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Berkeley, California

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ABSTRACT

Conditions under which Cabibbo's angle association with the axial vector current remains unchanged even in the symmetry-violating world are derived. It is also found that the approximate conservation of the axial vector current is consistent with the dispersion theoretical approach to the π-μ and K-μ decay amplitudes only under the condition $z_3 \ll 1$, z_3 being the wave function renormalization constant of the π- or K-meson field.

Recently Cabibbo has shown[1] that the assumption of SU(3) invariance for strong interactions[2] allows a general understanding of weak leptonic processes. He assumed that the weak current responsible for leptonic processes is a member of an octet, and can be written as[3]

$$J_\mu = \cos\theta\,[v_\mu^{(0)} + a_\mu^{(0)}] + \sin\theta\,[v_\mu^{(1)} + a_\mu^{(1)}] \quad , \tag{1}$$

where v_μ and a_μ are vector and axial vector currents, respectively, and the $\cos\theta$ term corresponds to the currents with $\Delta S = 0$, $\Delta Q = 1$; the $\sin\theta$ term corresponds to the currents with $\Delta S = 1$, $\Delta Q = 1$. According to this proposal, the π-μ decay constant $F^{(\pi)}$ is connected with the K-μ decay constant $F^{(K)}$ through $\tan\theta$[1]:

$$F^{(K)}/F^{(\pi)} = \tan\theta \quad . \tag{2}$$

This relation, needless to say is true only in the exact symmetry limit, and so its validity in the real world is unclear. In this note, therefore, we shall try to find the conditions under which relation (2) remains unchanged even to a symmetry-violating world. We shall take the viewpoint that to the symmetry limit the axial vector current $a_\mu^{(i)}$ is a member of an octet and is exactly conserved, but in the real world this symmetry is not exact and therefore the current is only "partially" conserved.[4,5] We shall also discuss whether this approximate conservation of $a_\mu^{(i)}$ is consistent with Ida's formulae[6] for $F^{(\pi)}$ and $F^{(K)}$, which involve a dispersion theoretical approach to the π-μ and K-μ decay processes and are generalizations of the Goldberger-Treiman formula for $F^{(\pi)}$.[7]

In the <u>real</u> world, let us define $L_m^{(i)}(s)$, a form factor of $a_\mu^{(i)}$ in the channel m (excluding single-particle states), by

$$t^{(i)}(\theta)\langle 0|\partial_\mu a_\mu^{(i)}|m\rangle = C_m L_m^{(i)}(s) \quad , \tag{3}$$

where $t^{(i)}(\theta)$ denotes functions

$$t^{(0)}(\theta) = \cos\theta \quad \text{and} \quad t^{(1)}(\theta) = \sin\theta \quad , \tag{4}$$

and C_m is an invariant in the channel m , e.g.,

$$C_{n\bar{p}} = \bar{u}(n)i\gamma_5 v(\bar{p}) \quad .$$

The form factor $L_m^{(i)}(s)$ is normalized at $s = 0$ by, e.g.,

$$L_{B'B}^{(i)}(0) = t^{(i)}(\theta) g_{B'B}^{(i)}(M_{B'} + M_B) \quad , \tag{5}$$

where $g_{B'B}^{(i)}$ is the axial vector coupling constant in the β decay of a baryon, and M_B denotes the baryon mass. The form factor $L_m^{(i)}(s)$ is assumed to satisfy the once-subtracted dispersion relation[8]

$$L_m^{(i)}(s) = L_m^{(i)}(0) + \frac{s}{\mu_i^2 - s} F^{(i)} G_m^{(i)} + \frac{s}{\pi} \sum_n \int ds' \frac{T_{mn}^*(s')\rho_n(s')L_n^{(i)}(s')}{s'(s' - s)} \quad , \tag{6}$$

where

μ_0 = pion mass,

μ_1 = kaon mass,

$F^{(i)}$ = "i"-meson decay constant,

$\rho_n(s)$ = phase-space volume,

$T_{mn}(s)$ = scattering amplitude from n channel to m channel in the pseudoscalar sector.

and $G_m^{(i)}$ is the coupling constant between the "i" meson and the m channel, e.g.,

$$G_{n\bar{p}}^{(0)} = 2(G)^{1/2} \quad ,$$

and is the boundary value of the function $G_m^{(i)}(s)$ which satisfies the dispersion relation

$$G_m^{(i)}(s) = G_m^{(i)} + \frac{s - \mu_i^2}{\pi} \sum_n \int ds' \frac{T_{mn}^*(s')\rho_n(s')G_n^{(i)}(s')}{(s' - \mu_i^2)(s' - s)} \quad . \tag{7}$$

The simplest solutions of Eqs. (6) and (7) are adapted here, viz.,

$$L_m^{(i)}(s) = M_m^{(i)}(s) + \frac{s}{\mu_i^2 - s} G_m^{(i)}(s)F^{(i)} \tag{8}$$

with

$$M_m^{(i)}(s) = \sum_n D^{-1}(s)_{mn} L_n^{(i)}(0) \tag{9}$$

and

$$G_m^{(i)}(s) = \sum_n D^{-1}(s)_{mn} G_n^{(i)}(0) \quad , \tag{10}$$

where D(s) is the denominator function of T in the usual N/D method, normalized to unity at s = 0. Now, according to the hypothesis of the approximate conservation of $a_\mu^{(i)}$, the approximate relation

$$F^{(i)} \approx M_m^{(i)}(s)/G_m^{(i)}(s) \tag{11}$$

should hold for any values of s and m in the solution (8).[9] Therefore, Eq. (11) reduces to the usual Goldberger-Treiman relation[7]

$$F^{(i)} \approx L_m^{(i)}(0)/G_m^{(i)}(0) \approx L_m^{(i)}(0)/G_m^{(i)} \quad , \tag{12}$$

e.g.,

$$F^{(\pi)} \approx L_{n\bar{p}}^{(0)}(0)/G_{n\bar{p}}^{(0)} = \sqrt{2}\, M_N g_A \cos\theta/G \quad , \tag{13}$$

where $g_A \equiv g_{n\bar{p}}^{(0)}$. This is a generalization of Nambu's method[5] to an arbitrary channel m. Relations (12) and (13) also give a relation between $G_m^{(0)}$ and $L_m^{(0)}(0)$,

$$G_m^{(0)} \approx \left(\frac{G}{\sqrt{2}\, M_v}\right)\left(\frac{L_m^{(0)}(0)}{g_{n\bar{p}}^{(0)} \cos\theta}\right) , \tag{14}$$

e.g.,

$$G_{B'B}^{(0)} \approx \sqrt{2}\, G \left(\frac{g_{B'B}^{(0)}}{g_A}\right)\left(\frac{M_{B'} + M_B}{2M_N}\right) , \tag{15}$$

which reduces to

$$G_{B'B}^{(0)} \approx G[D(1 - f) + Ff]_{B'B}^{\pi^-} \tag{15'}$$

in the exact symmetry limit. Here D and F are the well-known D and F coupling coefficients, and f is the mixing parameter in weak interactions [Eq. (15') implies $f_{weak} \approx f_{strong}$]. But we cannot obtain a relation like (14)

$$G_m^{(1)} \approx \left(\frac{G}{\sqrt{2}\, M_N}\right)\left(\frac{L_m^{(1)}(0)}{g_A \sin\theta}\right) , \tag{16}$$

unless Eq. (2) is assumed still to be true even in the symmetry-violating world. One can easily see that the relation (16) between $G_m^{(1)}$ and $L_m^{(1)}(0)$ is a necessary and sufficient condition for the validity of Eq. (2), even if the symmetry is violated in some form.[10]

Now let us examine whether the approximate conservation of $a_\mu^{(i)}$ is consistent with Ida's formula for $F^{(i)}$. Following Goldberger and Treiman, we also assume that the function $F^{(i)}(s)$ satisfies an unsubtracted dispersion relation

$$F^{(i)}(s) = \frac{1}{\pi}\int ds' \frac{\mathrm{Im}\, F^{(i)}(s')}{s' - s} \quad . \tag{17}$$

The imaginary part of $F^{(i)}(s)$ is given by

$$s\ \mathrm{Im}\ F^{(i)}(s) = \pi \sum_m L_m^{(i)*}(s)\rho_m(s)G_m^{(i)}(s)$$

$$= \pi \sum_m \{H_m^{(i)}(s) + [s/(\mu_i^2 - s)]F^{(i)}\}G_m^{(i)*}(s)\rho_m(s)G_m^{(i)}(s) \quad , \tag{18}$$

where

$$H_m^{(i)}(s) \equiv \mathrm{Re}\ [M_m^{(i)}(s)/G_m^{(i)}(s)] \tag{19}$$

and

$$F^{(i)} \equiv F^{(i)}(s = \mu_i^2) \quad . \tag{20}$$

Inserting Eq. (18) into Eq. (17) and putting $s = \mu_i^2$, we have

$$F^{(i)} = \int ds \frac{s - \mu_i^2}{s} \sum_m \sigma_m^{(i)}(s)H_m^{(i)}(s)/[1 + \int ds\sigma^{(i)}(s)] \quad , \tag{21}$$

where

$$\sigma_m^{(i)}(s) \equiv [G_m^{(i)*}(s)\rho_m(s)G_m^{(i)}(s)]/(s - \mu_i^2)^2 \tag{22}$$

and

$$\sigma^{(i)}(s) \equiv \sum_m \sigma_m^{(i)}(s) \quad , \tag{23}$$

if the denominator in Eq. (21), which is equal to the wave-function renormalization constant of the "i"-meson field, $[Z_3^{(i)}]^{-1}$, is finite.

This is Ida's formula.[6] The approximate conservation of $a_\mu^{(i)}$ has already assured the validity of Eq. (11), i.e., $H_m^{(i)}(s) \approx F^{(i)}$, so that Eq. (21) reduces to

$$1 \approx \int ds \frac{s - \mu_i^2}{s} \sigma^{(i)}(s)/[1 + ds\, \sigma^{(i)}(s)] \quad . \tag{24}$$

This equation is consistently valid only in the case when the denominator in Eq. (24) is much larger than unity, that is, $z_3^{(i)}$ is much smaller than unity, but does not vanish.

When $z_3^{(i)}$ vanishes, Ida's formula (21) is no longer valid. In order that the dispersion relation for $F^{(i)}(s)$ need no substractions, or in other words, in order that the expression

$$F^{(i)} = \int ds \left[\frac{s - \mu_i^2}{s} \sum_m \sigma_m^{(i)}(s) H_m^{(i)}(s) - \sigma^{(i)}(s) F^{(i)} \right] \tag{25}$$

be meaningful, we must have

$$\lim_{s\to\infty} \left[\sum_m \sigma_m^{(i)}(s)(H_m^{(i)}(s) - F^{(i)})/\sigma^{(i)}(s) \right] = 0 \quad . \tag{26}$$

This is also Ida's relation, and is consistent with our approximate formula, $H_m^{(i)}(s) \approx F^{(i)}$.[11]

In conclusion we have shown that

(i) the relation (16) between the strong and weak coupling constants is the condition for validity of Eq. (2) even in the symmetry-violating world;

(ii) the approximate conservation of the axial vector current is consistent with Ida's formula for $F^{(i)}$ only when $z_3^{(i)}$ is much smaller than unity, $z_3^{(i)} \ll 1$.

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FOOTNOTES AND REFERENCES

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+ On leave of absence from Department of Physics, Osaka University, Osaka, Japan.

1. N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

2. M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

3. Here we have assumed CP invariance for the leptonic decay processes. We have supposed that the 2π decay of K_2^0 may occur through another mechanism.

4. J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).

5. Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

6. M. Ida, Phys. Rev. 132, 401 (1963).

7. M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1478 (1958).

8. All discussion will be unchanged even if we adapt the viewpoint that all channels except for those of baryon pairs have no direct coupling to the lepton pair, and therefore $L_m^{(i)}(s)$ for channels indirectly coupled to the lepton pairs satisfies unsubtracted dispersion relation.

9. See Ref. 5. Here we have not supposed that even at the high-energy limit the approximate relation (11) tends to the exact relation. If Eq. (11) tends to the exact relation at the high-energy limit, we shall encounter a difficulty mentioned in footnote 11.

10. Note that the second brackets in Eqs. (14) and (16) are independent of θ by definition (3). If we take a viewpoint of Freund and Nambu, $L_m^{(i)}(0)$ is given by, for $m = B'B$,

$$L_{B'B}^{(i)}(0) = g_A t^{(i)}(\theta)(M_{B'} + M_B)[D(1 - f) + Ff]_{B'B}^{i}/\sqrt{2} \quad ,$$

and Eqs. (14) and (16) reduce to

$$G_{B'B}^{(i)} \approx G[D(1 - f) + Ff]_{B'B}^{i} \frac{M_{B'} + M_B}{2M_N} \quad ,$$

which shows a deparature from the exact SU(3) coupling scheme. P.G.O. Freund and Y. Nambu, Phys. Rev. Letters 13, 221 (1964).

11. This approximate relation will never tend to the exact relation at the high-energy limit. If we assume the exact relation, $\lim_{s\to\infty} H_m^{(i)}(s) = F^{(i)}$, the form factor $L_m^{(i)}(s)$ should exactly vanish at the high-energy limit. In this case the dispersion relation for $L_M^{(i)}(s)$ needs no subtractions, and we have the solution

$$L_m^{(i)}(s) = \frac{\mu_i^2}{\mu_i^2 - 2} F^{(i)} G_m^{(i)}(s) \quad ,$$

from which the Goldberger-Treiman relation is derived automatically. But inserting the above solution into Eq. (16) and putting $s = \mu_i^2$, we obtain the absurd result

$$1 = -\int ds \frac{\mu_i^2}{s} \sigma^{(i)}(s) < 0 \quad .$$

See also reference 6.

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