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ANGULAR DISPERSION OF COINCIDENT FISSION-FRAGMENT TRAJECTORIES ARISING FROM A DISK SOURCE

Berkeley, California

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ABSTRACT

The angular distribution of coincident fission-fragment trajectories arising from a disk source of arbitrary size is derived theoretically, when two disk detectors of equal size are equidistant from and coaxial with the source. Only the geometric effects are considered; the effects of prompt neutron emission are neglected. The angular distribution is found as a closed-form expression in terms of elementary functions, and graphs are given of the distribution for several size-to-spacing ratios of the source and detectors.

Corrections are frequently necessary for the angular dispersion introduced by the use of finite detectors and disk sources in the simultaneous detection of fission-fragment pairs from individual fission events. Two factors contribute to the distribution of angles which result in a coincident measurement, namely: (a) the geometry of the detection arrangement, and (b) the angular dispersion introduced by the recoil of prompt neutrons from the fission fragments. In this report, we ignore (b) and consider only the restricted distributions arising from detection geometries in which the two fission-fragment detectors are both equidistant from and coaxial with the fission source. These corrections are of particular interest in experiments concerned with the measurements of the various prompt radiations emitted in coincidence with fission.

Consider a uniform disk source with radius a of fissionable material for which the two fragments arising from a fission anywhere on the disk travel in exactly opposite directions along a line that may point in any direction with equal probability. That is, if θ is the angle that the line of travel of the two fragments makes with the normal to the plane of the disk, then, since the probability that this line lies in any solid angle $d\omega$ is $(1/2\pi)\, d\omega$, the probability that the line of travel has an angle between θ and $(\theta + d\theta)$ with the normal to the plane of the disk is $\sin\theta\, d\theta$.

Let two disk detectors of radius A be placed coaxial with the source, one on each side of it a distance d away. [See Fig. 1(a)]. In order to register the occurrence of a fission, both fragments must be detected, one in each detector. Thus only some of the fissions occurring at the source will be registered; the others will have only one, or possibly none, of the fragments detected. The problem is to find the probability, $P(\theta)\, d\theta$, that a fission with fragments leaving the source between angle θ and $(\theta + d\theta)$ with the normal will have both its fragments detected.

Consider any point on the source a distance y from the center, where $y \leq A$. (See Fig. 1.) Then all fissions for which $0 \leq d \tan\theta \leq A - y$ [region I of Fig. 1(b)] will be detected by both detectors. If the angle θ lies in the range $A - y < d \tan\theta < (A^2 - y^2)^{1/2}$, then only some of the fissions will be detected by both detectors [region II of Fig. 1(b)]. Straightforward geometric considerations show that in this range only the fraction $(2/\pi) \sin^{-1} \left[(A^2 - y^2 - d^2 \tan^2\theta)/2yd \tan\theta\right]$ of the fissions with angle θ will be detected

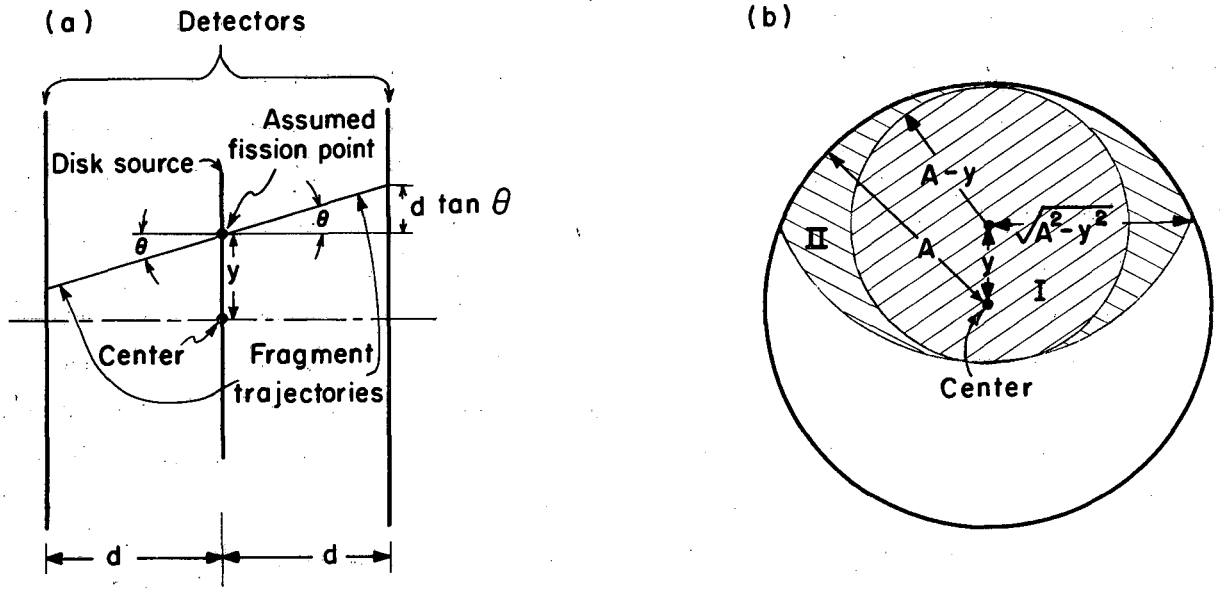

Fig. 1. Geometric configuration.
(a) Side view of detector-source configuration.
(b) End view of right detector showing regions of detection. Only those fragments striking regions I or II from fissions occurring at point y on the source will have their complementary fragment detected by the left detector also.

by both detectors. If $d \tan\theta \geq (A^2 - y^2)^{1/2}$ then none of the fissions can be detected by both detectors. Let $P_y(\theta)\, dy\, d\theta$ denote the probability that a fission occurs in the ring between distances y and $(y + dy)$ from the center of the source, has fragments leaving at an angle between θ and $(\theta + d\theta)$, and is detected by both detectors. Then the above considerations yield

$$P_y(\theta) = \frac{\sin\theta}{\pi a^2} \begin{cases} 2\pi y & , \quad (0 \leq d \tan\theta \leq A - y) \\ 4y \sin^{-1} \dfrac{A^2 - y^2 - d^2 \tan^2\theta}{2yd \tan\theta} & , \quad (A - y < d \tan\theta < (A^2 - y^2)^{1/2}) \\ 0 & , \quad (d \tan\theta \geq (A^2 - y^2)^{1/2}) \end{cases}$$

One can then obtain the desired probability $P(\theta)$ by integrating over the source from $y = 0$ to $y = a$, $P(\theta) = \int_0^a P_y(\theta)\, dy$. If $a \leq A$ (source smaller than detectors), the result is

$$P(\theta) = \begin{cases} \sin\theta & , \quad (\text{if } 0 \leq \tan\theta \leq R_2 - R_1) \qquad (1a) \\ \sin\theta \dfrac{(R_2 - \tan\theta)^2}{R_1^2} + \dfrac{2 \sin\theta}{\pi R_1^2} \cdot \Big\{ -\dfrac{\pi}{2} \tan^2\theta \\ + \pi R_2 \tan\theta - (R_2^2 - R_1^2) \sin^{-1} \dfrac{R_2^2 - R_1^2 - \tan^2\theta}{2R_1 \tan\theta} \\ - 1/2 \left[4R_1^2 \tan^2\theta - (R_2^2 - R_1^2 - \tan^2\theta)^2\right]^{1/2} \\ - R_2^2 \sin^{-1}\left(\dfrac{1}{2R_1R_2}\left[4R_1^2 \tan^2\theta - (R_2^2 - R_1^2 - \tan^2\theta)^2\right]^{1/2}\right)\Big\} , \\ \qquad \left[\text{if } R_2 - R_1 \leq \tan\theta \leq (R_2^2 - R_1^2)^{1/2}\right] \qquad (1b) \\ \sin\theta \dfrac{(R_2 - \tan\theta)^2}{R_1^2} + \dfrac{2 \sin\theta}{\pi R_1^2} \cdot \Big[-\pi/2 \tan^2\theta \\ + \pi R_2 \tan\theta - \tan\theta\, (R_2^2 - \tan^2\theta)^{1/2} \\ -R_2^2 \sin^{-1}\left(\dfrac{\tan\theta}{R_2}\right)\Big] & , \quad \left[\text{if } (R_2^2 - R_1^2)^{1/2} \leq \tan\theta \leq R_2\right] \quad (1c) \\ 0 & , \quad (\text{if } \tan\theta \geq R_2) \qquad (1d) \end{cases}$$

where $R_1 = a/d$ and $R_2 = A/d$ are dimensionless parameters.

If $a > A$ (source larger than detectors), then one obtains $P(\theta)$ after noticing that fissions occurring at points on the source at a distance greater than A from its center cannot be detected by both detectors. Thus the source is equivalent to a source of the same average strength per unit area, but of radius A, and one may "remove" the portion of the source with radius greater than A and use Eq. (1) for $P(\theta)$ with $a = A$ (i.e., $R_1 = R_2$). In that case, of course, the only nontrivial regions for $\tan\theta$ are Eqs. (1c) and (1d).

Figures 2-11 show graphs of $P(\theta)$ as given by Eq. (1) for $R_1 = 0.0(0.1)1.0$ and $R_2 = 0.1(0.1)1.0$. There is a separate figure for each value of R_2, and each figure contains the curves for all the values of R_1. In each case the top curve is the one for $R_1 = 0.0$ (point source) and the lower curves are the ones for the successive values $R_1 = 0.1, 0.2, \ldots, 0.9, 1.0$. The curves were calculated on the IBM 7094 and plotted on the CAL-COMP 1462 Plotter. A FORTRAN IV program is available to calculate curves and/or tables for $P(\theta)$ for general values of R_1 and R_2.

This problem was suggested by Dr. Stanley G. Thompson and Dr. S. S. Kapoor. We thank J. K. Poggenburg for his assistance with the CAL-COMP plotting program.

FOOTNOTE

* Work performed under the auspices of the U. S. Atomic Energy Commission.

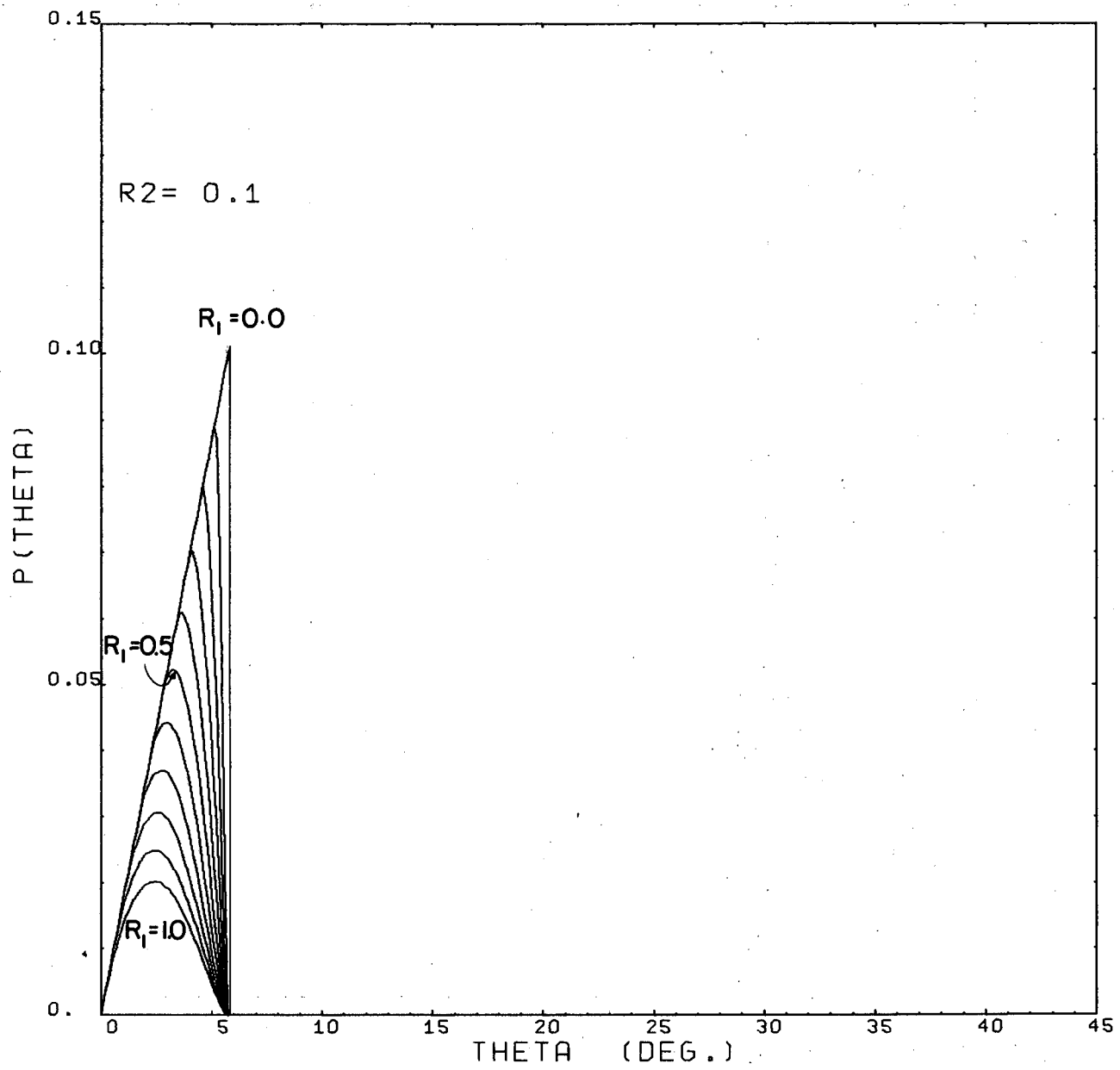

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Fig. 2. Angular distribution of fission-fragment trajectories with $R_2 = 0.1$.

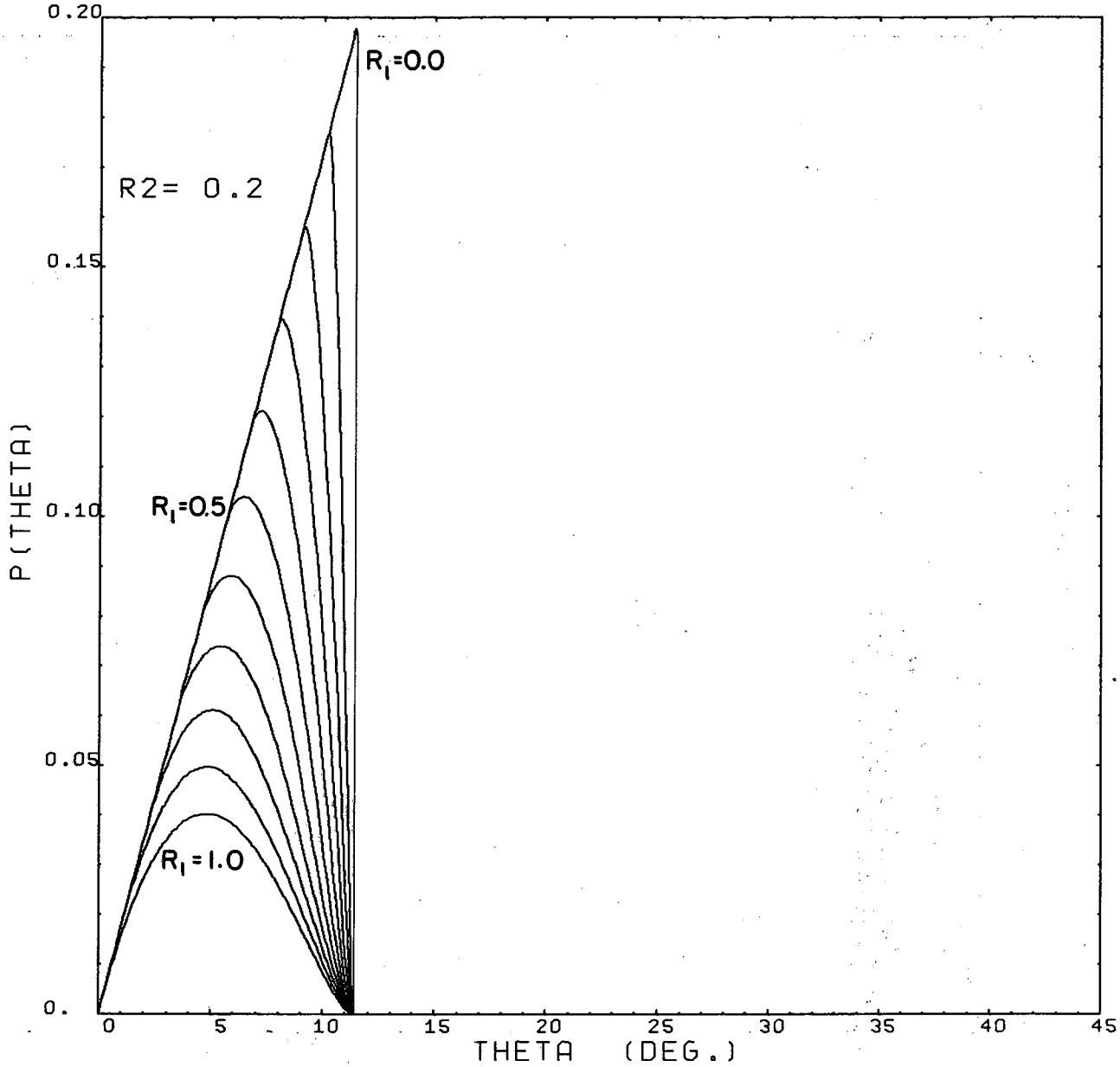

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Fig. 3. Angular distribution of fission-fragment trajectories with $R_2 = 0.2$.

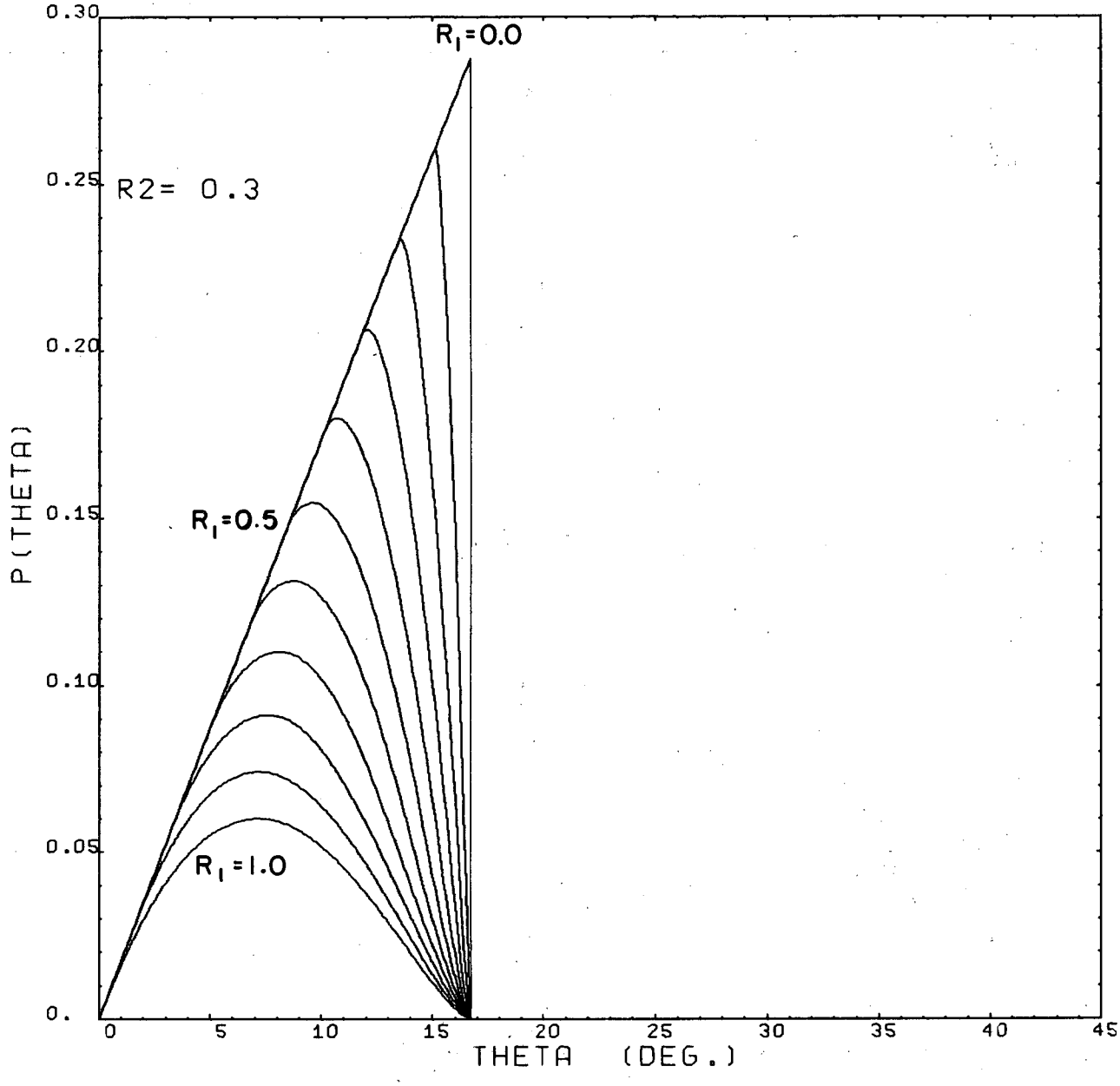

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Fig. 4. Angular distribution of fission-fragment trajectories with $R_2 = 0.3$.

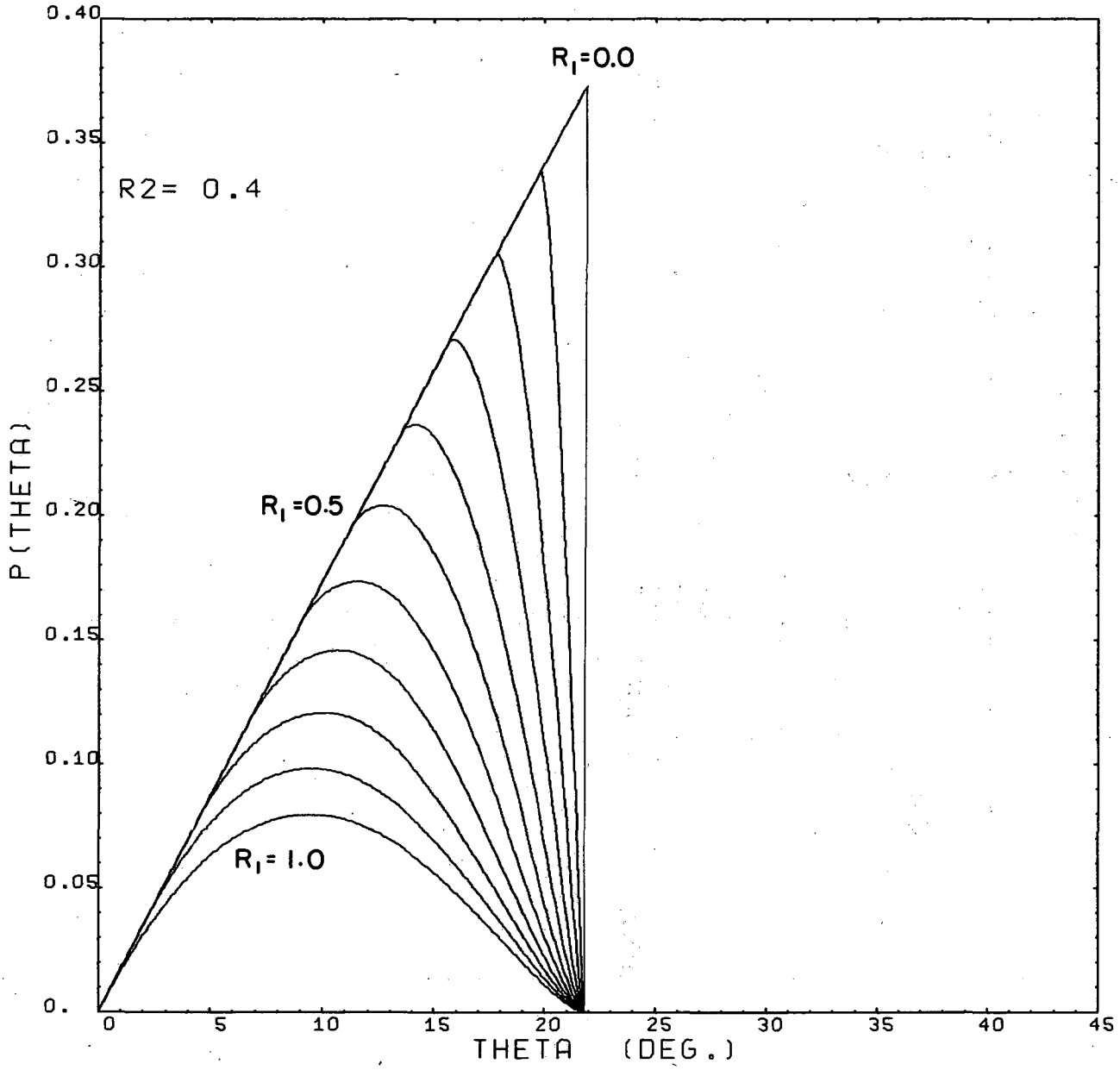

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Fig. 5. Angular distribution of fission-fragment trajectories with $R_2 = 0.4$.

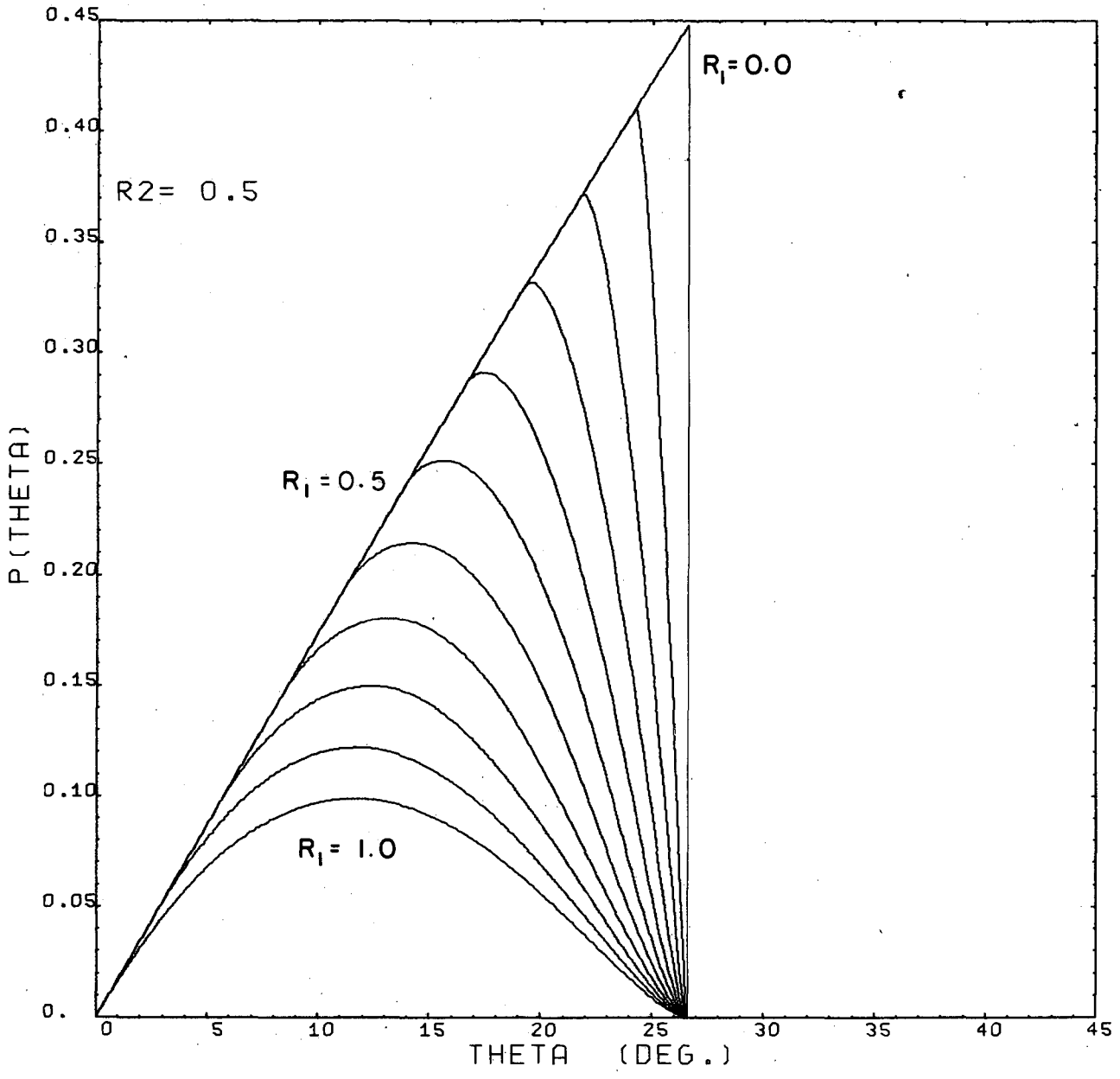

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Fig. 6. Angular distribution of fission-fragment trajectories with $R_2 = 0.5$.

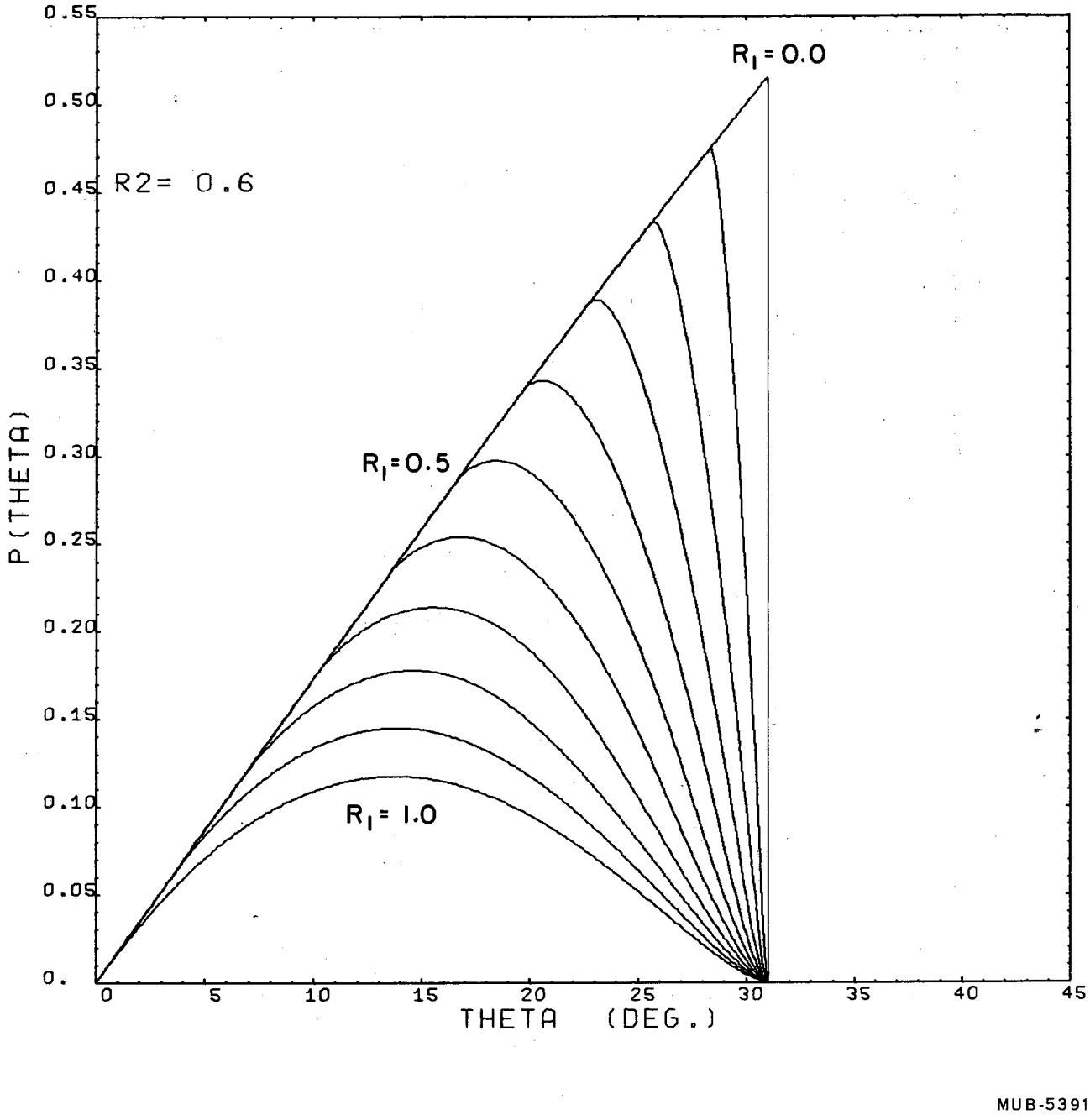

Fig. 7. Angular distribution of fission-fragment trajectories with R_2 = 0.6.

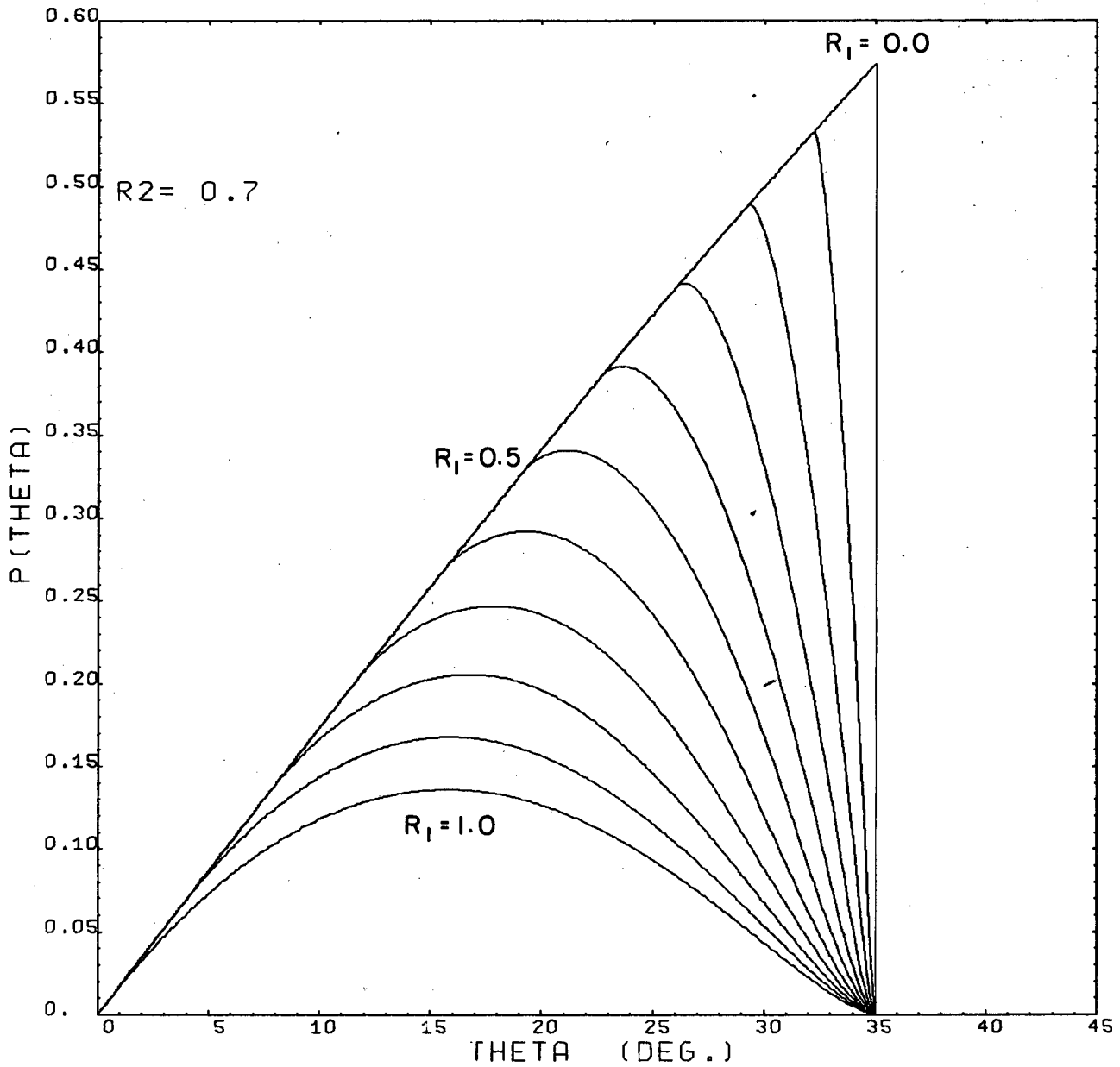

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Fig. 8. Angular distribution of fission-fragment trajectories with $R_2 = 0.7$.

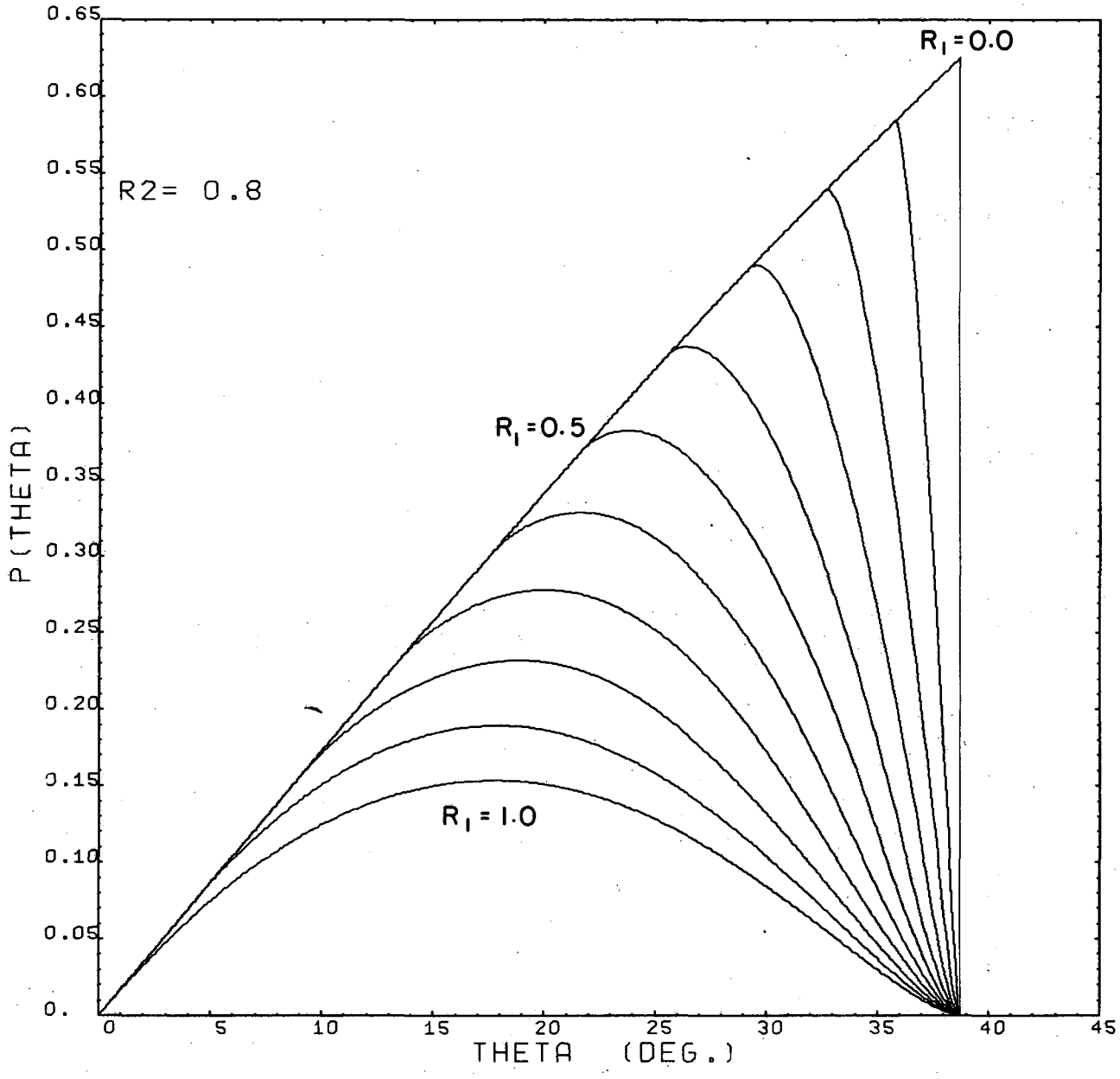

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Fig. 9. Angular distribution of fission-fragment trajectories with $R_2 = 0.8$.

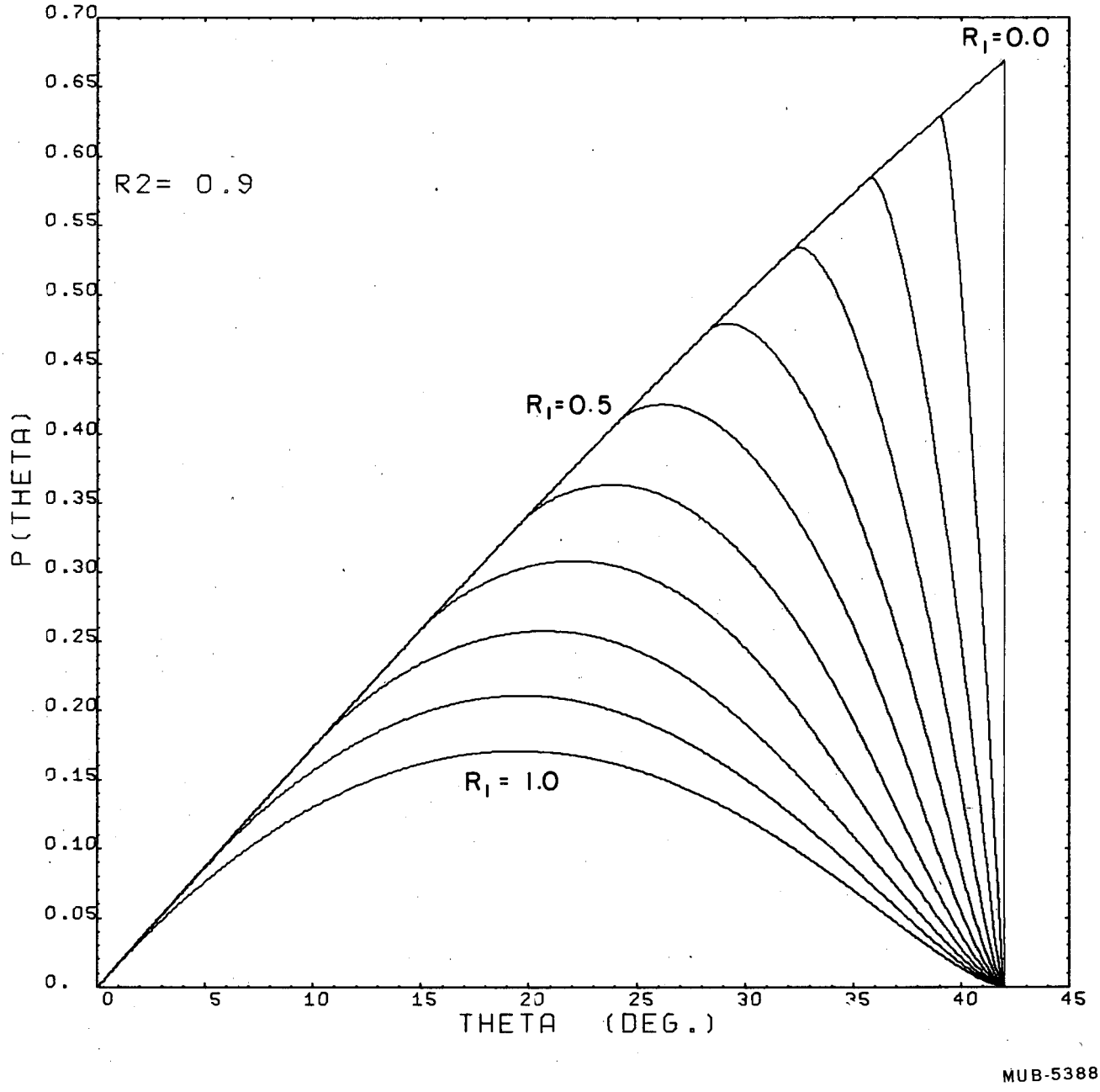

Fig. 10. Angular distribution of fission-fragment trajectories with $R_2 = 0.9$.

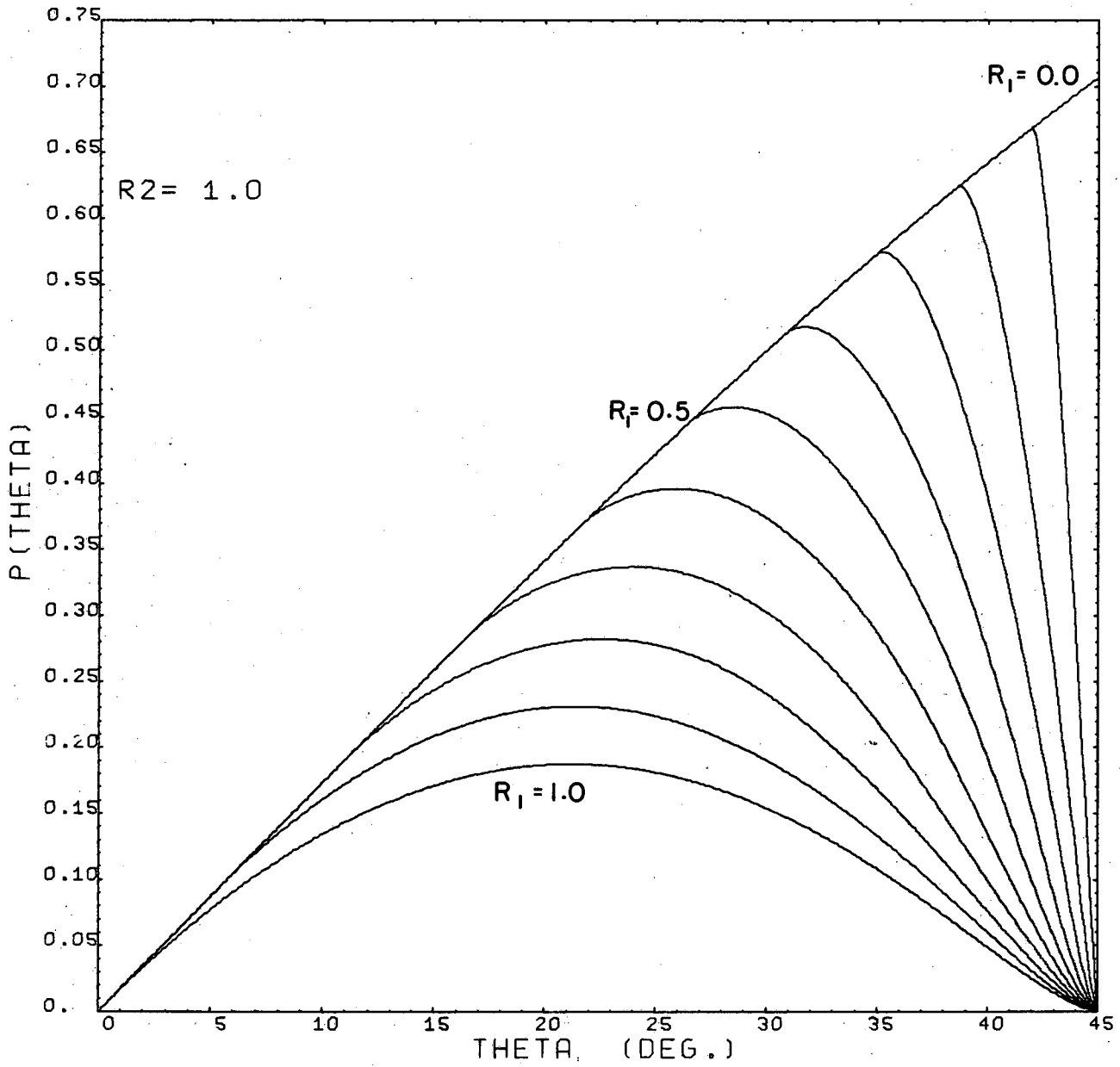

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Fig. 11. Angular distribution of fission-fragment trajectories with $R_2 = 1.0$.

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