $4f^7(^8S)6s6p$

Level	Method	Calculated A For Q_1	Calculated A For Q_2	Calculated A For Q_3	Calculated A Total	Observed
$^{10}P_{11/2}$	α	-1·50	30·04	2·16	30·70	31·13 ± 0·10
	β	-1·60	30·23	2·27	30·90	
$^{10}P_{9/2}$	α	-1·81	31·99	4·17	34·35	34·138 ± 0·008
	β	-1·94	32·21	4·35	34·62	
$z^8P_{9/2}$	α	-1·66	21·20	2·62	22·16	22·18 ± 0·01
	β	-1·78	20·71	2·76	21·69	
$y^8P_{9/2}$	α	-1·83	-9·81	3·32	-8·32	-7·68 ± 0·11
	β	-1·96	-9·26	3·51	-7·71	
$^{10}P_{7/2}$	α	-2·29	40·80	-5·96	32·55	32·32 ± 0·02
	β	-2·46	41·08	-6·26	32·36	
$z^8P_{7/2}$	α	-2·21	-13·55	5·53	-10·23	
	β	-2·37	-13·90	5·83	-10·44	
$y^8P_{7/2}$	α	-2·20	-6·50	1·13	-7·57	-7·1 ± 0·2
	β	-2·36	-5·75	1·15	-6·95	
$^6P_{7/2}$	α	-1·96	0·23	2·82	1·09	0 < A < 2·2
	β	-2·10	-0·32	2·96	0·54	
$z^8P_{5/2}$	α	-2·83	-14·71	-6·66	-24·20	
	β	-3·04	-15·37	-7·04	-25·44	
$y^8P_{5/2}$	α	-3·02	2.64	-4·30	-4·68	-5 < A < 0
	β	-3·24	3·70	-4·57	-4·11	
$^6P_{5/2}$	α	-2·68	-16·25	-0·05	-18·98	-19·72 ± 0·05
	β	-2·88	-16·83	-0·03	-19·74	
$^6P_{3/2}$	α	-4·23	-66·08	-4·76	-75·07	
	β	-4·54	-66·51	-5·00	-76·05	

All values are in mK

TABLE 3. BREAKDOWN OF CORE-POLARIZATION CONTRIBUTIONS

Configuration	Type of excitation ($n<6<n'$)	δa_f	δa_p	δa_s
EuI $4f^7 6s^2$	$(ns)^2(6s)^2 \rightarrow (ns)^2 6s(n's)$	A_1	-	-
	$(ns)^2(6s)^2 \rightarrow (ns)6s^2(n's)$	A_2	-	-
	$(ns)^2 6s \rightarrow (ns)^2(n's)$	$\frac{1}{2}A_1$	$\frac{1}{2}A_3$	$\frac{7}{2}A_1 + \frac{1}{2}A_3$
EuI $4f^7 6s6p$	$(ns)^2 6s \rightarrow (ns)6s(n's)$	A_2	A_4	0
	$(ns)^2 6s \rightarrow (ns)6s^2$	$\frac{1}{2}A_5$	$\frac{1}{2}A_6$	$-\frac{7}{2}A_5 - \frac{1}{2}A_6$
	$(ns)^2 6s \rightarrow (ns)^2(n's)$	$\frac{1}{2}A_1$	-	$\frac{7}{2}A_1$
EuII $4f^7 6s$	$(ns)^2 6s \rightarrow (ns)6s(n's)$	A_2	-	0
	$(ns)^2 6s \rightarrow (ns)6s^2$	$\frac{1}{2}A_5$	-	$-\frac{7}{2}A_5$
EuIII $4f^7$	$(ns)^2 \rightarrow (ns)(n's)$	A_2	-	-
	$(ns)^2 \rightarrow (ns)6s$	A_5	-	-

TABLE 4. HYPERFINE STRUCTURE ANOMALIES

Level	$\Delta/\Delta(s)$ (Theory)	Δ(Observed)	$\Delta(s)$ (Calculated)
$^{10}P_{11/2}$	0·93		
$^{10}P_{9/2}$	0·88	-0·57 ± 0·12%	-0.64 ± 0·14%
$z^{8}P_{9/2}$	0·88	-0·40 ± 0·20%	-0·45 ± 0·23%
$y^{8}P_{9/2}$	1·40		
$^{10}P_{7/2}$	1·18	-0·75 ± 0·12%	-0·64 ± 0·10%
$z^{8}P_{7/2}$	1·54		
$y^{8}P_{7/2}$	1·15		
$^{6}P_{7/2}$	-1·59		
$z^{8}P_{5/2}$	0·72		
$y^{8}P_{5/2}$	0·08		
$^{6}P_{5/2}$	1·00		
$^{6}P_{3/2}$	0·94		

TABLE 5. VALUE OF B FOR EuI $4f^7(^8S)6s6p$

Level	Ω	B for ^{151}Eu		B for ^{153}Eu	
		Theory	Experiment	Theory	Experiment
$^{10}P_{11/2}$	0·4501	13·19	13·5 ± 1·5	33·26	
$^{10}P_{9/2}$	-0·5650	-16·56	-16·79± 0·10	-41·75	-42·13 ± 0·12
$z^8P_{9/2}$	0·3415	10·01	9·65± 0·15	25·23	24·62 ± 0·15
$y^8P_{9/2}$	0·4689	13·75	8·6 ± 1·2	34·65	
$^{10}P_{7/2}$	0·1800	5·28	5·25± 0·20	13·30	13·27 ± 0·15
$z^8P_{7/2}$	-0·2280	-6·68		-16·85	
$y^8P_{7/2}$	-0·6189	-18·14	-15 ± 4	-45·73	
$^6P_{7/2}$	0·1268	3·72		9·37	
$z^8P_{5/2}$	0·0790	2·32		5·84	
$y^8P_{5/2}$	0·1643	4·82		12·14	
$^6P_{5/2}$	-0·4362	-12·78	-11·8± 0·4	-32·23	
$^6P_{3/2}$	0·0900	2·64		6·65	

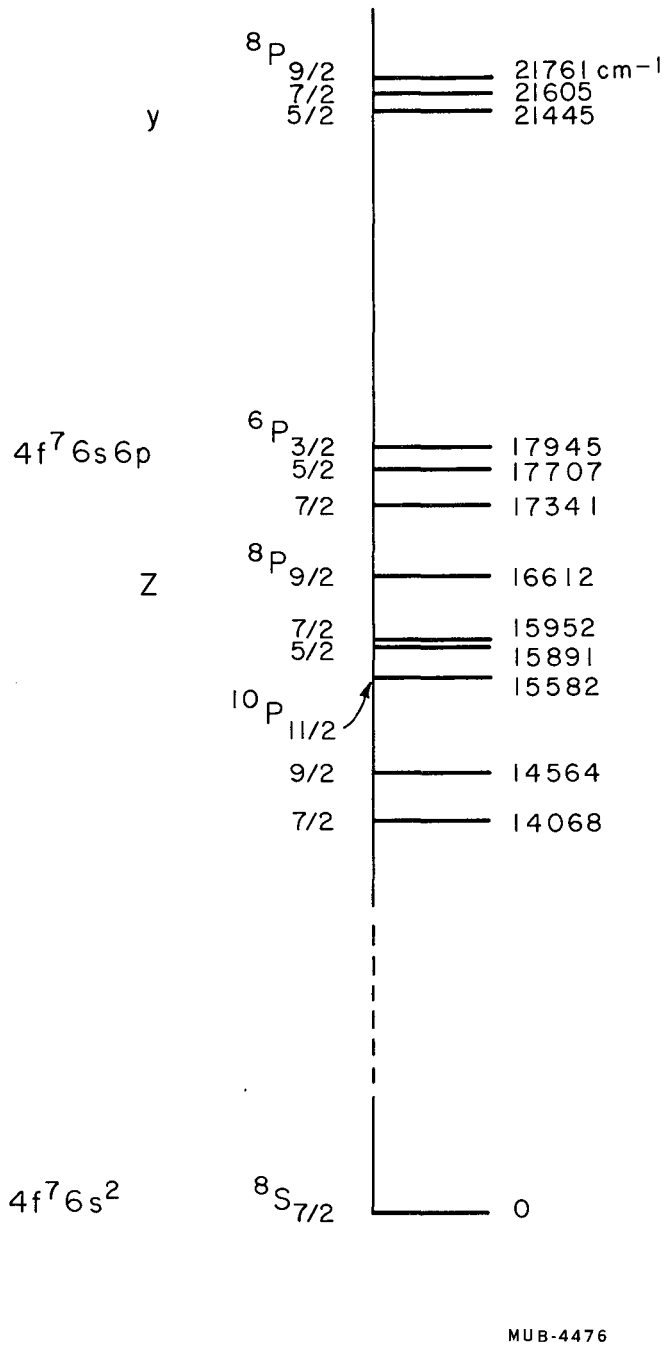

Fig. 1. Energy-level scheme for EuI $4f^7(^8S)6s6p$.

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