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SPACE AND TIME IN S-MATRIX THEORY

Berkeley, California

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# SPACE AND TIME IN S-MATRIX THEORY

Henry P. Stapp

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## ABSTRACT

It is shown how macroscopic space-time correlations emerge from a strictly mass-shell scattering theory. The $i\epsilon$ rule for the resolution of pole singularities is derived from a macroscopic mass-shell causality condition. The experimental and theoretical significance of form factors are discussed from the point of view of analytic S-matrix theory. They are defined as constructs of mass-shell functions, and the conditions under which they can be ascribed approximate experimental significance are examined. The possibility that asymptotic field theory is contained within analytic S-matrix theory is briefly discussed.

# I. INTRODUCTION

The S matrix describes correlations between limits of observations that extend in the limit into the infinite past or future. Because experience has aspects associated with finite time differences the question arises whether S-matrix theory is complete in the sense that all calculable correlations between experiences reside in, and can be calculated from, the mass-shell scattering matrix.

The principal object of this work is to examine this question and to attempt to show how the space-time structure of experience is contained in, and is derivable from, the mass-shell scattering amplitudes. As an incidental result the $i\epsilon$ prescription for the integration over physical region poles is derived from a strictly mass-shell causality condition.

A second object concerns form factors. Form factors have been used in dispersion theoretic treatments of electron-proton scattering. The question arises whether the success of these calculations indicates an inadequacy of pure S-matrix theory. This question is examined and it is shown how form factors arise naturally within the S-matrix framework. The experimental and theoretical significance of form factors is discussed from the S-matrix viewpoint, and a preliminary look is given to the question of whether these form-factor equations may be used to construct the local fields of field theory from the mass-shell scattering amplitudes.

In the following three sections some of the basic ingredients of S-matrix theory are summarized. These provide the basis for the work given in the succeeding sections.

## 2. SPACE-TIME DISPLACEMENTS[1]

The S matrix is a function of momentum-energy vectors:

$$S = S(K) \quad , \tag{2.1}$$

where

$$K = \{k_i\} \quad . \tag{2.2}$$

The energies are fixed by the mass constraints

$$k_i^{\,2} = m_i^{\,2} \quad , \tag{2.3}$$

together with the convention that vectors of K for initial particles shall be the negatives of the physical energy-momentum vectors.

By the superposition principle[2] the transition amplitude associated with a superposition $\phi(K)$ of initial and of final momentum values is

$$A \equiv A_\phi \equiv \int S(K)\,\phi(K)dK \quad . \tag{2.4}$$

Here the integration is over the covariant momentum-space factor

$$dK \equiv \prod_i \frac{d^3\underset{\sim}{k}_i}{(2\pi)^3} \frac{m_i}{|k_i^{\,0}|} \quad . \tag{2.5}$$

The basic property regarding space and time is that space-time displacements are represented by the usual exponential factor. In particular, if particle i is displaced by $x_i$, then the transition amplitude $A_\phi$ is changed to

$$A_\phi(X) = \int S(K)\,\exp(-iKX)\,\phi(K)dK \quad , \tag{2.6}$$

where

$$KX = \sum k_i^{\mu} x_{i\mu} = \sum_i (k_i^0 t_i - \underset{\sim}{k}_i \cdot \underset{\sim}{x}_i) \quad . \tag{2.7}$$

All quantum effects related to Planck's constant are apparent reflections of this basic connection between momentum-energy and space-time displacements. It is the connection formulated by de Broglie that has remained a basic element in quantum mechanics ever since. The S-matrix approach skips back over Schroedinger to de Broglie and Planck, when space and time are considered.

The concept of a displacement is not the same as that of a position; one can renounce the idea that a "particle at a point" makes sense and still hold the idea that the displacement of the particle by a certain amount makes sense. For instance, if a particle is represented by a solution of the free-particle wave equation then it is defined over all of space-time. One cannot restrict the wave function to finite times and still have a solution to the free wave equation. In this sense a free particle cannot be localized in time. Yet there is no trouble displacing it by any arbitrary amount; multiplication by the displacement operator defined above would do just that.

One can define a space-time S matrix by

$$\tilde{S}(X) = \int S(K) \exp(-iKX) dK \quad . \tag{2.8}$$

It satisfies the free-wave equation. Because of the invariant form of the phase-space factor, and of $(KX)$, the invariance equation[2]

$$S(K) = \Lambda_s^{-1} S(\Lambda K) \tag{2.9}$$

implies

$$\tilde{S}(X) = \Lambda_s^{-1} \tilde{S}(\Lambda X) \quad . \tag{2.10}$$

The function $\tilde{S}(X)$ defined in this way is the analog of the (free) field quantities of field theory.

It is, however, not the analog of the coordinate space wave function. The wave function has the property that its absolute value squared integrated over three-dimensional space gives probability. The field functions, on the other hand, have the simple transformation properties. In a relativistic theory these are generally different (unless one introduces redundant components) because integration over three-dimensional space is not an invariant concept.

The S-matrix analog of the wave function is

$$\tilde{S}_p(X) \equiv \int S(K) \exp(-iKX) \left(\frac{K^0}{M}\right)^{1/2} dK \quad , \tag{2.11}$$

where

$$\left(\frac{K^0}{M}\right)^{1/2} = \prod_i \left(\frac{|k_i^0|}{m_i}\right)^{1/2} \quad . \tag{2.12}$$

Defining, in a similar way,

$$\tilde{\phi}_p(X) \equiv \int \phi(K) \exp(iKX) \left(\frac{K^0}{M}\right)^{1/2} dK \quad , \tag{2.13}$$

one obtains

$$A_\phi \equiv \int \phi(K)\, S(K) dK = \int \tilde{\phi}_p(X)\, \tilde{S}_p(X) d^3X \quad . \tag{2.14}$$

Thus the transition amplitude is represented by an integration over $d^3X$. The space integration can be performed at any chosen time, because of the translation invariance following from the opposite signs in (2.11) and (2.13).

The factor $|m_i/k_i^0|^{1/2}$ relating the field function and the wave function is just the familiar factor that occurs in (free) field theory[3] when one expands the field in terms of $a(k)$, which is the momentum-space wave function. The distinction between fields and wave functions can also be regarded as the distinction between coordinate space and "position" space. Wigner and Newton[4] construct a position operator in coordinate space, on the basis of certain general requirements. The transformation from coordinate space to the space built on position eigenfunctions is just the transformation from the field to the wave function.

Spins have not been explicitly mentioned. If they are treated in the manner described in References 2, 5, and 6, then the above formulas are unaltered; one simply interprets the products to represent a covariant contraction on the spinorial indices.

This is an appropriate place to describe the viewpoint here adopted with regard to space-time functions. The fundamental assumption in S-matrix theory is that the elementary observables correspond to freely moving physical particles. Freely moving physical particles are represented by solutions of the free-wave equation. Thus the elementary observables of S-matrix theory are represented by solutions to the free-wave equation. By Fourier transformation one can obtain equivalent momentum-energy functions. It is the momentum-energy S matrix that is the theoretically interesting function, because of its apparent analyticity properties. The coordinate-space functions are the ones most closely connected to experience.

Solutions of the free-wave equation extend to infinite times in a well-defined way. Any attempt to localize them with more than a $|t|^{-3/2}$ fall-off introduces off-mass-shell components. Correspondingly, any attempt to observe a physical particle in a finite time interval introduces off-mass-shell components. Thus the observation of a physical particle should be considered as a limit of observations that take place over intervals that extend in the limit into the infinite future or past. The result of such a measurement is represented by a mass-shell function, which can be considered a solution of the free-wave equation. The experimental significance of these space-time functions and the way in which they are related to asymptotic measurements can be determined by an analysis of the measurement process, as we shall see in Section V. More specifically, it will be shown in Section V that the space-time functions defined by Fourier transformation of the momentum-space functions have, as a consequence of weak analyticity properties in momentum space, certain characteristics that accord with our usual notion of the meaning of space-time wave functions, and that these characteristics serve to impress on experience correlations corresponding to the familiar space-time structure of physical phenomena. This correspondence between the space-time functions defined by Fourier transformation and space-time aspects of experience provides a justification both for the identification of the $x_i$ occurring in (2.6) as displacements and for the viewpoint regarding space-time functions set forth in these last two paragraphs.

## III. THE DECOMPOSITION PRINCIPLE

The assumption that $A_{\emptyset}(X)$ is invariant under translation (all $x_i$ set equal) implies energy-momentum conservation:

$$\left(\sum k_i\right) S(K) = 0 \quad . \tag{3.1}$$

Experience suggests that an interaction involving many particles has parts that can be regarded as interactions between subgroups of particles, and that the interaction between a subgroup is independent of the other particles in the senses that (a) it is independent of the position of the subgroup relative to the other particles, and (b) it is independent of what the particles <u>not</u> in the subgroup happen to be. It is these apparent features of elementary particle phenomena that allow us to make any sense at all of the situation, either experimentally or theoretically.

The assertion that the S matrix has parts satisfying these cluster decomposition properties is now generally taken as a basic assumption in S-matrix theory.[7,8,9] Since each part is translationally invariant it also must satisfy a conservation law analogous to (3.1) .

Because of the constraint (3.1) the S matrix must contain, as a factor, a momentum-energy delta function, in order that the integrations contribute anything. The parts of the S matrix corresponding to interactions of subgroups of particles are translationally invariant and must also have corresponding conservation delta functions for the individual subgroups of particles. The part of S that has only the single overall conservation law is called the connected part of S and is written as $S_c$ . The decomposition postulate states that

$$S(K) = \sum_p \prod_j S_c(K_{jp}) \quad , \tag{3.2}$$

where $K_{jp}$ is the jth subset of the pth partition of $K$. There could be a phase factor $\alpha^p(K)$ but this can be taken to be unity by writing the variables in "normal order", provided the usual connection between spin and statistics is maintained.[6,7]

If the connected part of $S$ is represented by a circle with external lines coming from the right and going to the left for initial and final particles, respectively, then the decomposition principle can be represented diagrammatically. For instance, for a three-particle reaction, with the same particles initially as finally, one has Fig. 1.

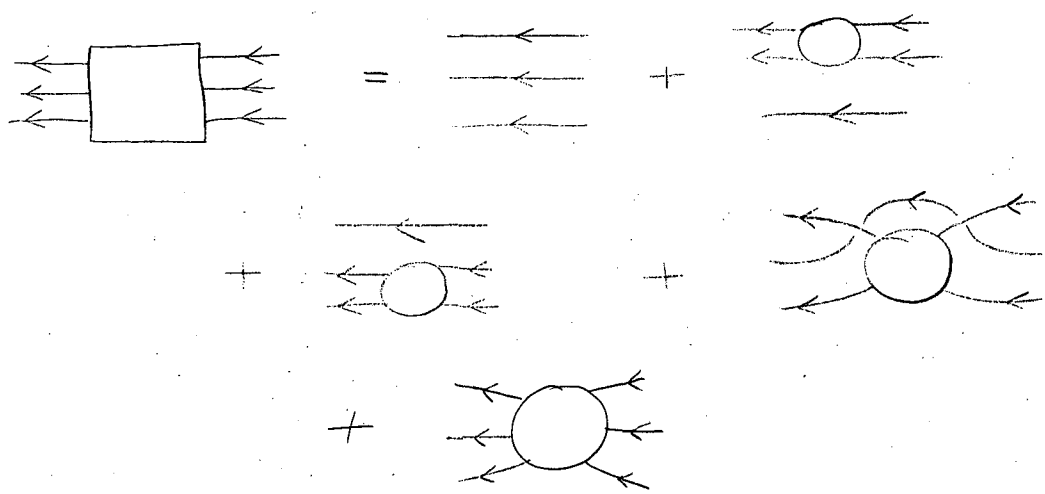

Fig. 1. Decomposition of $S$ for a simple case.

## IV. POLES AND PARTICLES

The connection of the S matrix to physical phenomena at finite times is closely tied to the pole-particle association inherent in S-matrix theory. The pole-particle association was well know from earlier theories. It also emerges quite naturally in a pure S-matrix theory. The relationship is contained in the pole-factorization theorem, which states that if the external variables of a multiparticle reaction are such that a one-particle exchange reaction of the kind indicated in Fig. 2 is physically possible,

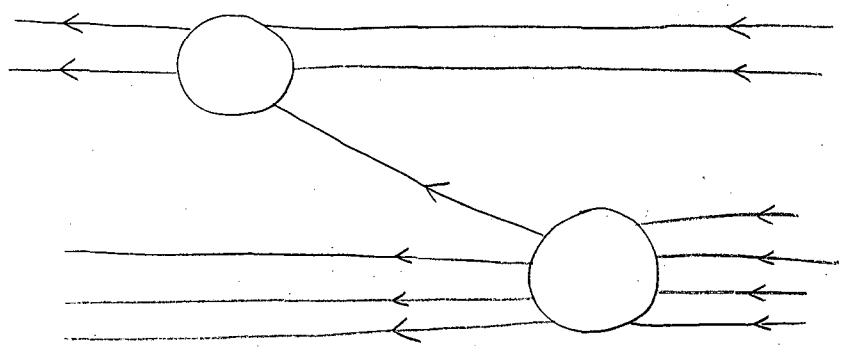

Fig. 2. Physical one-particle exchange reaction.

then in a neighborhood of a point $k^2 = \mu^2$ that has no singularities other than those associated with Landau single-particle exchange diagrams, the scattering amplitude is the sum of an analytic function plus a pole contribution that is just the product of the scattering amplitudes represented by the two individual reactions represented in Fig. 2 times a pole factor

$$P_\mu(k^2) = \frac{2i\mu}{k^2 - \mu^2 + i\epsilon} \quad . \tag{4.1}$$

Here $\mu$ is the mass of the exchanged particle and $k$ is the sum of the momenta associated with external lines of either of the two bubbles in Fig. 2. This factorization property is a consequence of the unitarity relation, the decomposition principle, and the assumption that the physical scattering function is to be regarded as the limit of the function from the upper half $k^2$ plane, near this point $k^2 = \mu^2$. A proof of this theorem was first given by Olive,[9] for simple cases. A general proof is given in Reference 6.

The importance of these pole contributions, with residues given by the product of the two scattering amplitudes, to the physical interpretation of the theory has often been emphasized.[9,10,11] They evidently correspond to the contribution to the scattering amplitude associated with the possibility of two independent sequential scatterings with the exchange of a physical particle, as illustrated in Fig. 2.

That the scattering amplitude should be considered as the limit from $\mathrm{Im}\, k^2 > 0$ (i.e., that in an integration over the physical region one should distort the contour slightly upward around the singularity) is presumably associated with the fact that the sequential scattering can occur only if the two reactions have the proper temporal relationship. It has often been suggested that to understand such questions regarding causality conditions it is necessary to go slightly off the mass shell, in order to obtain the temporal localization of the wave packets needed to define the time order. This idea is used in a recent paper by Eden and Landshoff,[12] who, in order to discuss causality conditions in S-matrix theory, abandon the strict mass-shell constraints. However, it is not necessary to go off the mass shell to obtain the connection between the pole $i\epsilon$ prescription and causality requirements. This is shown in the next section, in the general context of a study of the space-time correlations residing in the mass-shell scattering matrix.

## V. CAUSALITY AND SPACE-TIME CORRELATIONS

Suppose a one-particle exchange reaction of the type shown in Fig. 3 is kinematically possible.

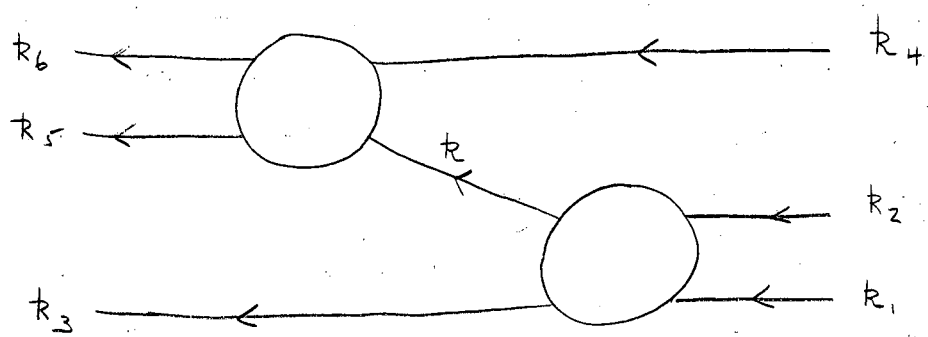


Fig. 3. A kinematically possible reaction.

That is, one can satisfy the relations

$$k_1 + k_2 + k_3 + k = 0 \tag{5.1}$$

and

$$-k + k_4 + k_5 + k_6 = 0 \quad , \tag{5.2}$$

with

$$k_i^2 = m_i^2 \tag{5.3}$$

and

$$k^2 = \mu^2 \quad . \tag{5.4}$$

The four-vectors $k_1$, $k_2$, and $k_4$ are the negatives of the physical energy-momentum vectors of the initial particles.

Our physical idea of causality requires that if initial particle wave packets with momenta centered around $k_1$ and $k_2$ come together and overlap appreciably only in the neighborhood of the origin of space-time and if the final particle with momentum $k_3$ is observed, then the intermediate particle should move out from near the origin of space-time with a velocity centered about $\underset{\sim}{v} = \underset{\sim}{k}/|k_0|$ . If then the measurements on the final particles 5 and 6 define free-particle wave packets with momenta centered respectively around $k_5$ and $k_6$ diverging from a point $(\underset{\sim}{x}, t)$ , then the transition probability should be appreciable only if $\underset{\sim}{x} \approx \underset{\sim}{v}t$ and $t > 0$ . It will now be verified that the contribution from the pole term has, in fact, just the expected properties.

The problem can be formulated in the following way. Let $\varphi(\underset{\sim}{p})$ be a function in momentum space that is infinitely differentiable and of compact support. Let the support be a very small neighborhood of the origin. Let $\varphi(\underset{\sim}{p})$ be real and shaped so that its Fourier transform is a smooth function in coordinate space centered at the origin. Because $\varphi(\underset{\sim}{p})$ is infinitely differentiable its transform falls off faster than any inverse power.

The support in $\underset{\sim}{p}$ space defines a velocity cone. The solution of the free wave equation corresponding to $\varphi(\underset{\sim}{p})$ has, for large $t$ and $(\underset{\sim}{x}, t) \equiv (\underset{\sim}{v}t, t)$ , the following behavior: it decreases faster than any inverse power of $t$ if $\underset{\sim}{v}$ is outside the velocity cone and it decreases like $|t|^{-3/2}$ if $\underset{\sim}{v}$ is inside the velocity cone. The properties are obvious from the consideration of $|\phi(x, t)|^2$ as a classical probability distribution. They have been proved by Ruelle[13] in quantum theory.

The function

$$\phi(\underset{\sim}{k}_i') = \varphi(\underset{\sim}{k}_i' - \underset{\sim}{k}_i)$$

corresponds, then, to a wave packet centered about $\underset{\sim}{k}_i$ which passes through the origin in space time. The action of the displacement operator displaces the origin to a new position.

If we write

$$k_i' = p_i + k_i \qquad \text{for } i = 1, 2, 5, 6 \tag{5.5}$$

and

$$k' = p + k \quad , \tag{5.6}$$

then the amplitude of interest can, with the help of (5.1) and (5.2), be written

$$A = \int \prod_{i=1,2,5,6} \left[ \frac{d^3 \underset{\sim}{p}_i}{(2\pi)^3} \varphi(\underset{\sim}{p}_i) \exp(-ip_i x_i) \right] \prod_{j=1}^{6} \exp(-i\, k_j\, x_j)$$

$$\times \int \frac{d^4 p}{(2\pi)^4} (2\pi)^4 \delta^4(p_1 + p_2 + p)(2\pi)^4 \delta^4(p_5 + p_6 - p)$$

$$\times \quad M_1 \frac{2i\mu}{(k + p)^2 - \mu^2 + i\epsilon} M_2 \tag{5.7}$$

$$= \int \frac{d^3(\underset{\sim}{p}_1 - \underset{\sim}{p}_2)}{2(2\pi)^3} \frac{d^3(\underset{\sim}{p}_5 - \underset{\sim}{p}_6)}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \prod_i \left[ \varphi(\underset{\sim}{p}_i) \exp(-ip_i x_i) \right]$$

$$\times \left[ \prod_j \exp(-ik_j x_j) \right] (2\pi)\, \delta(p_1^{\,0} + p_2^{\,0} + p^0) \quad (2\pi)\, \delta(p_5^{\,0} + p_6^{\,0} - p^0)$$

$$\times \quad \frac{M_1\, 2i\mu\, M_2}{(k + p)^2 - \mu^2 + i\epsilon} \quad . \tag{5.8}$$

The $M_1$ and $M_2$ represent the two (mass-shell) amplitudes corresponding to the subprocesses. In the second equation the $\underset{\sim}{p}_i$ are considered to be functions of $\underset{\sim}{p}_1 - \underset{\sim}{p}_2$, $\underset{\sim}{p}_5 - \underset{\sim}{p}_6$ and of

$$\underset{\sim}{p} = \underset{\sim}{p}_1 + \underset{\sim}{p}_2 = \underset{\sim}{p}_5 + \underset{\sim}{p}_6 \quad . \tag{5.9}$$

Because the $\underset{\sim}{p}_i$ are all very small we can use

$$\begin{aligned} k_i^{0'} &\equiv \left[ (\underset{\sim}{k}_i + \underset{\sim}{p}_i)^2 + m_i^2 \right]^{1/2} \\ &= k_i^0 + \frac{\underset{\sim}{k}_i}{k_i^0} \cdot \underset{\sim}{p}_i + \cdots \\ &= k_i^0 + \underset{\sim}{v}_i \cdot \underset{\sim}{p}_i + \cdots \quad . \end{aligned} \tag{5.10}$$

That is,

$$p_i^0 \approx \underset{\sim}{v}_i \cdot \underset{\sim}{p}_i \quad . \tag{5.11}$$

Thus we have

$$\exp(-i\, p_i\, x_i) \approx \exp\left[ i\, \underset{\sim}{p}_i \cdot (\underset{\sim}{x}_i - \underset{\sim}{v}_i t) \right] \quad , \tag{5.12}$$

and also

$$p_1^0 + p_2^0 \approx \frac{1}{2} \left[ \underset{\sim}{p} \cdot (\underset{\sim}{v}_1 + \underset{\sim}{v}_2) + (\underset{\sim}{p}_1 - \underset{\sim}{p}_2) \cdot (\underset{\sim}{v}_1 - \underset{\sim}{v}_2) \right] \tag{5.13}$$

and

$$p_5^0 + p_6^0 \approx \frac{1}{2} \left[ \underset{\sim}{p} \cdot (\underset{\sim}{v}_5 + \underset{\sim}{v}_6) + (\underset{\sim}{p}_5 - \underset{\sim}{p}_6) \cdot (\underset{\sim}{v}_5 - \underset{\sim}{v}_6) \right] \quad . \tag{5.14}$$

Inserting these expressions into (5.8), for the special case

$$x_i = y_1 \qquad \text{for } i = 1, 2, 3 \tag{5.15}$$

and

$$x_i = y_2 \qquad \text{for } i = 4, 5, 6 \tag{5.16}$$

one obtains

$$A \approx \int \frac{d^4p}{(2\pi)^4} (2i\mu) \frac{f_2(p) \exp\left[-i(k+p)(y_2 - y_1)\right] f_1(p)}{(k+p)^2 - \mu^2 + i\epsilon} \tag{5.17}$$

where

$$f_1(p) = \int \frac{d^3(\underset{\sim}{p}_1 - \underset{\sim}{p}_3)}{(2\pi)^3} \, 2\pi\, \delta\left[(\underset{\sim}{v}_1 + \underset{\sim}{v}_2) \cdot \underset{\sim}{p} + (\underset{\sim}{v}_1 - \underset{\sim}{v}_2) \cdot (\underset{\sim}{p}_1 - \underset{\sim}{p}_2) + 2p_0\right]$$

$$\times\, \varphi(\underset{\sim}{p}_1)\, \varphi(\underset{\sim}{p}_2) M_1 \quad , \tag{5.18}$$

and

$$f_2(p) = \int \frac{d^3(\underset{\sim}{p}_5 - \underset{\sim}{p}_6)}{(2\pi)^3} \, 2\pi\, \delta\left[(\underset{\sim}{v}_5 + \underset{\sim}{v}_6) \cdot \underset{\sim}{p} + (\underset{\sim}{v}_5 - \underset{\sim}{v}_6) \cdot (\underset{\sim}{p}_5 - \underset{\sim}{p}_6) - 2p_0\right]$$

$$\times\, \varphi(\underset{\sim}{p}_5)\, \varphi(\underset{\sim}{p}_6) M_2 \quad . \tag{5.19}$$

The functions $M_1$ and $M_2$ are evaluated on their mass shells at points in a small neighborhood of $(k_1, k_2, k_3, k)$ and $(k_4, k_5, k_6, -k)$ that is determined by the supports of $\varphi(\underset{\sim}{p}_1)\, \varphi(\underset{\sim}{p}_2)$ and $\varphi(\underset{\sim}{p}_5)\, \varphi(\underset{\sim}{p}_6)$ respectively. If the $M_1$ and $M_2$ are analytic in these neighborhoods then the functions $f_1(p)$ and $f_2(p)$ will be infinitely differentiable functions of compact support, provided

$$(\underset{\sim}{v}_1 - \underset{\sim}{v}_2) \neq 0 \neq (\underset{\sim}{v}_5 - \underset{\sim}{v}_6) \quad . \tag{5.20}$$

These conditions are implied by the physical formulation of the problem; the wave packets were required to be converging or diverging from their region of overlap. This means their relative velocities are not zero.

Combining $f_1$ and $f_2$ into a single function $f$, one has

$$A \approx A' \equiv \int \frac{d^4p}{(2\pi)^4} (2i\mu)\, f(p) \frac{\exp\left[-i(k+p)(y_2 - y_1)\right]}{(k+p)^2 - \mu^2 + i\epsilon} \quad , \tag{5.12}$$

where $f(p)$ is a $C^\infty$ function of small compact support centered about $p = 0$. Invoking the convolution theorem for the energy part, one obtains

$$A' = \int \frac{d^3\underset{\sim}{p}}{(2\pi)^3} \int dt'\, \tilde{f}(\underset{\sim}{p}, t')\, \tilde{g}(\underset{\sim}{p}, t - t') \exp(-ik_0 t') \times \exp\left[+i(\underset{\sim}{k} + \underset{\sim}{p}) \cdot (\underset{\sim}{y}_2 - \underset{\sim}{y}_1)\right], \tag{5.22}$$

where

$$\tilde{f}(\underset{\sim}{p}, t') = \int \frac{dp^0}{2\pi} f(p) \exp(-i\, p^0\, t') \quad , \tag{5.23}$$

and

$$\tilde{g}(\underset{\sim}{p}, t) = \int \frac{dp^0}{2\pi} \frac{2i\mu \exp\left[-i(k^0 + p^0)t\right]}{(k+p)^2 - \mu^2 + i\epsilon} \quad , \tag{5.24}$$

and

$$t = y_2^{\,0} - y_1^{\,0} \quad . \tag{5.25}$$

Going, for simplicity, to a frame in which $k$ is purely timelike, one obtains

$$\tilde{g}(\underset{\sim}{p},t) = \frac{\mu}{\omega}\left[\theta(t)\exp(-i\omega t) + \theta(-t)\exp(i\omega t)\right] , \tag{5.27}$$

where

$$\omega = (\mu^2 + \underset{\sim}{p}^2)^{1/2} . \tag{5.28}$$

Thus

$$A' = \int \frac{d^3\underset{\sim}{p}}{(2\pi)^3}\frac{\mu}{\omega}\exp(i\,\underset{\sim}{p}\cdot\underset{\sim}{x})\int dt'\,\tilde{f}(\underset{\sim}{p},t')$$

$$\times\left\{\theta(t-t')\exp(-i\omega t)\exp\left[-i(k^0-\omega)t'\right] + \theta(t'-t)\exp(+i\omega t)\exp\left[-i(k^0+\omega)t'\right]\right\} \tag{5.29}$$

where

$$\underset{\sim}{x} = \underset{\sim}{y}_2 - \underset{\sim}{y}_1 . \tag{5.30}$$

The original wave packets in space-time have tails falling off faster than any inverse power. Thus the causal properties, and the space-time correlations that we expect, should hold only to within tails that fall off faster than any inverse power. Since $\tilde{f}(\underset{\sim}{p},t')$ falls off faster than any inverse power in $t'$ one can, for large positive times, keep only the first term in the square bracket, and for large negative times keep only the second term; the other term in each case is an integral over just the tail of $\tilde{f}(\underset{\sim}{p},t')$. By the same approximation one

can in the term that is retained ignore the $\theta$ function; the extra tail added to the integral contributes only terms that drop off faster than any inverse power of $t$. With the $\theta$ functions omitted the $t'$ integration can be performed to give

$$A' \approx \int \frac{d^3\underset{\sim}{p}}{(2\pi)^3} \frac{\mu}{\omega} \exp\ i(\underset{\sim}{p}\cdot\underset{\sim}{x} \mp \omega t)\ f(\underset{\sim}{p}, \pm\omega - k^0) \qquad \text{for} \quad \pm t \gg 0 . \tag{5.31}$$

In the case $-t \gg 0$ the integral is identically zero because $f$ is zero; $f$ has compact support in the variable $p_0$, and this support is near the origin. For the case of large positive $t$ the factor

$$\omega - k^0 = \omega - \mu \approx \underset{\sim}{p}^2/2\mu \tag{5.32}$$

is approximately zero in the small $p$ limit. Thus $A(x)$ is, in this approximation, just the wave function of the freely moving particle of mass $\mu$ whose momentum-space wave function is $f(\underset{\sim}{p}, \underset{\sim}{p}^2/2\mu)$. This gives exactly the dependence on the displacement vector $x = y_2 - y_1$ that we expect. The only approximations have been to omit terms that fall off faster than any inverse power of $t$ and to drop higher-order terms in $\underset{\sim}{p}$ in slowly varying functions of $\underset{\sim}{p}$. The latter is justified by the fact that the support in $p$ space can be taken arbitrarily small.

The use of these approximations was not required by principle; the neglected terms are given by definite expressions that could be calculated numerically, if not analytically. Thus the mass-shell functions provide for a detailed calculation of the dependence on $x$, not just for the asymptotic and limiting forms. For small values of $t$ one would have to include contributions other than the simple pole term, of course.

Although this calculation is for the simplest example, the method is quite general. The result shows clearly how space-time correlations of the type observed in the macroscopic world are contained in, and described by, the pure mass-shell scattering matrix. In particular, we see that the scattering matrix for a very-many-particle situation would be dominated by contributions corresponding to many sets of interactions of subgroups of particles separated by freely moving particles. That is, the asymptotic transition probabilities would be dominated by terms that display the space-time structure of macroscopic experience.

We see also that the sign of $i\epsilon$ in the pole denominator is determined by a macroscopic causality condition, within a completely mass-shell framework; a $-i\epsilon$ term would give an acausal macroscopic connection between the space-time positions of the two scattering centers. The $i\epsilon$ prescription determined here is for the calculation of physical matrix elements of the S matrix. It refers therefore also to the evaluation of the S matrix in the unitarity equations.

It is now possible to examine the original assumption that the initial and final particles are represented by solutions to the free-wave equation, and to understand the experimental significance of these solutions. The larger reaction that includes the measurement processes has a multi-pole contribution with a pole for each of the initial and final particles of the original reaction. As the measurement process regions tend to infinity the overall reaction is dominated by this multi-pole contribution. A direct generalization of the above analysis gives, as the generalization of (5.31), the dominant asymptotic contribution

$$A'_{\phi} = \int S(K)\, \phi(K)\, dK \quad . \tag{5.33}$$

Here $S(K)$ is the S matrix for the original process and $\phi(K)$ is a product of functions $\phi_i(k_i)$. The function $\phi_i(k_i)$ is the generalization of the function $\left[f_i(k_i)\exp(-ik_ix_i)\right]$ of (5.21), where $f_i$ is the function defined in (5.18) for initial [or in (5.19) for final] particles. [The sign of $k_i$ is fixed now by the "original" reaction.]

The exponential factor in $\phi_i(k_i)$ can be reincorporated into the wave functions of the measurement process. This takes them to their actual forms, which are centered in coordinate space about $x_i$. Reconverting (5.18) [or (5.19)] back to the original form one obtains [see (5.7)]

$$\phi_i(k_i) = \int\left[\prod_j \frac{dk_j'}{(2\pi)^3}\phi_j(k_j')\right] (2\pi)^4\,\delta(\Sigma k_j' - k_i)M_i$$

$$= \int\left[\prod_j \frac{d^3k_j'}{(2\pi)^3}\phi_j(k_j')\right] M_i\int d^4x\,\exp\left[-i\,(\Sigma k' - k_i)\,x\right]$$

$$\approx M_i\int d^4x\,\left[\prod_j \phi_j(x)\right]\exp(+ik_ix)\quad, \tag{5.34}$$

where in the last line $M_i$ is assumed to be almost constant over the small momentum region of integration. The function $\phi_i(k_i)$ is, then, essentially the Fourier transform of the product of the solutions of the free-wave equation corresponding to the particles that participate in the measurement of particle $i$. The intersection region of these solutions acts as the effective source region of particle $i$.

The space-time regions defined by intersections of solutions of the free-wave equation will be called "locations." These locations have experimental significance because the overall reaction proceeds only if the location defined by the intersection of two of these solutions lies

within the wave packets of the other particles with which they interact. One may use two solutions whose intersection defines a rather small space-time region to probe the wave function of particles with larger wave packets, etc. In this way space-time relationships emerge from S-matrix theory even though the locations having experimental significance are the space-time regions of intersections of solutions to the free-wave equation rather than space-time points. It is easy to show that the location of a particle, as defined by the region of intersection of its space-time wave packet with that of another particle, is never spatially confined in its rest frame to less than its Compton wavelength.

In summary, then, the result of the asymptotic measurement of a particle is represented by a mass-shell function. This function is expressed in terms of the functions that describe the particles that participate in the measurement process. The coordinate-space forms of the mass-shell functions, which can be considered as defined by Fourier transformation, have the significance that a process involving the particles proceeds only if the space-time regions defined by these solutions to the free-wave equations overlap. Moreover, the overlap regions associated with the various subprocesses must be related to one another by a causal space-time connection. These results about the space-time structure of the dominant multipole contributions follow from weak analyticity assumptions about the momentum-space functions; no initial assumption regarding the physical significance of the coordinate-space functions is needed.

The wave functions representing the initial and final particles of the original reaction are expressed in terms of the initial and final wave functions of the larger process. If these latter are known then one has an expression for the contribution to the overall process. The meaning and form of the initial and final wave functions for this larger

process is explored by considering a still larger process in which the initial and final particles of the original larger process are observed. This situation in which the significance of the wave functions at each stage is determined by examining reactions in which the corresponding particles are measured is a completely reasonable and natural one. The connection to experience is obtained by including the observer in the system under examination. It should then be possible to terminate the infinite regression, because the quantities to be calculated, which are correlations in the memory pattern of the observer, should not depend on the experimental significance of the initial (or final) wave function associated with a particle whose source (or destiny) is unknown to the observer. That is, the interactions that provide the source of these particles, being unknown, are not correlated to the memory of the observer, and hence the meaning of the wave functions explored by the source reaction should not be relevant.

To see this more clearly consider the model shown in Fig. 4 on the following page for the measurement of an electron-electron scattering experiment. The observer particles at A and B are assumed to be in wave packets whose forms are correlated to the memory pattern of the observer (e.g., through retina position). If these wave packets are considered known then their intersection regions A and B are defined, and hence the momentum of electron AB is essentially known. The wave packet of the electron from B to x is therefore also determined. Similarly the direction of the incident wave packet is determined. Thus the dependence on the source S is only through a factor that determines the overall normalization. That is, the observer may or may not "see" the electron at both A and B, but if we normalize to the case that he does see the electron at A and B then the dependence on S drops out.

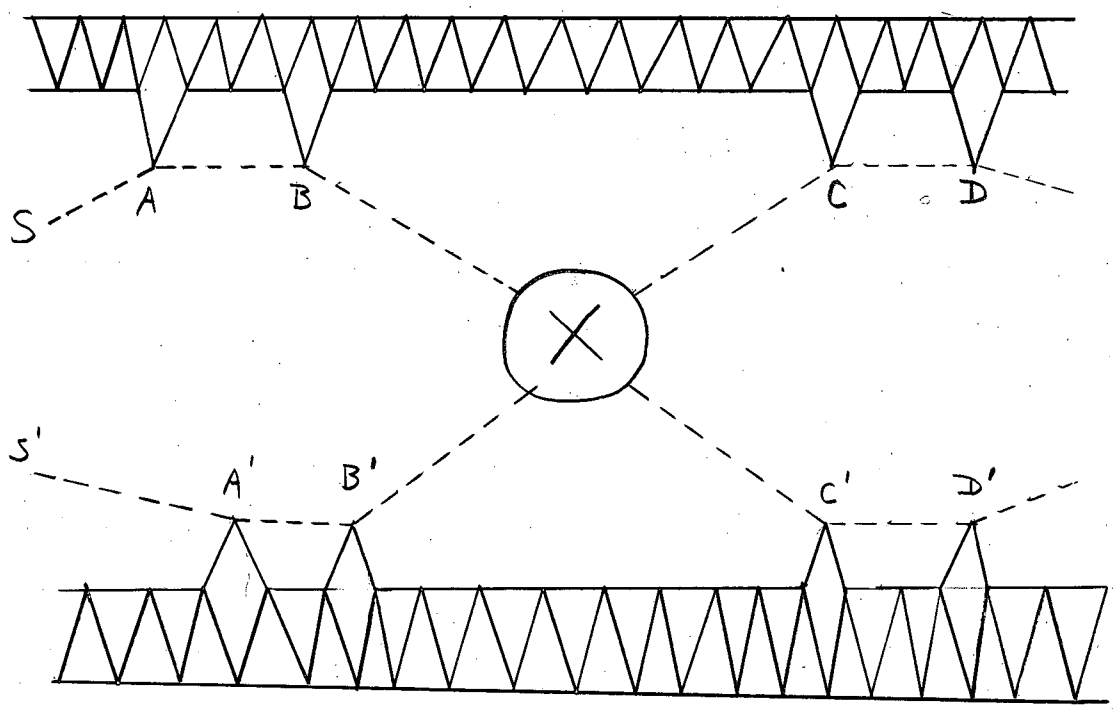


Fig. 4. Diagram for S-matrix theory of measurement of electron-electron scattering. The dotted lines represent the two electrons. The solid lines represent the observer. The zigzag lines in the observer, who is broken into two pieces for diagrammatic convenience, represent his memory and physiological clock.

This may be stated the other way around: if one considers the overall reaction, then the connection of the momentum variable of the electron incident on A to the experience of the observer is not obtained by examining the reaction that is the source of the particle. Rather it is directly correlated to the observer experience: if the observer sees

the electron at both A and B then the momentum variable is essentially determined. There is no need to invoke the idea of an initial wave function for this particle at all, provided we consider only the subset of memory patterns in which the observer definitely sees the electron at both A and B, for this fact effectively redefines the initial wave function anyway. The same goes for A' and B'.

Suppose the final interactions are such that the observer records various alternative possibilities. If the observer is subjected to a succession of a large number of such experiments then the ratios of the numbers of occurrences of the various possible alternatives, as recorded in his memory pattern, will, by virtue of the space-time correlations discussed above, be determined by the e-e scattering matrix in the usual way. Final electron wave functions, like initial ones, play no role; one deals only with the S matrix itself, and its internal space-time structure.

No attempt will be made here to trace the flow of information into the memory pattern of the observer. If the observer were diffuse so that pole contributions were dominant then the space-time correlations of the type discussed above would evidently carry over to the observer. Since the long-range photon interaction is in fact the dominant one, one might expect results similar to those obtained in the pole approximation.

In order to apply S-matrix theory to the calculation of the memory pattern one would in principle evidently have to let the particles constituting the observer disperse at least to the extent that the pole approximation becomes valid. But since the memory pattern correlations in question should be independent of the parts of the history of the observer before and after the period of the experiment, one would expect the <u>relative</u> probabilities in question to be independent of those parts

of the observer's history. That is, those parts should become irrelevant by the same renormalization procedure that removed the dependence on the external electron variable. It may therefore be possible to calculate these relative probabilities without actually tracing the observer's history to the stage where the particles constituting the observer are dispersed.

It should be stressed that in calculating the relative probabilities of memory patterns one does not invoke the idea of a temporal development of memory. All one deals with is the S matrix, which refers to infinite times. A memory is such that it contains a record of its history, including a record of the order in which the events of its history occurred. What one calculates is not a detailed history of the development of the memory in time but rather certain correlations in a memory pattern as disclosed by a pole approximation. That is, if the pole approximation is valid for a certain particle then the S matrix can be expressed by folding the S matrix for a subprocess into a wave function for this particle. If at some stage each particle is treated in a pole approximation then the entire S matrix is represented by folding a final wave function into some internal S matrix. Certain correlations in the memory pattern will be determined solely by this internal S matrix. They will, because of the gross space-time properties of the S matrix discussed above, reflect the gross causal features they imply. Because memories contain a record of their histories the pertinent information regarding the history should be contained in, and derivable from, the information contained in the memory at the later stage. Thus what one calculates in S-matrix theory is not the detailed development of the memory with time but only a record of that development as revealed by correlations in the memory pattern in situations where the pole approximation is valid. The peculiar aspect of experience that makes events appear to be "happening in time" is not explained.

## IV. FORM FACTORS

The electron-proton scattering experiments have been interpreted in terms of form factors which have been calculated by Frazer and Fulco[14] and others using dispersion relations suggested by field theory. The apparent success of these calculations has been cited as evidence for the validity of the field concept. Their equations follow, however, also from a purely mass-shell approach.

Let us, for definiteness, consider electron-proton scattering in the two-pion approximation used by Frazer and Fulco; the intermediate state summation will be approximated by the photon and two-pion contributions. And let us for reasons to be discussed presently ignore the contributions from the cross cuts. Then the dispersion relation for e - p scattering is represented in Fig. 5 and given in Eq. (6.1) .

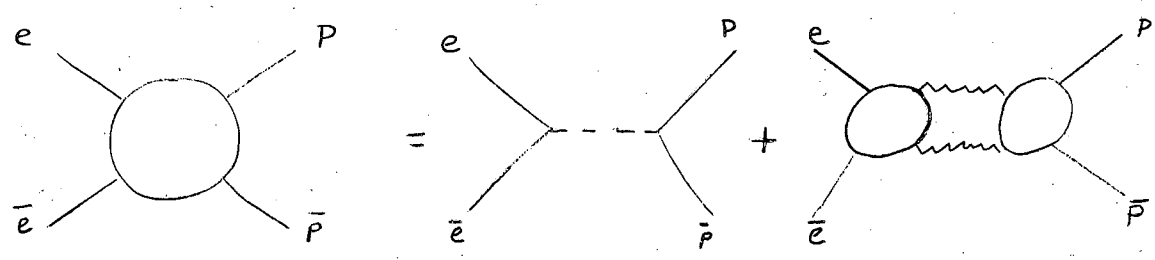


Fig. 5. Representation of terms in e-p dispersion relation.

$$T_{ep}(S) = \frac{\gamma_e \Gamma_p}{S - S_p} + \int_{(2\pi)^2} \frac{T_{e\pi}(S_1^-)\, T_{\pi p}(S_1^+)}{S - S_1}\, dS_1 \tag{6.1}$$

The $S_p$ would be zero for a photon pole, and $dS_1$ incorporates a factor $(2\pi)^{-1}$. These T's correspond to $(-2 \sin\delta\, e^{i\delta})$ .

Similarly, one has

$$T_{e\pi}(S_1) = \frac{\gamma_e \Gamma_\pi}{S_1 - S_p} + \int_{(2\pi)^2} \frac{T_{e\pi}(S_2^-)\, T_{\pi\pi}(S_2^+)}{S_1 - S_2}\, dS_2 \quad . \tag{6.2}$$

Equation (6.2) can be rewritten as

$$T_{e\pi}(S_1) = \frac{\gamma_e\, \Gamma_\pi(S_1)}{S_1 - S_p} \quad , \tag{6.3}$$

where $\Gamma_\pi(S_1)$ is defined to be a solution of the equation

$$\Gamma_\pi(S_1) = \Gamma_\pi + (S_1 - S_p) \int_{(2\pi)^2} \frac{\Gamma_\pi(S_2^-)}{(S_2 - S_p)} \frac{T_{\pi\pi}(S_2^+)}{(S_1 - S_2)}\, dS_2 \quad . \tag{6.4}$$

One sees this by multiplying (6.4) on the left by $\gamma_e(S_1 - S_p)^{-1}$ and comparing to (6.2) .

Inserting (6.3) into (6.1), one obtains

$$T_{ep}(S) = \frac{\gamma_e\, \Gamma_p}{S - S_p} + (S - S_p) \int_{(2\pi)^2} \frac{\gamma_e\, \Gamma_\pi(S_1^-)}{(S_1 - S_p)} \frac{T_{\pi p}(S_1^+)}{(S - S_1)}\, dS_1 \quad , \tag{6.5}$$

which can be written as

$$T_{ep}(S) = \frac{\gamma_e\, \Gamma_p(S)}{S - S_p} \quad , \tag{6.6}$$

with $\Gamma_p(S)$ defined by

$$\Gamma_p(S) = \Gamma_p + (S - S_p) \int_{(2\pi)^2} \frac{\Gamma_\pi(S_1^-)}{(S_1 - S_p)} \frac{T_{\pi p}(S_1^+)}{(S - S_1)}\, dS_1 \quad . \tag{6.7}$$

Equations (6.3) , (6.4) , (6.6) , and (6.7), projected onto a partial wave state, are the form factor equations used by Frazer and Fulco. Since they are simply formal re-expressions of the mass-shell equations (6.1) and (6.2) the success of the Frazer-Fulco calculations is not evidence for the inadequacy of the mass-shell framework but evidence rather for the validity of the approximations represented by (6.1) and (6.2) .

The restriction to two-pion intermediate states is completely inessential to the above discussion. The essential approximation is the neglect of cross cuts.

The cross cut gives the $t$ dependence. Experimentally there is little $t$ dependence. Thus the approximation can be supported by experimental considerations. The reason for the absense of $t$ dependence is in the dominance of the one-photon exchange contribution; which has no $t$ dependence. The dominance of the one-photon exchange contribution is due to certain perculiarities of the electron interaction--its weakness and its selection rules.

To see what is happening one can look at the iterative solution to (6.1) or (6.2) . It reads, in a generalized notation,

$$T_{ef}^{+}(S) = \frac{\gamma_e \Gamma_f}{S - S_p} + \int \frac{\gamma_e \Gamma_n}{(S_1^- - S_p)} \frac{T_{nf}(S_1^+)}{(S - S_1)} dS_1$$

$$+ \int \frac{\gamma_e \Gamma_n}{(S_2^- - S_p)} \frac{T_{nm}(S_2^+)}{(S_1^- - S_2)} \frac{T_{mf}(S_1^+)}{(S - S_1)} dS_1 \, dS_2$$

$$+ \cdots$$

$$+ \int \frac{\gamma_e \Gamma_n}{(S_m^- - S_p)} \frac{T_{np}(S_m^+)}{(S_{m-1}^- - S_m)} \cdots \frac{T_{qf}(S_1^+)}{(S - S_1)} dS_1 \cdots dS_m$$

$$+ \cdots$$

$$\equiv \sum_m \int dS_m \frac{\gamma_e \Gamma_n}{(S_m^- - S_p)} \left[ T_{nf}^{(m)}(S_m, S) \right]_+ \quad . \tag{6.8}$$

The iteration formula can be looked at as the result of displaying explicitly the photon-pole contribution to $T_{en}$, wherever it occurs. The form factor approximation says, in effect, that the entire electron interaction is traceable in this way to the interaction through the photon pole, and furthermore that only the first-order contributions matter. This last restriction is evidently just an approximation resulting from the smallness of the coupling constant. That the electron interaction over a wide range of energies is actually well represented by contributions traceable to the photon pole is an empirical fact, which may ultimately be understood in terms of consistency requirements. But at present the dominance of photon contributions must be taken as the working assumption regarding the electron interaction, whether one starts from a field theoretic or S-matrix framework.

The formulas (6.3) and (6.5) for $T_{e\pi}$ and $T_{ep}$ are not symmetric on the two sides; the $\gamma_e$ remains the simple vertex whereas the $\Gamma$ on the right becomes $\Gamma(S)$. This is because the photon pole is always required to involve the electron. If one had considered a contribution coming from a pole in $T_{\pi\pi}$ or $T_{\pi p}$ on the right then the remaining factor $T_{e\pi}$ would make the contribution of higher order, since $T_{e\pi}$ is itself small of order $\alpha$.

On the other hand if one were to look at the photon-pole contribution to $T_{\pi\pi}$ then no such selection rule would intervene; in collecting all terms first order in the pole contribution one could take the pole contribution from either side. One might expect, in this case, that the net conribution first order in the pole contribution would have the form

$$T_{\pi\pi}{}^{p}(S) = \frac{\gamma_\pi(S)\,\Gamma_\pi(S)}{S - S_p} , \tag{6.9}$$

where $\Gamma_\pi(S)$ is the form factor determined by (6.4), and $\gamma_\pi(S)$ is the similar quantity that faces left. This expectation is, in fact, borne out.

To see how this works, consider the general form of the dispersion integral

$$T_{if}(S) = \frac{\gamma_i\,\Gamma_f}{S - S_p} + \int \frac{T_{in}(S_1^-)\,T_{nf}(S_1^+)}{S - S_1}\,dS_1 + R , \tag{6.10}$$

where $R$ is the part coming from the cross cuts and other parts of the contour. The object is to isolate all contributions that are first order in the photon pole. As one iterates he could always substitute into both T's appearing on the right. At some stage one of the T's is replaced by its pole part. To get rid of the various terms containing $R$, and hence

to get expressions involving only T's , one can "de-iterate" and reconstitute the T's that were not along the path leading to the T that was finally replaced by the pole part. One is left then with an expression containing only T's . Some of these will be to the right of the pole and others to the left. There are generally many ways that one could have arrived at contributions with, say, n T's on the left and m T's on the right of the pole. One would expect that if one adds these contributions (having n on the left and m on the right) then one would obtain just the contribution to the quantity

$$\frac{\gamma_i(S)\,\Gamma_f(S)}{S - S_p}$$

that has n T's in $\gamma_i(S)$ and m T's in $\Gamma_f(S)$ . This is indeed the case, as will now be shown.

Let $P_{nm}$ be the contribution to T(S) first order in the pole and having n T's on the left of the pole and m T's on the right of the pole. For example, one obtains from the iteration procedure

$$P_{11} = \int \frac{T(S_1^-)}{S - S_1} \frac{\gamma\,\Gamma}{S_2^- - S_p} \frac{T(S_2^+)}{S_1^+ - S_2} dS_1\, dS_2$$

$$+ \int \frac{T(S_2^-)}{S_1^- - S_2} \frac{\gamma\,\Gamma}{S_2^+ - S_p} \frac{T(S_1^+)}{S - S_1} dS_1\, dS_2$$

$$= \int T(S_1^-)\,\gamma\,\Gamma\,T(S_2^+) \left[ \frac{1}{(S - S_1)} \frac{1}{(S_2^- - S_p)} \frac{1}{(S_1^+ - S_2)} \right.$$

$$\left. + \frac{1}{(S_2^- - S_1)} \frac{1}{(S_1^+ - S_p)} \frac{1}{(S - S_2)} \right] dS_1\, dS_2$$

$$= \int \frac{T(S_1^-)\,\gamma\,(S - S_p)\,\Gamma\,T(S_2^+)}{(S - S_1)\,(S_1^+ - S_p)\,(S_2^- - S_p)\,(S - S_2)} dS_1\, dS_2$$

$$\equiv \int \frac{T(S_1'^-)\,\gamma\,(S - S_p)\,\Gamma\,T(S_1''^+)}{(S - S_1')\,(S_1'^+ - S_p)\,(S_1''^- - S_p)\,(S - S_1'')} dS_1'\, dS_1'' \quad . \tag{6.11}$$

Thus this term is all right.

The general proof is by induction. Suppose we know that for $n + m \leq N$

$$P_{nm}(S) = \frac{T_-^{(n)}\,\gamma\,(S - S_p)\,\Gamma\,T_+^{(m)}}{(S_n'^+ - S_p)\,(S_m''^- - S_p)} \quad , \tag{6.12}$$

where $T_{\pm}^{(i)}$ is the ith-order term with appropriately arranged denominators. [cf. (6.8)] This means that the replacement of the (n + 1)th term $T(S_{n+m})$ by a pole and summation gives

$$T(S_{n+m}) \rightarrow \frac{\gamma\,\Gamma}{S_{n+m} - S_p} \overset{\Sigma}{\rightarrow} \frac{\gamma(S - S_p)\,\Gamma}{(S_n'^{+} - S_p)(S_m''^{-} - S_p)} \quad , \tag{6.13}$$

where the second arrow represents the effect of summation over all ways of achieving the separation $(n,m)$ . The outer variables are also appropriately arranged by the summation. The prime and double prime variables are variables relabeled so as to correspond to those associated with the T's appearing to the left and right of the pole term, respectively, as for example, in the last line of (6.11).

The $P_{nm}(S)$ is a sum of two terms:

$$P_{nm}(S) = C_{n,m-1} + D_{n-1,m} \quad . \tag{6.14}$$

The term $C_{n,m-1}$ is the result of replacing the (n + 1)th term $T(S_{n+m-1})$ in the manner

$$T(S_{n+m-1}) \rightarrow \frac{\gamma\,\Gamma}{S_{n+m}^{-} - S_p} \frac{T(S_{n+m}^{+})}{S_{n+m-1} - S_{n+m}} \tag{6.15}$$

and summing over all ways of achieving the separation $(n,m-1)$ . Since $S_{n+m}$ is fixed in this summation one obtains from (6.13) , with $S_{n+m} \equiv S''_m$ in place of $S_p$ ,

$$C_{n,m-1} = \int \frac{T_{-}^{(n)}\,\gamma\,(S - S''_m)\,\Gamma\,T(S_m''^{+})\,T_{+}^{(m-1)}}{(S_n'^{+} - S''_m)\,(S_m''^{-} - S_p)\,(S_{m-1}''^{-} - S''_m)} \quad . \tag{6.16}$$

Similarly,

$$D_{n-1,m} = \int \frac{T_-^{(n-1)} T(S_n') \gamma (S - S_n') \Gamma T_+^{(m)}}{(S_{n-1}'^{+} - S_n')(S_n'^{+} - S_p)(S_m''^{-} - S_n')} \quad . \tag{6.17}$$

The result then follows from the identity

$$\frac{(S - S_m'')}{(S_n' - S_m'')(S_m'' - S_p)} + \frac{(S - S_n')}{(S_n' - S_p)(S_m'' - S_n')}$$

$$= \frac{(S - S_p)}{(S_n' - S_p)(S_m'' - S_p)} \quad . \tag{6.18}$$

This result is not dependent on the validity of the iterative expansion. What is desired is the term first order in $\lambda$, at $\lambda = 1$, of the solution of the integral equation

$$T(S) = \lambda \frac{\gamma \Gamma}{S - S_p} + \int \frac{T(S_1^-) T(S_1^+)}{S - S_1} dS_1 + R \quad . \tag{6.19}$$

Taking a derivative on $\lambda$, at $\lambda = 1$, one obtains

$$T'(S) = \frac{\gamma \Gamma}{S - S_p} + \int \frac{T(S_1^-) T'(S_1^+) dS_1}{S - S_1} + \int \frac{T'(S_1^-) T(S_1^+) dS_1}{S - S_1} \quad . \tag{6.20}$$

One easily confirms that

$$T'(S) = \frac{\gamma(S) \Gamma(S)}{S - S_p} \quad , \tag{6.21}$$

with $\gamma(S)$ and $\Gamma(S)$ satisfying their integral equations, is a solution

of (6.20) . $\Big[$ After substitution use the identities $\Gamma(S_1^+) = \Gamma(S) + \left[\Gamma(S_1^+) - \Gamma(S)\right]$ and $\gamma(S_1^-) = \gamma + \left[\gamma(S_1^-) - \gamma\right]$ $\Big]$.

This derivation of the form-factor expression shows the significance of form factors within an S-matrix framework: in addition to the primary pole contribution to the dispersion integral there are indirect pole contributions coming from the pole terms in the functions that determine the multiparticle discontinuities, in the pole-energy channel. If one systematically collects all the contributions, direct and indirect, that contain as a factor the pole contribution in this channel, then the result is equivalent to replacing the mass-shell factors $\gamma_f \Gamma_i$ by the form factors $\gamma_f(S)\, \Gamma_i(S)$ , where these factors are the solutions of the form-factor integral equations. It is reasonable to call the form-factor expression $\gamma_f(S)\, \Gamma_i(S)(S - S_p)^{-1}$ the "complete pole contribution."

The complete pole contribution represents a certain part of the dispersion formula expression for the scattering amplitude. The question is: what experimental or theoretical significance does this particular contribution have?

In the case of $e - p$ scattering the complete photon-pole contribution would be expected to have an approximate experimental significance. This derives from the fact that the pole contribution is small yet dominant. Because of the smallness, the term first order in $\alpha$ will dominate the terms of the higher order in $\alpha$ . But in spite of its smallness the pole contribution is a large part of the complete $e - p$ interaction.

This experimental significance is of an approximate rather than fundamental character. Since the twin characteristics of smallness and dominance do not appear to be satisfied in the strong interaction situations, an experimental significance for form-factor expressions in strong interactions would not be expected. Stated differently, there seems no particular

reason to believe that the part of the multiparticle contributions coming from the pole term in a factor that determines the multiparticle discontinuity should contribute a large part of the corresponding multiparticle contribution. And to the extent that there is large "t dependence" the possibility is experimentally ruled out.

A theoretical significance for the form-factor expressions could arise in various ways. If the complete pole contribution were a good representation of the entire amplitude over a certain range of variables then the expression might be useful in practical calculations. Such a significance would again be of an approximate rather than fundamental kind.

A possible fundamental significance of the form-factor expression arises from considering the question of <u>why</u> the complete pole contribution dominates e - p scattering. This dominance can be understood as an expression of the requirement that the electron interaction be purely electromagnetic (aside from weak interactions). The S-matrix formulation of this requirement would be the statement that all electron interactions can be "ultimately traced" to the electron interaction via the photon pole. This is, if one systematically displays the photon pole contribution in every factor of a dispersion formula for electron scattering then the entire function consists of terms in which the electron ultimately appears only in photon-pole terms.

Whether the electron interaction is indeed purely electromagnetic in this sense is not known. But the first few terms of the iterative expansion based on this assumption give a good representation of the electron interaction over a wide range of variables.

If this selection rule is indeed valid then the operation of displaying and collecting the contributions that are traceable to the photon pole has a basic theoretical significance; it is through such operations that

the basic selection rule for the electron interaction is expressed. The form factors, being the result of collecting terms in which the photon pole occurs once, would acquire an important theoretical status as the vehicle for expressing the selection rule.

Whether all lepton interactions are ultimately traceable to just a few basic interactions is problematical. For strong interactions the possibility that interactions are traceable to a few basic ones seems even less likely. The basic interactions in question refer to physical particles, which are understood as eigenstates. Even if there were somehow some basic interactions expressible in terms of "basic entities" it is not clear that this would imply that the physical particle interactions would be traceable to a few interactions between certain physical particles. If there are not such basic physical particle reactions to which others can be traced then the form-factor expressions would not acquire the corresponding basic theoretical status.

In the bootstrap philosophy, one discards the idea of tracing interactions back to certain fundamental ones but regards them as mutually self-supporting. This view would not provide form factors a status of the kind described above.

A second way in which the form factors could acquire theoretical status would be through causality conditions. The integral equations defining the form factors are just those suggested by local field theory. If it is possible to solve the form-factor equations for all particles simultaneously, and hence to define the simultaneous extensions of all amplitudes off their mass shells, then one would obtain by Fourier transformation just the basic space-time functions of asymptotic field theory. It seems not implausible that the off-mass-shell momentum-space functions would, if they exist, continue to satisfy the maximal analyticity property,

extended now off the mass shell. That is, they might have no physical-sheet singularities other than those of the terms of the perturbation solution. But since the perturbation terms satisfy locality their singularities cannot be in the regions forbidden to functions satisfying the coordinate-space microscopic causality condition. Hence the space-time functions would satisfy the microscopic causality property. Thus, if the existence of extensions satisfying the extended maximal analyticity property could be proved (assuming maximal analyticity on the mass shell), then local asymptotic field theory would be shown to be contained within analytic S-matrix theory. There seems no strong reason to believe that such solutions do not exist, since there are no divergence problems in these dispersion equations and the linear integral equations to be solved are evidently simpler than the bilinear equations that the mass shell functions must satisfy.[15] The pursuit of this interesting but difficult-looking question is not within the scope of this paper.

It is probably unnecessary to mention that asymptotic field theories are built on a Hilbert space corresponding to the physical particles; every physical particle whether composite or not is represented by a state orthogonal to those representing every other physical particle; the idea of some particles' being built out of others is not reflected by a corresponding relationship between states. Such a theory is rather similar to S-matrix theory in that both build upon physical particles. That asymptotic field theory should be contained in S-matrix theory, and that its locality property should follow from maximal analyticity, would not be completely unreasonable. The idea of basic underlying fields and of a more basic Hilbert space, of the kind occurring in Lagrangian theories, and generally visualized in axiomatic field theory, is not implied or suggested by the above considerations.

In the next section the form-factor equations for channels containing several particles are considered and their formal solution discussed.

## VII. THE CONSTRUCTION OF FIELDS OF COMMUNICATING PARTICLES

The form-factor equations are

$$\frac{\Gamma_{if}(S)}{S - S_i} = \frac{\Gamma_{if}(S_i)}{S - S_i} + \int \frac{\Gamma_{in}(S_n^-)\, T_{nf}(S_n^+)}{(S_n - S_i)(S - S_n)}\, dS_n \tag{7.1a}$$

and

$$\frac{\gamma_{fi}(S)}{S - S_i} = \frac{\gamma_{fi}(S_i)}{S - S_i} + \int \frac{T_{fn}(S_n^-)\, \gamma_{ni}(S_n^+)}{(S - S_n)(S_n - S_i)}\, dS_n \quad . \tag{7.1b}$$

If there are several stable particles in the S channel then these will be exhibited explicitly:

$$\frac{\Gamma_{if}(S)}{S - S_i} = \frac{\Gamma_{if}(S_i)}{S - S_i} + \sum_{j \neq i} \frac{\Gamma_{ij}(S_j)\, T_{jf}(S_j)}{(S_j - S_i)(S - S_j)} + \int \frac{\Gamma_{i\alpha}(S_\alpha^-)\, T_{\alpha f}(S_\alpha^+)}{(S_\alpha - S_i)(S - S_\alpha)}\, dS_\alpha \tag{7.2a}$$

$$\frac{\gamma_{fi}(S)}{S - S_i} = \frac{\gamma_{fi}(S_i)}{S - S_i} + \sum_{j \neq i} \frac{T_{fj}(S_j)}{(S - S_j)} \frac{\gamma_{ji}(S_j)}{(S_j - S_i)} + \int \frac{T_{f\alpha}(S_\alpha^-)\, \gamma_{\alpha i}(S_\alpha^+)}{(S - S_\alpha)(S_\alpha - S_i)}\, dS_\alpha \quad . \tag{7.2b}$$

The functions $T_{\alpha\beta}(S) \equiv T(\Omega_\alpha, S, \Omega_\beta)$ are functions of $S$ and of dimensionless parameters specifying the angles and ratios of kinetic energies of the initial and final particles. All T's must satisfy

the dispersion formula (6.10) in order that the arguments establishing the mathematical significance of the form factor be valid. Now the $T_{j\alpha}$ and $T_{\alpha j}$ are originally defined only at $S = S_j$, since this constraint is imposed by the mass shell conditions. However, they can be considered defined at $S \neq S_j$ by the form-factor equations (7.2a) and (7.2b) with $j$ in place of $i$ (and $j'$ in place of $j$). They will then satisfy (6.10), as is required, because the form-factor equations are just a special case of (6.10). [The term $R$ in (6.10) contains the contribution from the contour at infinity, and hence any necessary subtraction terms.]

The equations contain a set of constants $\Gamma_{ij}(S_j)$ and $\gamma_{ji}(S_j)$. Conditions on these constants are imposed by the equations themselves with $f = j$. These latter equations have additional constants $\Gamma_{ij}(S_i)$ and $\gamma_{ji}(S_i)$.

If one uses the identities

$$\Gamma_{if}(S_f) \equiv T_{if}(S_f) \tag{7.3a}$$

and

$$\gamma_{fj}(S_f) \equiv T_{fj}(S_f) \quad , \tag{7.3b}$$

and the fact

$$T_{jj}(S_j) = 0 \tag{7.4}$$

(which is true because the unity part of the S matrix has been subtracted to give $T$), then (7.2) gives

$$\Gamma_{ij}(S_j) - \gamma_{ij}(S_j) = \Gamma_{ij}(S_i) - \gamma_{ij}(S_i) \quad . \tag{7.5}$$

The two sides of these equations vanish if one requires that

$$\Gamma_{jf}(S_j) = T_{jf}(S_j) \tag{7.6a}$$

and

$$\gamma_{fj}(S_j) = T_{fj}(S_j) \quad . \tag{7.6b}$$

These equations are true by definition when $f$ is a several-particle index. By taking them to be definitions of the left side of the equation for single-particle states $f$ one ensures the validity of the consistency requirements (7.5) .

In terms of the functions

$$\Phi_{if}(S) \equiv \frac{\Gamma_{if}(S)}{S - S_i} \qquad (i \neq f) \tag{7.7a}$$

and

$$\phi_{fi}(S) \equiv \frac{\gamma_{fi}(S)}{S - S_i} \qquad (i \neq f) \tag{7.7b}$$

and the definition

$$\Phi_{ii}(S_i) = \phi_{ii}(S_i) = 1 \quad , \tag{7.7}$$

Eq. (7.7) become, for $i \neq f$ ,

$$\Phi_{if}(S) = \sum_j \Phi_{ij}(S_j)T_{jf}(S_j)/(S - S_j) + \int \frac{\Phi_{i\alpha}(S_\alpha^-)T_{\alpha f}(S_\alpha^+)}{S - S_\alpha} \, dS_\alpha \quad , \tag{7.8a}$$

and

$$\phi_{fi}(S) = \sum_j T_{fj}(S_j)\phi_{ji}(S_j)/(S - S_j)$$

$$+ \int \frac{T_{f\alpha}(S_\alpha^-)\phi_{\alpha i}(S_\alpha^+)}{S - S_\alpha} dS_\alpha \quad . \tag{7.8b}$$

For multiparticle states $f = \beta$ Eq. (6.8) can be written

$$\Phi_{i\beta}(S) = F_{i\beta}(S) + \int \frac{\Phi_{i\alpha}(S_\alpha^-)T_{\alpha\beta}(S_\alpha^+)}{S - S_\alpha} dS_\alpha \tag{7.9a}$$

and

$$\phi_{\beta i}(S) = f_{\beta i}(S) + \int \frac{T_{\beta\alpha}(S_\alpha^-)\phi_{\alpha i}(S_\alpha^+)}{S - S_\alpha} dS_\alpha \quad , \tag{7.9b}$$

where

$$F_{i\beta}(S) \equiv \sum_j \Phi_{ij}(S_j)T_{j\beta}(S_j)/(S - S_j) \tag{7.10a}$$

and

$$f_{\beta i}(S) \equiv \sum_j T_{\beta j}(S_j)\phi_{ji}(S_j)/(S - S_j) \quad . \tag{7.10b}$$

These are equations of the Omnes type. Solutions are given by

$$\Phi_{i\beta}(S) = F_{i\beta}(S) + \int \frac{F_{i\alpha}(S_\alpha^-)T_{\alpha\gamma}(S_\alpha^+)\underset{\sim}{\Phi}_{\gamma\delta}^{-1}(S_\alpha^+)dS_\alpha}{S - S_\alpha} \underset{\sim}{\Phi}_{\delta\beta}(S) \tag{7.11a}$$

and

$$\phi_{\beta i}(S) = f_{\beta i}(S) + \underset{\sim}{\phi}_{\beta\delta}(S) \int \frac{\underset{\sim}{\phi}_{\delta\gamma}^{-1}(S_\alpha^-)T_{\gamma\alpha}(S_\alpha^-)f_{\alpha i}(S_\alpha^+)}{S - S_\alpha} dS_\alpha \quad , \tag{7.11b}$$

where $\underset{\sim}{\bar{\phi}}$ and $\underset{\sim}{\phi}$ are solutions of the homogeneous equations $(F, f = 0)$.

Inserting (7.10) into (7.11), one obtains

$$\bar{\phi}_{i\beta}(S) = \sum_j \phi_{ij} G_{j\beta}(S) \tag{7.12a}$$

and

$$\phi_{\beta i}(S) = \sum_j g_{\beta j}(S) \phi_{ji} \quad , \tag{7.12b}$$

where

$$\bar{\phi}_{ij} \equiv \bar{\phi}_{ij}(S_j) \quad , \tag{7.13a}$$

and

$$\phi_{ji} \equiv \phi_{ji}(S_j) \quad . \tag{7.13b}$$

The functions $G_{j\beta}(S)$ and $g_{\beta j}(S)$ are functions of the mass-shell functions $T_{j\gamma}(S_j)$ and $T_{\alpha\gamma}(S_\alpha^+)$, and $T_{\gamma j}(S_j)$ and $T_{\gamma\alpha}(S_\alpha^-)$, respectively.

The solution (7.11) is not unique. There is a question of whether the homogeneous solutions are unique. And one can add solutions of the homogeneous equation. We shall suppose that analyticity requirements and asymptotic conditions will specify the physically interesting solution and that, as in the case treated by Omnes, the solution is still linear in the inhomogeneous term, so that (7.12) remains valid.

The insertion of (7.12) into (7.8) gives, using (7.4),

$$\bar{\phi} H \phi = N \, , \tag{7.14}$$

where

$$H_{ij} = \delta_{ij} + \int G_{i\alpha}(S_\alpha^-) g_{\alpha j}(S_\alpha^+) dS_\alpha \tag{7.15}$$

and $N$ is a diagonal matrix with elements

$$N_{ii} = 1 - \lim_{S \to S_i} \frac{\Gamma_{ii}(S)}{S - S_i}$$

$$= 1 - \lim_{S \to S_i} \frac{\gamma_{ii}(S)}{S - S_i} \quad . \tag{7.16}$$

Inspection of (7.2) shows that

$$N_{ii} \geqslant 1 \quad . \tag{7.17}$$

The Hermitian analyticity property of the $T$'s ,

$$T_{\alpha f}(S_\alpha^+) = T_{f\alpha}^*(S_\alpha^-) \quad , \tag{7.18}$$

implies

$$g_{\alpha j}(S_\alpha^+) = G_{j\alpha}^*(S_\alpha^-) \quad , \tag{7.19}$$

provided the homogeneous solutions are chosen to satisfy the analogous property

$$\underset{\sim}{\phi}_{\alpha\beta}(S_\alpha^+) = \underset{\sim}{\bar{\phi}}_{\beta\alpha}^*(S_\alpha^-) \quad . \tag{7.20}$$

The matrices H and N are then positive-definite Hermitian matrices, and one can write

$$H = AA^{\dagger} = (UD)(D^{\dagger}U^{\dagger}) \tag{7.21}$$

and

$$N = BB^{\dagger} \quad , \tag{7.22}$$

where the dagger denotes Hermitian conjugate. The matrix D is a diagonal matrix with positive elements and U is the unitary matrix that transforms the positive-definite Hermitian matrix H from diagonal form.

If the $\bar{\phi}$ and $\phi$ are assumed to satisfy

$$\phi = \bar{\phi}^{\dagger} \quad , \tag{7.23}$$

which is the analog of (7.18) , then (7.14) becomes

$$(B^{-1}\bar{\phi}A)(A^{\dagger}\bar{\phi}^{\dagger}B^{-1\dagger}) = I \quad . \tag{7.24}$$

The solution of this is

$$\bar{\phi} = BUA^{-1} \quad , \tag{7.25}$$

where U is an arbitrary unitary transformation. The A is completely determined by the mass-shell scattering amplitudes and B is a diagonal matrix defined by

$$B_{jj}^{-1} = (UA^{-1})_{jj} \quad , \tag{7.26}$$

which is the normalization condition

$$\tilde{\phi}_{jj} = 1 = \phi_{jj} \quad . \tag{7.27}$$

The ambiguity associated with the U in (7.25) can be eliminated by identifying this U with the U in (7.21). If one takes this particular solution of the problem then

$$B^{-1}\tilde{\phi} = UD^{-1}U^{-1} \quad . \tag{7.28}$$

This makes the "unrenormalized field" $B^{-1}\tilde{\phi}$ a positive definite Hermitian matrix, and gives

$$B^{-1}\tilde{\phi} = \phi B^{-1} \quad . \tag{7.29}$$

Thus for this particular solution the "unrenormalized" $\tilde{\phi}$ and $\phi$ are identical.

The term "unrenormalized" comes from comparing the above results to corresponding field theoretic ones. The relationship to field theory is given by the identifications

$$\tilde{\phi}_{if}(S_f) \equiv \langle 0|\tilde{\phi}_i(0)|f(S_f)\rangle \tag{7.30a}$$

and

$$\phi_{fi}(S_f) \equiv \langle f(S_f)|\phi_i(0)|0\rangle \quad , \tag{7.30b}$$

where $\tilde{\phi}_i(x)$ and $\phi_i(x)$ are the renormalized field operators of field theory. The normalization conditions

$$\langle 0|\tilde{\phi}_i(0)|i\rangle = 1 = \langle i|\phi_i(0)|0\rangle \tag{7.31}$$

on the renormalized fields are conditions (7.27). The ith diagonal matrix element of (7.14) is

$$\tilde{\phi}_{ij}(s_j)\phi_{ji}(s_j) + \int \tilde{\phi}_{i\alpha}(s_\alpha^-)\phi_{\alpha j}(s_\alpha^+)ds_\alpha = N_{ii} = (Z_i)^{-1} \quad , \tag{7.32}$$

where $Z_i$ is just the wave-function renormalization constant of field theory.

The interesting equations are the off-diagonal elements of (7.14). They say that if the field matrix elements are constructed by our form-factor equations then

$$\langle 0|\tilde{\phi}_i(0)\phi_j(0)|0\rangle = 0 \qquad (i \neq j) \quad . \tag{7.33}$$

That is, the fields for different particles are not just multiples of the same field, but are fundamentally different, and in the sense of (7.33), orthogonal. That is, if the action of $\tilde{\phi}_i(0)$ and $\phi_j(0)$ on the vacuum is taken to <u>define</u> (renormalized) bare particle states

$$\langle 0|\tilde{\phi}_i(0) = \langle i_b| \tag{7.34a}$$

and

$$\phi_j(0)|0\rangle = |j_b\rangle \quad , \tag{7.34b}$$

then these states are orthogonal for $i \neq j$. It had seemed from a consideration of Zimmermann's work that a single field, apart from normalization factors, should serve as the field for all particles with the same set of quantum numbers. But we see that if the fields are constructed from the mass-shell quantities by means of our form-factor equations then the fields for different particles will be essentially different.

Although we have started from a mass-shell theory involving only physical quantities, the form factor equations have brought in, quite naturally, the wave function renormalization constants $Z_i$. These equations evidently make sense only if $Z_i > 0$ for all $i$. This suggests that fields could be constructed from the mass-shell S matrix if the renormalization constants were finite. What happens when some of the $Z_i$ are zero has not been examined from the present viewpoint.[16]

The question arises, what happened to the expected solution in which the fields for different particles are just multiples of each other. Equations (7.8a) appear to be linear in the $\phi_{ij}$ and hence one would expect that a solution would be given by taking all $\phi_{kj}$ to be multiples of a single solution $\phi_{ij}$. The asymmetry comes in the normalization condition $\phi_{ii} = 1$. The $\phi_{ij}$ for $i \neq j$ are the limits as $S$ goes to $S_j$ of $\phi_{ij}(S)$. This is not true for $i = j$. In this case the limit is in fact necessarily negative. The asymmetry in these definitions means that a multiple of $\phi_{ij}$ will not satisfy the equation for $\phi_{kj}$ for $k \neq i$.

The origin of this asymmetry lies in the fact that the subtracted dispersion relations for the $\Gamma$'s ensure that $\Gamma_{ij}(S_j)$ is the limit as $S$ goes to $S_j$ of $\Gamma_{ij}(S)$. The same does not hold for $\phi_{ij}(S) = \Gamma_{ij}(S)/(S - S_j)$ for the case $i = j$. The same situation would occur in a field theoretic

treatment, provided the $\Gamma_{ij}$ dispersion relations require only one subtraction. This requirement is essentially equivalent to the requirement that the fields be "orthogonal" in the sense (7.33) .

The conditions $\Gamma_{ij}(S_i) = \gamma_{ij}(S_i)$ and $\Gamma_{ij}(S_j) = \gamma_{ij}(S_j)$ that we have imposed would generally be assumed also in field theory, as $\Gamma_{ij}(S)$ and $\gamma_{ij}(S)$ are both once subtracted versions of the same function

$$F_{ij}(S) \equiv \sum_R \langle 0|J_i(0)|k\rangle(k|J_j{}^\dagger 0|0\rangle(-i)/(S - S_k)$$

$$+ \int ds_\alpha \langle 0|J_i(0)|\alpha\rangle\langle\alpha|J_j{}^\dagger(0)|0\rangle(-i)/(S - S_\alpha) + \text{Const}, \tag{7.35}$$

which is the negative of momentum-space form of the time-ordered product of the currents $J_i(x)$ and $J_j{}^\dagger(y)$ .

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