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RANGE-ENERGY CURVES

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University of California
Radiation Laboratory

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Introduction

The range-energy curves which are presented in this report were computed for the University of California Radiation Laboratory range-energy program. The rate of energy loss given was computed from the theoretical formula given by Livingston and Bethe, Rev. of Mod. Phys. 9:263 (1937), and the range values were determined by numerical integration of the reciprocal of the rate of energy loss with respect to the energy, with low energy values being based on experimental rather than theoretical data.

Two tables are presented, one for protons and one for electrons, which enable the computation of the rate of energy loss, given the value of the mean excitation potential. While the value of this latter constant has been taken, in this paper, as being given by

$$I = \alpha Z = 11.5\ Z \text{ ev.},$$

the validity of such a simplistic formulation has been increasingly questioned, several recent experiments yielding results considerably different from those expected. The range-energy curves therefore are tentative pending a better understanding of the mean excitation potential.

The energy-loss considered in this paper derives only from collision effects. The losses due to bremstrahlung and meson and pair production result from an essentially different process. The calculation of the Fermi effect is ill-advised pending a better determination of the constants.

W. A. Aron
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THEORETICAL RANGE-ENERGY VALUES

The construction of heavy particle accelerators giving energies in the hundred million-volt region has made imperative the extension of range and rate of energy loss calculations to energies not previously used. This has been done for aluminum and air by J. H. Smith[1]. The work here presented extends these results to other stopping media.

The values computed have been the rate of energy loss and range corresponding to loss of energy by ionization only. Effects such as nuclear interactions, meson production, and bremstrahlung, which will play an increasingly important role at higher energies, have been neglected. The energy loss correction due to the polarization effects of a dense media have also been omitted[2].

On this basis, the rate of energy loss is given by[3]

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} NZ \left[\log_e \frac{2mv^2}{I} - \log_e (1-\beta^2) - \beta^2 \right] \qquad (1)$$

in which

e = electronic charge,
m = electronic mass,
ze = charge of heavy particle,
NZ = number of electrons per unit volume of stopping material,
$\beta = v/c$; where c is the velocity of light,
I = the mean excitation potential.

This formula holds when $v \gg u_k$ where u_k is the velocity of the orbital electrons in the K shell of the atoms of the stopping medium.

The quantity, I, is given by

$$Z \log_e I = \sum_{n,l} f_{n,l} \log_e A_{n,l} \qquad (2)$$

where $f_{n,l}$ are the oscillator strengths appearing in spectra, and $A_{n,l}$ the corresponding excitation energies. The $f_{n,l}$'s are subject to the

condition

$$\sum_{n,l} f_{n,l} = Z . \tag{2a}$$

The definition of I by equation (2) is not generally useful, the evaluation being too difficult for systems of many electrons. On the basis of the Fermi-Thomas model of the atom with several electrons, F. Bloch[4] has shown that $I = kZ$, where k is a constant to be evaluated empirically. Measurements by R. R. Wilson[5] have indicated a value of $k = 11.5$ ev. This is the value used in these calculations, although recent measurements have indicated this value may be considerably in error.

In cases where the condition $v \gg u_k$ is not satisfied, correction terms are needed in equation (1). For elements in the atomic number range from carbon to aluminum, these corrections are given by Livingston and Bethe[3]. For the elements of higher atomic number, the correction was made in the following manner:

Quantities Z^* and I^* were defined by the relations

$$Z \log_e I = Z^* \log_e I^* + \sum_{\lambda=k,l,m} N_\lambda \log_e A_\lambda ,$$

$$Z = Z^* + \sum_{\lambda=k,l,m} N_\lambda ,$$

where N_λ, N_l and N_m are the number of dispersion electrons in the K, L, and M shells respectively and A_k, A_l, and A_m are the corresponding excitations potentials. The values of the N's used are those given by Honl[6] and Livingston and Bethe[1]. Then the equation takes the form:

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} N \left[Z^* \log_e \frac{2mv^2}{I^*} + \sum_\lambda{}' N_\lambda \log_e \frac{2mv^2}{A_\lambda} + (Z^* + \sum_\lambda{}' N_\lambda)\,(\log_e(1-\beta^2) + \beta^2 \right]$$

where the $\sum_\lambda{}'$ indicate sums over those λ for which $\log_e \frac{2mv^2}{A_\lambda} \geqslant 0$.

The range, R, of the particles was then obtained by numerical integration.

$$R(E) = \int_{E_0}^{E} \left(\frac{dx}{dE}\right) dE + R(E_0)$$

It was necessary to determine $R(E_0)$ from experimental data[5,7] since at very low energies the theory cannot be satisfactorily corrected. This value, $R(E_0)$, was determined from the range of the particle in air at the same energy and the experimentally measured ratio of the stopping power in air and in the medium in question.

Stopping Medium	Stopping Power at Initial Point
Hydrogen	4.71 cm H_2 = 1 cm air
Helium	5.91 cm He = 1 cm air
Lithium	1.84 mg cm^{-2} = 1 cm air
Beryllium	1.16 mg cm^{-2} = 1 cm air
Boron	1.16 mg cm^{-2} = 1 cm air
Carbon	1.13 mg cm^{-2} = 1 cm air
Nitrogen	1.001 cm N_2 = 1 cm air
Air	
Oxygen	.917 cm O_2 = 1 cm air
Argon	1.010 cm A = 1 cm air
Aluminum	1.48 mg cm^{-2} Al = 1 cm air
Copper	2.02 mg cm^{-2} Cu = 1 cm air
Silver	1.84 mg cm^{-2} Ag = 1 cm air
Lead	3.54 mg cm^{-2} Pb = 1 cm air

In making the computation, it was found convenient to introduce a variable, $u = \frac{E}{Mc^2}$, into eq. (1), where M equals the rest mass of the particle. A table of the various functions which recurred in the calculations is appended.

The range and energy loss values for deuterons were obtained from those for protons by the relations:

$$\left(\frac{dE}{dx}\right)_{deuterons} = \left(\frac{dE}{dx}\right)_{protons} \quad \text{at the same velocity.}$$

$$R_{deuteron}(E) = \frac{Md}{Mp} R_{proton}\left(\frac{Mp}{Md} E\right)$$

The range and energy loss of alpha particles were computed directly at low energies and obtained from the proton values at high energies by the relations:

$$\left(\frac{dE}{dx}\right)_{alpha} = 4\left(\frac{dE}{dx}\right)_{proton} \quad \text{at the same velocity.}$$

$$R_\alpha(E) - R_\alpha(E_0) = \frac{z_p^2 M_\alpha}{z_\alpha^2 M_p}\left[R_p\left(\frac{M_p}{M_\alpha} E\right) - R_p\left(\frac{M_p}{M_\alpha} E_0\right)\right]$$

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(1) Smith, J. H.; Phys. Rev. 71:32 (1947).
(2) Fermi, E.; Phys. Rev. 57:485 (1940)
Halpern, O. and Hall, H.: Phys. Rev. 73:477 (1948)
Kramers, H. A.: Physica 13:401 (1947)
(3) Livingston, M. S. and Bethe, H.: Rev. Mod. Phys. 9:263 (1937)
(4) Bloch, F.: Zeits. f. Phys. 81:363 (1933)
(5) Wilson, R. R.: Phys. Rev. 60:749 (1941)
(6) Honl, H.: Zeits. f. Phys. 84:1 (1933)
(7) Mano, G.: Ann. de Phys. 1:407 (1934)

PROTON RANGES

TABLE I. The relativistic equation for the energy loss of a heavy charged particle going through matter and due to excitation and ionization (corrections due to non-contributing shells, the Fermi effect, Bremstrahlung, etc., are not included) is given by

$$-\frac{dT}{dx} = \frac{4\pi e^4 Z^2 N}{mc^2\left(\frac{u^2+2u}{(u+1)^2}\right)}\left[Z\left(\log_e \frac{2mc^2}{I} + \log_e (u^2+2u) - \frac{u^2+2u}{(1+u)^2}\right)\right]$$

$u = T/aMc^2$, where M is the rest mass of a proton, and aM is the rest mass of the particle in question. T is the relativistic kinetic energy. (For the table below, a = 1).

E in Mev Range-Energy Parameters for Protons.	$\frac{u^2+2u}{(1+u)^2}$	$\log_e(u^2+2u) - \frac{u^2+2u}{(1+u)^2}$	$\log_e 2mc^2 + \log_e(u^2+2u) - \frac{u^2+2u}{(1+u)^2}$
0.0			
1	0.00212839	- 6.1523843	7.6845609
2	0.00425000	- 5.4608265	8.3761185
3	0.00636485	- 5.0569439	8.7800013
4	0.00847297	- 4.7708382	9.0661070
5	0.01057438	- 4.5492642	9.2876810
6	0.01266913	- 4.3685061	9.4684391
7	0.01475723	- 4.2159123	9.6210329
8	0.01683871	- 4.0839314	9.7530138
9	0.01891360	- 3.9676929	9.8692523
10	0.02098193	- 3.8638703	9.9730749
11	0.02304372	- 3.7700914	10.0668538
12	0.02509901	- 3.6846069	10.1523385
13	0.02714782	- 3.6060833	10.2308619
14	0.02919018	- 3.5334886	10.3034566
15	0.03122611	- 3.4660027	10.3709425
16	0.03325565	- 3.4029653	10.4339799
17	0.03527882	- 3.3438351	10.4931101
18	0.03729564	- 3.2881656	10.5487796
19	0.03930615	- 3.2355810	10.6013642
20	0.04131036	- 3.1857645	10.6511807
25	0.05123796	- 2.9699154	10.8670298
30	0.06101215	- 2.7947394	11.0422058
35	0.07063607	- 2.6475956	11.1893496
40	0.08011279	- 2.5209284	11.3160168
45	0.08944529	- 2.4098721	11.4270731
50	0.0986365	- 2.3111034	11.5258418
55	0.1076893	- 2.2222538	11.6146914
60	0.1166030	- 2.1415707	11.6953745
65	0.1253903	- 2.0677364	11.7692088
70	0.1340440	- 1.9997099	11.8372353
75	0.1425699	- 1.9366773	11.9002679
80	0.1509704	- 1.8779803	11.9589649
85	0.1592482	- 1.8230809	12.0138643
90	0.1674054	- 1.7715335	12.0654117
95	0.1754445	- 1.7229659	12.1139793
100	0.1833677	- 1.6770629	12.1598823
125	0.2213216	- 1.4793030	12.3576422
150	0.2566896	- 1.3199357	12.5170095
175	0.2897017	- 1.1865350	12.6504102
200	0.3205624	- 1.0717519	12.7651933
225	0.3494548	- 0.9708911	12.8660541
250	0.3765425	- 0.8807920	12.9561532
275	0.4019731	- 0.7992338	13.0377114
300	0.4258788	- 0.7245645	13.1123807
325	0.4483793	- 0.6556005	13.1813447

E in Mev	$\frac{u^2 + 2u}{(1 + u)^2}$	$\log_e (u^2+2u) - \frac{u^2+2u}{(1+u)^2}$	$\log_e 2mc^2 + \log_e(u^2+2u) - \frac{u^2 + 2u}{(1 + u)^2}$
350	0.4695824	- 0.5914031	13.2455421
375	0.4895862	- 0.5312475	13.3056977
400	0.5084793	- 0.4745588	13.3623864
425	0.5263425	- 0.4208749	13.4160708
450	0.5432495	- 0.3698183	13.4671269
475	0.5592670	- 0.3210793	13.5158659
500	0.5744564	- 0.2743994	13.5625458
550	0.6025710	- 0.1863816	13.6505636
600	0.6279887	- 0.1043906	13.7325546
650	0.6510438	- 0.0274132	13.8095320
700	0.6720268	0.0453165	13.8822617
750	0.6911606	0.1143904	13.9513356
800	0.7086731	0.1802752	14.0172204
850	0.7247372	0.2433457	14.0802909
900	0.7395083	0.3039063	14.1408515
950	0.7531216	0.3622095	14.1991545
1000	0.7656950	0.4184652	14.2554104
1500	0.8519401	0.8979588	14.7349040
2000	0.8980442	1.2776356	15.1145808
2500	0.9255420	1.5946031	15.4315483
3000	0.9432485	1.8674006	15.7043458
3500	0.9553153	2.1070978	15.9440430
4000	0.9639061	2.3209639	16.1579091
4500	0.9702381	2.5140747	16.3510199
5000	0.9750390	2.6901265	16.5270717
5500	0.9787655	2.8518960	16.6888412
6000	0.9817158	3.0015465	16.8384917
6500	0.9840913	3.1407613	16.9777065
7000	0.9860323	3.2709064	17.1078516
7500	0.9876385	3.3930947	17.2300399
8000	0.9889828	3.5082414	17.3451866
8500	0.9901192	3.6171167	17.4540619
9000	0.9910884	3.7203675	17.5573127
9500	0.9919217	3.8185474	17.6554926
10,000	0.9926434	3.9121312	17.7490764

$2m_0c^2 = 1.021666 \times 10^6$ ev.

M_0c^2 for protons $= 938.1747 \times 10^6$ ev.

$\log_e 2m_0c^2 = 13.8369452$

I to be entered in ev.

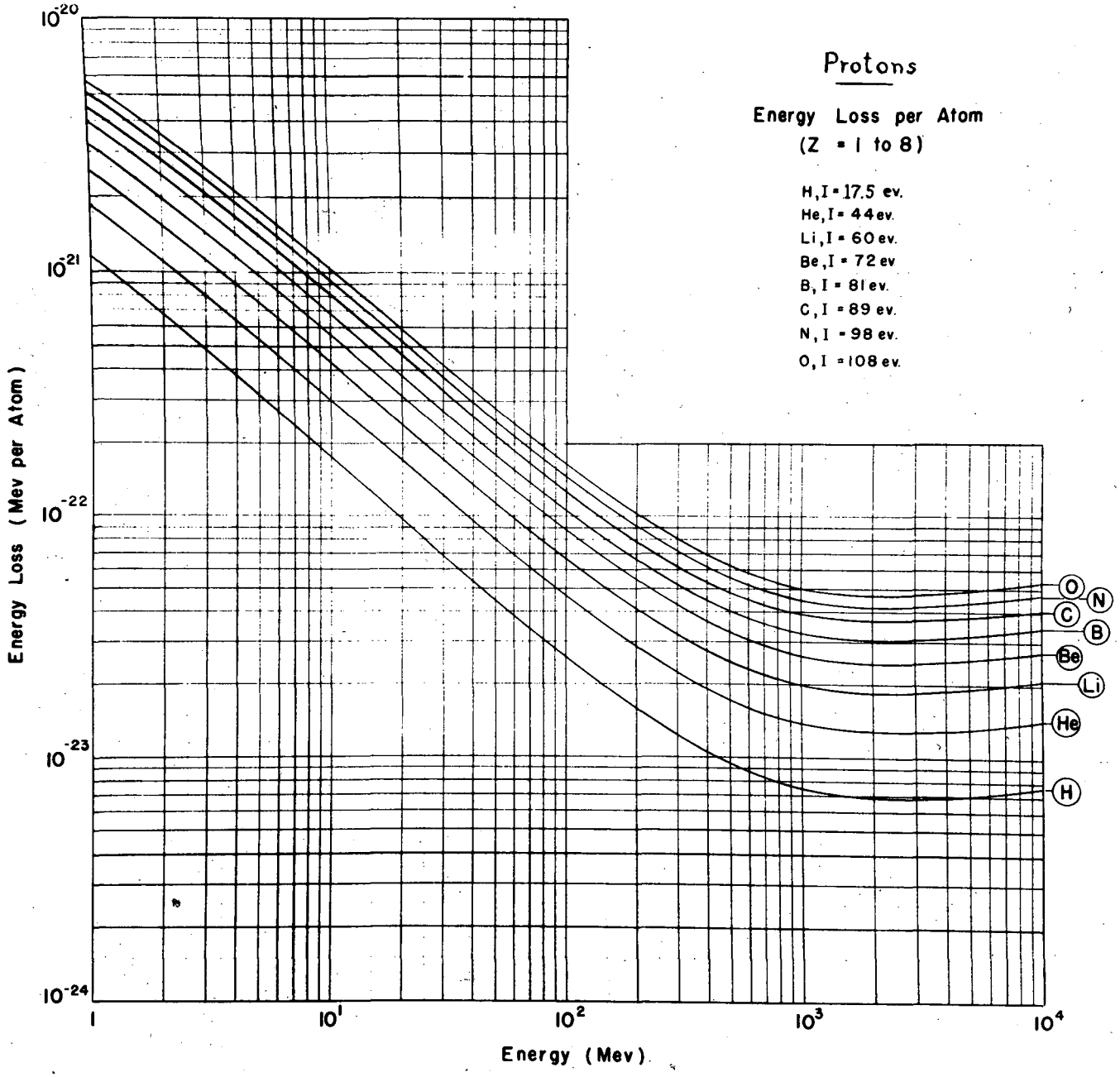
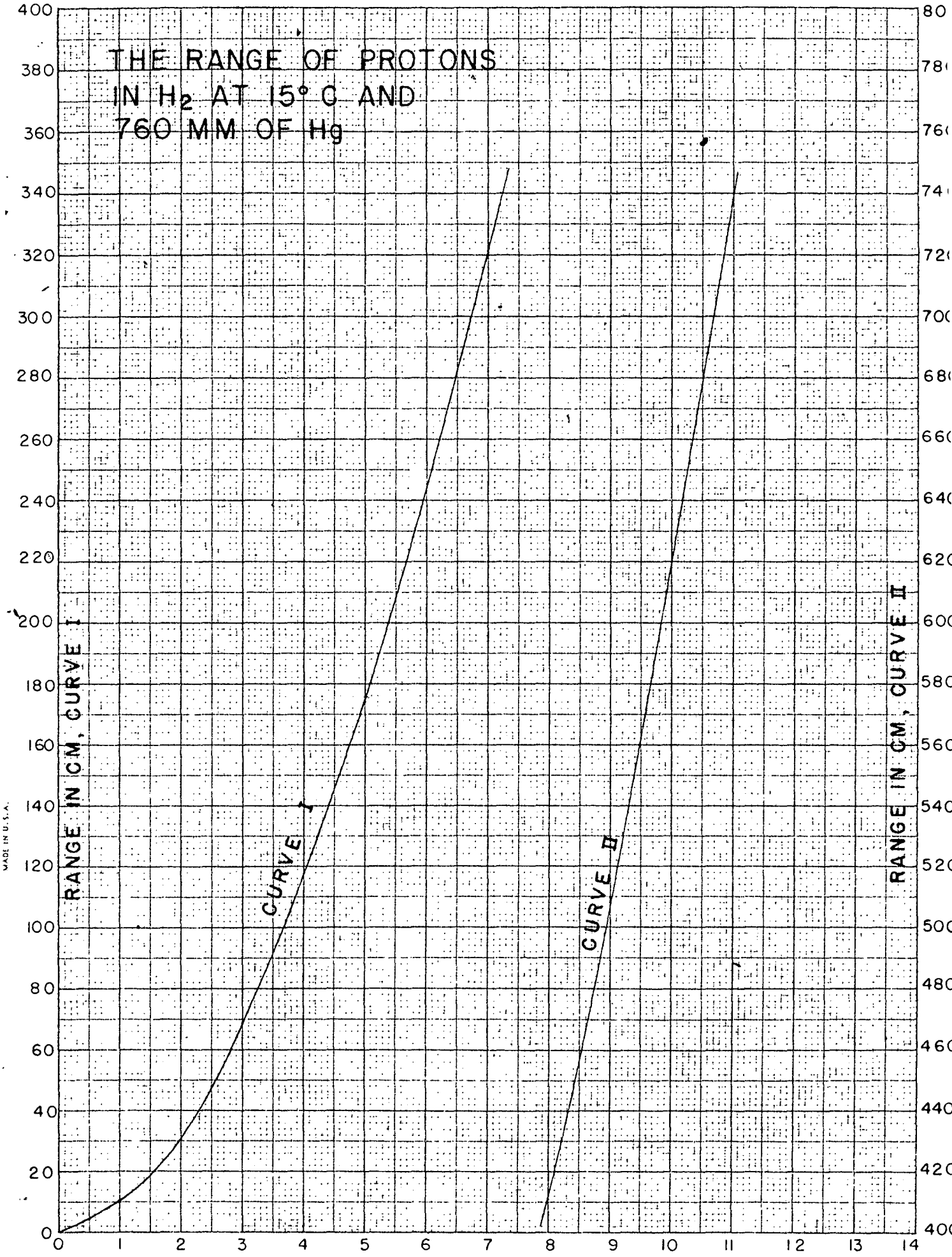

Protons
Energy Loss per Atom
(Z = 1 to 8)
H, I = 17.5 ev.
He, I = 44 ev.
Li, I = 60 ev.
Be, I = 72 ev
B, I = 81 ev.
C, I = 89 ev.
N, I = 98 ev.
O, I = 108 ev.
O
N
C
B
Be
Li
He
H
Energy Loss (Mev per Atom)
Energy (Mev)
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
1
10^{1}
10^{2}
10^{3}
10^{4}

Table II. Ranges of Protons in Hydrogen and Helium (in cm) and Energy Losses (Mev/cm) for ~~0°~~ 15°C and 760 mm. of Hg.

E in Mev	Hydrogen (I = 17.5 ev) Energy Loss $(-\frac{dE}{dx})$	Hydrogen Range	Helium (I = 44 ev) Energy Loss $(-\frac{dE}{dx})$	Helium Range
0	0	0	0	0
1	5.88476×10^{-2}	1.0025×10^{1} cm	4.75985×10^{-2}	1.3030×10^{1} cm
2	3.36971		2.80637	
3	2.41487	6.9061	2.00607	8.4160
4	1.90174		1.61918	
5	1.57824	1.7410×10^{2}	1.35183	2.0778×10^{2}
6	1.35435	2.4270	1.16537	2.8769
7	1.18957	3.2169	1.02733	3.7930
8	1.06288	4.1076	9.20701×10^{-3}	4.8229
9	9.62240×10^{-3}	5.0979	8.35660	5.9645
10	8.80238	6.1855	7.66136	7.2155
11	8.12051		7.08158	
12	7.54400	8.6480	6.59015	1.0039×10^{3}
13	7.04979		6.16793	
14	6.62113	1.1485×10^{3}	5.80097	1.3281
15	6.24557		5.47889	
16	5.91365	1.4687	5.19375	1.6931
17	5.61804		4.93944	
18	5.35301	1.8246	4.71110	2.0980
19	5.11395		4.50487	
20	4.89716	2.2157	4.31764	2.5419
25	4.05773		3.59050	
30	3.48225	4.6776	3.08987	5.3241
35	3.06191		2.72299	
40	2.74078	7.9416	2.44195	8.9943
45	2.48706×10^{-3}		2.21941×10^{-3}	
50	2.28132	1.1961×10^{4}	2.03861	1.3498×10^{4}
55	2.11097		1.88867	
60	1.96757	1.6697	1.76226	1.8791
65	1.84498		1.65406	
70	1.73905	2.2115	1.56046	2.4736
75	1.64654		1.47862	
80	1.56505	2.8188	1.40644	3.1597
85	1.49262		1.34229	
90	1.42789	3.4886	1.28488	3.9046
95	1.36965		1.23320	
100	1.31697	4.2186	1.18642	4.7154
125	1.11433		1.00617	
150	9.76921×10^{-4}	9.0290	8.83659×10^{-4}	9.6737
175	8.77559		7.94925	
200	8.02397	1.4386×10^{5}	7.27717	1.5955×10^{5}
225	7.43533		6.75029	
250	6.96260	2.1104	6.32684	2.3356
275	6.57481		5.97928	
300	6.25129	2.8704	5.68919	3.1712

E in Mev	Hydrogen		Helium	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
325	5.97754		5.44364	
350	5.74314	3.7065	5.23335	4.0893
375	5.54040		5.05144	
400	5.36349	4.6086	4.89271	5.0787
425	5.20796		4.75315	
450	5.02865	5.5697	4.62963	6.1303
475	4.94771×10^{-4}		4.51967×10^{-4}	
500	4.83799	6.5814×10^{5}	4.42128	7.2363×10^{5}
550	4.65020		4.25293	
600	4.49589	8.7302	4.11470	9.5859
650	4.36739		3.99967	
700	4.25914	1.1022×10^{6}	3.90293	1.2085×10^{6}
750	4.16719		3.82084	
800	4.08836	1.3418	3.75057	1.4701
850	4.02034		3.69004	
900	3.96131	1.5904	3.63760	1.7410
950	3.90981		3.59196	
1000	3.86469	1.8461	3.55206	2.0194
1500	3.61964		3.33866	
2000	3.54362	4.5895	3.27707	4.9940
2500	3.52729		3.26866	
3000	3.53620	7.4212	3.28242	8.0499
3500	3.55670		3.30613	
4000	3.58263	1.0232×10^{7}	3.33429	1.1074×10^{7}
4500	3.61094		3.36423	
5000	3.64006	1.3001	3.39457	1.4046
5500	3.68530		3.42457	
6000	3.69769	1.5719	3.45387	1.6967
6500	3.72551		3.48228	
7000	3.75246	1.8403	3.50971	1.9839
7500	3.77849		3.53614	
8000	3.80360×10^{-4}	2.1050×10^{7}	3.56157×10^{-4}	2.2666×10^{7}
8500	3.82779		3.58604	
9000	3.85111	2.3663	3.60960	2.5456
9500	3.87358		3.63227	
10000	3.89525	2.6244	3.65412	2.8209



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Engraving 7 X 10 in.
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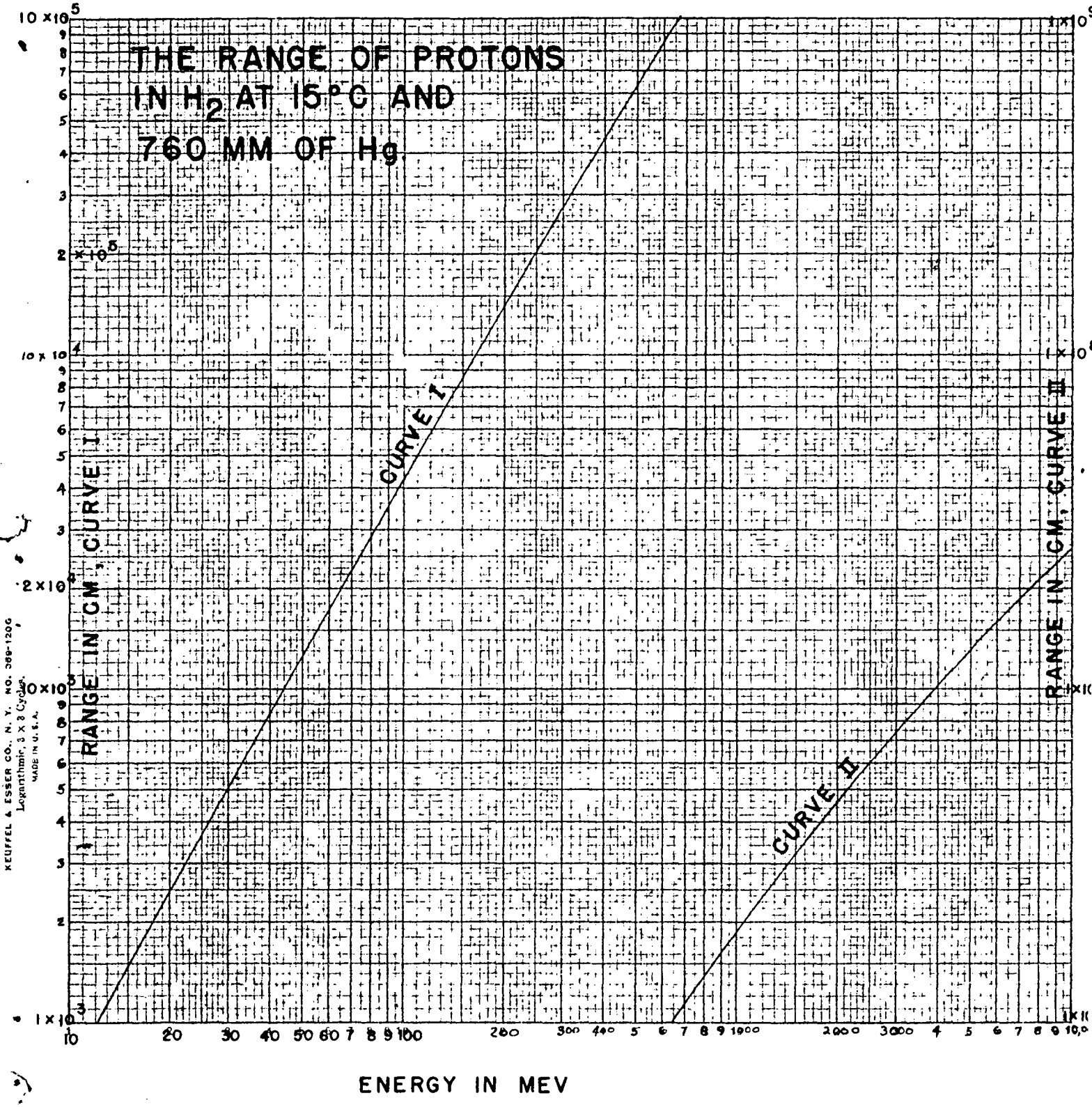
THE RANGE OF PROTONS
IN H_2 AT 15°C AND
760 MM OF Hg.
CURVE I
CURVE II
RANGE IN CM, CURVE I
RANGE IN CM, CURVE II
ENERGY IN MEV
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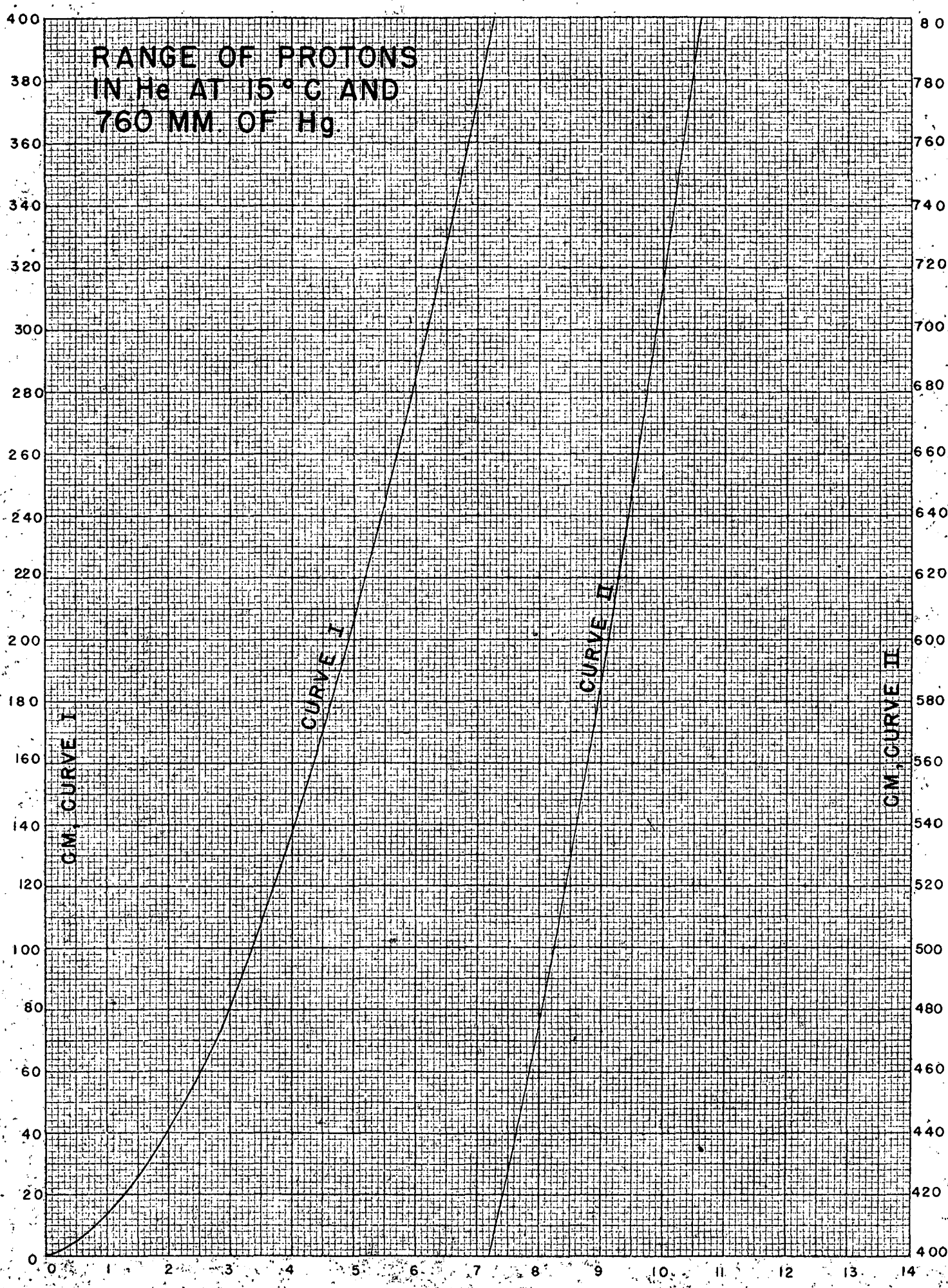

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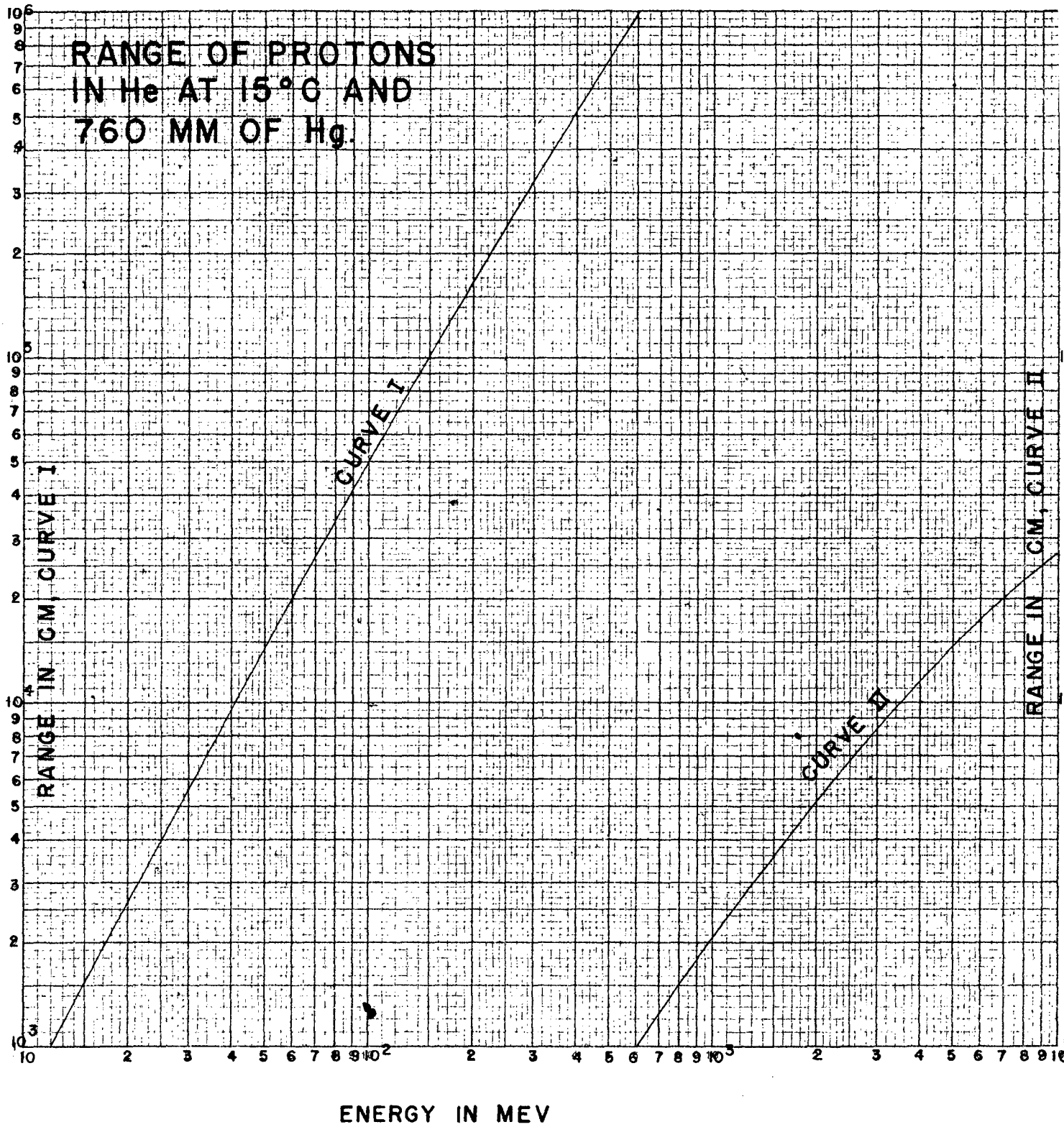
RANGE OF PROTONS
IN He AT 15°C AND
760 MM OF Hg.
CURVE I
CURVE II
RANGE IN CM, CURVE I
RANGE IN CM, CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 X 3 Cycles.
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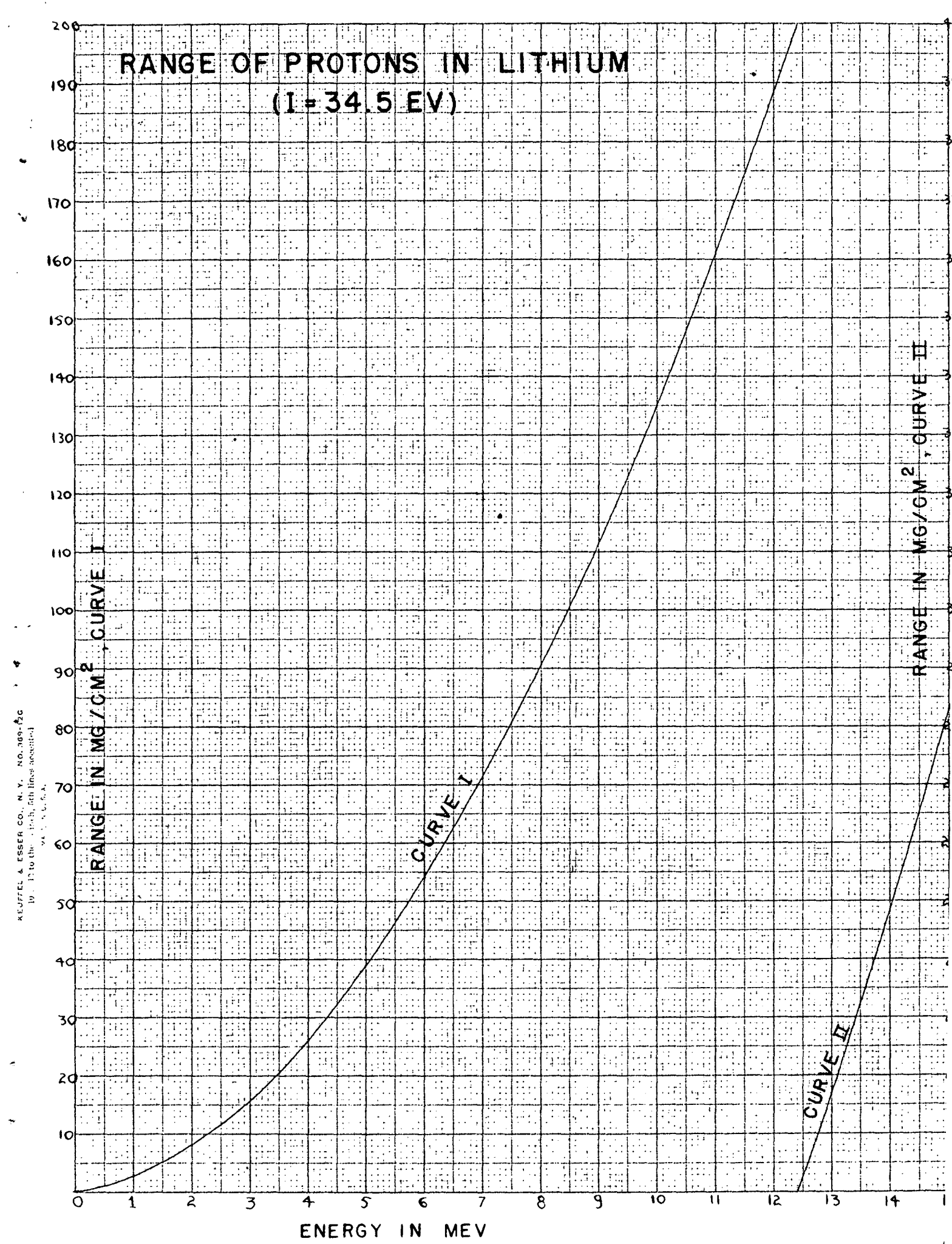
Table III Ranges of Protons in Lithium and Beryllium

(in Mg/cm^2) and Energy Losses (in Mev/mg/cm^2)

E in Mev	Lithium (I = 34.5 ev)		Beryllium (I = 46 ev)	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
0		0.0000		0.0000
1	2.58707×10^{-1}	2.8738	2.46973×10^{-1}	2.9100
2	1.51183	8.1422	1.45866	8.4276
3	1.09382	1.60295×10	1.06050	1.65443×10
4	8.66545×10^{-2}	2.63834	8.42671×10^{-2}	2.71890
5	7.22183	3.90797	7.03775	4.02471
6	6.21736	5.40540	6.06860	5.56018
7	5.47503	7.12294	5.35088	7.31884
8	4.90241	9.05655	4.79629	9.29626
9	4.44626	1.12012×10^{2}	4.35391	1.14873×10^{2}
10	4.07372	1.35535	3.99217	1.38884
11	3.76331	1.61096	3.69045	1.64959
12	3.50041	1.88670	3.43468	1.93067
13	3.27435	2.18225	3.21490	2.23179
14	3.07860	2.49751	3.02387	2.55269
15	2.90660	2.83183	2.85618	2.89312
16	2.75440	3.18540	2.70771	3.25285
17	2.61871	3.55788	2.57528	3.63168
18	2.49694	3.94910	2.45636	4.02940
19	2.38700	4.35880	2.34896	4.44583
20	2.28721	4.78701	2.25143	4.88079
25	1.90004		1.87264	
30	1.63380	1.00445×10^{3}	1.61178	1.02152×10^{3}
35	1.43889		1.42058	
40	1.28969	1.69902	1.27409	1.72503
45	1.17162×10^{-2}		1.15808×10^{-2}	
50	1.07576	2.55219×10^{3}	1.06382	2.58817×10^{3}
55	9.96287×10^{-3}		9.85637×10^{-3}	
60	9.29321	3.55561	9.19723	3.60244
65	8.72019		8.63296	
70	8.22467	4.70211	8.14481	4.76052
75	7.79157		7.71801	
80	7.40969	5.98531	7.34156	6.05595
85	7.07034		7.00694	
90	6.76674	7.39944	6.70749	7.48288
95	6.49347		6.43788	
100	6.24616	8.93925	6.19383	9.03599
125	5.29376		5.25348	
150	4.64686	1.83635×10^{4}	4.61426	1.85324×10^{4}
175	4.17853		4.15124	
200	3.82395	3.03134	3.80050	3.05609
225	3.54605		3.52554	
250	3.32275	4.44008	3.30454	4.47303
275	3.13950		3.12314	
300	2.98657	6.03164	2.97173	6.07293

E in Mev	Lithium (I = 34.5 ev)		Beryllium (I = 46 ev)	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
325	2.85714		2.84357	
350	2.74630	7.78077	2.73381	7.83041
375	2.65042		2.63886	
400	2.56675	9.66654	2.55601	9.72443
425	2.49320		2.48317	
450	2.42807	1.16714×10^5	2.41870	1.17374×10^5
475	2.37013×10^{-3}		2.36131×10^{-3}	
500	2.31826	1.37804×10^5	2.30995	1.38542×10^5
550	2.22951		2.22209	
600	2.15662	1.82624	2.14995	1.83512
650	2.09596		2.08993	
700	2.04490	2.30310	2.03943	2.31336
750	2.00157		1.99659	
800	1.96446	2.80251	1.95993	2.81402
850	1.93249		1.92835	
900	1.90476	3.31983	1.90099	3.33245
950	1.88062		1.87719	
1000	1.85950	3.85146	1.85638	3.86504
1500	1.74605		1.74518	
2000	1.71259	9.53948	1.71322	9.55613
2500	1.70722		1.70900	
3000	1.71361	1.53906×10^6	1.71635	1.54013×10^6
3500	1.72531		1.72887	
4000	1.73941	2.11859	1.74371	2.11847
4500	1.75451		1.75947	
5000	1.76986	2.68852	1.77542	2.68683
5500	1.78645		1.79119	
6000	1.79998	3.24846	1.80659	3.24515
6500	1.81444		1.82151	
7000	1.82840	3.79956	1.83592	3.79418
7500	1.84187×10^{-3}		1.84980×10^{-3}	
8000	1.85484	4.34252×10^6	1.86316	4.33482×10^6
8500	1.86732		1.87601	
9000	1.87934	4.87808	1.88837	4.86790
9500	1.89091		1.90028	
10000	1.90207	5.40695	1.91175	5.39416


RANGE OF PROTONS IN LITHIUM
(I = 34.5 EV)
RANGE IN MG/CM², CURVE I
RANGE IN MG/CM², CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
10
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30
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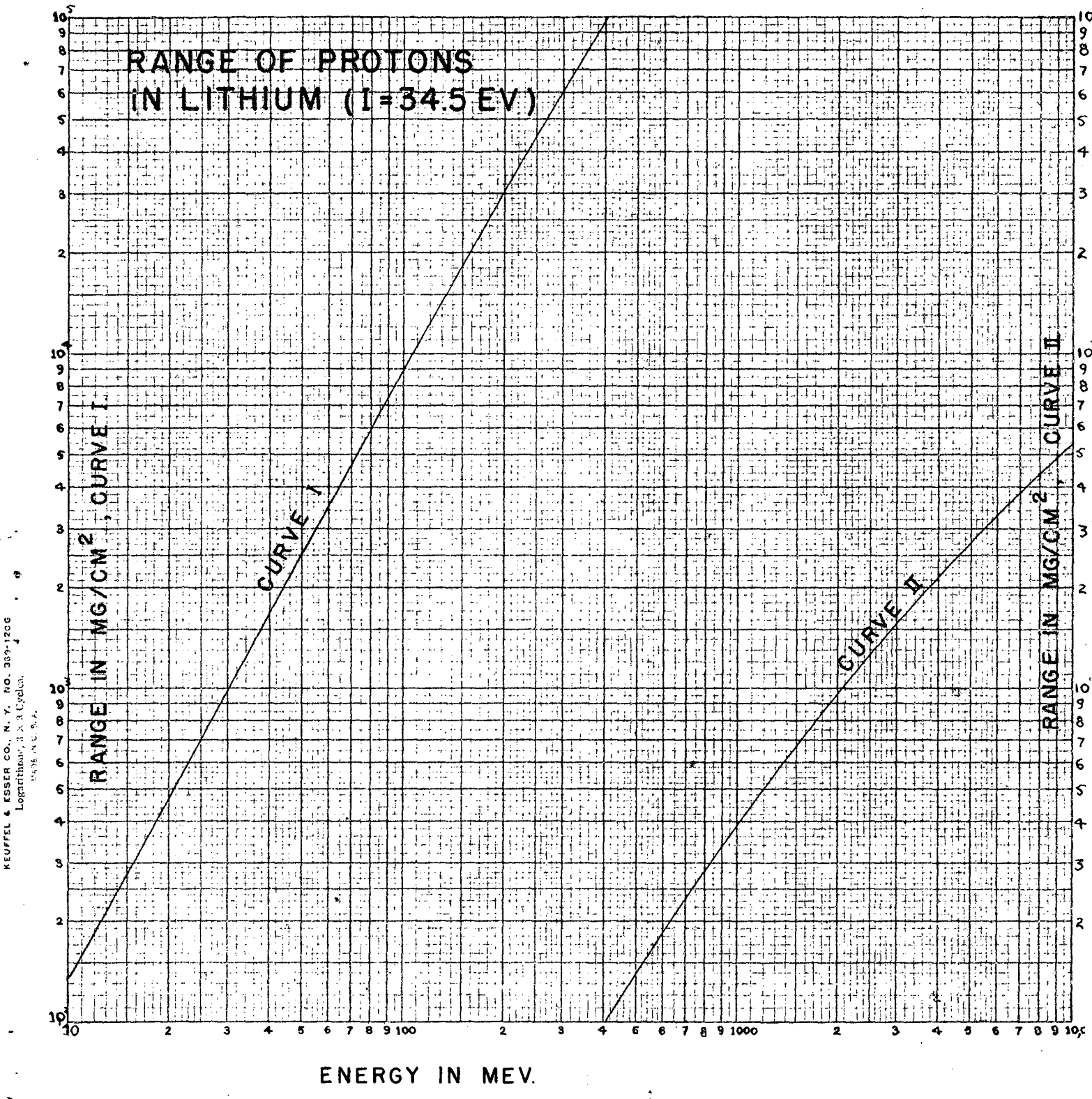
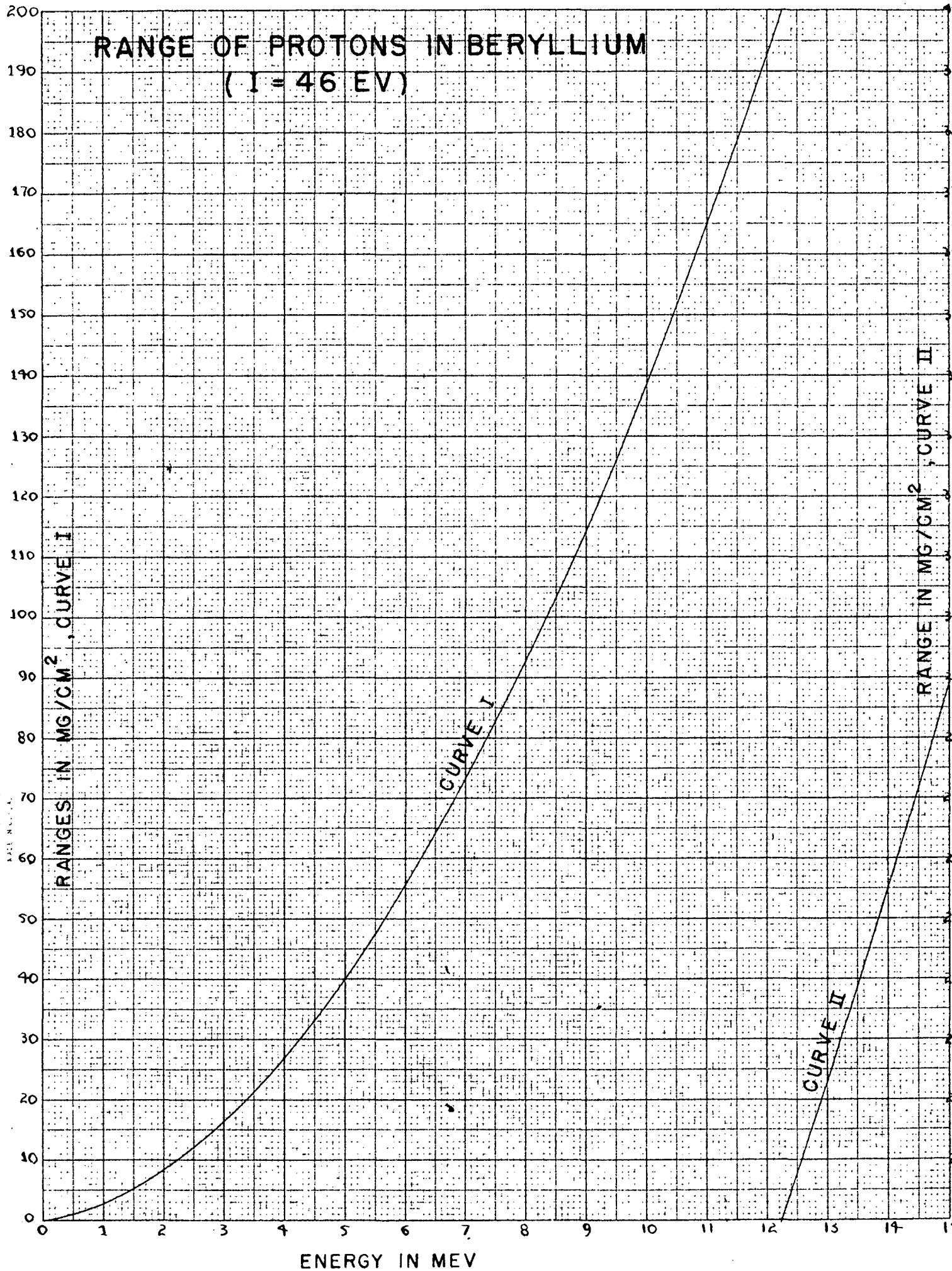
200


RANGE OF PROTONS
IN LITHIUM (I=34.5 EV)
RANGE IN MG/CM², CURVE I
RANGE IN MG/CM², CURVE II
CURVE I
CURVE II
ENERGY IN MEV.
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 x 3 Cycles


RANGE OF PROTONS IN BERYLLIUM
(I = 46 EV)
RANGES IN MG/CM^2, CURVE I
RANGE IN MG/CM^2, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
0
1
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3
4
5
6
7
8
9
10
11
12
13
14

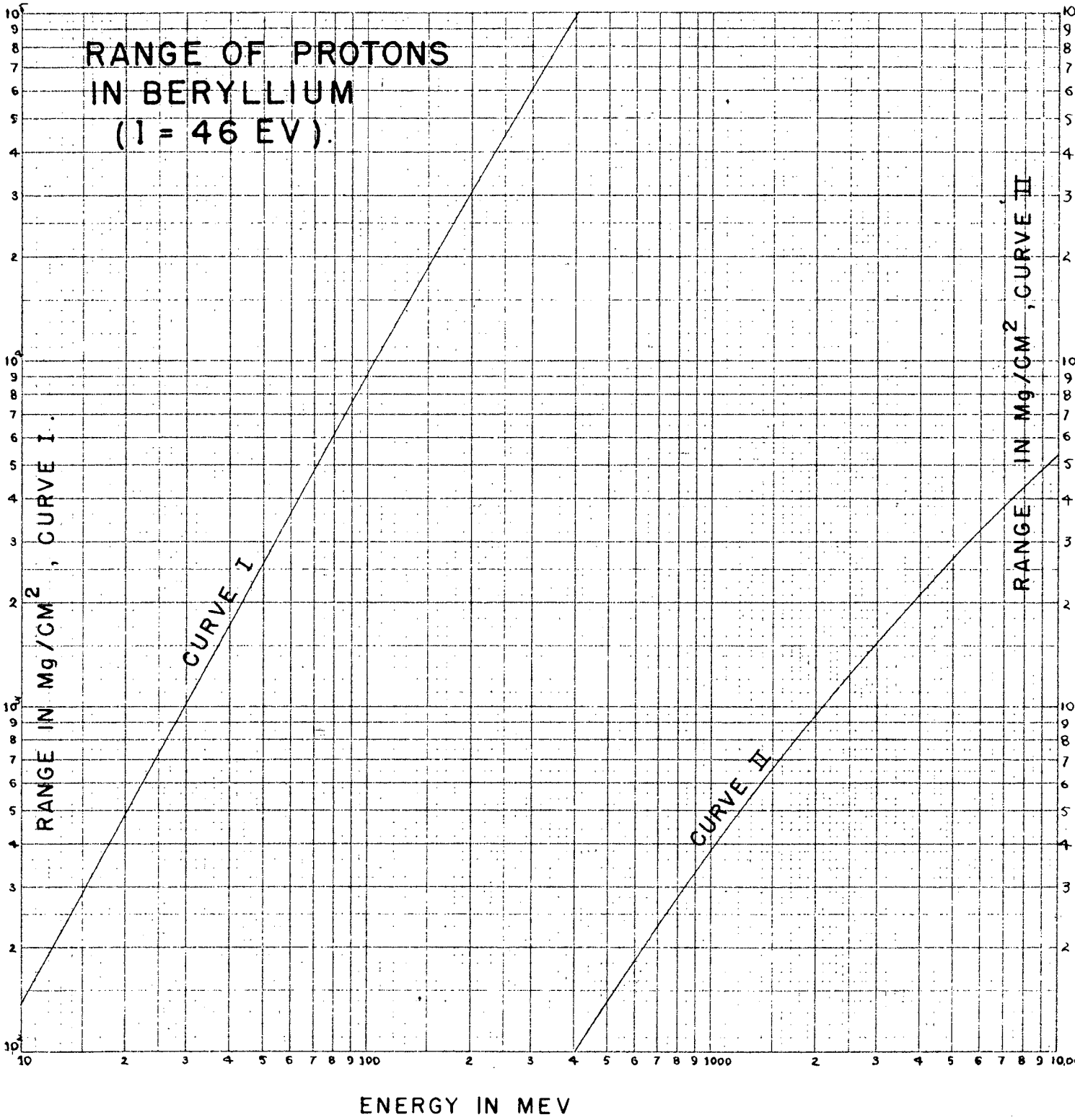
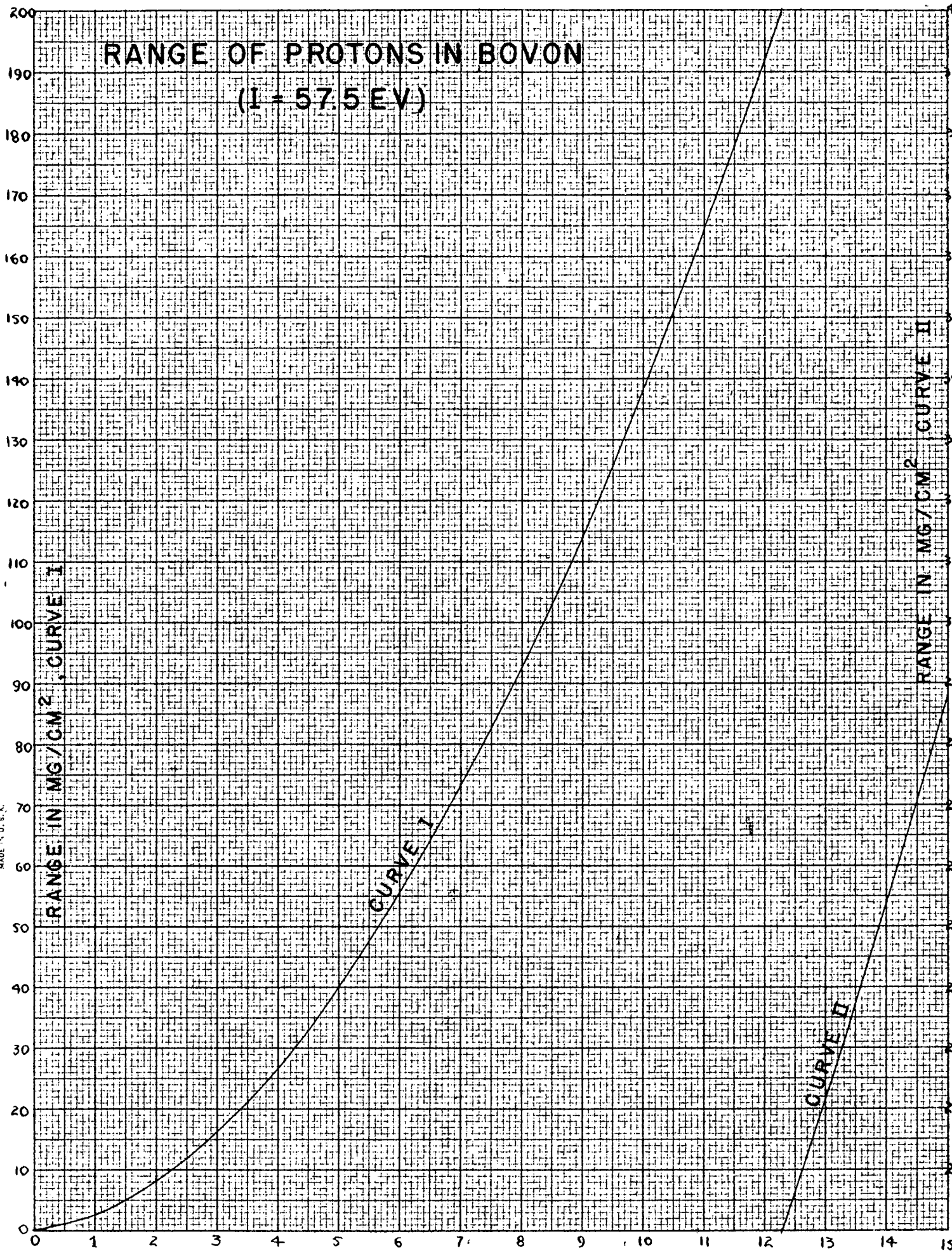

RANGE OF PROTONS
IN BERYLLIUM
(I = 46 EV).
RANGE IN Mg/CM^2, CURVE I.
RANGE IN Mg/CM^2, CURVE II
CURVE I
CURVE II
ENERGY IN MEV

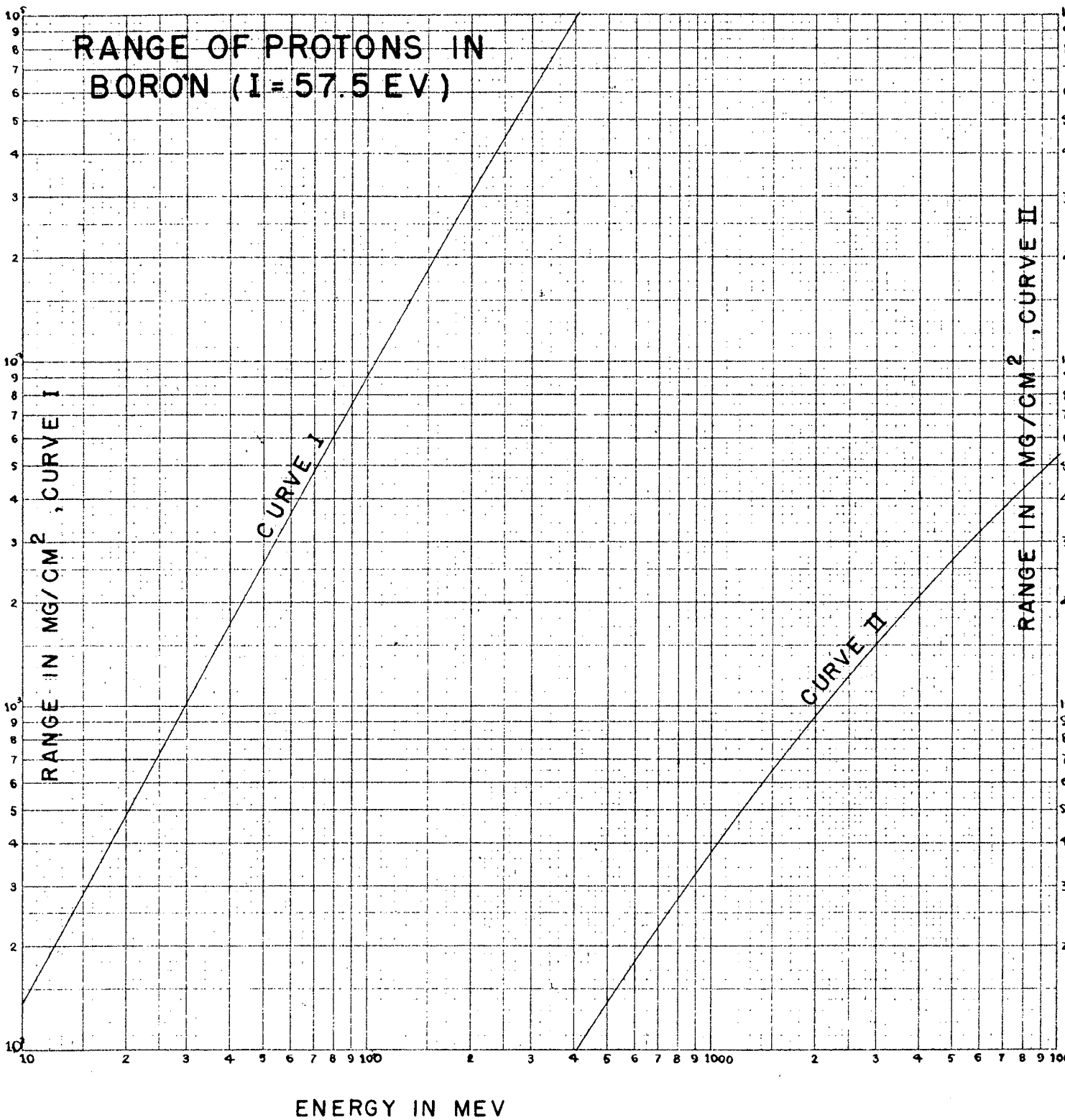
Table IV. Ranges of Protons in Boron and Carbon (in Mg/cm^2) and Energy Losses (in $Mev/mg/cm^2$)

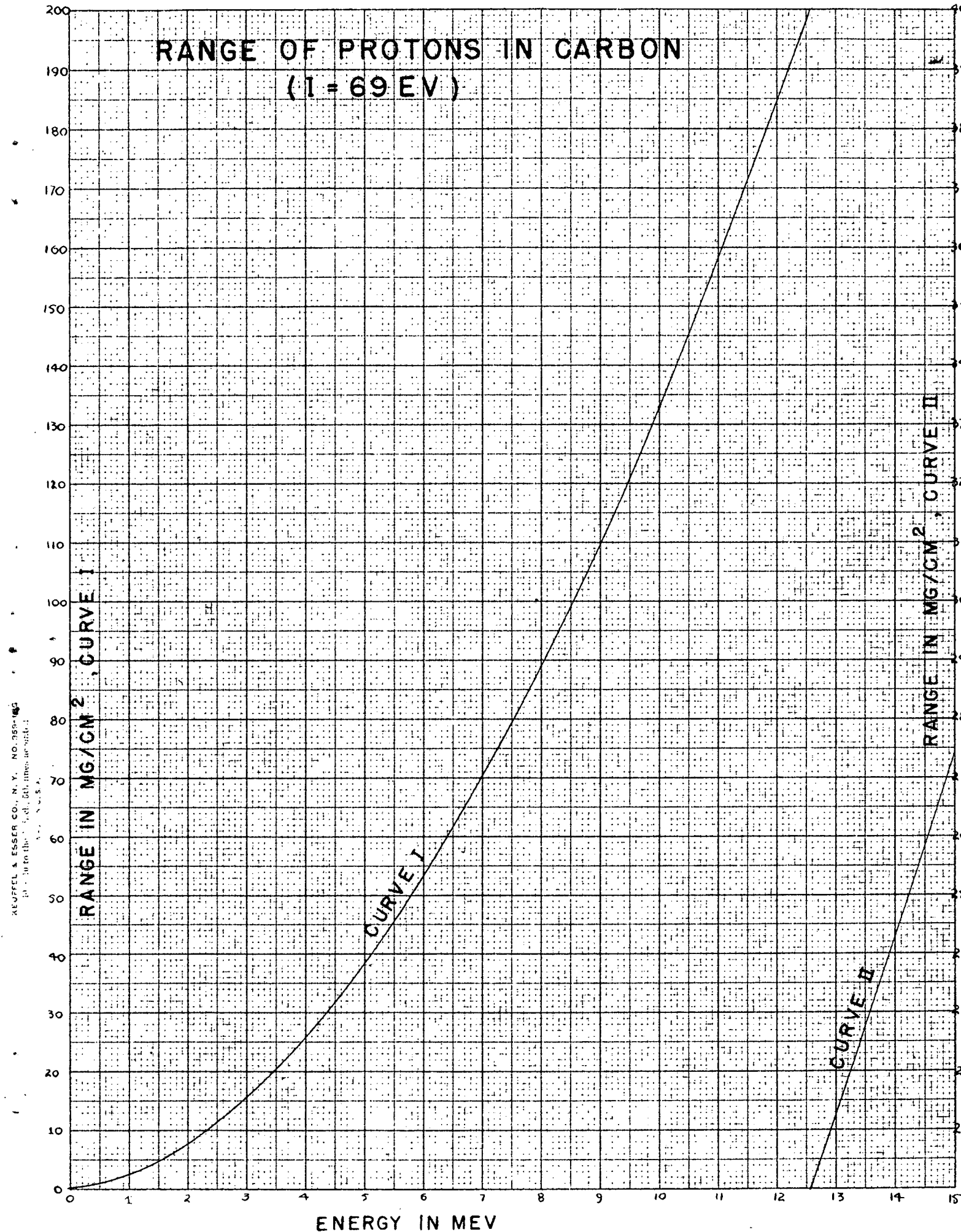
E in Mev	Boron (I = 57.5 ev)		Carbon (I = 69 ev)	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
0		0		0
1	2.42467×10^{-1}	2.890	2.41758×10^{-1}	2.760
2	1.44542	8.4650	1.49676	8.1130
3	1.05529	1.66485×10	1.09761	1.59838×10
4	8.40700×10^{-2}	2.73305	8.75831×10^{-2}	2.62925
5	7.03397	4.04064	7.33961	3.88859
6	6.07363	5.57560	6.34518	5.35789
7	5.36113	7.33162	5.60616	7.03867
8	4.80635	9.30499	5.03354	8.92378
9	4.36942	1.14905×10^{2}	4.57573	1.10106×10^{2}
10	4.00899	1.38826	4.20066	1.32935
11	3.70811×10^{-2}	1.64782	3.88732	
12	3.45285	1.92749	3.62130	1.84374
13	3.23335	2.22697	3.39243	
14	3.04245	2.54597	3.19326	2.43321
15	2.87479	2.88425	3.01826	
16	2.72627	3.24160	2.86317	3.09577
17	2.59374	3.61779	2.72471	
18	2.47468	4.01262	2.60039	3.82973
19	2.36710	4.42591	2.48783	
20	2.26939	4.85748	2.38565	4.63358
25	1.88953		1.98812	
30	1.62761	1.01441×10^{3}	1.71372	9.65798
35	1.43545		1.51222	
40	1.28811×10^{-2}	1.71063×10^{3}	1.35762×10^{-2}	1.62667×10^{3}
45	1.17135		1.23504	
50	1.07642	2.56400	1.13533	2.43603
55	9.97656×10^{-3}		1.05256	
60	9.31220	3.56604	9.82726×10^{-3}	3.38580
65	8.74326		9.22900	
70	8.25089	4.70951	8.71124	4.46908
75	7.82082		8.25809	
80	7.44035	5.98793	7.85829	5.67979
85	7.10259		7.50276	
90	6.80024	7.39565	7.18446	7.01242
95	6.52797		6.89777	
100	6.28146	8.92731	6.63817	8.46197
125	5.33120		5.63703	
150	4.68484	1.82853×10^{4}	4.95568	1.73119×10^{4}
175	4.21641		4.46169	
200	3.86146	3.01278	4.08726	2.85450
225	3.58311		3.79356	
250	3.35934	4.40695	3.55741	4.16715
275	3.17563		3.35463	
300	3.02229	5.98040	3.20164	5.65534

E in Mev	Boron (I = 57.5 ev)		Carbon (I = 69 ev)	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
325	2.89247		3.06459	
350	2.78129	7.70817	2.94721	7.28607
375	2.68510		2.85566	
400	2.60117	9.56958	2.75705	9.03835
425	2.52738		2.67915	
450	2.46208×10^{-3}	1.15473×10^{5}	2.61020×10^{-3}	1.09040×10^{5}
475	2.40394		2.54883	
500	2.35192	1.36267	2.49391	1.28652
550	2.26294		2.39999	
600	2.18989	1.80424	2.32290	1.70288
650	2.12914		2.25880	
700	2.07803	2.27367	2.20489	2.14536
750	2.03470		2.15920	
800	1.99763	2.76500	2.12012	2.60832
850	1.96571		2.08649	
900	1.93808	3.27353	2.05739	3.08746
950	1.91405		2.03209	
1000	1.89306	3.79587	2.01001	3.57945
1500	1.781369		1.89296	
2000	1.74998	9.37126	1.86071	8.82617
2500	1.74663		1.85803	
3000	1.75493	1.50904×10^{6}	1.86757	1.42023×10^{6}
3500	1.76841		1.88251	
4000	1.78417	2.07442	1.89981	1.95134
4500	1.80080		1.91798	
5000	1.81758	2.62974	1.93626	2.47274
5500	1.83414		1.95427	
6000	1.85028	3.17499	1.97432	2.98436
6500	1.86591		1.98878	
7000	1.88099×10^{-3}	3.71096×10^{6}	2.00513×10^{-3}	3.48711×10^{6}
7500	1.89550		2.02087	
8000	1.90947	4.23856	2.03601	3.98199
8500	1.92290		2.05041	
9000	1.93577	4.75864	2.06455	4.46971
9500	1.94825		2.07802	
10000	1.96023	5.27195	2.09099	4.95096



KEUFFEL & ESSER CO., N. Y. NO. 359-120
10 to the ½ inch, 5th lines accented
MADE IN U.S.A.


RANGE OF PROTONS IN
BORON (I = 57.5 EV)
RANGE IN MG/CM^2, CURVE I
RANGE IN MG/CM^2, CURVE II
CURVE I
CURVE II
ENERGY IN MEV


RANGE OF PROTONS IN CARBON
(I = 69 EV)
RANGE IN MG/CM², CURVE I
RANGE IN MG/CM², CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
0
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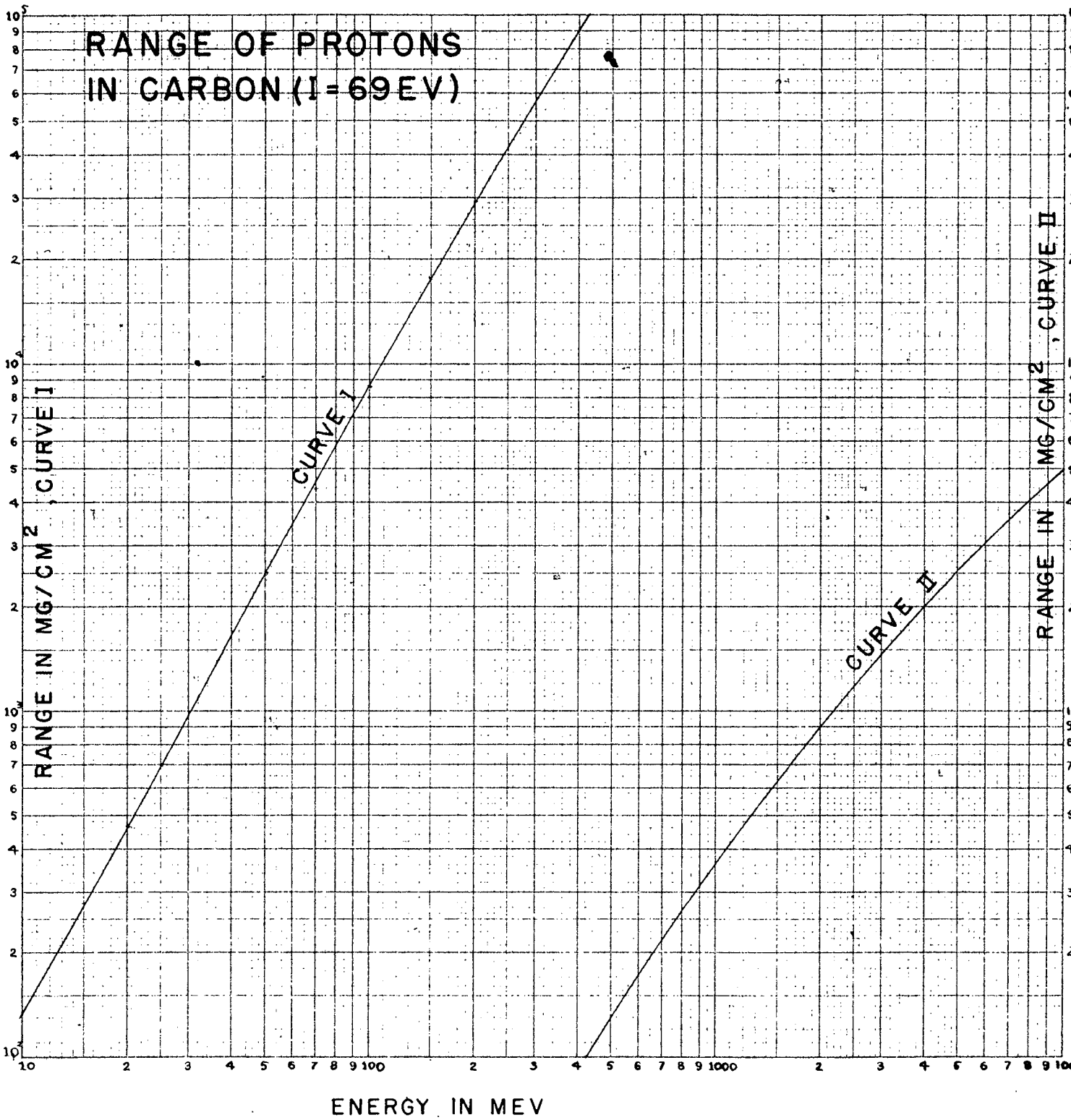
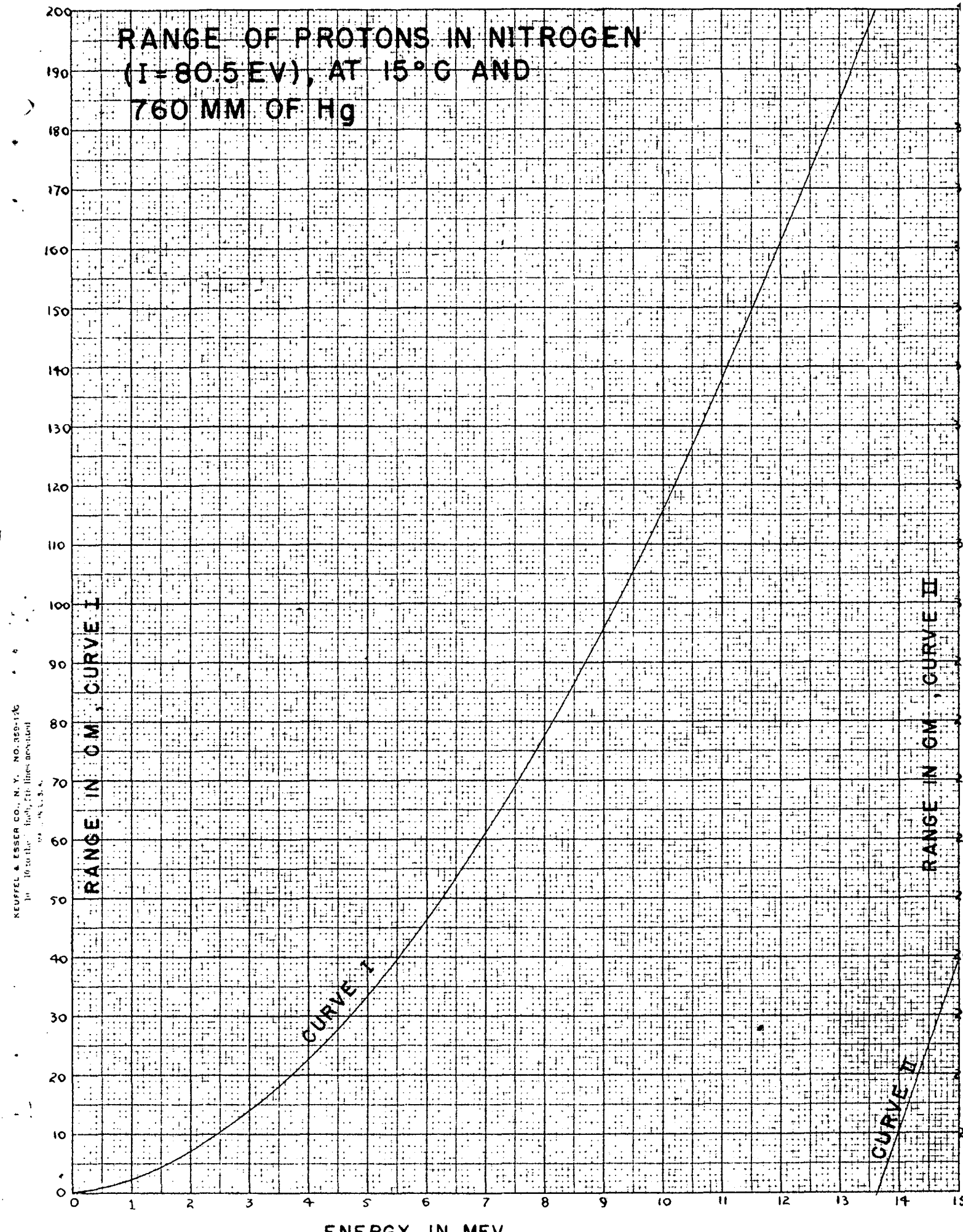


KEUFFEL & ESSER CO., N. Y. NO. 359-60G

Table V. Ranges of Protons in Nitrogen and Oxygen (in cm) and Energy Losses (in Mev/cm) for ~~0°~~ 15° C and 760 mm. of Hg.

E in Mev	Nitrogen (I = 80.5 ev)		Oxygen (I = 92 ev)	
	Energy Loss $(-\frac{dE}{dx})$	Range	Energy Loss $(-\frac{dE}{dx})$	Range
0		0		0
1	2.81579×10^{-1}	2.220	3.08769×10^{-1}	1.990
2	1.70599	7.4883	1.88442	
3	1.25422	1.3877×10	1.39013	1.25434×10
4	1.00377	2.2841	1.11442	2.06241
5	8.42394×10^{-2}	3.3775	9.36501×10^{-2}	3.04653
6	7.29050	4.6577	8.11301	4.19685
7	6.44692	6.1192	7.17990	5.51021
8	5.79250	7.7584	6.45524	6.98123
9	5.26878	9.5707	5.87478	8.60730
10	4.83937	1.15531×10^{2}	5.39848	1.03847×10^{2}
11	4.48037	1.37025	5.00002	1.23112
12	4.17540	1.60162	4.66135	1.43840
13	3.91288	1.84917	4.36967	1.66012
14	3.68432	2.11268	4.11561	1.89604
15	3.48340	2.39194	3.89218	2.14601
16	3.30528	2.68676	3.69403	2.40982
17	3.14674	2.99697	3.51701	2.68733
18	3.00320	3.32232	3.35784	2.97843
19	2.87391	3.66287	3.21389	3.28295
20	2.75641	4.01819	3.08302	3.60069
25	2.29893		2.57320	
30	1.98284	8.36328	2.22063	7.48263
35	1.75056		1.96136	
40	1.57223×10^{-2}	1.40722×10^{3}	1.76220×10^{-2}	1.25780×10^{3}
45	1.43076		1.60414	
50	1.31565	2.10586	1.47547	1.88092
55	1.22005		1.36858	
60	1.13936	2.92525	1.27833	2.61139
65	1.07022		1.20098	
70	1.01036	3.85941	1.13399	3.44384
75	9.57973×10^{-3}		1.07537	
80	9.11738	4.90309	1.02361	4.37358
85	8.70613		9.77534×10^{-3}	
90	8.33789	6.05152	9.36328	5.39637
95	8.00616		8.99176	
100	7.70574	7.30039	8.65525	6.50835
125	6.54676		7.35665	
150	5.75759	1.49208×10^{4}	6.47202	1.32898×10^{4}
175	5.18522		5.83020	
200	4.75126	2.45506	5.34346	2.18543
225	4.41080		4.96152	
250	4.13700	3.58761	4.65432	3.19226
275	3.91216		4.40202	
300	3.72444	4.86483	4.19136	4.32736

E in Mev	Nitrogen (I = 80.5 ev)		Oxygen (I = 92 ev)	
	Energy Loss $\left(-\frac{dE}{dx}\right)$	Range	Energy Loss $\left(-\frac{dE}{dx}\right)$	Range
325	3.56551		4.01299	
350	3.42937	6.26646	3.86019	5.57270
375	3.31159		3.72800	
400	3.20881	7.77573	3.61265	6.95639
425	3.11845		3.50910	
450	3.03849×10^{-3}	9.37860×10^{4}	3.42149×10^{-3}	8.38053×10^{4}
475	2.96731		3.34160	
500	2.90363	1.10632×10^{5}	3.27013	9.87644
550	2.79471		3.14791	
600	2.70533	1.46387	3.04763	1.30508×10^{5}
650	2.63102		2.96427	
700	2.56855	1.84375	2.89420	1.64225
750	2.51561		2.83485	
800	2.47035	2.24112	2.78411	1.99487
850	2.43142		2.74048	
900	2.39774	2.65229	2.70276	2.35970
950	2.36848		2.66999	
1000	2.34294	3.07441	2.64141	2.73416
1500	2.20809		2.49096	
2000	2.17159	7.57273	2.45093	6.72149
2500	2.16934		2.44927	
3000	2.18120	1.21778×10^{6}	2.46339	1.08000×10^{6}
3500	2.19927		2.48441	
4000	2.22001	1.67243	2.50837	1.48244
4500	2.24171		2.53335	
5000	2.26350	2.11853	2.55840	1.87720
5500	2.28493		2.58300	
6000	2.30578	2.55621	2.60691	2.26434
6500	2.32593		2.63001	
7000	2.34534	2.98618	2.65226	2.64462
7500	2.36403×10^{-3}		2.67366×10^{-3}	
8000	2.38198	3.40922×10^{6}	2.69421	3.01867×10^{6}
8500	2.39924		2.71325	
9000	2.41584	3.82604	2.73296	3.38722
9500	2.43180		2.75123	
10000	2.44718	4.23728	2.76882	3.75071


RANGE OF PROTONS IN NITROGEN
(I = 80.5 EV), AT 15° C AND
760 MM OF Hg
RANGE IN CM, CURVE I
RANGE IN CM, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
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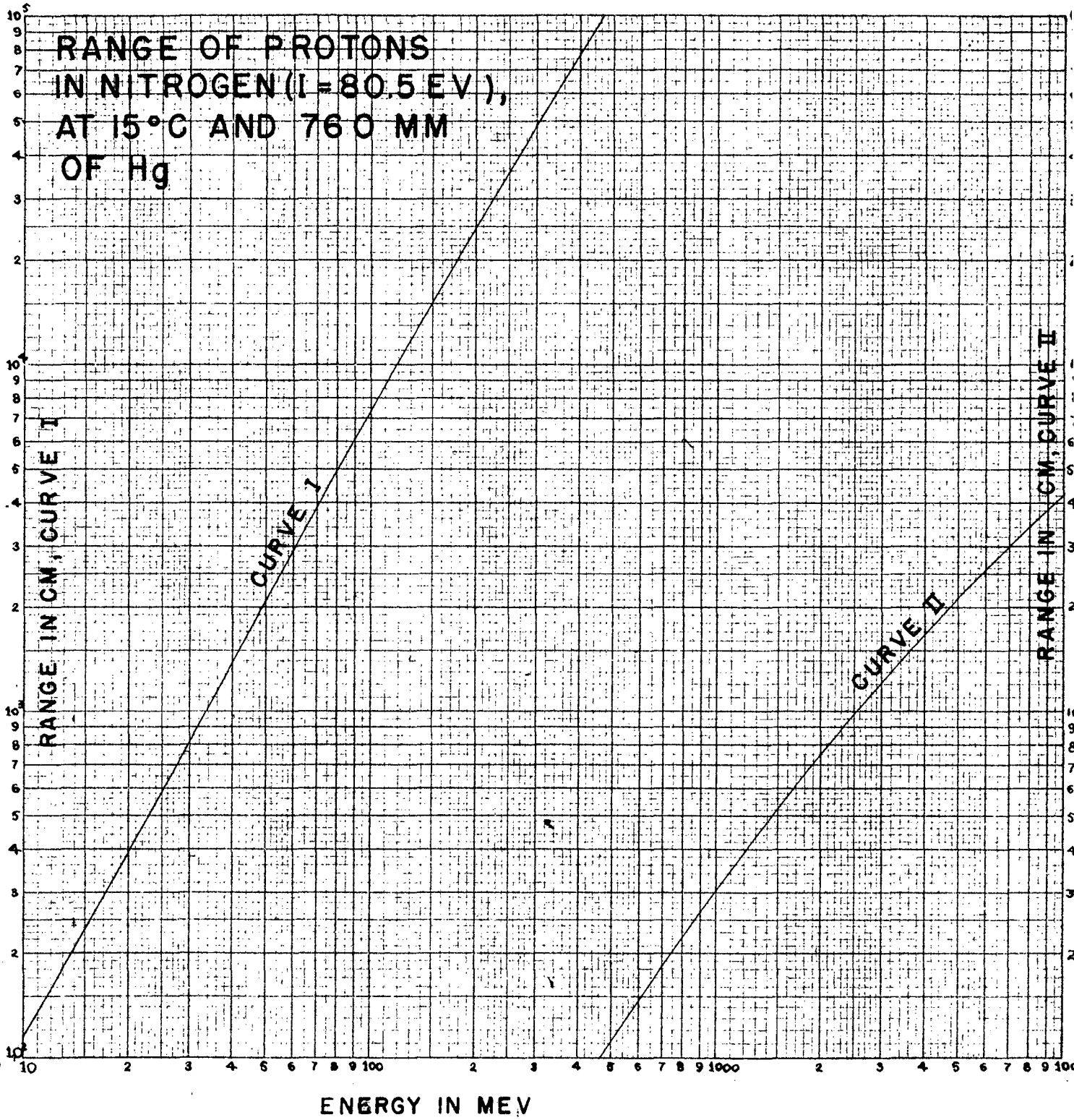
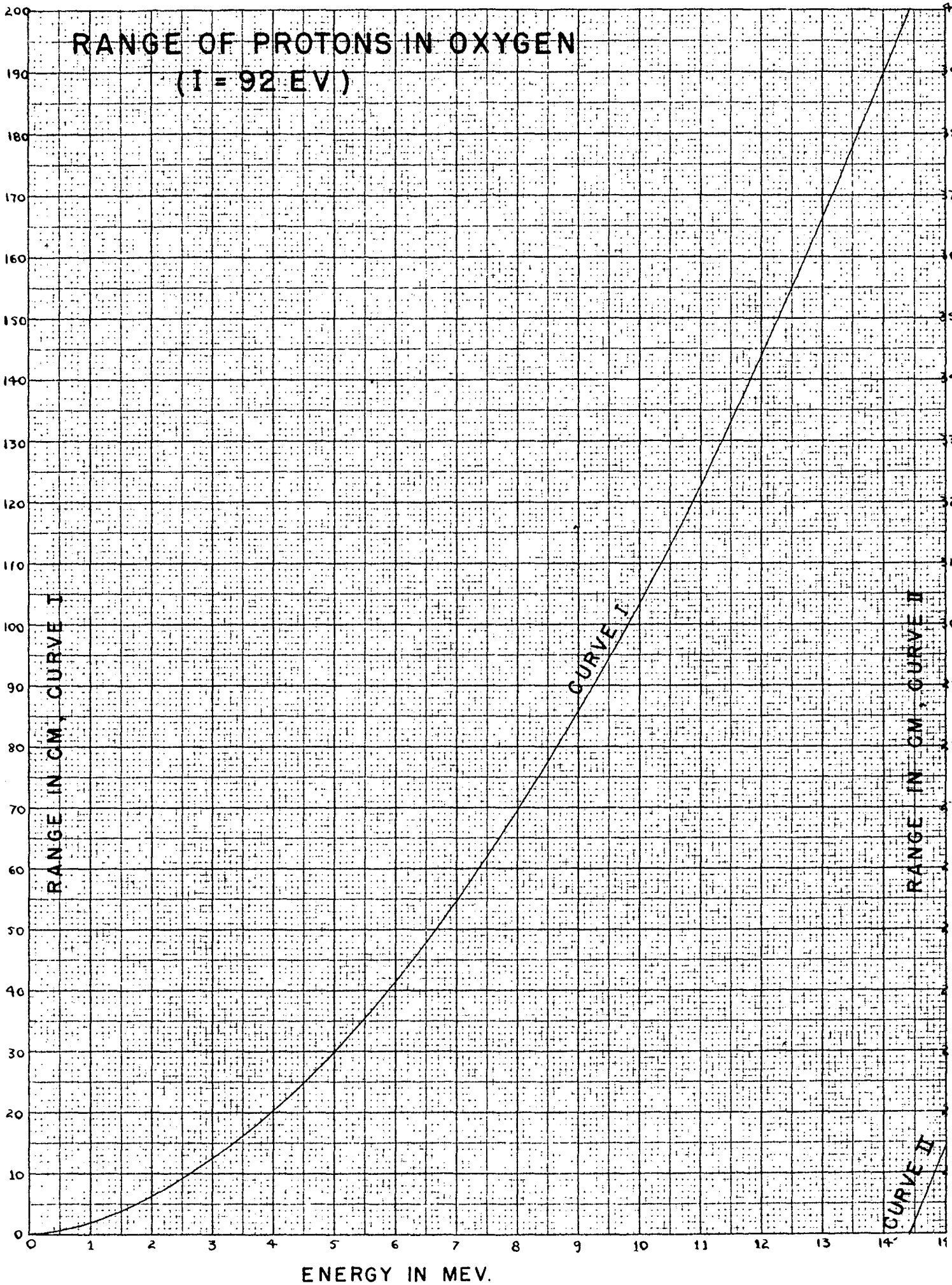
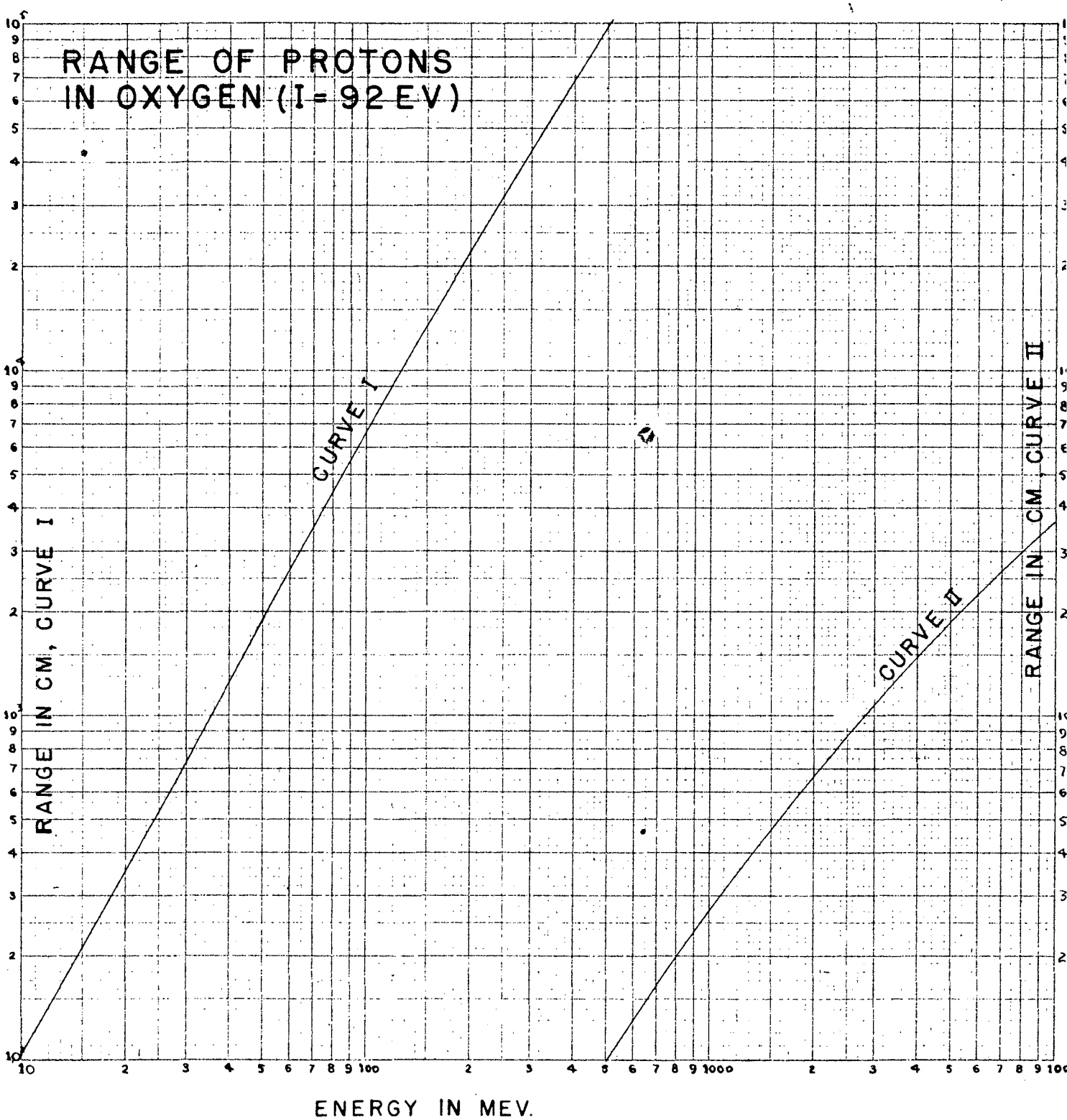


KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 x 3 Cycles


RANGE OF PROTONS IN OXYGEN
(I = 92 EV)
RANGE IN CM. CURVE I
RANGE IN CM. CURVE II
CURVE I
CURVE II
ENERGY IN MEV.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
1
2
3
4
5
6
7
8
9
10
11
12
13
14


RANGE OF PROTONS
IN OXYGEN (I = 92 EV)
CURVE I
CURVE II
RANGE IN CM, CURVE I
RANGE IN CM, CURVE II
ENERGY IN MEV.

Range of Protons in Air

The curves for the range of protons in air which follow have been drawn from data found in Parkinson, Herb, Bellany, and Hudson[1] ($0 < E < 1.5$ Mev), Livingston and Bethe[2] ($1.5 < E < 15$ Mev), and Smith[3] ($15 < E < 10.000$ Mev) and give the range due to energy loss by collision effects only.

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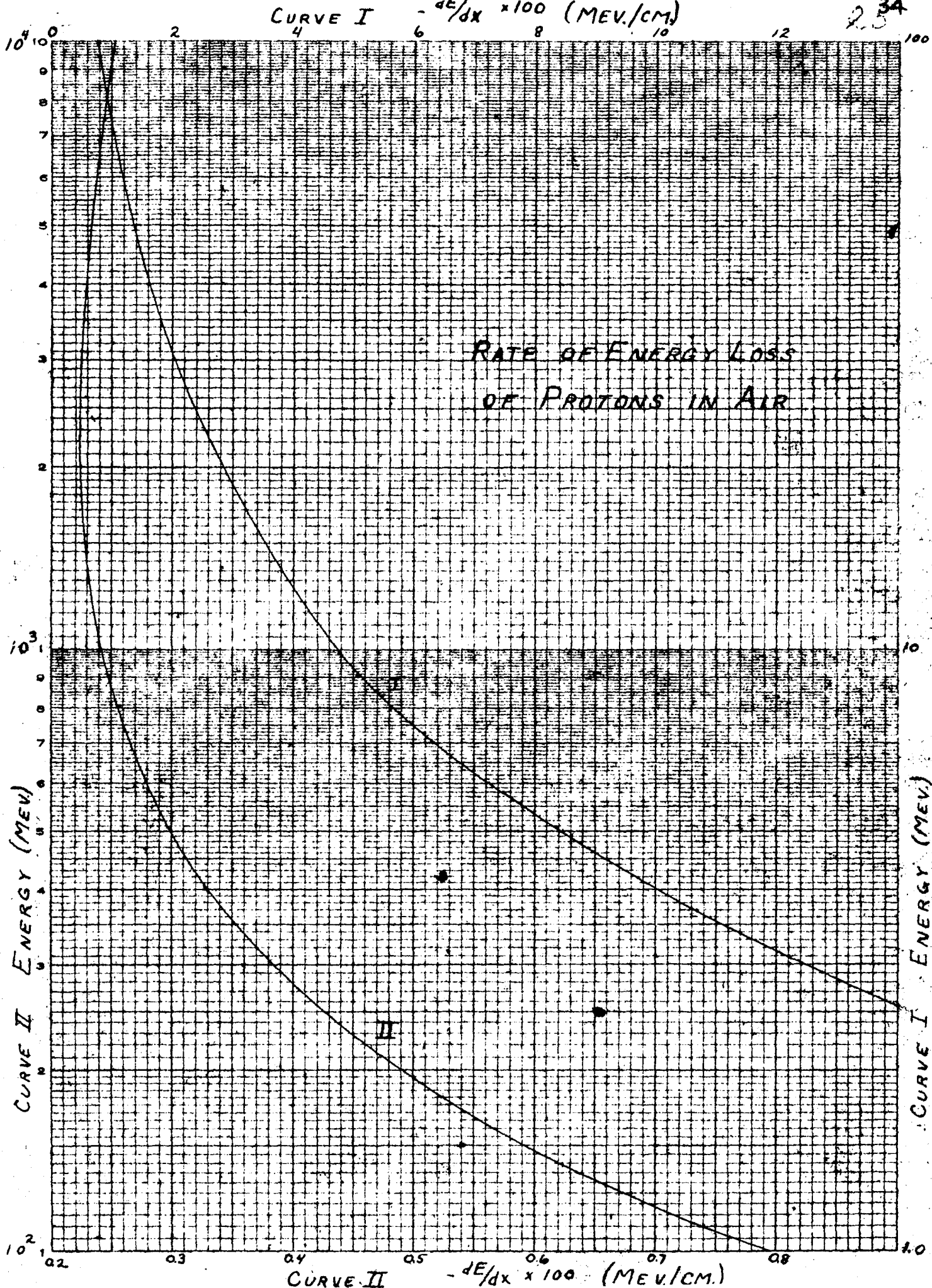
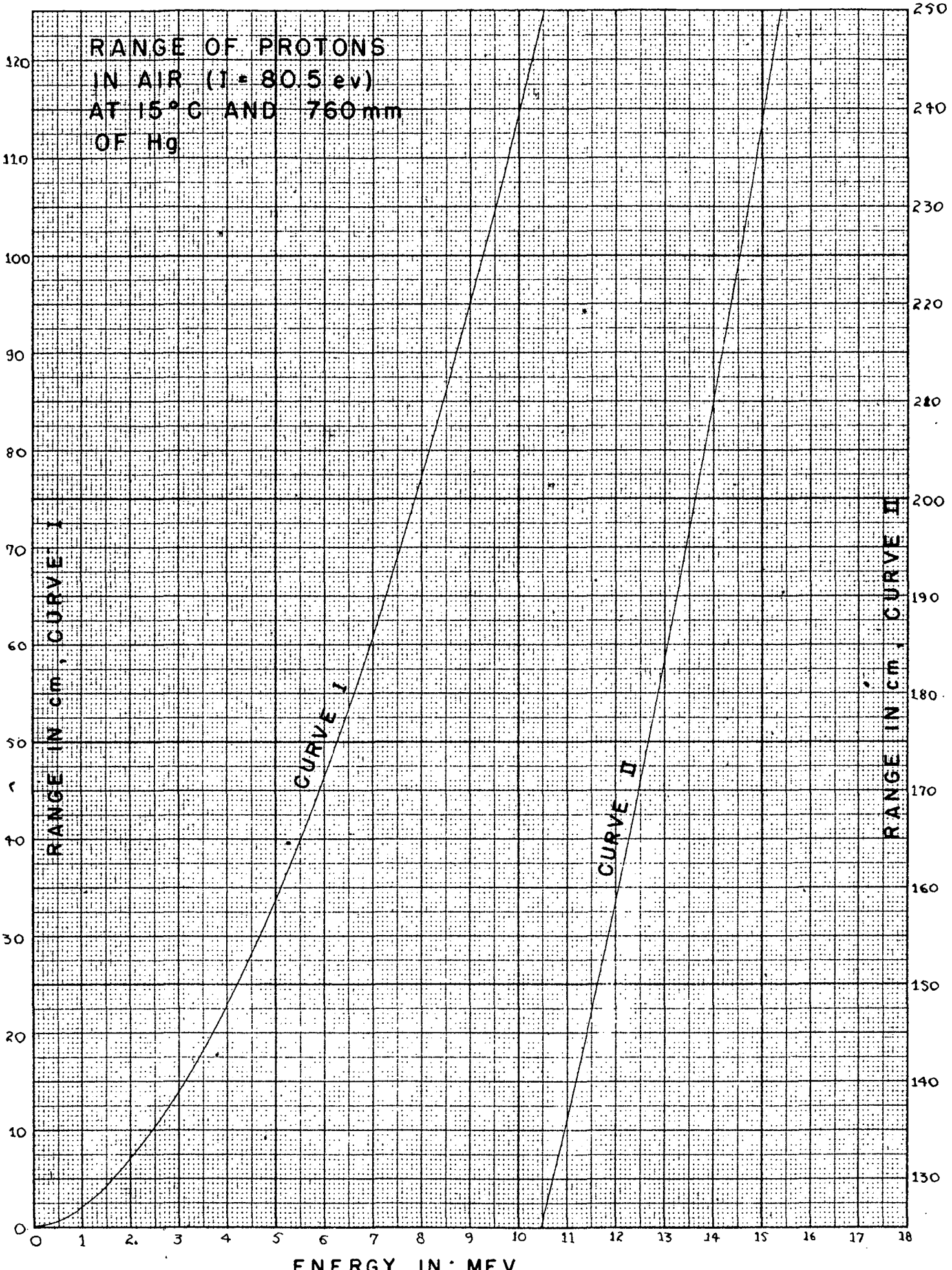
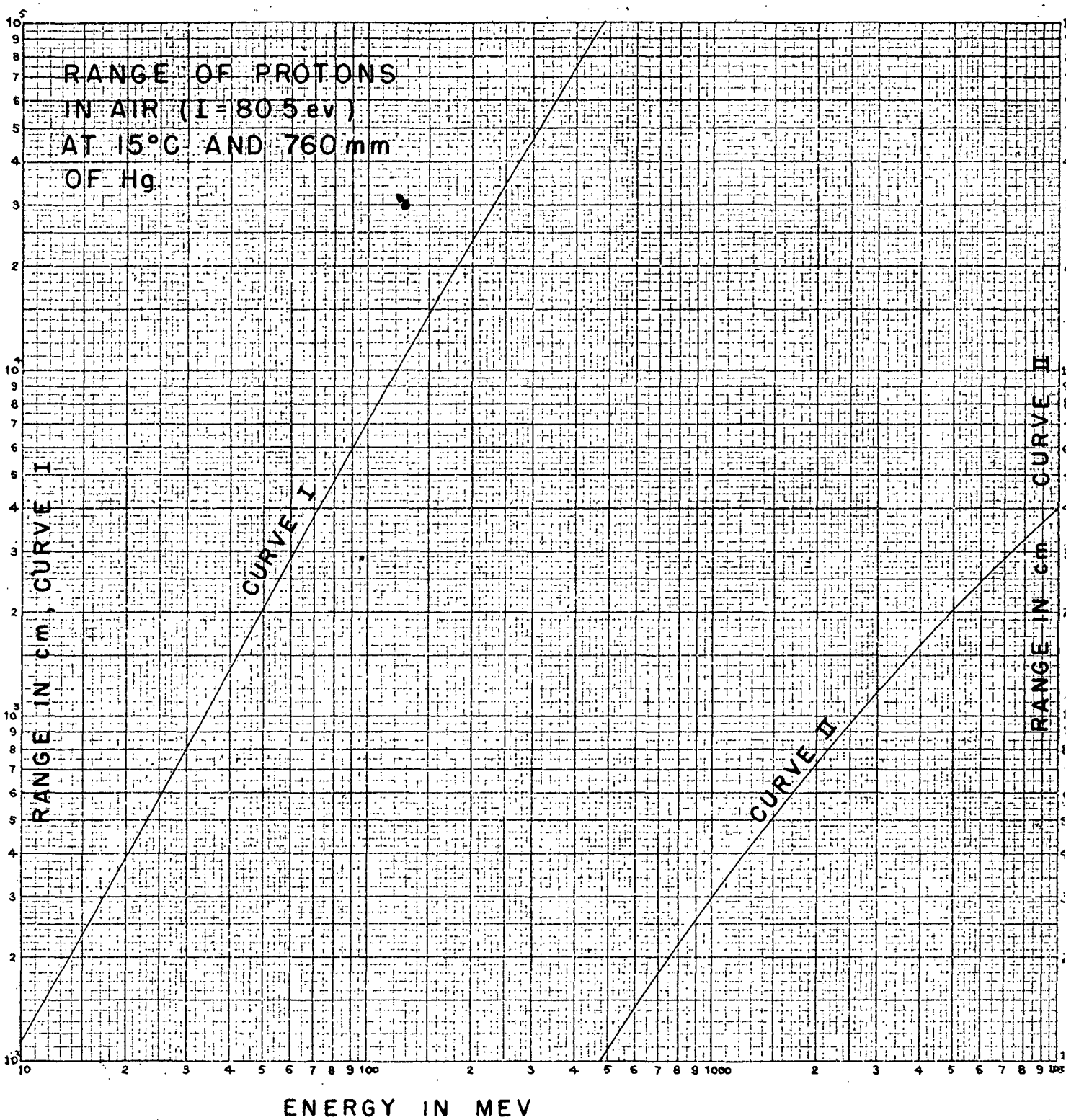
[1] Parkinson, D. B.; Herb, R. G.; Bellany, J. C.; Hudson, C. M.: The Range of Protons in Aluminum and in Air; Phys. Rev. 52:75 (1947).

[2] Livingston, M. S. and Bethe, H.: Nuclear Physics: Part C, Nuclear Dynamics, Experimental; Rev. Mod. Phys. 9:269 (1937).

[3] Smith, J. H.: Theoretical Range-Energy Values for Protons in Aluminum and Air; Phys. Rev. 71:32 (1947).


CURVE I -dE/dx x100 (MEV./CM.)
RATE OF ENERGY LOSS
OF PROTONS IN AIR
CURVE II ENERGY (MEV.)
CURVE I ENERGY (MEV.)
CURVE II -dE/dx x 100 (MEV./CM.)
I
II
EUGENE DIETZGEN CO.
MADE IN U.S.A.
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF PROTONS
IN AIR (I = 80.5 ev)
AT 15°C AND 760 mm
OF Hg
CURVE I
CURVE II
RANGE IN cm. CURVE I
RANGE IN cm. CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250

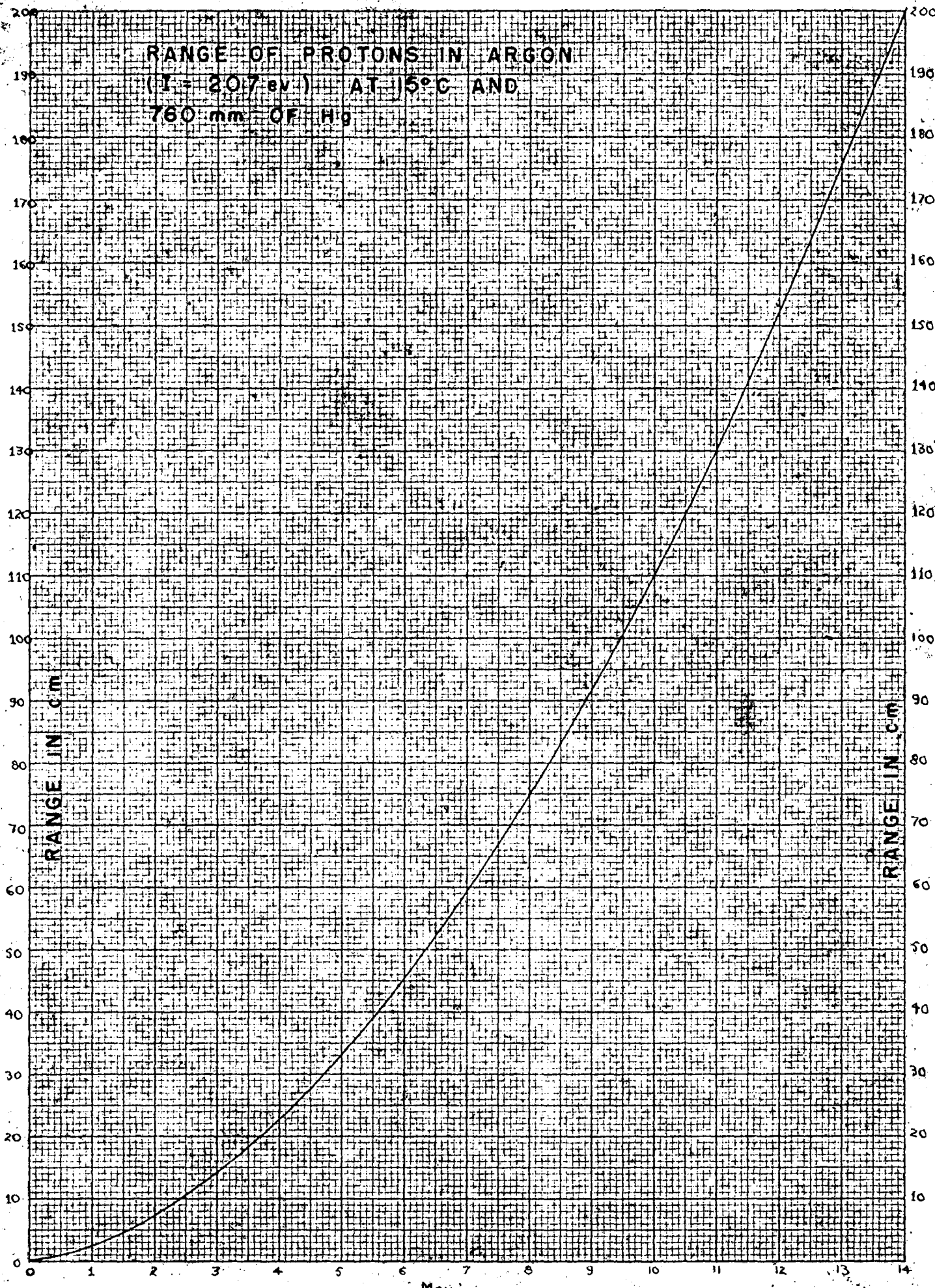


KEUFFEL & ESSER CO., N. Y. NO. 359-120
Logarithmic, 3 Cycles.
MADE IN U. S. A.

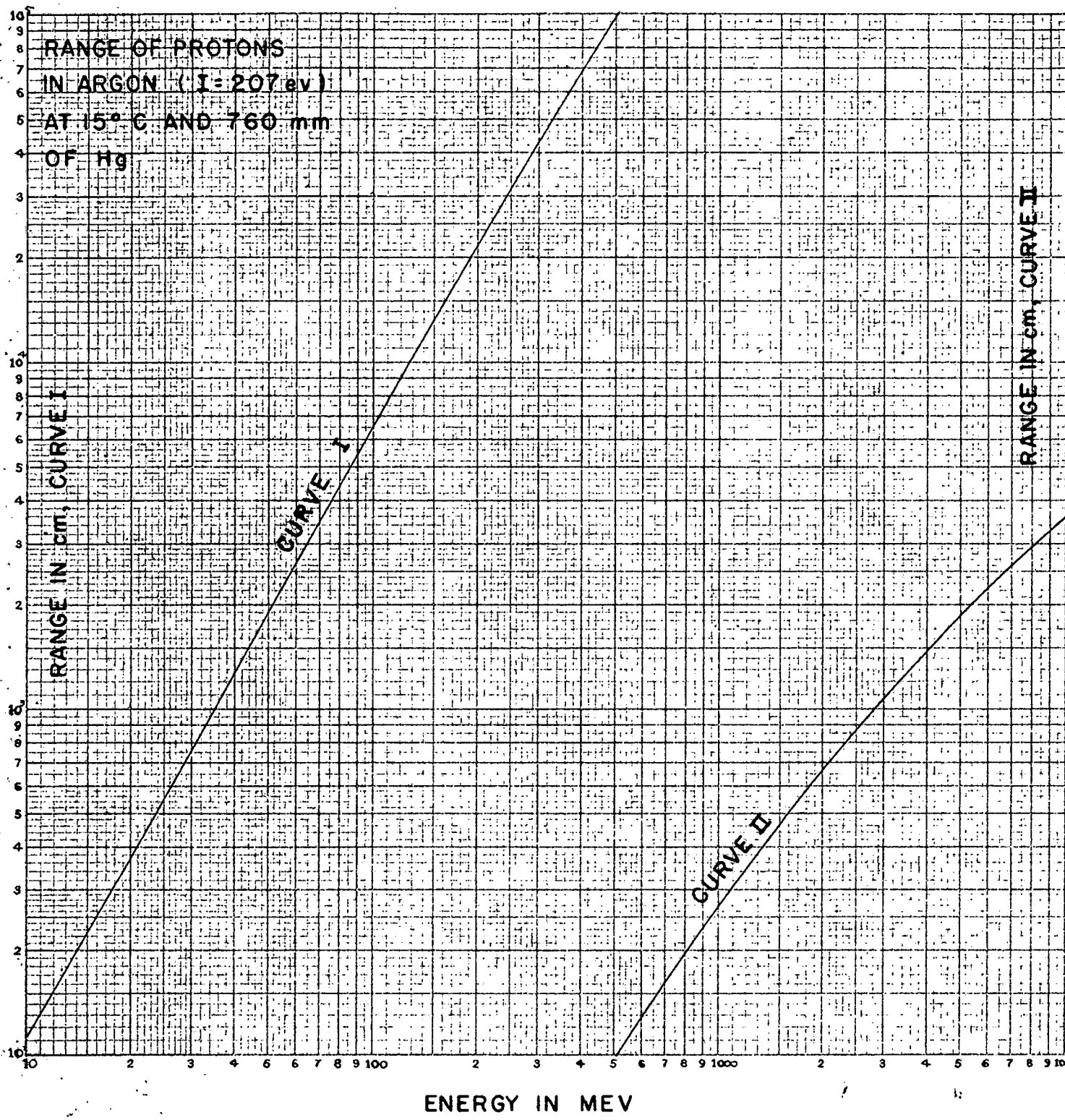
Table VI. Ranges of Protons in Argon (in cm.)
and Energy Losses (in Mev cm^{-1})
at 15°C and 760 mm of Hg.

E in Mev	Argon (I = 207 ev) Energy Loss	Argon (I = 207 ev) Range
0		0
1	2.58308×10^{-1}	
2	1.67398	7.2727 cm.
3	1.26611	1.41780×10
4	1.03003	2.30311
5	8.74316×10^{-2}	3.36041
6	7.63107	4.58858
7	6.79302	5.97995
8	6.13654	7.53138
9	5.60701	9.23804
10	5.16996	1.10973×10^{2}
11	4.80252	1.31057
12	4.48887	1.52610
13	4.21772	1.75604
14	3.98075	2.00022
15	3.77173	2.26051
16	3.58586	2.53042
17	3.41940	2.81609
18	3.26938	3.11526
19	3.13343	3.42778
20	3.00960	3.75350
25	2.52495	
30	2.18757	7.70948
35	1.93822	
40	1.74590	1.28656×10^{3}
45	1.59276	
50	1.46775	1.91413
55	1.36366	
60	1.27559	2.64722
65	1.19996	
70	1.13435	3.48038
75	1.07685	
80	1.02602	4.40884
85	9.80748×10^{-3}	
90	9.40157	5.42830
95	9.03549	
100	8.70360	6.53490
125	7.41992	
150	6.54270	1.32585×10^{4}
175	5.90479	
200	5.42004	2.17148
225	5.03939	
250	4.73280	3.16276
275	4.48081	
300	4.27028	4.27790

E in Mev	Argon (I = 207 ev) Energy Loss	Argon (I = 207 ev) Range
325	4.09194	
350	3.93914	5.49921
375	3.80691	
400	3.69152	6.81210
425	3.59008	
450	3.50032	8.20440
475	3.42044	
500	3.34900	9.66584
550	3.22689	
600	3.12680	1.27625×10^{5}
650	3.04371	
700	2.97401	1.60463
750	2.91501	
800	2.86471	1.94758
850	2.82155	
900	2.78434	2.30186
950	2.75211	
1000	2.72409	2.66514
1500	2.57989	
2000	2.54627	6.51561
2500	2.55068	
3000	2.57041	1.04323×10^{6}
3500	2.59659	
4000	2.62532	1.42830
4500	2.65471	
5000	2.68385	1.80600
5500	2.71227	
6000	2.73976	2.17373
6500	2.76621	
7000	2.79162	2.53527
7500	2.81600	
8000	2.83939	2.89042
8500	2.86183	
9000	2.88339	3.23987
9500	2.90411	
10000	2.92403	3.58423



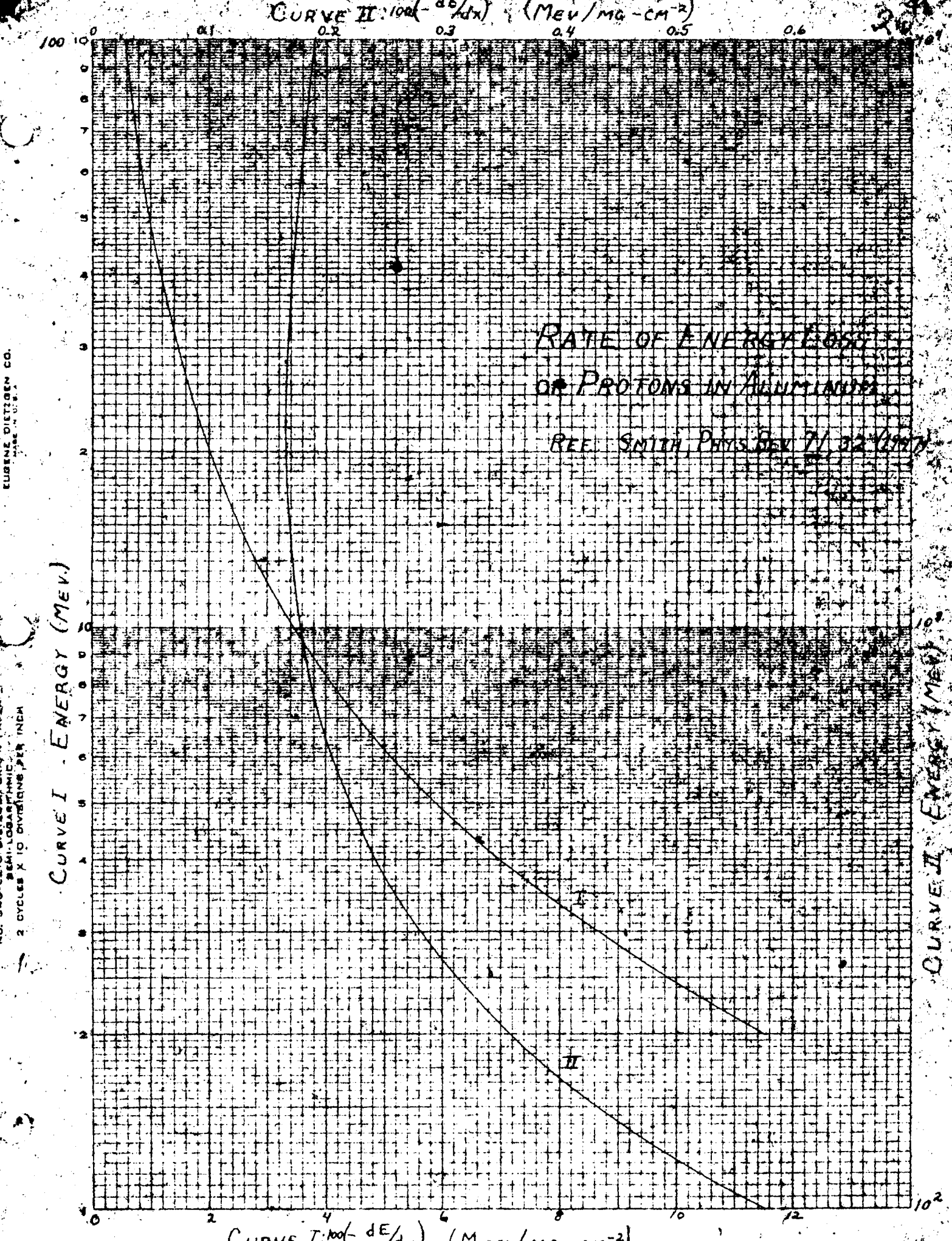
KEUFFEL & ESSER CO., N. Y. NO. 359-11
10 x 10 to the ½ inch, 5th lines accented.
Enameling 7 x 10 in.
MADE IN U.S.A.

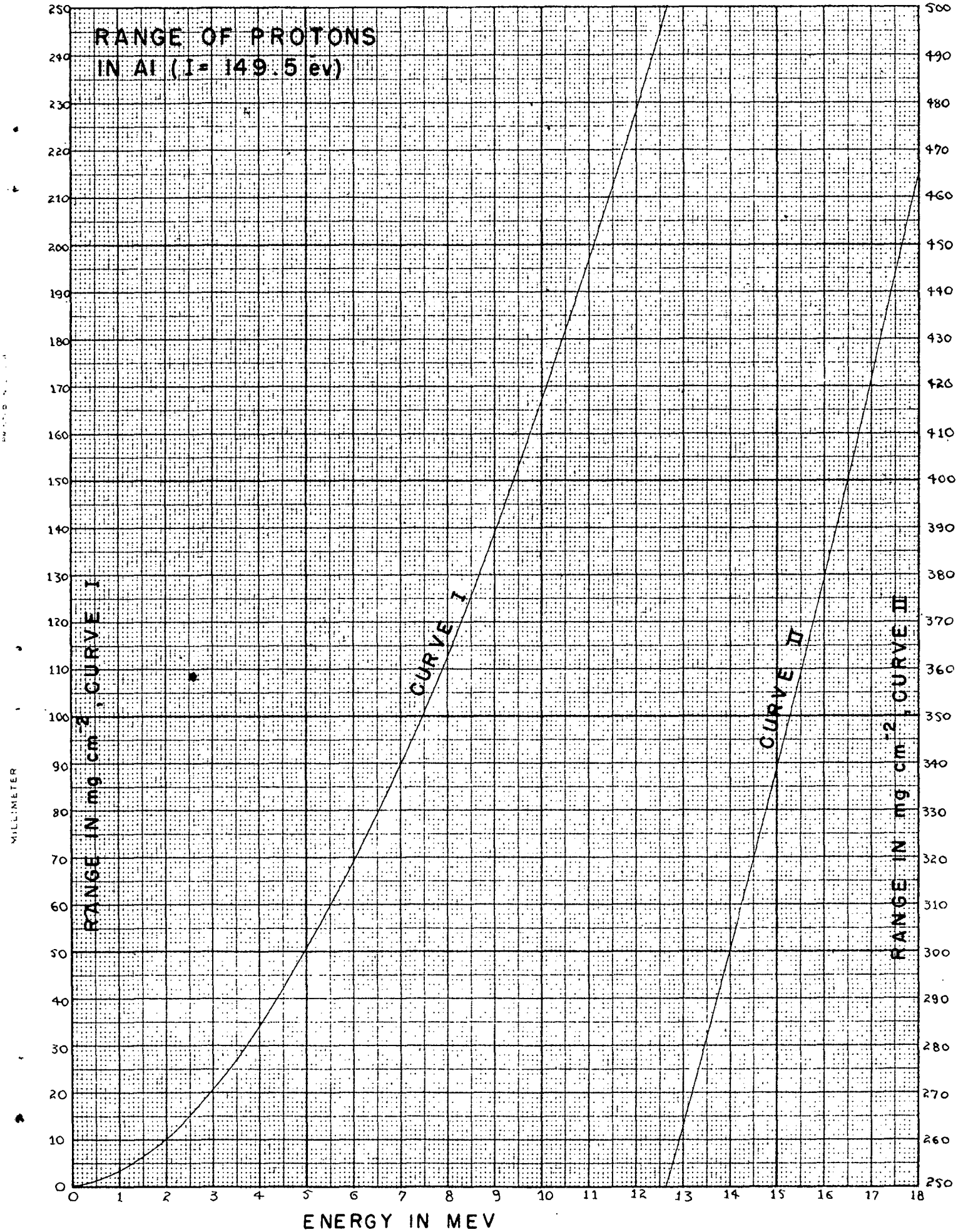


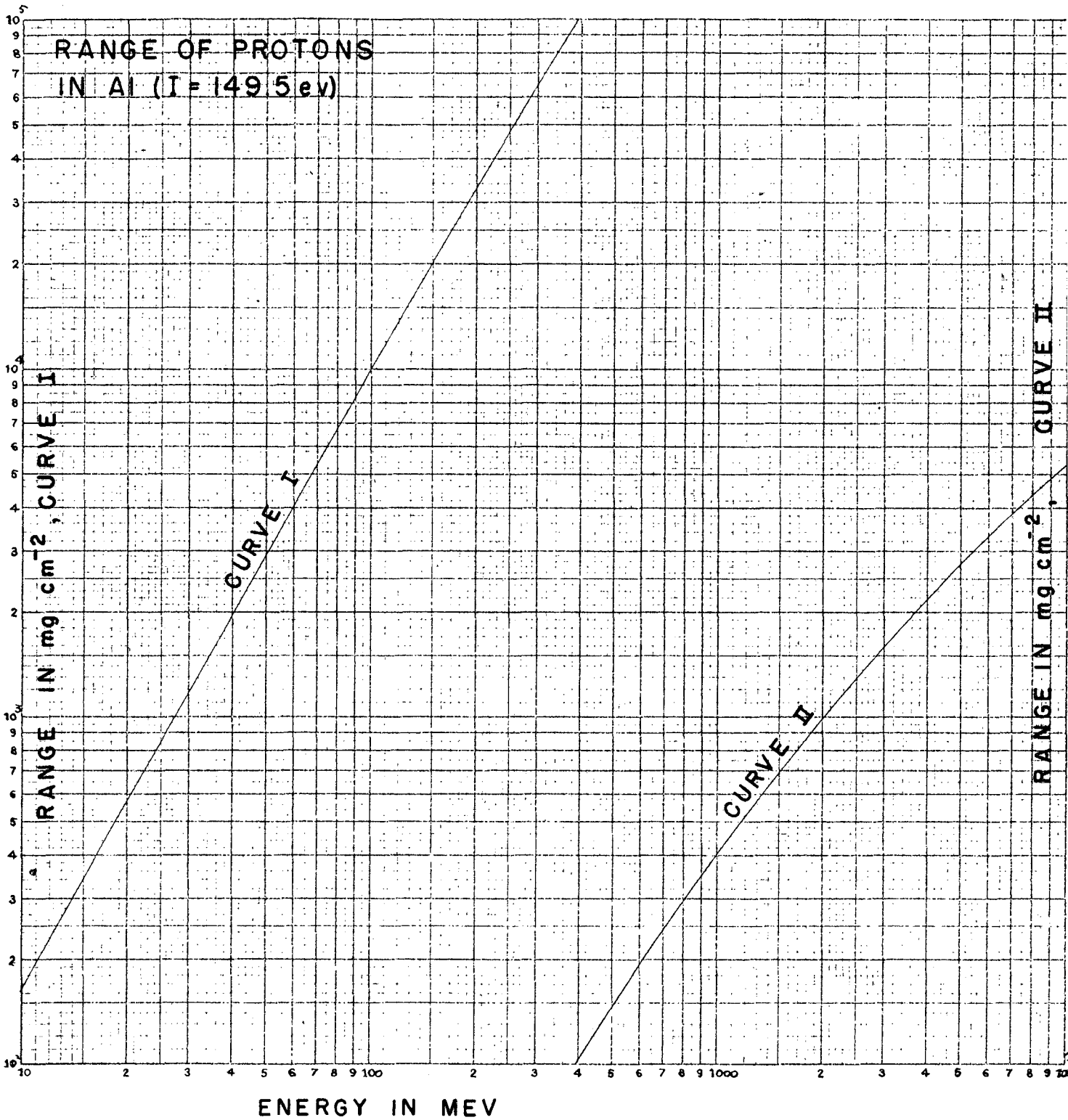
KEUFFEL & ESSER CO., N. Y. NO. 359-120.
Logarithmic, 3 X 3 cycles.
MADE IN U.S.A.

Range of Protons in Aluminum

The curves for the range of protons in aluminum which follow have been drawn from the data given by J. H. Smith (Theoretical Range-Energy Values for Protons in Aluminum and Air: Phys. Rev. 71:32 (1947)).


CURVE II: 100(-dE/dx) (MEV/MG-CM⁻²)
0
0.1
0.2
0.3
0.4
0.5
0.6
100
10
RATE OF ENERGY LOSS
OF PROTONS IN ALUMINUM
REF. SMITH, PHYS. REV. 71, 32 (1947)
CURVE I - ENERGY (MEV.)
CURVE II - ENERGY (MEV.)
II
0
2
4
6
8
10
12
10²
EUGENE DIETZGEN CO.
MADE IN U.S.A.
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF PROTONS
IN Al (I = 149.5 ev)
RANGE IN mg cm⁻², CURVE I
RANGE IN mg cm⁻², CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440
450
460
470
480
490
500

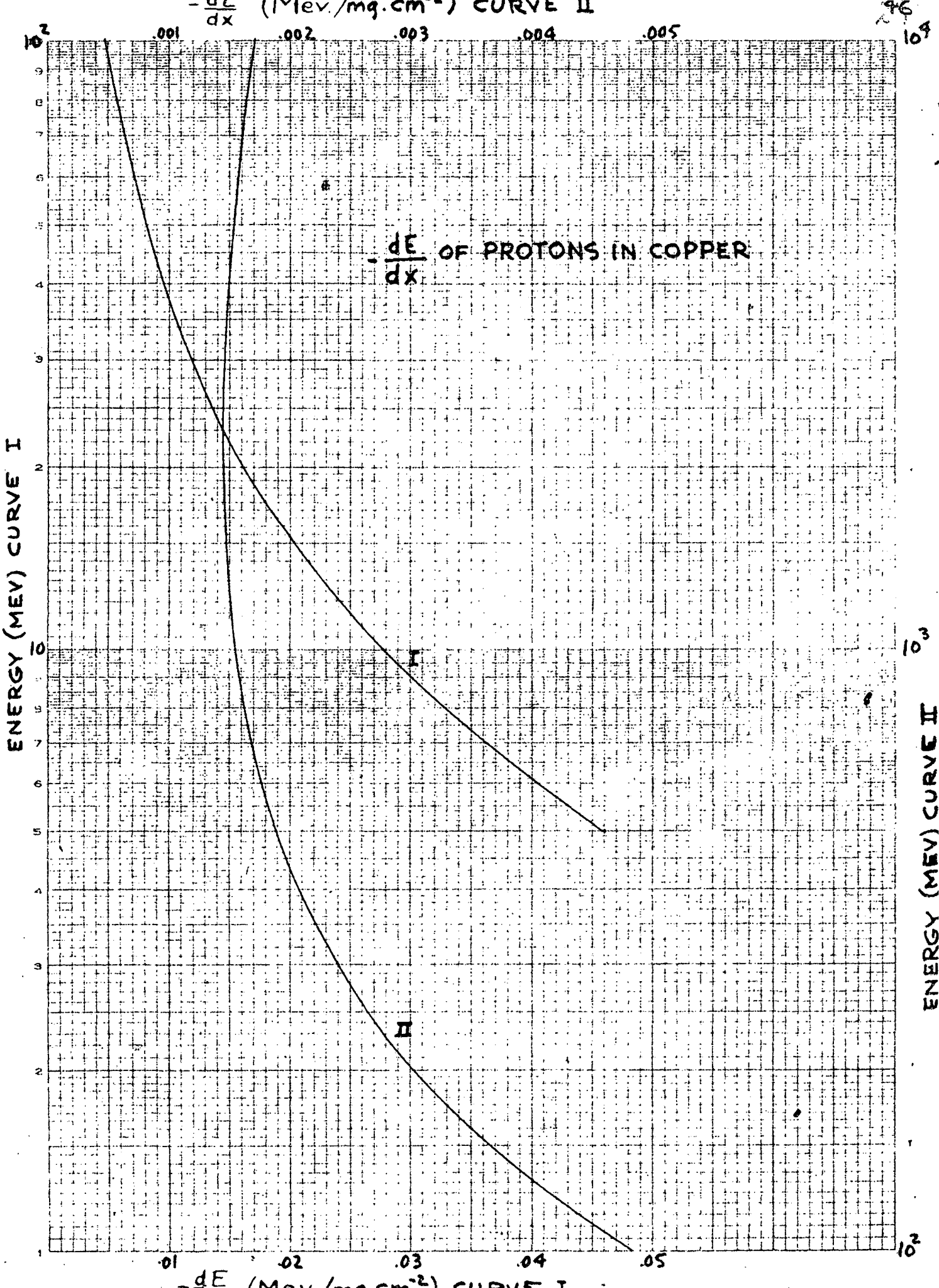


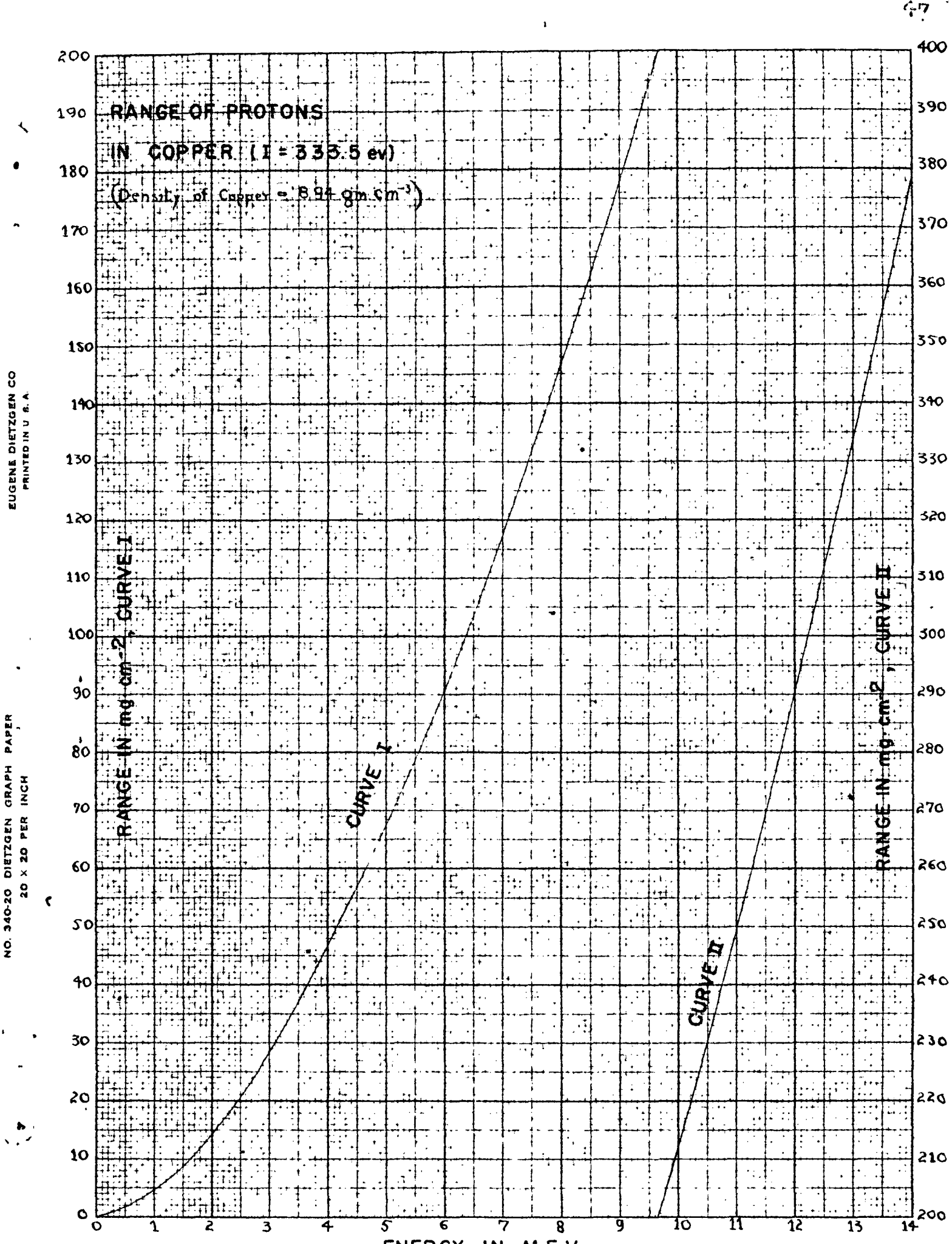
KEUFFEL & ESSER CO., N. Y. NO. 359-120
Logarithmic 3 Cycles
MADE IN U.S.A.

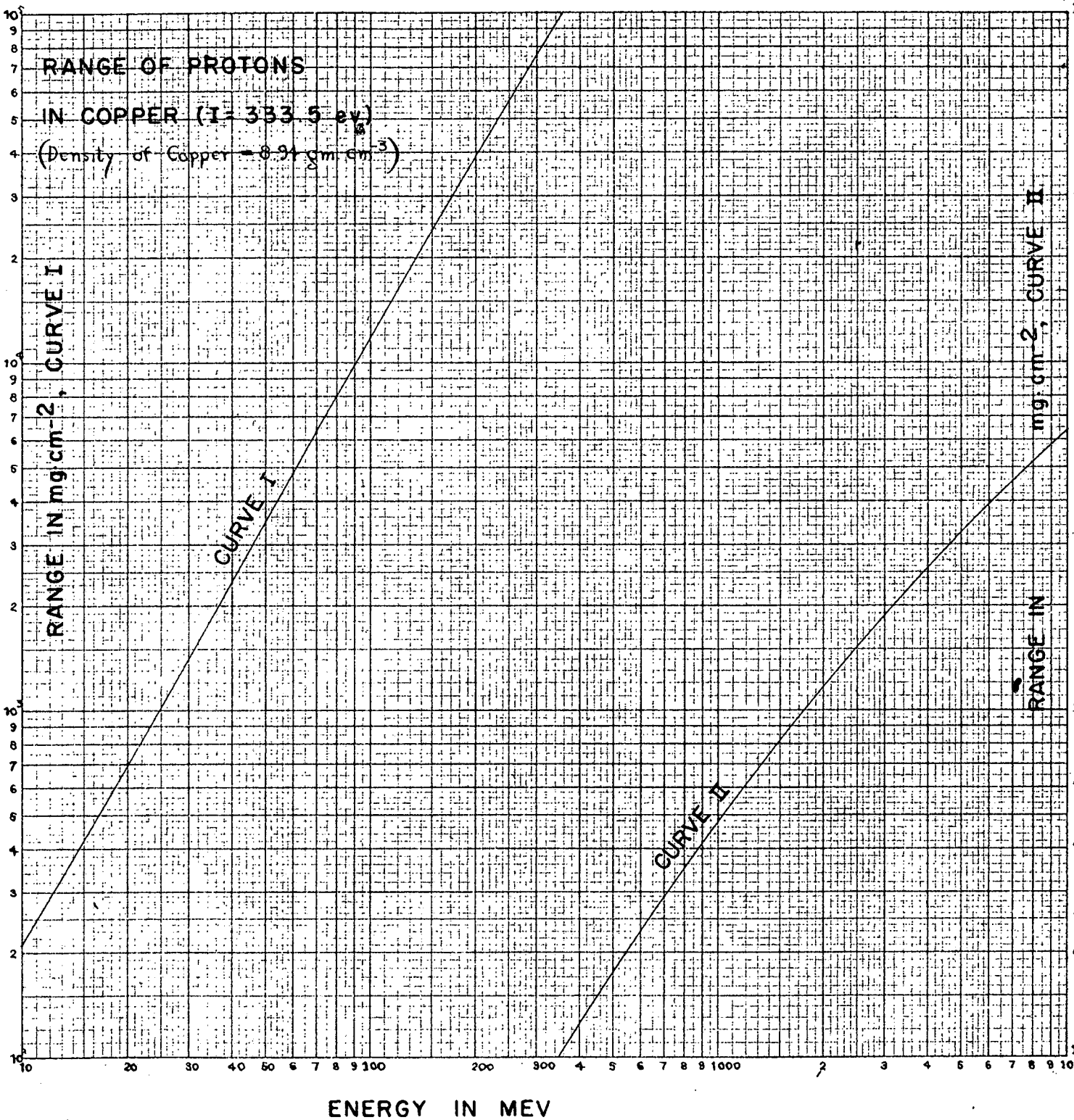
Table VII. Ranges of Protons in Copper and Silver (in mg cm^{-2}) and Energy Losses (in Mev/mg cm^{-2})

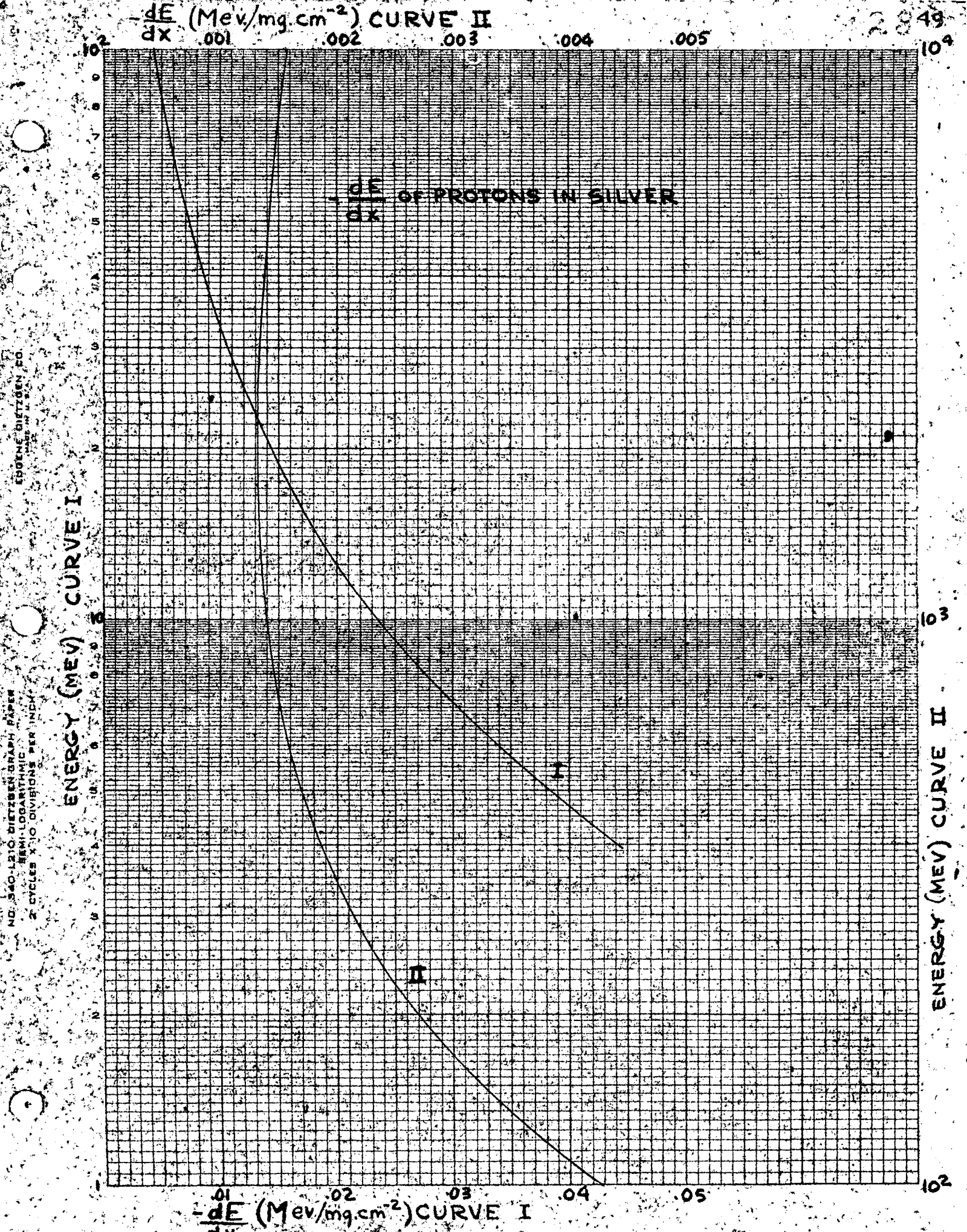
E in Mev	Copper (I = 333.5 ev)		Silver (I = 540.5 ev)	
	Energy Loss	Range	Energy Loss	Range
1		4.50		
2		14.30		
3		28.40		
4		46.66	44.2295×10^{-3}	62.60
5	46.08×10^{-3}	68.48		
6	40.46	91.70	33.7182	114.96
8	32.81	146.95	27.5711	180.94
10	27.80	213.45	23.4934	259.82
12	24.24	290.71	20.5746	351.04
14	21.57	378.36	18.3836	454.07
16	19.48	476.08	16.6613	568.51
18	17.80	583.61	15.2683	694.05
20	16.42	700.71	14.1161	830.40
22	15.26	827.17	13.1457	977.33
24	14.27	962.81	12.3161	1134.61
26	13.42	1107.44	11.5981	1302.04
28	12.67	1260.91	10.9699	1479.44
30	12.02	1423.1	10.4153	1666.62
35	10.67	1865.6	9.27491	2176.39
40	9.629	2359.7	8.38928	2744.09
45	8.798	2903.7	7.68009	3367.76
50	8.119	3495.9	7.09841	4045.60
55	7.552	4135.0	6.61207	4776.0
60	7.072	4819.7	6.19901	5557.5
65	6.659	5548.8	5.84353	6388.7
70	6.300	6321.2	5.53418	7268.4
75	5.985	7135.9	5.26238	8.1953×10^{3}
80	5.706	7991.8	5.02157	9.1683
85	5.458	8888.1	4.80667	10.1864
90	5.235	9823.8	4.61365	11.2484
95	5.034	10978.0	4.43929	12.3535
100	4.852	11810.0	4.28096	13.5007
110	4.533	1.3944×10^{4}	4.00422	15.918
120	4.264	1.6220	3.77030	18.494
130	4.034	1.8633	3.56989	21.221
140	3.835	2.1177	3.39622	24.094
150	3.661	2.3847	3.24426	27.108
160	3.507	2.6639	3.11017	30.257
170	3.371	2.9548	2.99096	33.537
180	3.249	3.2571	2.88428	36.942
190	3.139	3.5703	2.78828	40.470
200	3.040	3.8941	2.70142	44.114
225	2.829	4.7477	2.51668	53.713
250	2.659	5.6601	2.36765	63.963
275	2.519	6.6268	2.24501	74.815
300	2.402	7.6437	2.14245	86.220

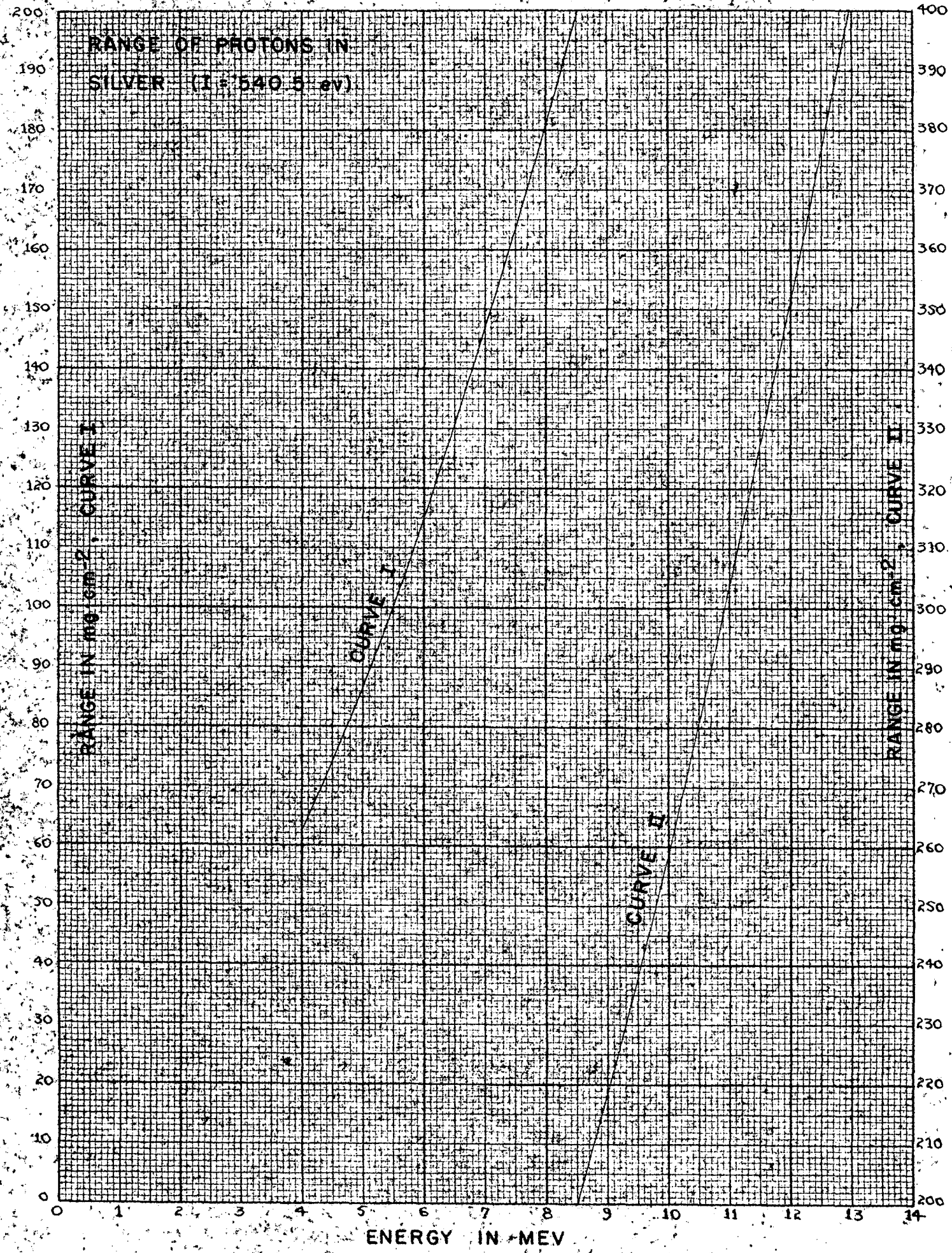
E in Mev	Copper (I = 333.5 ev)		Silver (I = 540.5 ev)	
	Energy Loss	Range	Energy Loss	Range
325	2.303	8.7070	2.05551	98.139
350	2.218	9.8135	1.98099	110.533
375	2.145	10.960	1.91649	123.368
400	2.081	12.144	1.86020	136.613
425	2.025	13.362	1.81071	150.238
450	1.975	14.613	1.76693	164.218
475	1.930	15.893	1.72798	178.528
500	1.891 x 10^{-3}	17.202 x 10^{4}	1.69317 x 10^{-3}	193.146 x 10^{3}
550	1.823	19.897	1.63371	223.226
600	1.768	22.684	1.58505	254.311
650	1.721	25.551	1.54474	286.277
700	1.683	28.490	1.51100	319.014
750	1.650	31.491	1.48253	352.429
800	1.623	34.547	1.45833	386.441
850	1.599	37.653	1.43765	420.978
900	1.578	40.801	1.41989	455.979
950	1.561	43.987	1.40458	491.389
1.0 x 10^3	1.545	47.207	1.39135	527.160
1.1	1.520	53.734	1.36993	599.62
1.2	1.501	60.355	1.35377	673.07
1.3	1.487	67.050	1.34161	747.29
1.4	1.476	73.801	1.33250	822.09
1.5	1.468	80.597	1.32579	897.34
1.6	1.462	87.425	1.32098	972.91
1.7	1.457	94.278	1.31769	1048.71
1.8	1.454	101.15	1.31562	1124.67
1.9	1.452	108.03	1.31455	1200.71
2.0	1.452	114.92	1.31429	1276.79
2.25	1.453	132.14	1.31634	1466.90
2.50	1.456	149.33	1.32106	1656.50
2.75	1.462	166.46	1.32748	1845.30
3.00	1.470	183.52	1.33496	2033.11
3.25	1.478	200.48	1.34311	2219.82
3.50	1.486 x 10^{-4}	2.1735 x 10^6	1.35167 x 10^{-3}	2.40537 x 10^6
3.75	1.495	2.3413 x 10^{6}	1.36044	2.58973
4.00	1.504	2.5080	1.36930	2.77290
4.25	1.513	2.6737	1.37818	2.95488
4.50	1.522	2.8385	1.38700	3.13570
4.75	1.531	3.0022	1.39572	3.31588
5.00	1.540	3.1651	1.40432	3.49395
5.5	1.557	3.4879	1.42109	3.84787
6.0	1.574	3.8073	1.43721	4.19772
6.5	1.590	4.1234	1.45267	4.54375
7.0	1.605	4.4364	1.46747	4.88619
7.5	1.620	4.7464	1.48163	5.22526
8.0	1.634	5.0538	1.49520	5.56118
8.5	1.648	5.3585	1.50819	5.89413
9.0	1.661	5.6608	1.52066	6.22428
9.5	1.673	5.9607	1.53262	6.55179
10.0	1.685	6.2585	1.54412	6.87681


$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.001
.002
.003
.004
.005
$-\frac{dE}{dx}$ OF PROTONS IN COPPER
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
.01
.02
.03
.04
.05
$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE I
EUGENE DIETZGEN CO.
MADE IN U.S.A.
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF PROTONS
IN COPPER (I = 333.5 ev)
(Density of Copper = 8.94 gm cm⁻³)
RANGE IN mg cm⁻², CURVE I
RANGE IN mg cm⁻², CURVE II
CURVE I
CURVE II
ENERGY IN MeV
EUGENE DIETZGEN CO
PRINTED IN U.S.A.
NO. 340-20 DIETZGEN GRAPH PAPER
20 X 20 PER INCH


RANGE OF PROTONS
IN COPPER (I = 333.5 ev)
(Density of Copper = 8.91 gm cm^{-3})
RANGE IN mg cm^{-2}, CURVE I
CURVE I
RANGE IN mg cm^{-2}, CURVE II
CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-129G
Logarithmic, 3 X 3 Cycles.
MADE IN U.S.A.


$-\frac{dE}{dx}$ (Mev/mg.cm^{-2}) CURVE II
.001
.002
.003
.004
.005
$-\frac{dE}{dx}$ OF PROTONS IN SILVER
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
.01
.02
.03
.04
.05
$-\frac{dE}{dx}$ (Mev/mg.cm^{-2}) CURVE I
10^2
10^3
10^4
EUGENE DIETZGEN CO.
NO. 340-L210 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH



NO. 340-20 DIETZGEN GRAPH PAPER
20" X 20 PER INCH
EUGENE DIETZGEN CO.
PRINTED IN U.S.A.

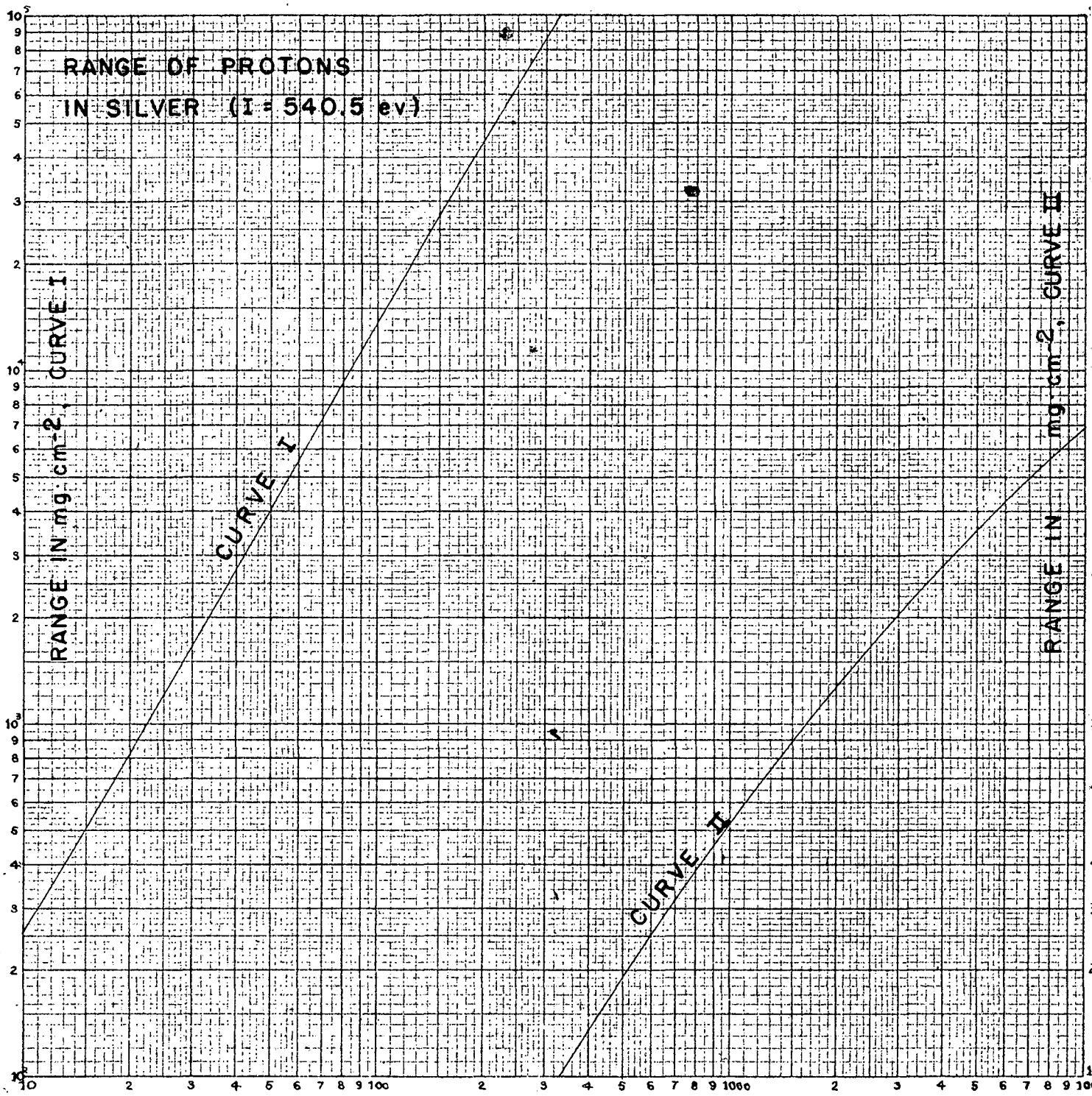
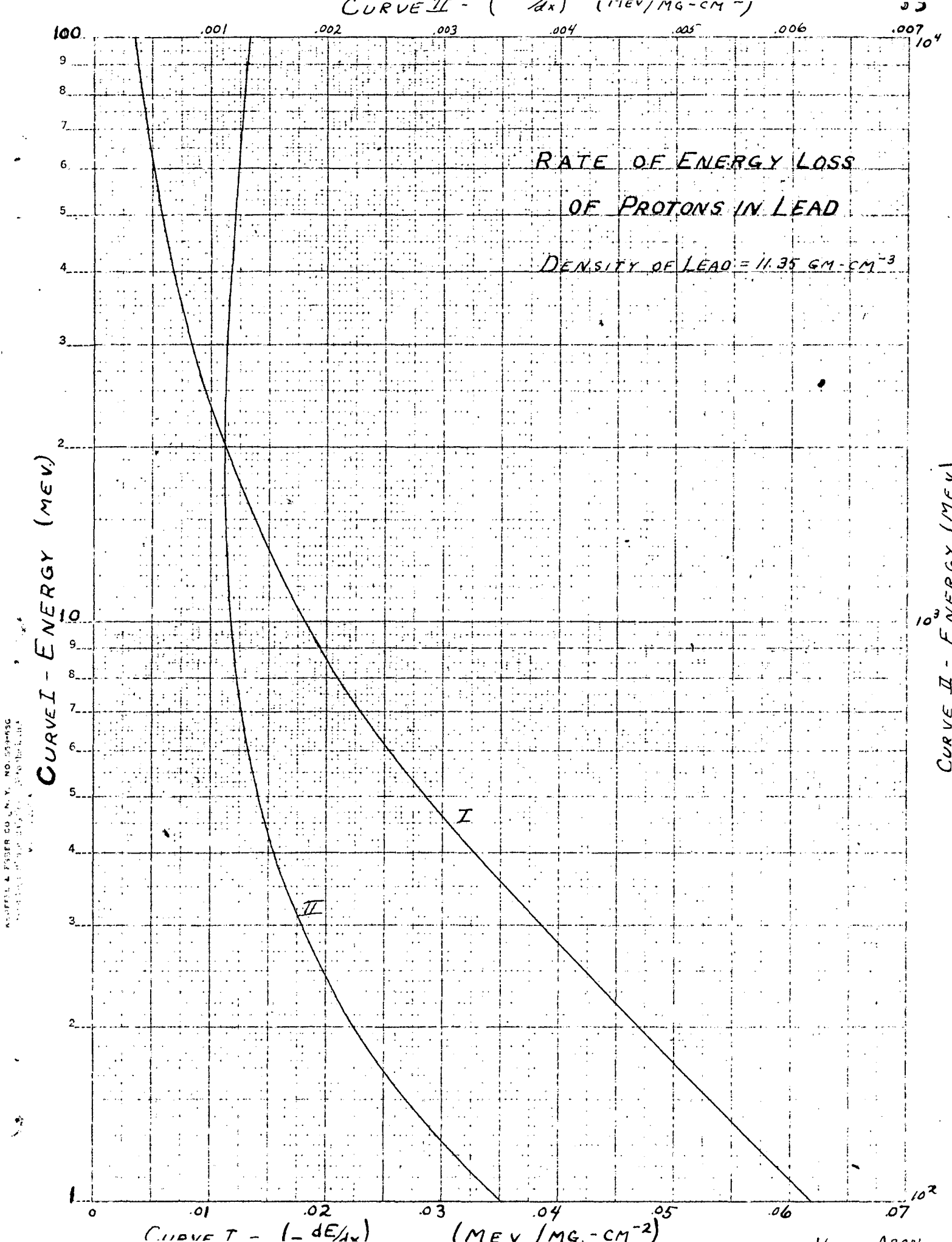
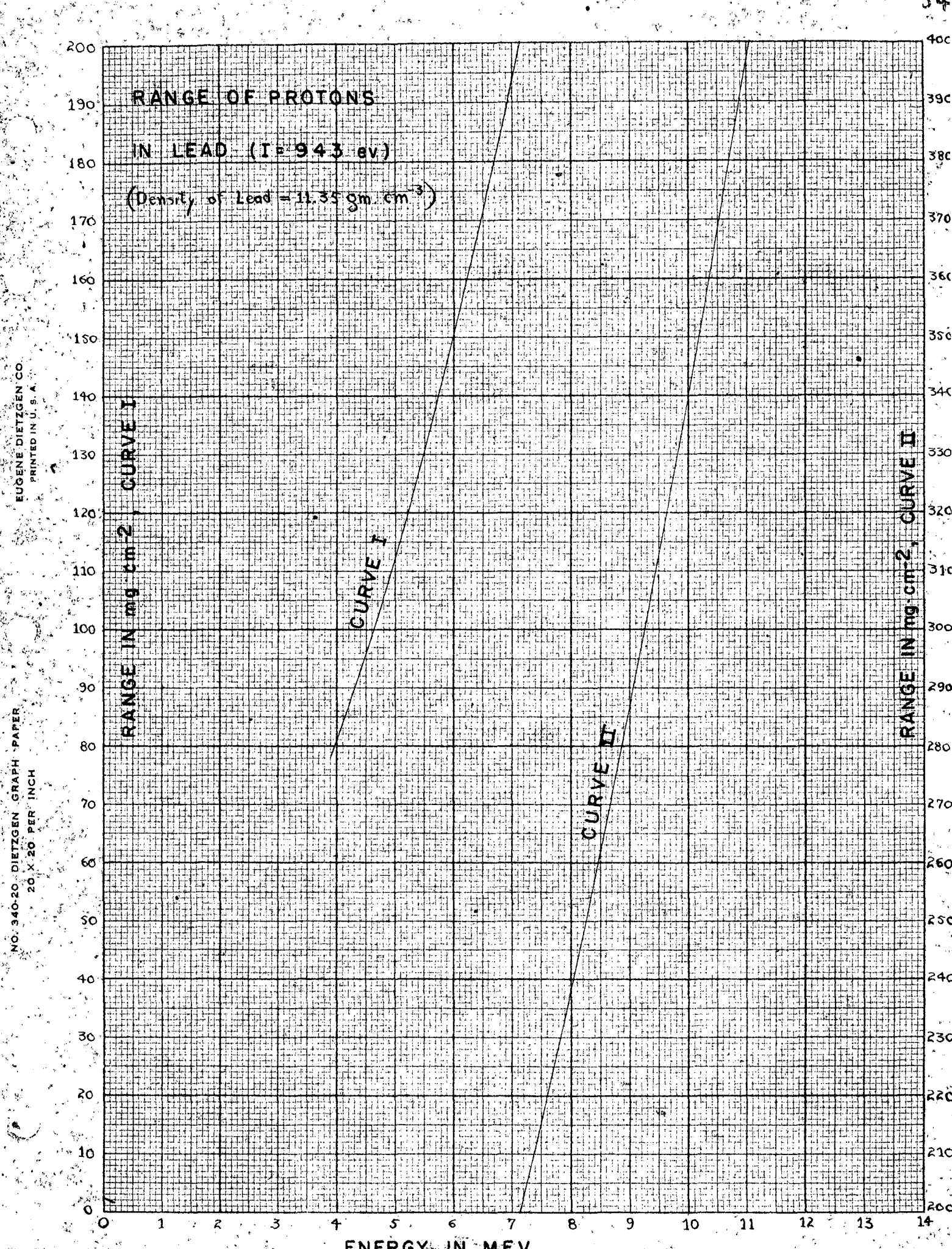

RANGE OF PROTONS
IN SILVER (I = 540.5 ev)
RANGE IN mg·cm-2, CURVE I
CURVE I
RANGE IN mg·cm-2, CURVE II
CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 X 3 Cycles.
MADE IN U.S.A.

Table VIII. Ranges of Protons in Lead
(in mg/cm^2) And Energy Losses (in Mev/mg cm^{-2})

E in Mev	Lead (I = 943 ev) Energy Loss	Range	E in Mev	Lead (I = 943 ev) Energy Loss	Range
1	61.8101 x 10^{-3}	7.90	425	1.51623	180.79
2	46.9850	25.05	450	1.48046	197.48
3	38.5336	49.8	475	1.44865	214.55
4	32.7565	81.67	500	1.42022	231.98
6	25.4906	151.52			
8	21.0857	238.23	550	1.37171	267.83
10	18.2072	340.55	600	1.33205	304.83
12	16.0765	457.72	650	1.29925	342.85
14	14.4437	589.20	700	1.27185	381.76
16	13.1483	734.52	750	1.24878	421.44
18	12.0929	893.29	800	1.22921	461.81
20	11.2147	1065.2	850	1.21254	502.77
22	10.4714	1249.9	900	1.19827	544.26
24	9.83323	1447.1	950	1.18602	586.20
26	9.27880	1656.6	1000	1.17547	628.56
28	8.79219	1878.1	1.1 x 10^3	1.15854	714.28
30	8.36134	2111.5	1.2	1.14591	801.09
35	7.47177	2745.3	1.3	1.13657	888.73
40	6.77739	3449.0	1.4	1.12975	977.00
45	6.21992	4220.0	1.5	1.12487	1065.71
50	5.76202	5056.0	1.6	1.12155	1.154.8
55	5.37791	5954.9	1.7	1.11948	1244.0
60	5.05074	6914.8	1.8	1.11839	1333.4
65	4.76848	7934.2	1.9	1.11812	1422.8
70	4.52231	9011.4	2.0	1.11851	1512.2
75	4.30559	10145.0	2.25	1.12163	1735.5
			2.50	1.12689	1957.9
80	4.11327	11.333 x 10^3	2.75	1.13348	2179.1
85	3.94134	12.576	3.00	1.14089	2399.0
90	3.78672	13.870			
95	3.64685	15.216	3.25	1.14878	2617.3
100	3.51969	16.612	3.50	1.15696	2834.2
110	3.29708	19.550	3.75	1.16528	3049.5
120	3.10856	22.675	4.00	1.17362	3263.3
130	2.94677	25.981	4.25	1.18192	3475.5
140	2.80638	29.460	4.50	1.19014	3686.3
150	2.68338	33.106	4.75	1.19825	3895.7
160	2.57472	36.912	5.00	1.20623	4103.6
170	2.47803	40.872	5.50	1.22172	4515.5
180	2.39142	44.981	6.0	1.23658	4922.2
190	2.31342	49.233	6.5	1.25078	5324.3
200	2.24280	53.624	7.0	1.26435	5721.8
225	2.09244	65.177	7.5	1.27734	6115.3
250	1.97099	77.498	8.0	1.28975	6504.8
275	1.87096	90.526	8.5	1.30163	6890.7
300	1.78724	104.21	9.0	1.31302	7273.1
325	1.71625	118.49	9.5	1.32395	7652.4
350	1.65337	133.32	10.0 x 10^3	1.33444	8028.5
375	1.60266	148.68			
400	1.55667	164.51			


CURVE II - (dE/dx) (MEV/MG-CM⁻²)
RATE OF ENERGY LOSS
OF PROTONS IN LEAD
DENSITY OF LEAD = 11.35 GM-CM⁻³
CURVE I - ENERGY (MEV)
CURVE II - ENERGY (MEV)
I
II
CURVE I - (- dE/dx) (MEV/MG.-CM⁻²)


RANGE OF PROTONS
IN LEAD (I = 943 ev)
(Density of Lead = 11.35 gm. cm^{-3})
RANGE IN mg cm-2, CURVE I
RANGE IN mg cm-2, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
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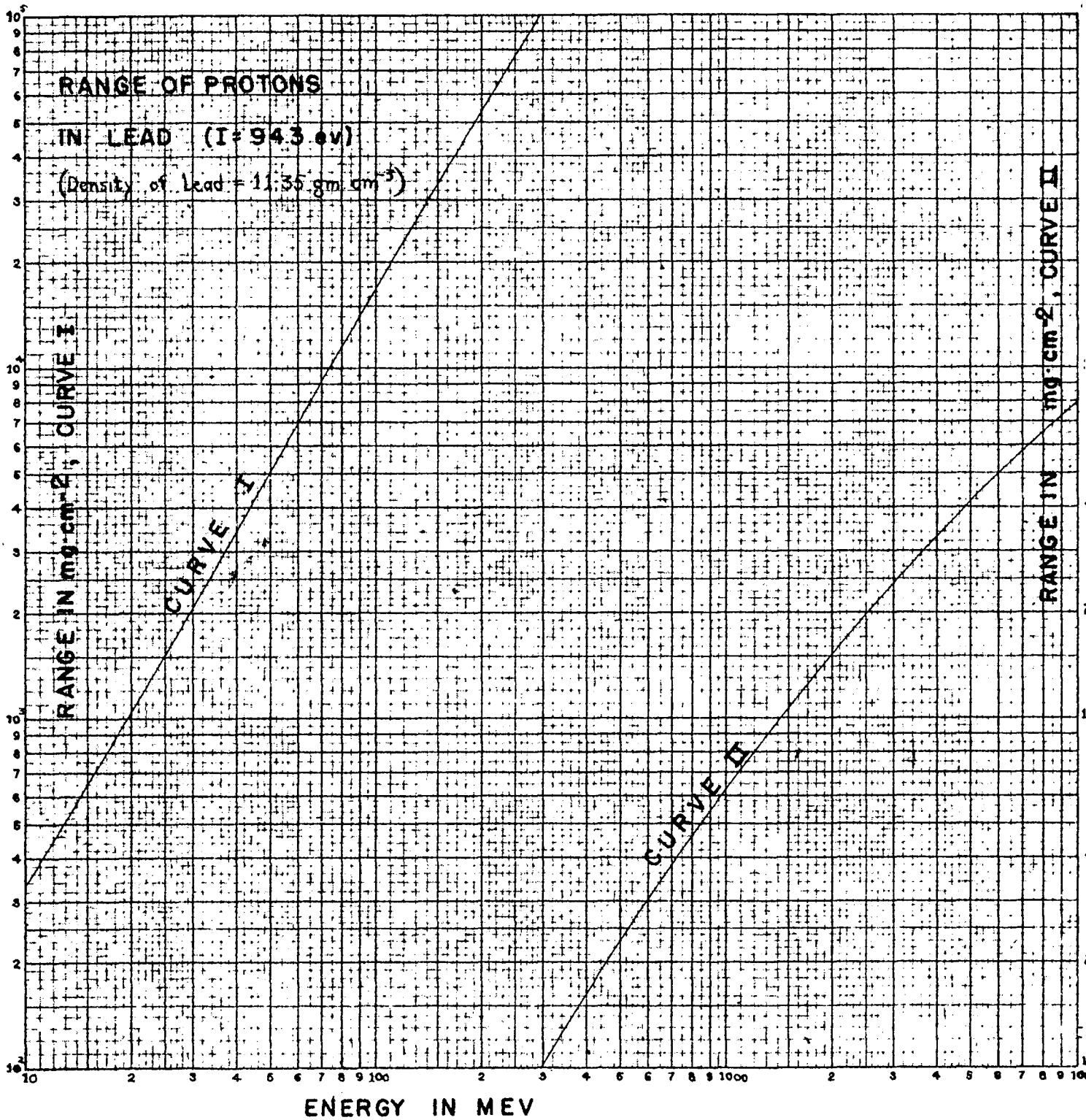


KEUFFEL & ESSER CO., N. Y. NO. 359-126G
Logarithmic, 3 X 3 Cycles

DEUTERON RANGES

Deuteron Ranges

The range of deuterons in various materials may be rather simply obtained from the ranges of protons in the same materials. Given two heavy particles, 1 and 2, we may write:

$$\frac{\left.\frac{dT}{dx}\right._{coll_1}}{\left.\frac{dT}{dx}\right._{coll_2}} = \frac{z_1^2\, v_2^2\, N_1\, Z_1 \log_e \left(\frac{2mv_1^2}{I_1} - \log_e (1-\beta^2)_1 - \beta_1^2\right)}{z_2^2\, v_1^2\, N_2\, Z_2 \log_e \left(\frac{2mv_2^2}{I_2} - \log_e (1-\beta^2)_2 - \beta_2^2\right)} \qquad (1)$$

where $\left.\frac{dT}{dx}\right._{coll_1}$ = collision energy loss for particle 1,

$\left.\frac{dT}{dx}\right._{coll_2}$ = collision energy loss for particle 2,

z_1, z_2 = atomic number for particles 1 and 2, respectively,

v_1, v_2 = velocities of particles 1 and 2, respectively,

N_1, N_2 = number of atoms per unit volume of stopping materials for particles 1 and 2, respectively,

Z_1, Z_2 = atomic numbers of stopping materials for particles 1 and 2, respectively,

I_1, I_2 = excitation potentials of stopping materials for particles 1 and 2, respectively.

Given the conditions that

$$v_1 = v_2 \,, \qquad (2)$$

and

$$z_1 \neq z_2 \,, \qquad (3)$$

and

$$\left.\begin{aligned} N_1 &= N_2 \\ Z_1 &= Z_2 \\ I_1 &= I_2 \end{aligned}\right\} \qquad (4)$$

equation (1) reduces to the expression

$$\frac{\left.\frac{dT}{dx}\right|_{coll_1}}{\left.\frac{dT}{dx}\right|_{coll_2}} = \frac{z_1^2}{z_2^2} \quad . \tag{5}$$

From equation (5) we get the relation

$$R_1\left(\frac{M_1}{M_2} E\right) = \frac{Z_2^2 \, M_1}{Z_1^2 \, M_2} \; R_2(E)_1 \tag{6}$$

where $R_2(E)$ signified the range of particle 2 at the energy E, and $R_1\left(\frac{M_1}{M_2} E\right)$ signified the range of particle 1 at the energy $\frac{M_1}{M_2}$ E.

For our specific case particle 1 may be taken as a deuteron and particle 2 as a proton. Then

$$R_1(1.99901 \; E) = 1.99901 \; R_2(E) \tag{7}$$

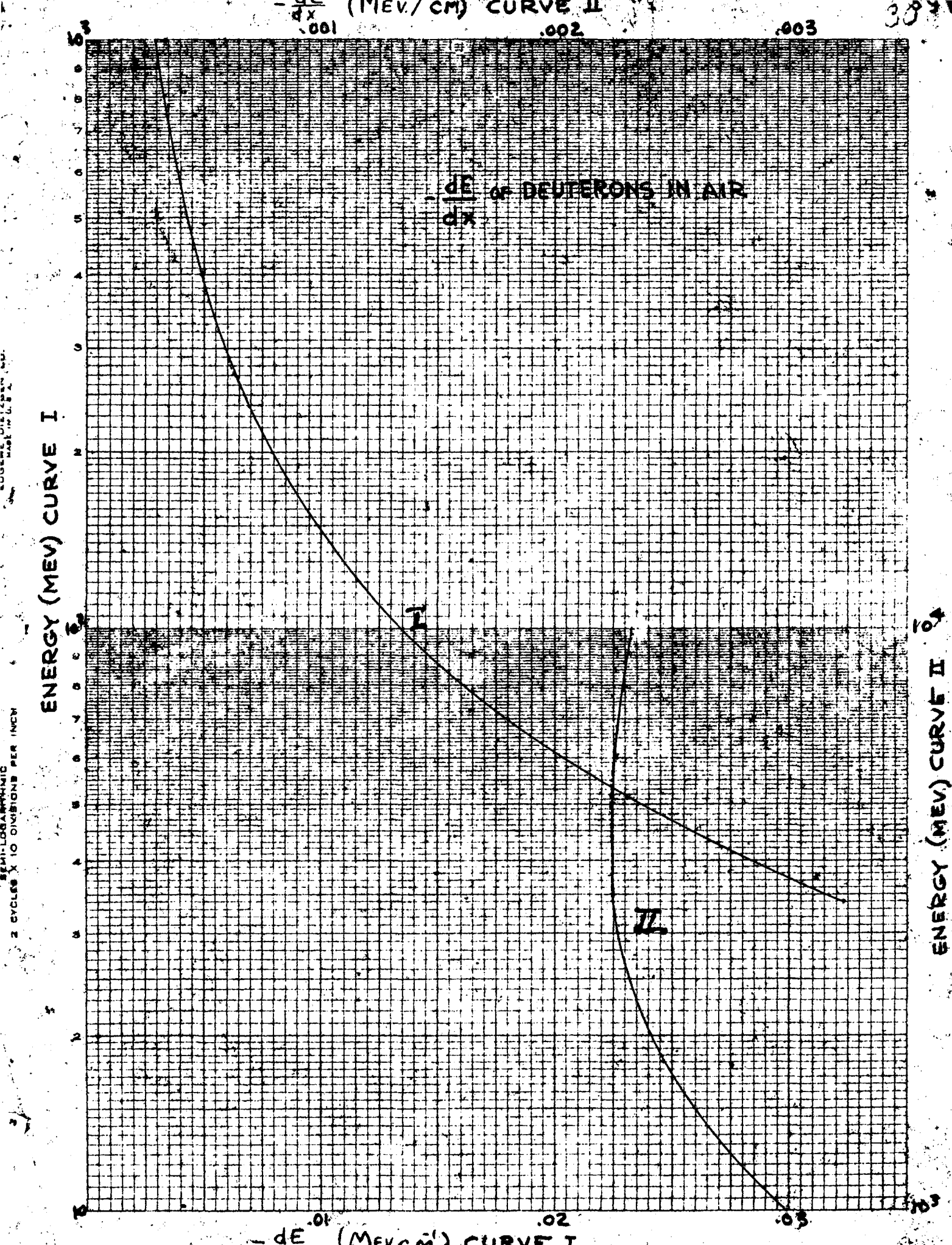
or

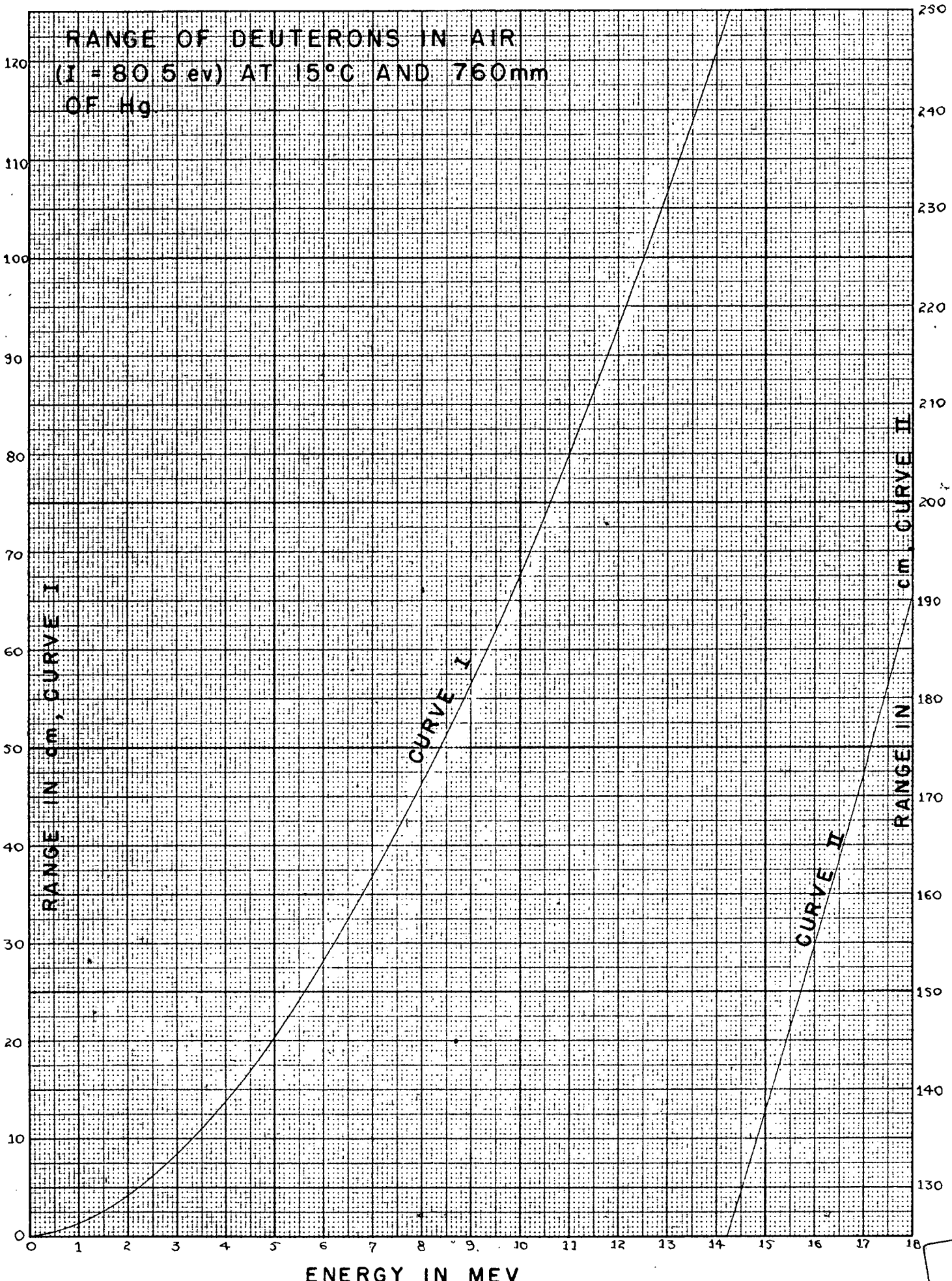
$$R_D \; (2E) = 2 \; R_p \; (E) \tag{8}$$

The equation (7) or (8) is valid for those energies for which

$$\eta = \frac{Em}{M \; z_{eff}^2 \; Ry} \gg 1 \; ,$$

or, approximately, for energies $>$ 2 Mev (for the deuteron).


$-\frac{dE}{dx}$ (MEV/CM) CURVE II
.001
.002
.003
$-\frac{dE}{dx}$ OF DEUTERONS IN AIR
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
.01
.02
.03
$-dE$ (MEV/cm) CURVE I
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF DEUTERONS IN AIR
(I = 80.5 ev) AT 15°C AND 760mm
OF Hg.
RANGE IN cm, CURVE I
RANGE IN cm, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
10
20
30
40
50
60
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150
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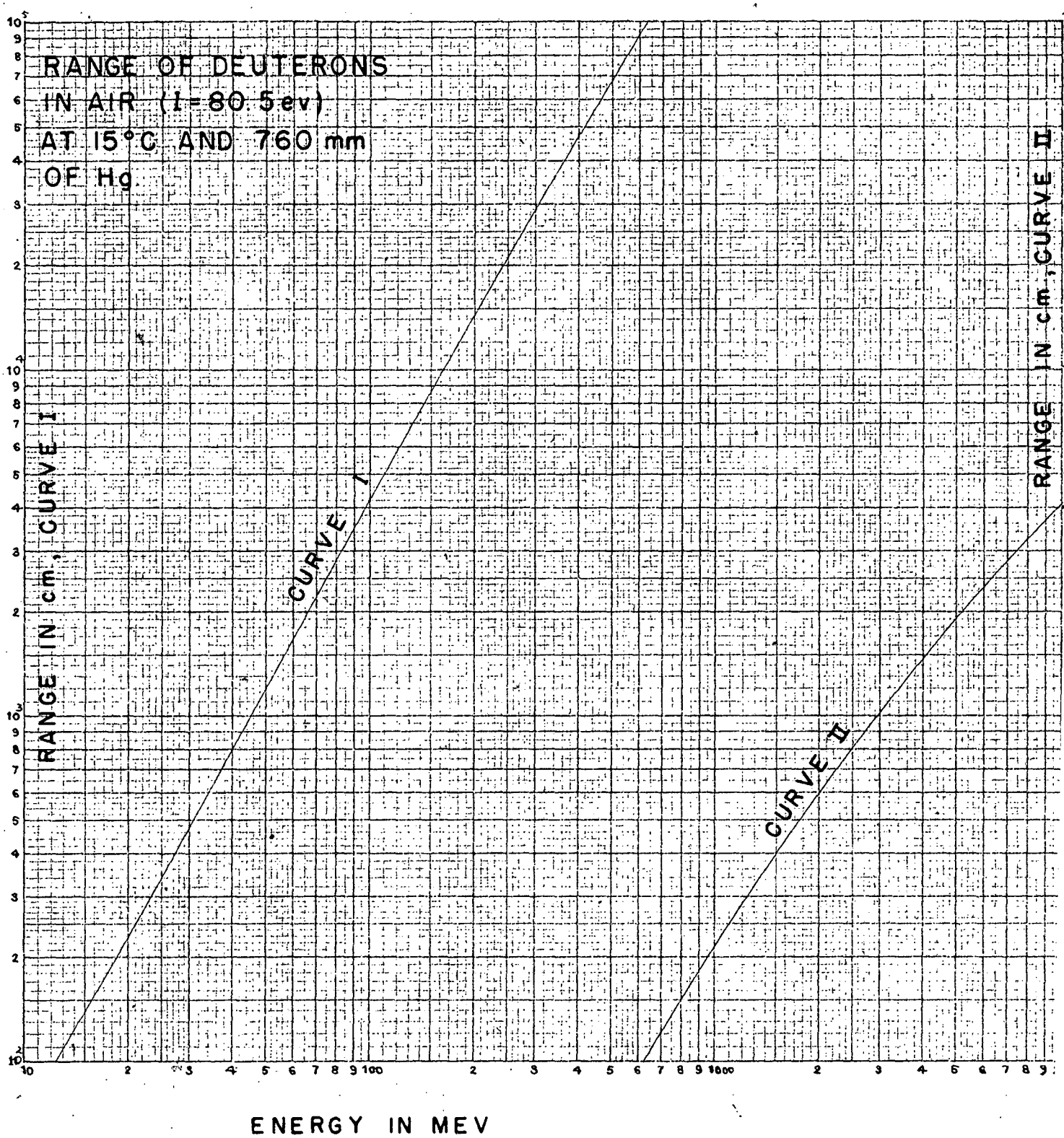
170
180
190
200
210
220
230
240
250


RANGE OF DEUTERONS
IN AIR (I = 80.5 ev)
AT 15°C AND 760 mm
OF Hg
RANGE IN cm, CURVE I
RANGE IN cm, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 355-120
Logarithmic, 3 Y cycles.
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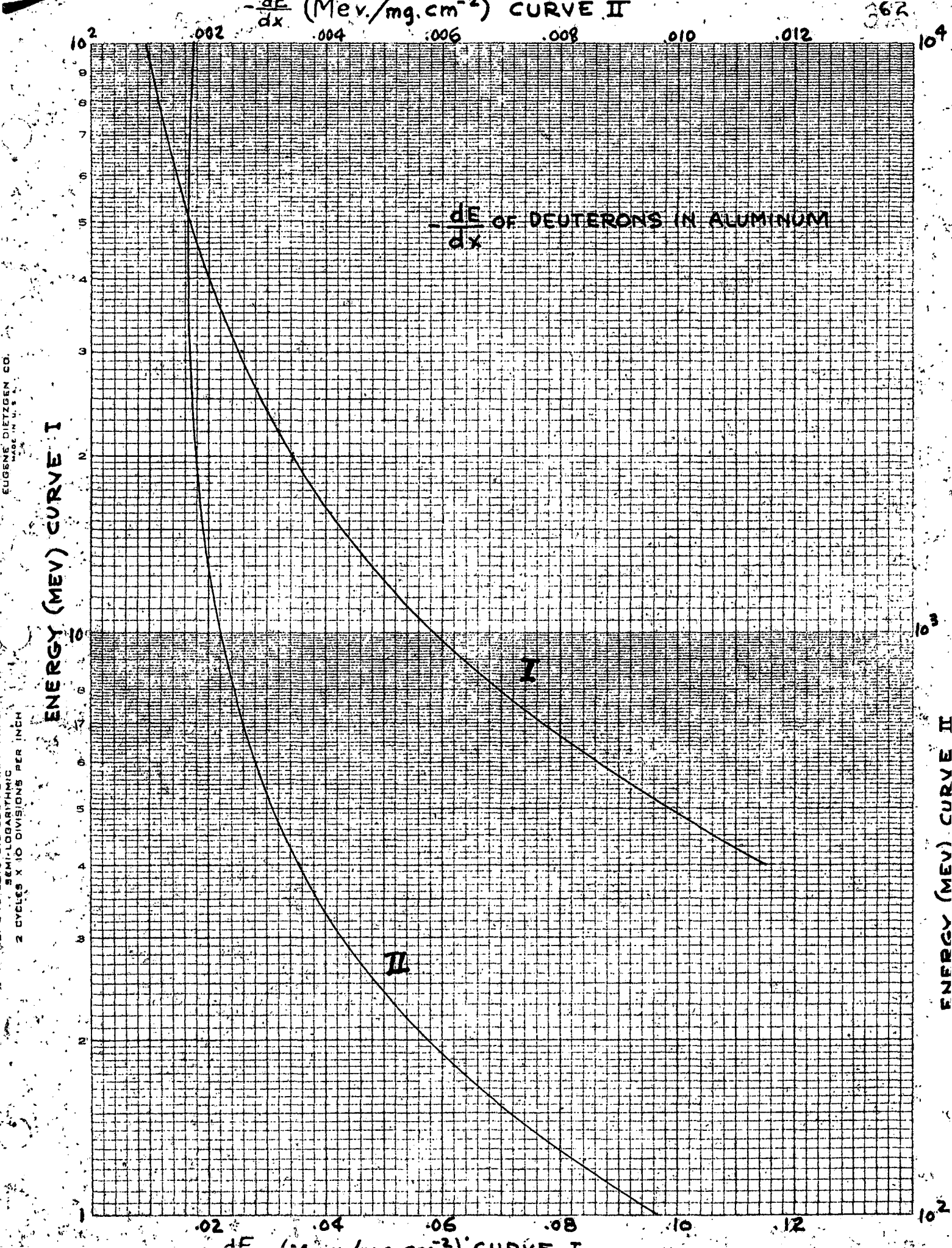
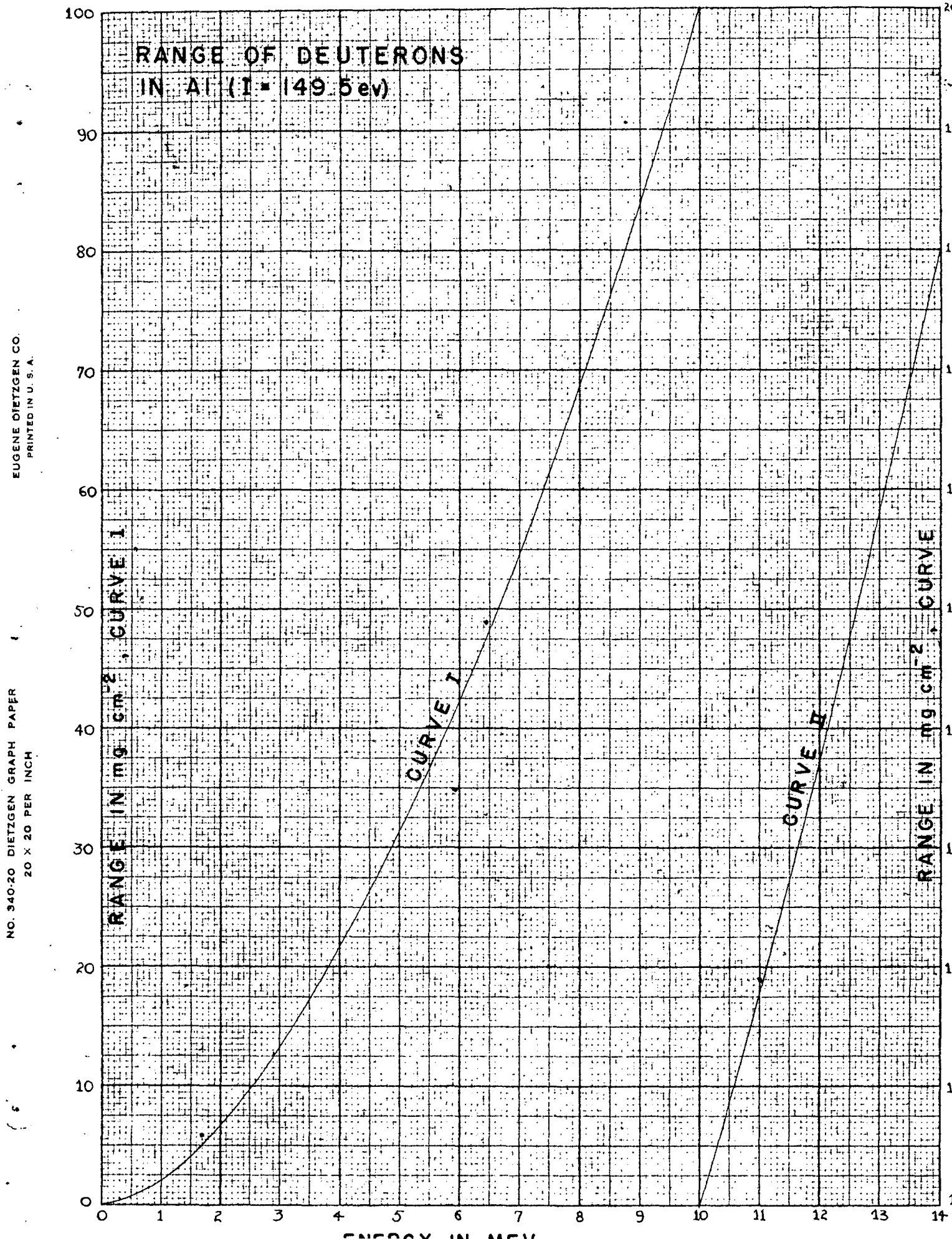

$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.002
.004
.006
.008
.010
.012
$-\frac{dE}{dx}$ OF DEUTERONS IN ALUMINUM
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
.02
.04
.06
.08
.10
.12
10^2
10^3
10^4
EUGENE DIETZGEN CO.
MADE IN U.S.A.
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF DEUTERONS
IN Al (I = 149.5 ev)
RANGE IN mg cm^{-2}, CURVE I
RANGE IN mg cm^{-2}, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
1
2
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6
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10
11
12
13
14
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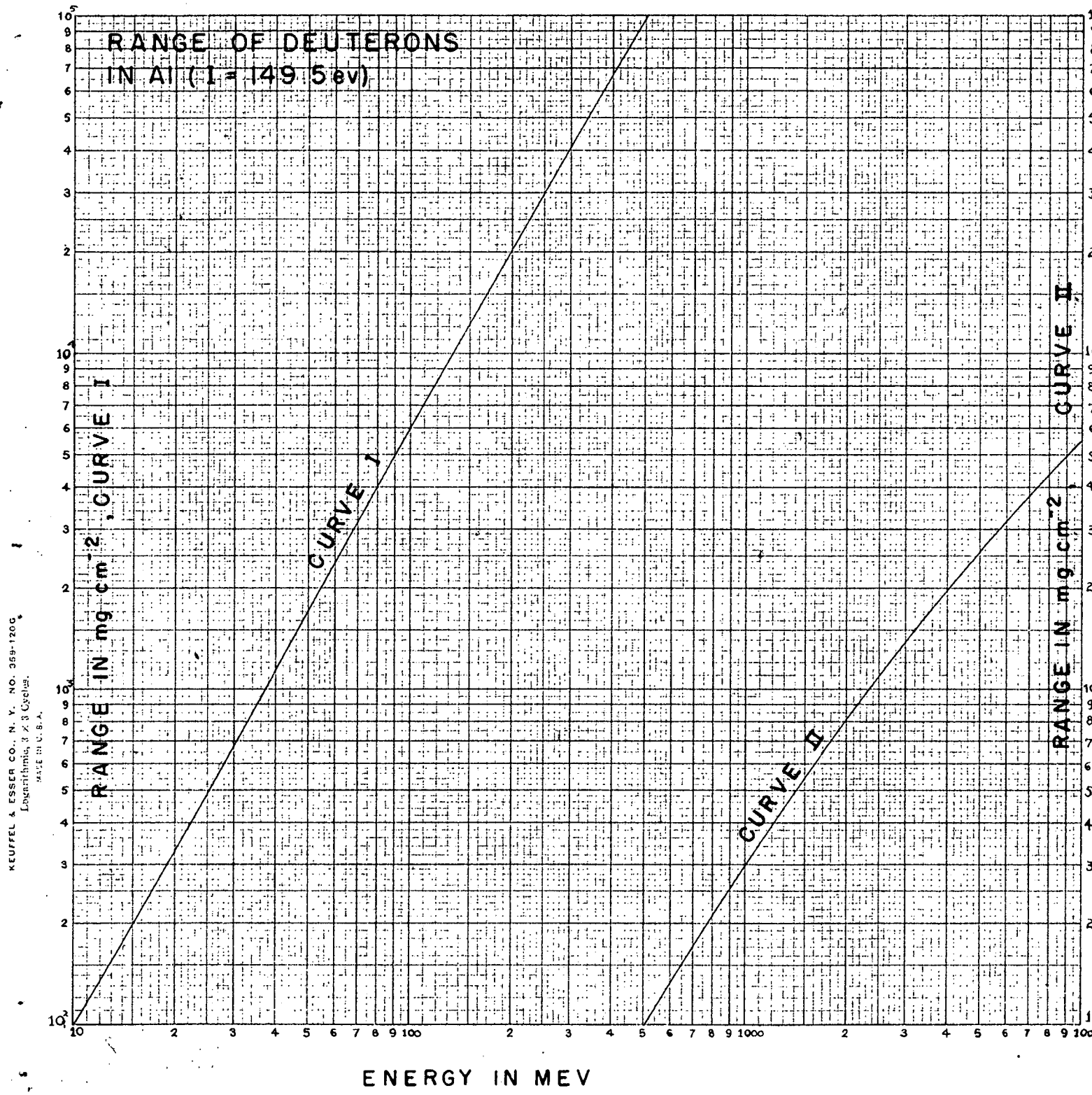
RANGE OF DEUTERONS
IN Al (I = 149.5 ev)
RANGE IN mg cm⁻², CURVE I
CURVE I
CURVE II
RANGE IN mg cm⁻², CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 x 3 Cycles.
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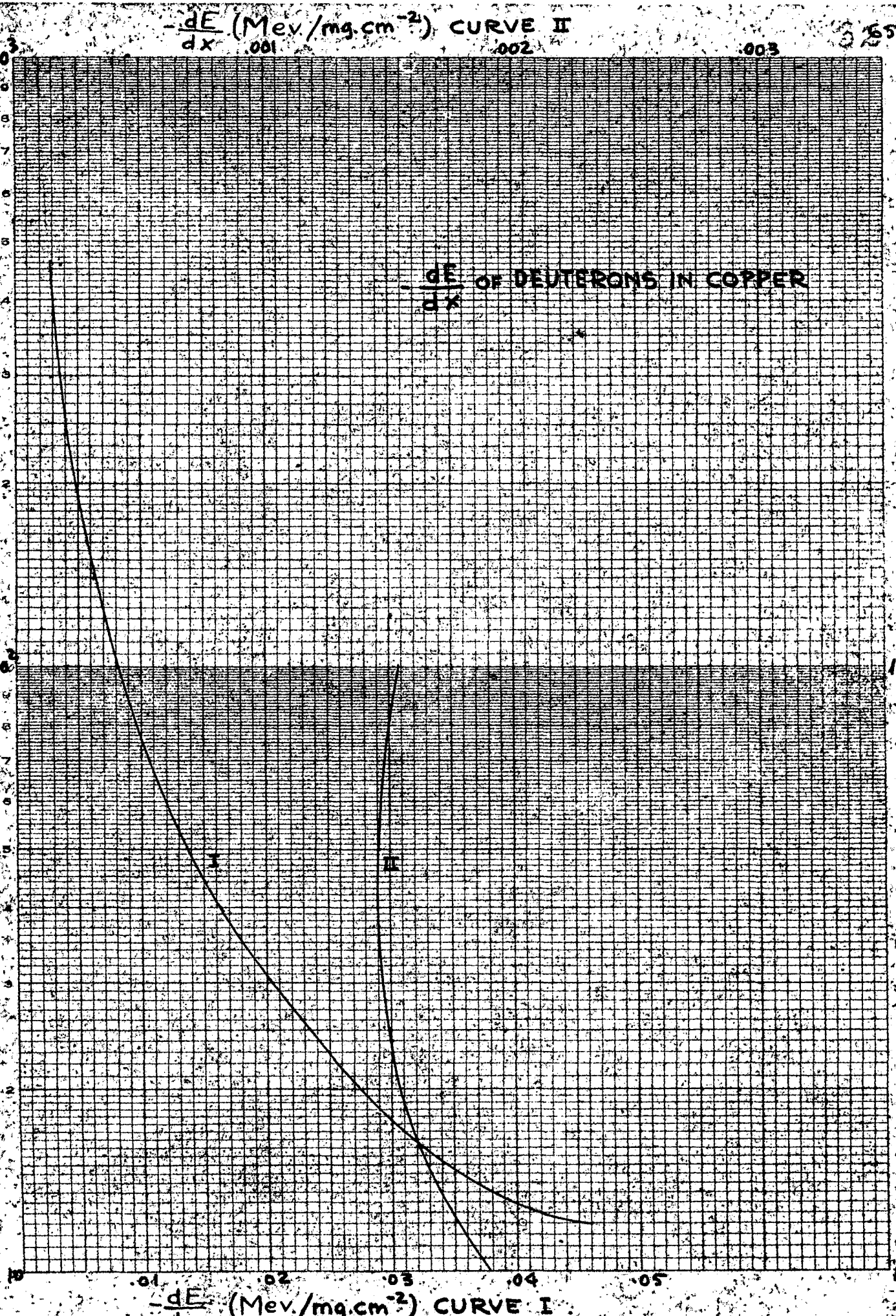
$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.001
.002
.003
$-\frac{dE}{dx}$ OF DEUTERONS IN COPPER
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
H
D
.01
.02
.03
.04
.05
$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE I
EUGENE DIETZGEN CO.
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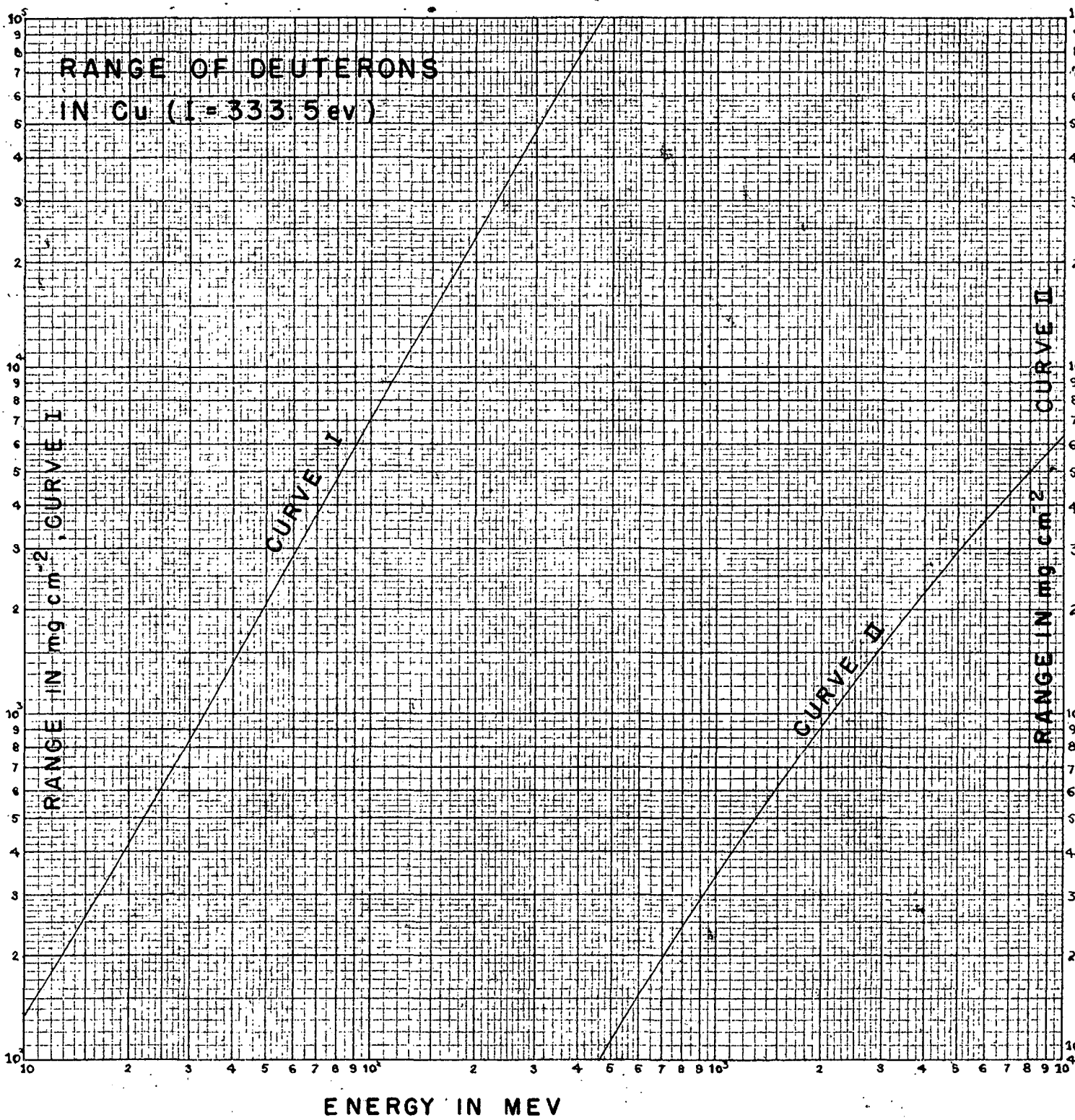
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF DEUTERONS
IN Cu (I = 333.5 ev)
RANGE IN mg cm⁻², CURVE I
CURVE I
CURVE II
CURVE II
RANGE IN mg cm⁻², CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 X 3 Cycles.
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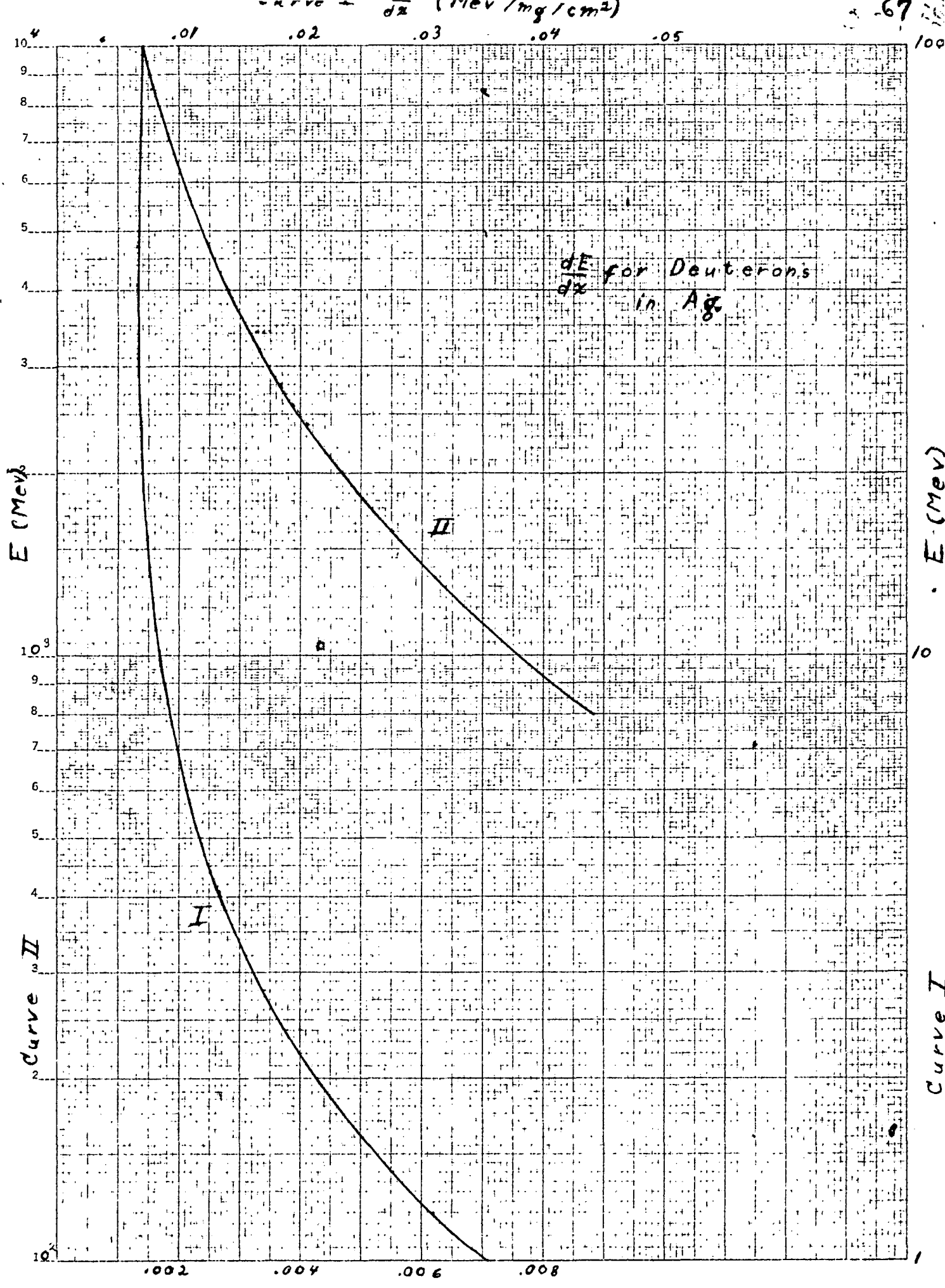
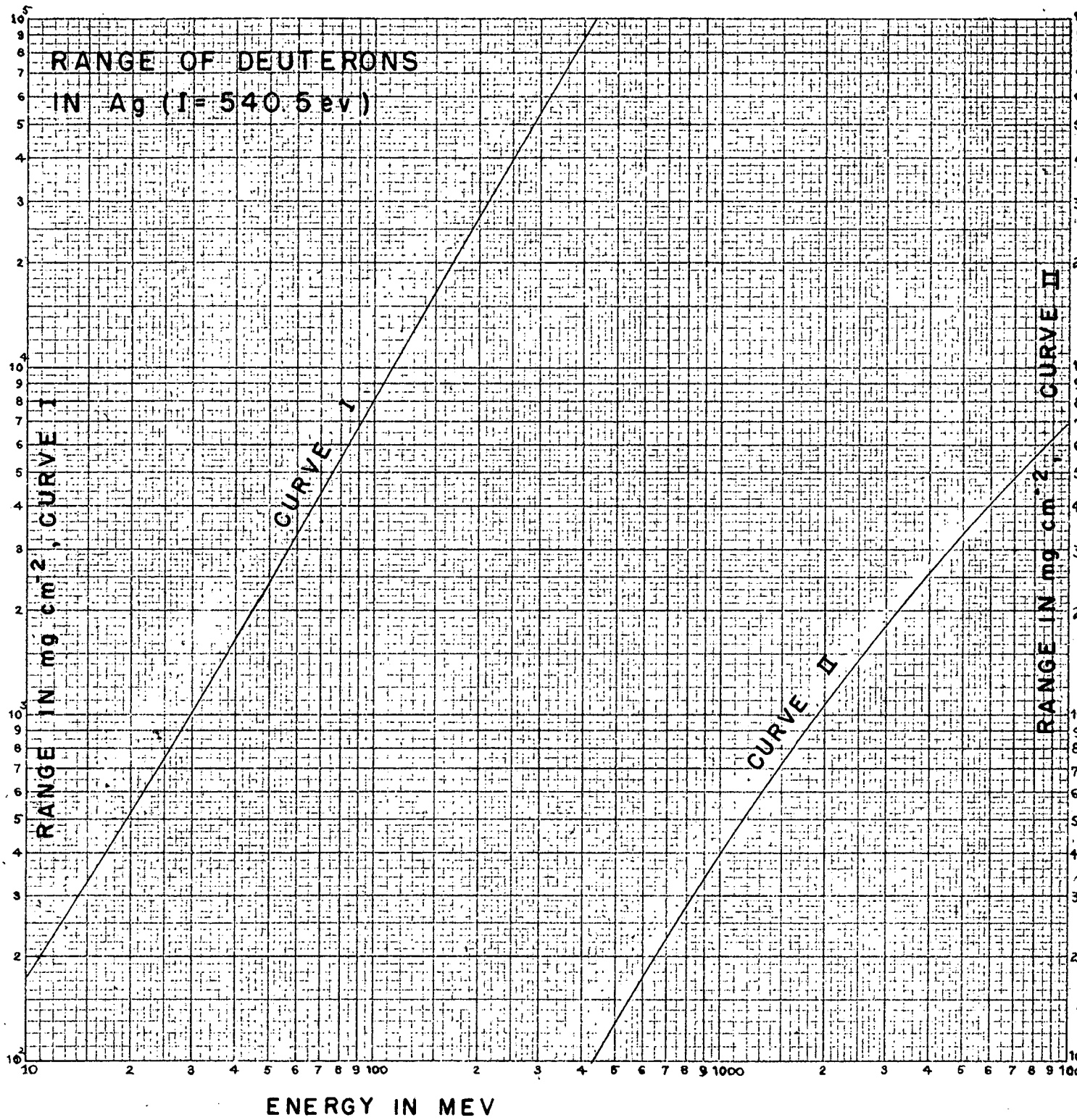

Curve I $\frac{dE}{dx}$ (Mev/mg/cm²)
.01
.02
.03
.04
.05
$\frac{dE}{dx}$ for Deuterons in Ag
E (Mev)
II
I
Curve II
Curve I
.002
.004
.006
.008
10⁴
10³
10²
100
10
1



KEUFFEL & ESSER CO., N. Y. NO. 359-120
Logarithmic, 3 x 3 cycles.
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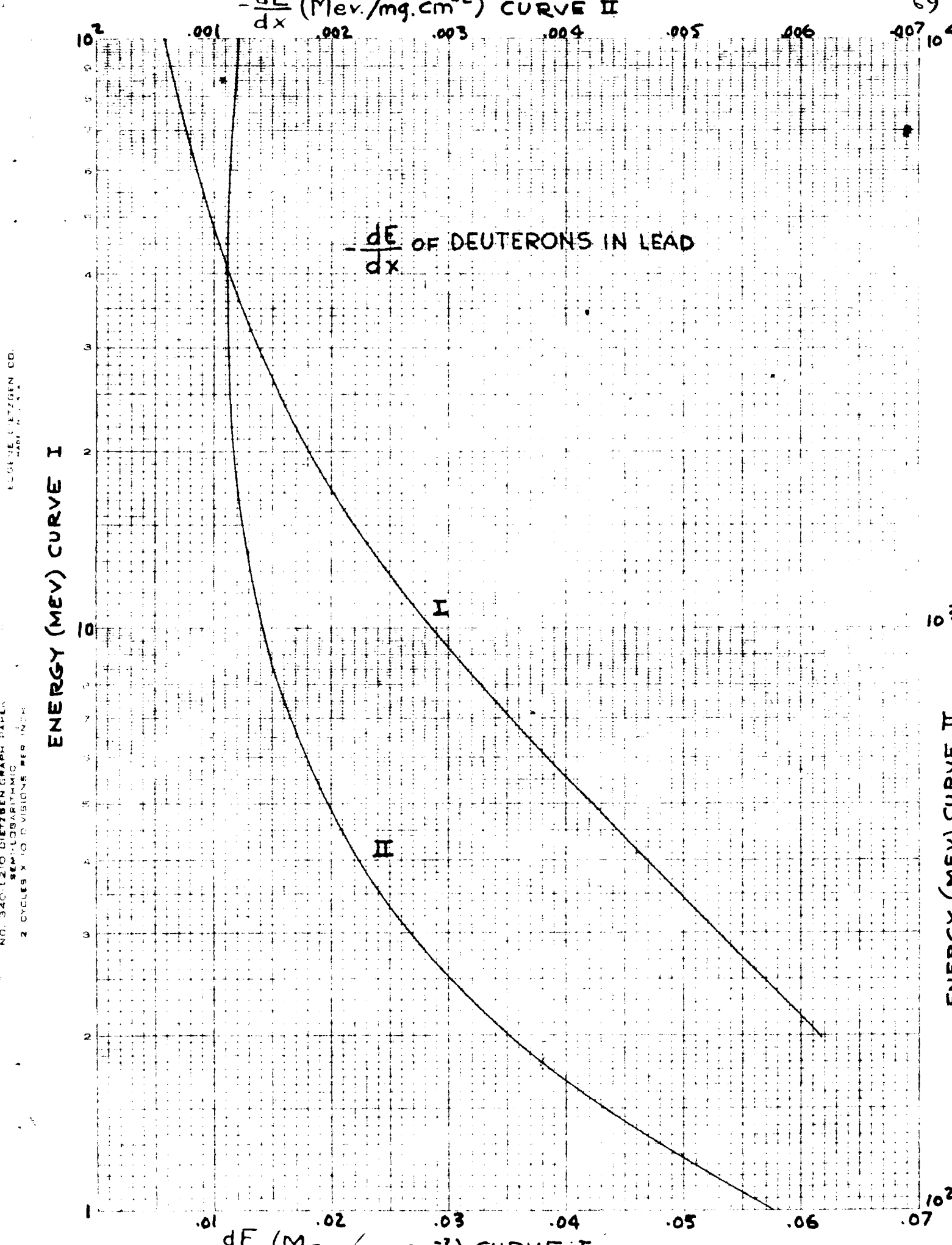
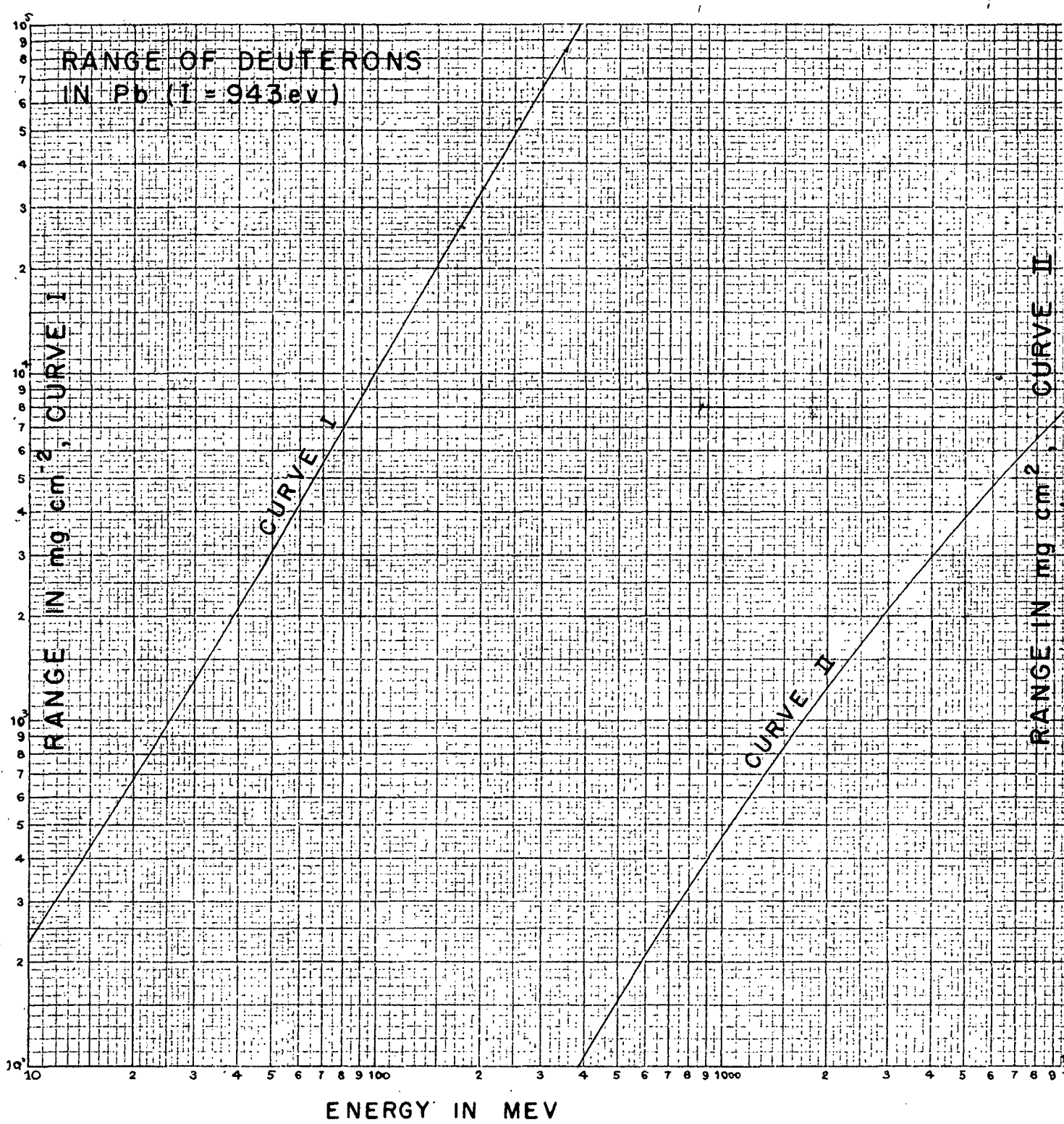
1650
770
270


$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.001
.002
.003
.004
.005
.006
.007
10^2
10^4
$-\frac{dE}{dx}$ OF DEUTERONS IN LEAD
ENERGY (MEV) CURVE I
10
1
I
II
.01
.02
.03
.04
.05
.06
.07
10^2


RANGE OF DEUTERONS
IN Pb (I = 943ev)
RANGE IN mg cm⁻², CURVE I
CURVE I
CURVE II
RANGE IN mg cm⁻², CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-120
Logarithmic, 3 x 3 cycles.
MADE IN U.S.A.

ALPHA-PARTICLE RANGES

Alpha-Particle Ranges

The range of alpha-particles in stopping materials may be calculated by integration of the equation

$$-\frac{dT}{dx} = \frac{4\pi e^4 z^2 NZ}{mv^2}\left[\log_e \frac{2mv^2}{I} - \log_e (1-\beta^2) - \beta^2\right] \qquad (1)$$

or, more approximately, from the proton ranges in the same stopping materials using the expression[1]

$$R_1\left(\frac{M_1}{M_2} E\right) = \frac{Z_2^2 M_1}{Z_1^2 M_2} R_2 (E) , \qquad (2)$$

which in our case reduces to,

$$R_1(3.9715 E) = .99287 R_2 (E) \qquad (3)$$

or, approximately,

$$R_\alpha (4E) = Rp (E) . \qquad (4)$$

The alpha-particle ranges which follow have been computed by equation (1) up to energies of about 200 Mev, and by equation (3) for energies $200 < E < 10{,}000$. Correction has been made for the dropping of electron shells in the stopping medium for low energies of the alpha-particle.

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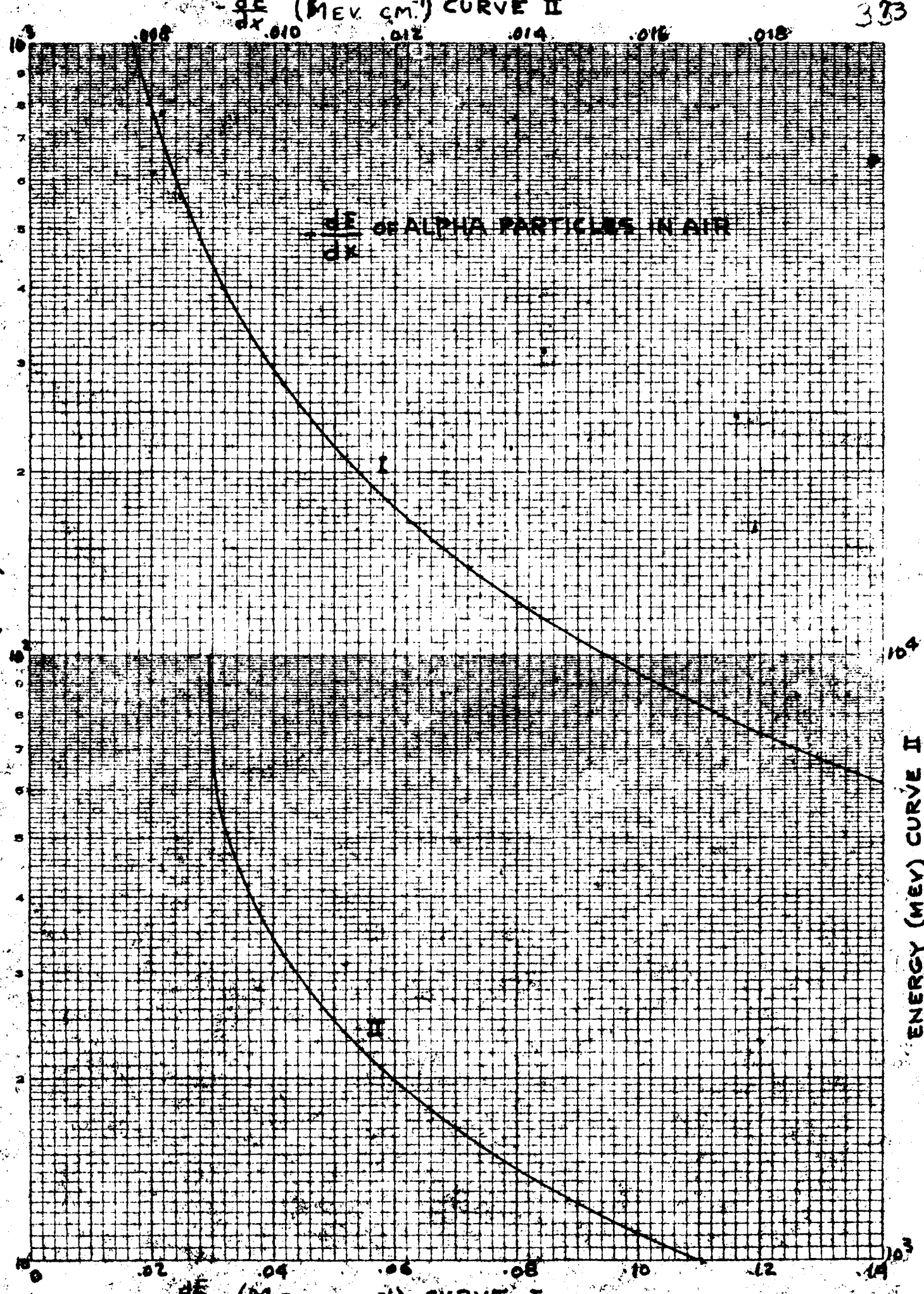
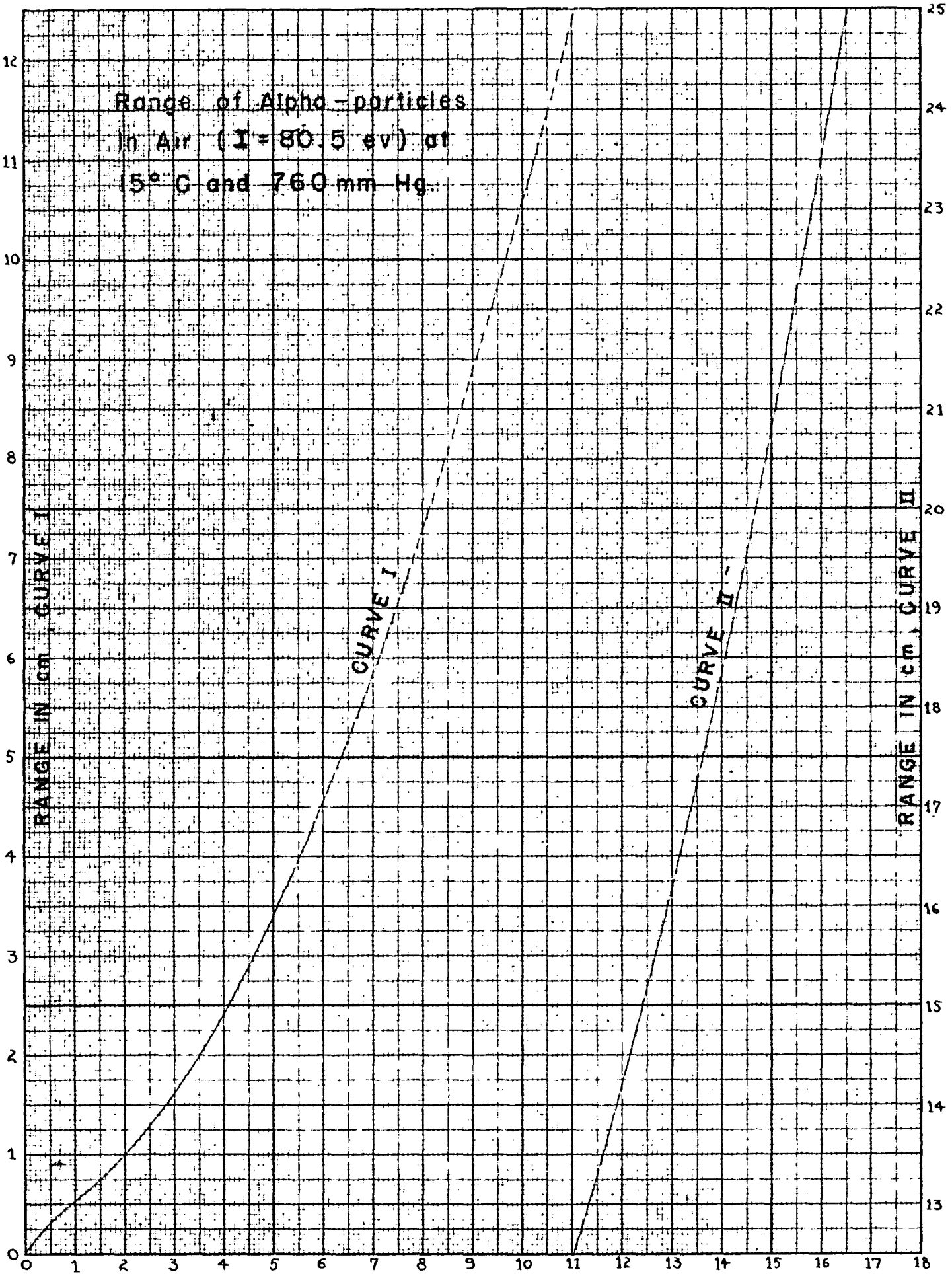
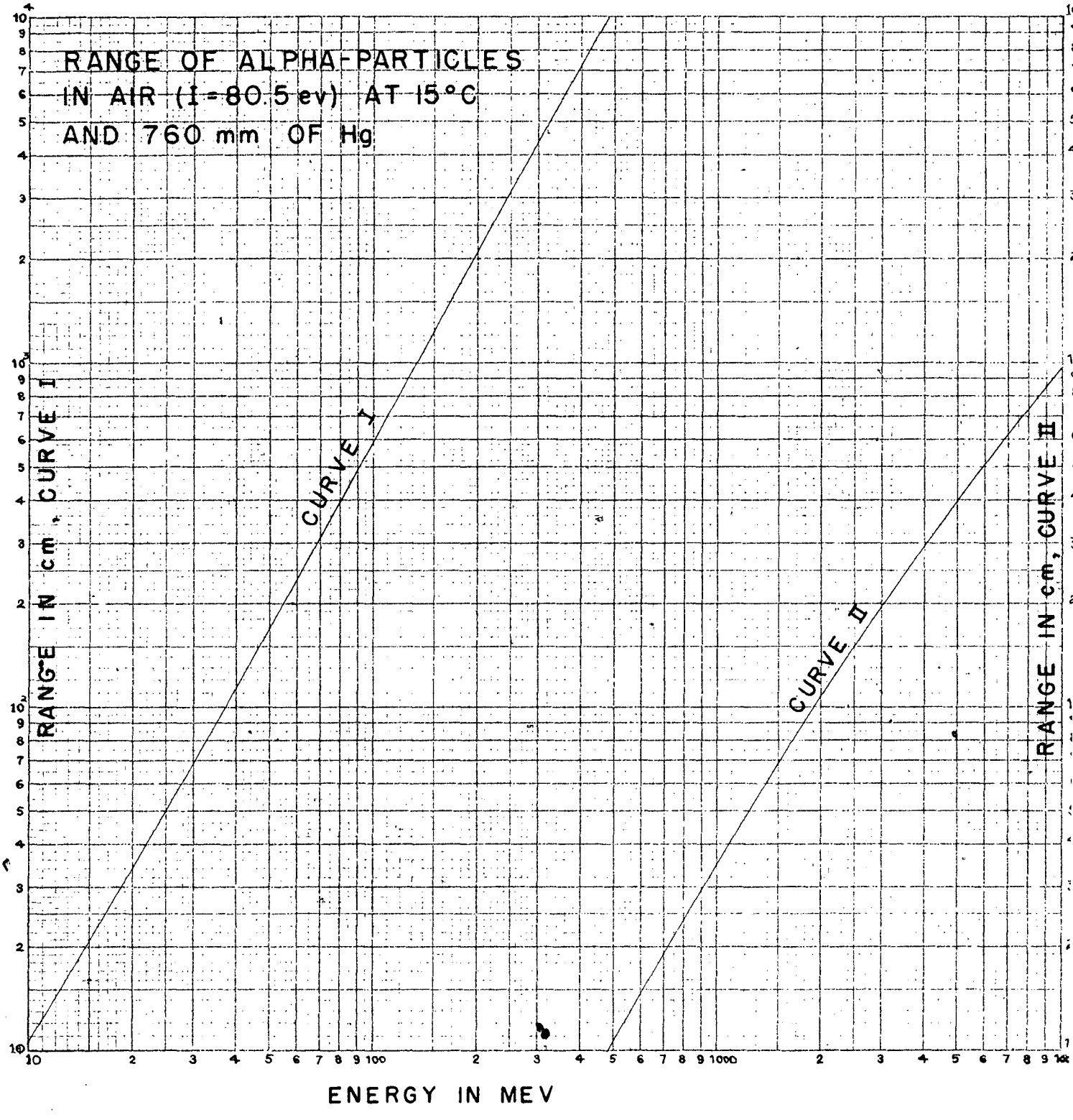
[1] See pages 57 to 58


$-\frac{dE}{dx}$ (MEV. CM.) CURVE II
.008
.010
.012
.014
.016
.018
$-\frac{dE}{dx}$ OF ALPHA PARTICLES IN AIR
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
10^4
10^3
0
.02
.04
.06
.08
.10
.12
.14
$\frac{dE}{dx}$ (MEV. CM.) CURVE I
EUGENE DIETZGEN CO.
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


Range of Alpha-particles
in Air (I = 80.5 ev) at
15° C and 760 mm Hg.
RANGE IN cm, CURVE I
RANGE IN cm, CURVE II
CURVE I
CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25



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Logarithmic, 3 cycles
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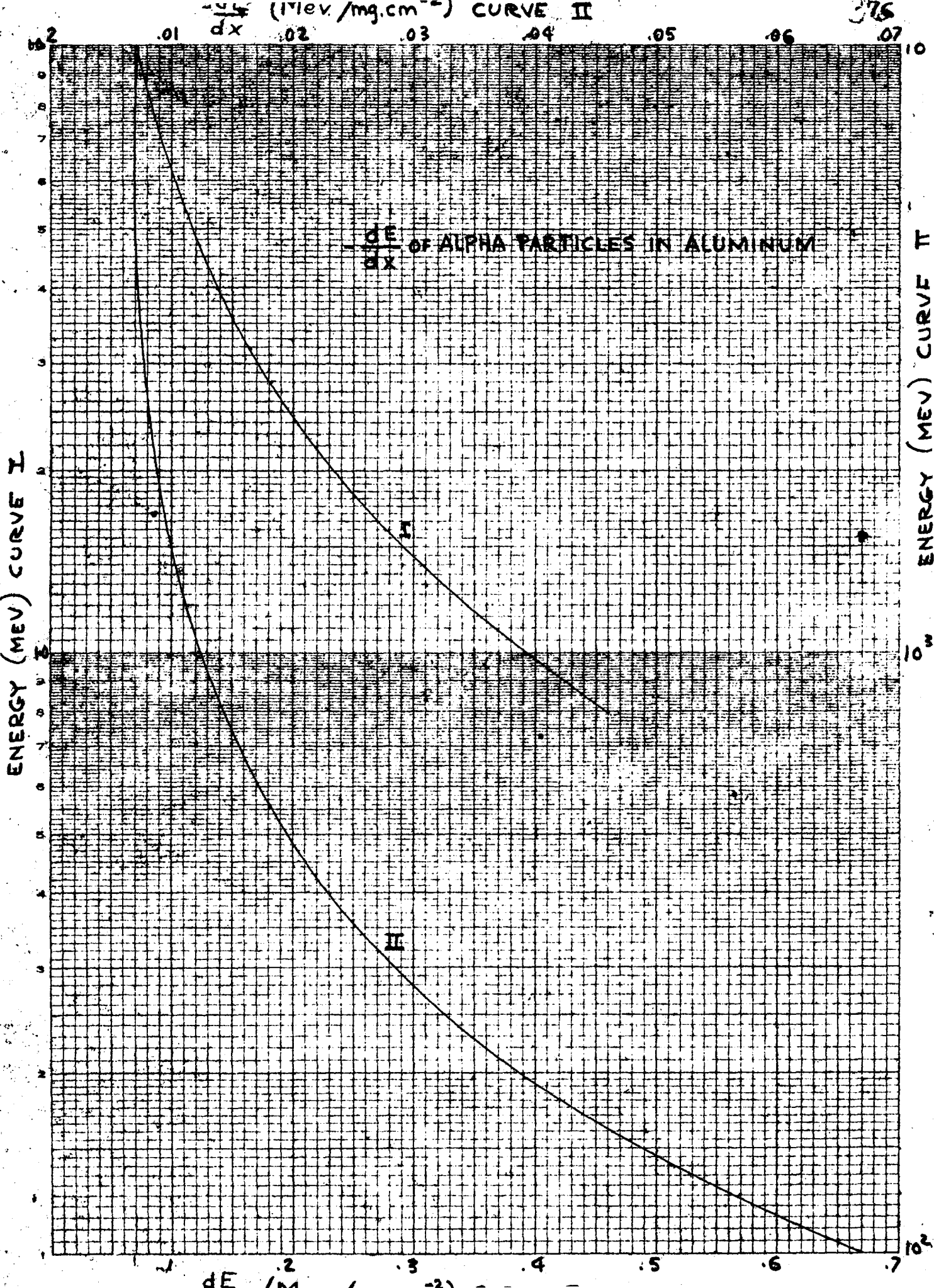
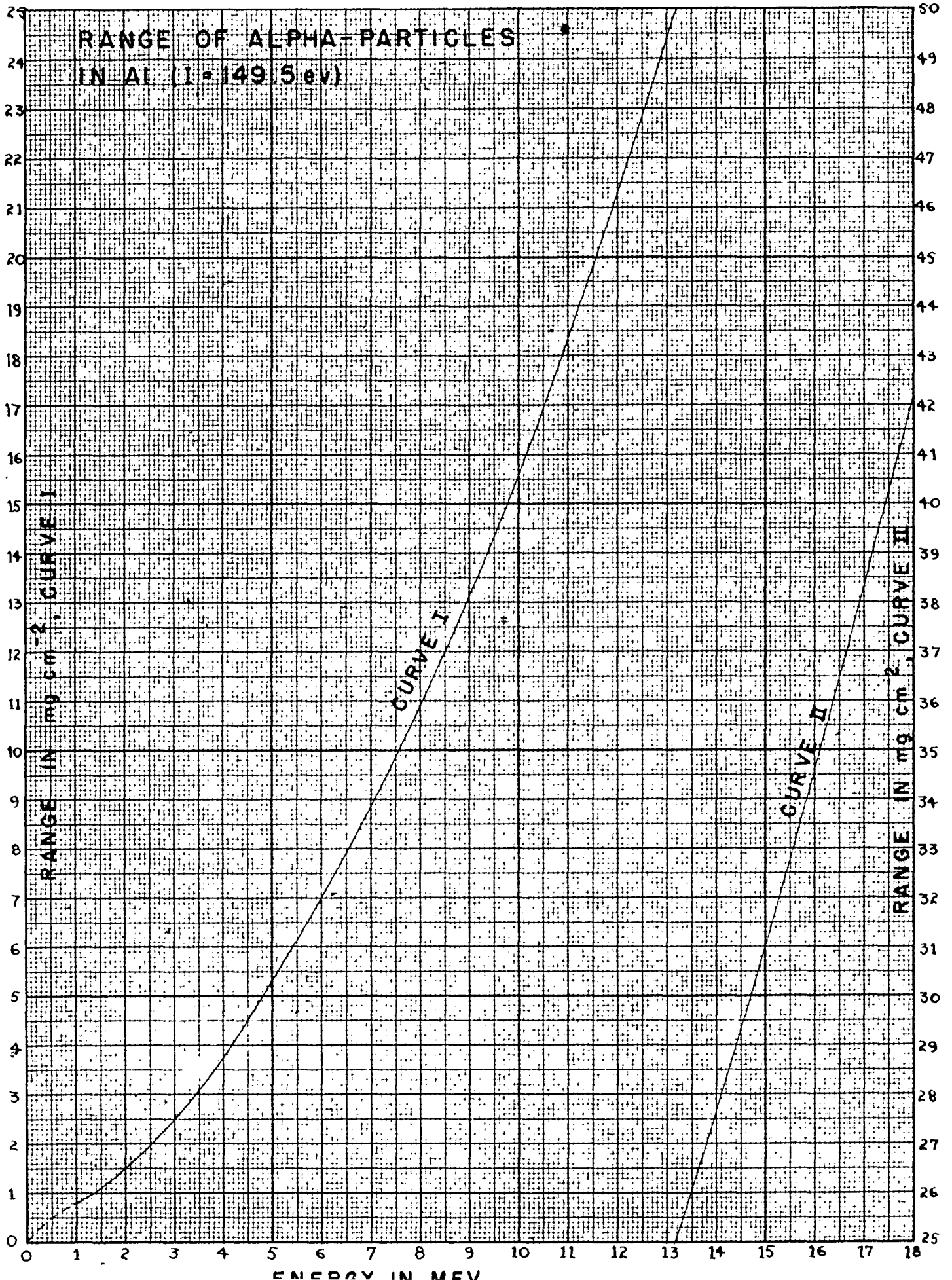
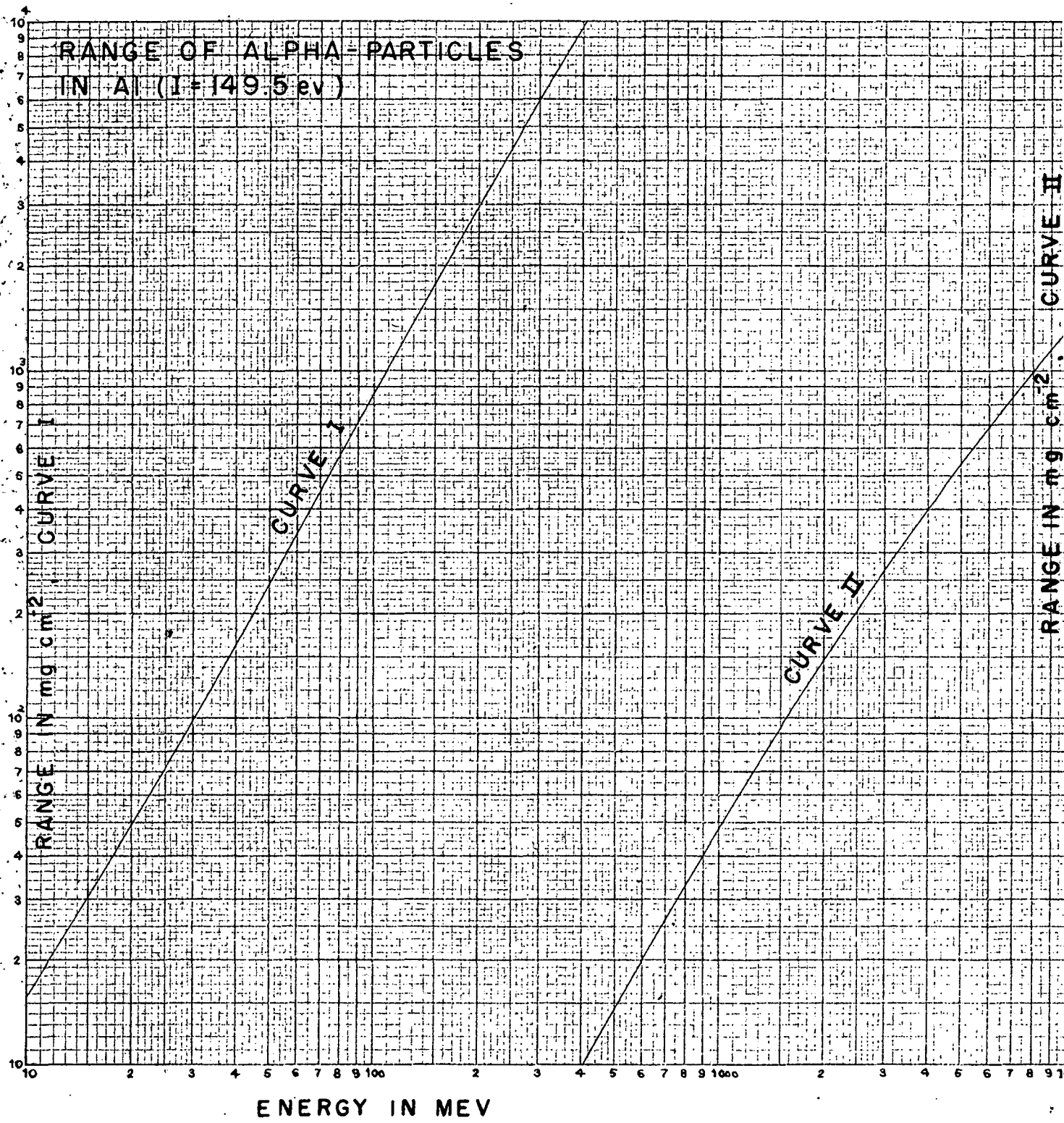

$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.01
.02
.03
.04
.05
.06
.07
$-\frac{dE}{dx}$ OF ALPHA PARTICLES IN ALUMINUM
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
10
10^2
.1
.2
.3
.4
.5
.6
.7
dE (Mev./mg.cm^{-2})


RANGE OF ALPHA-PARTICLES
IN AL (I = 149.5 ev)
CURVE I
CURVE II
RANGE IN mg cm⁻², CURVE I
RANGE IN mg cm⁻², CURVE II
ENERGY IN MEV
MILLIMETER



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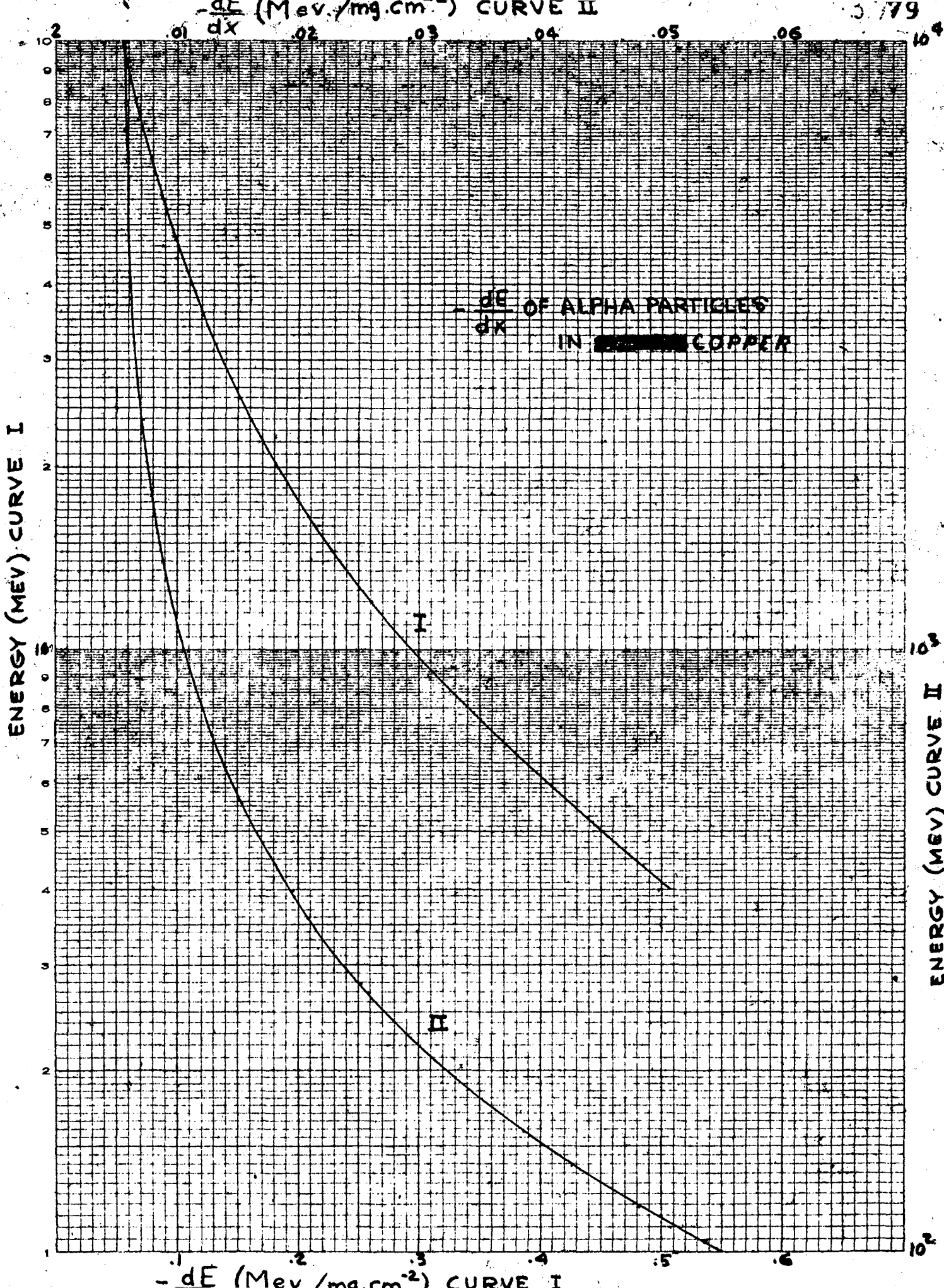

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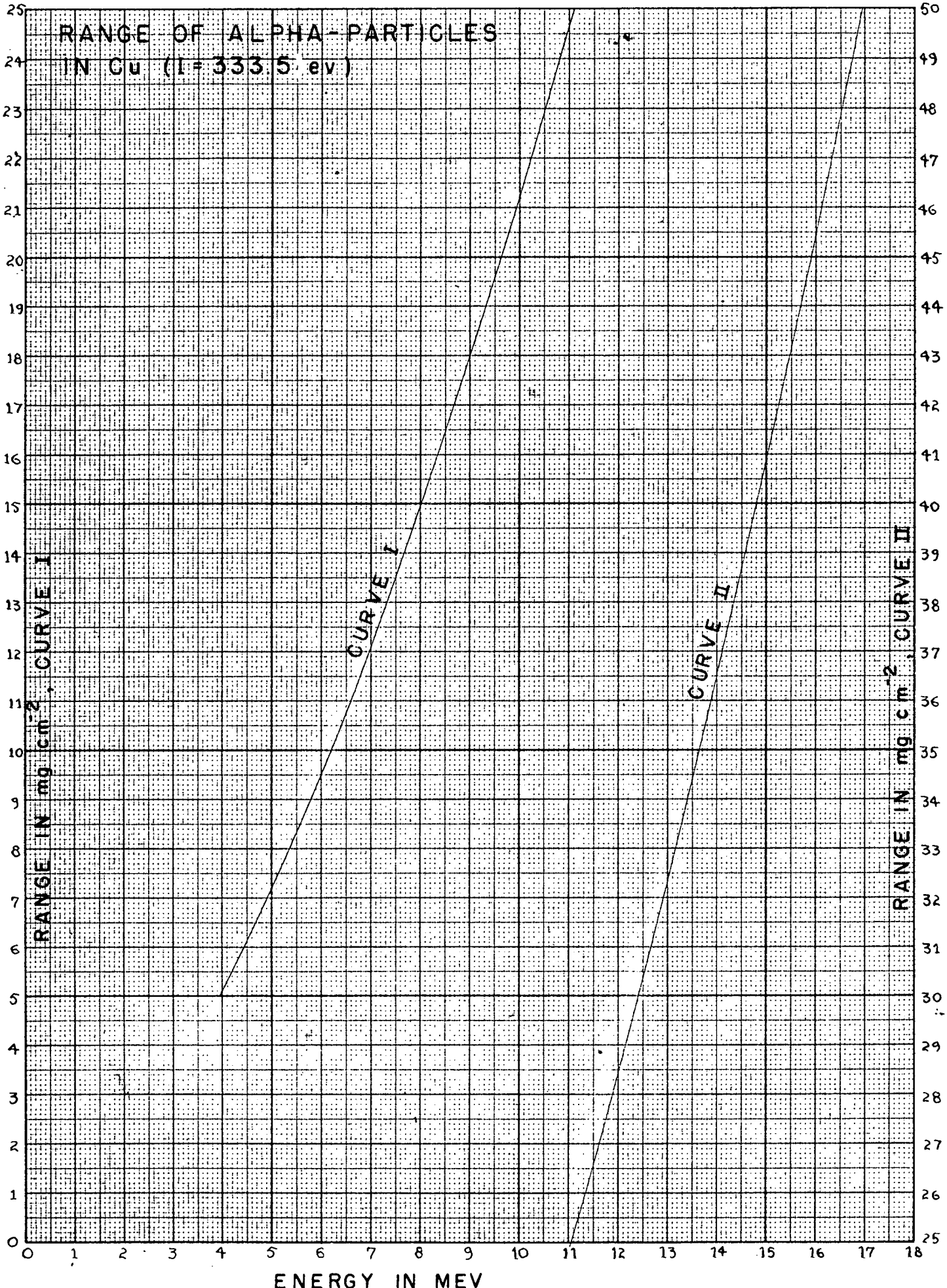
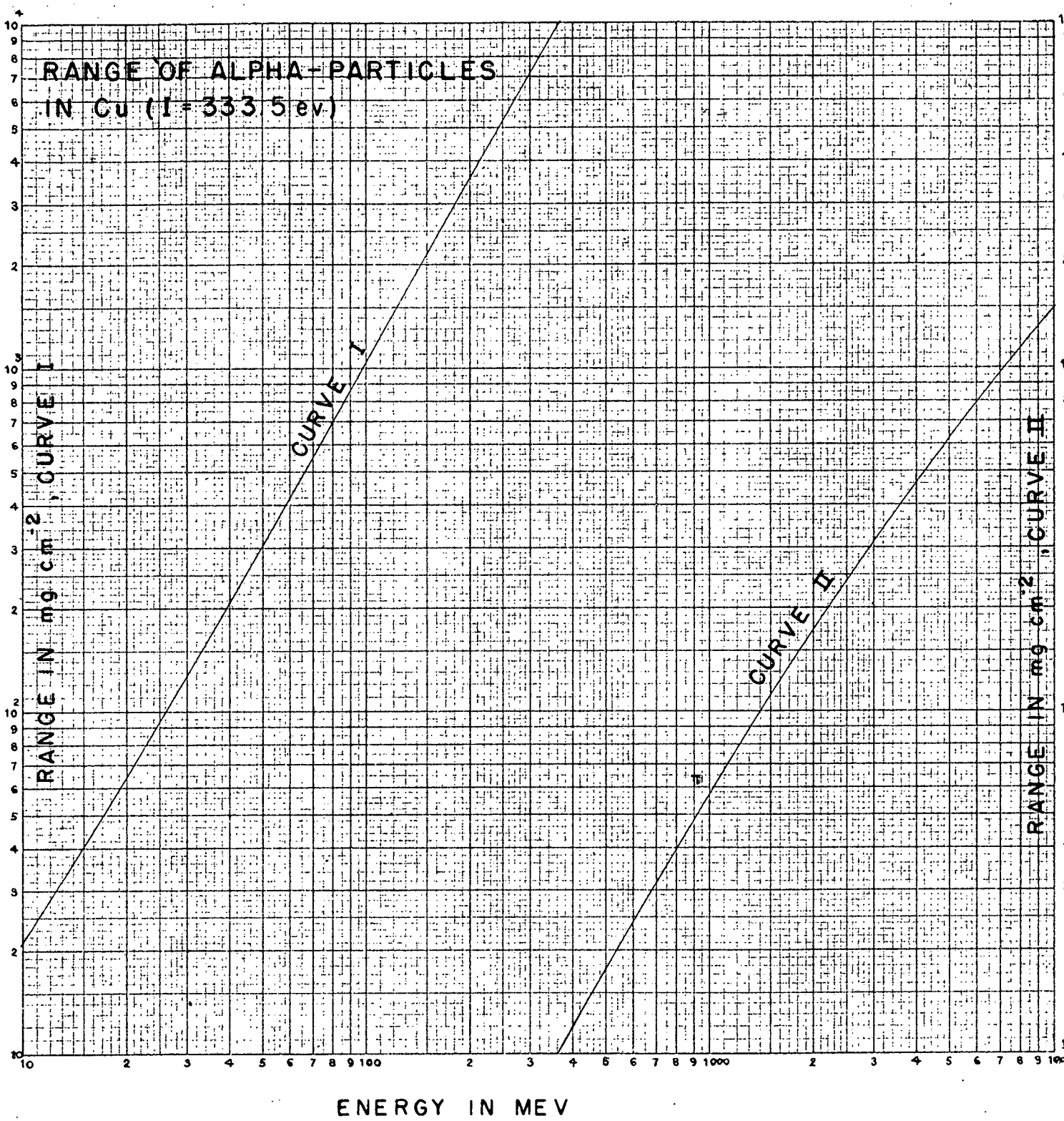
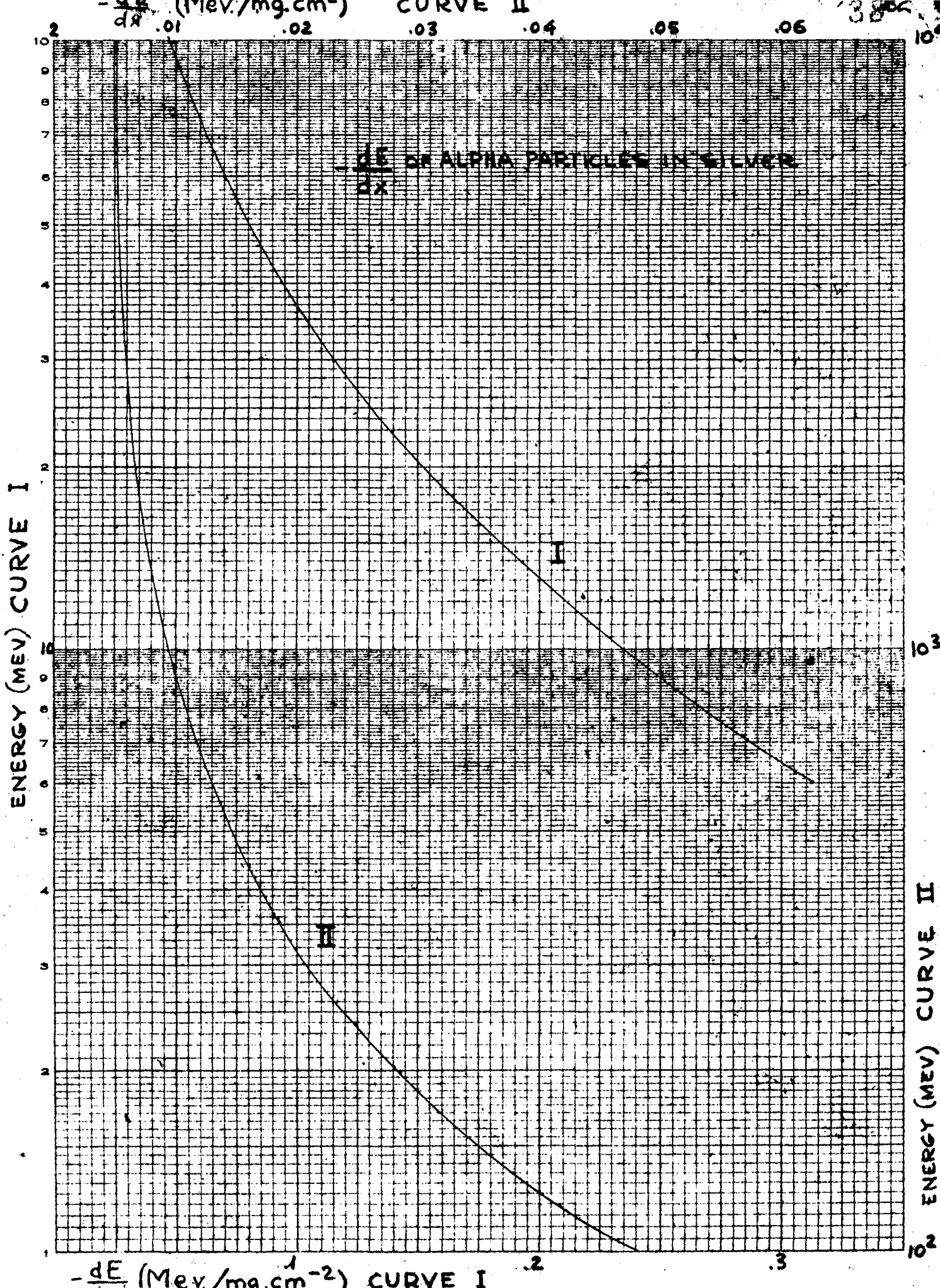
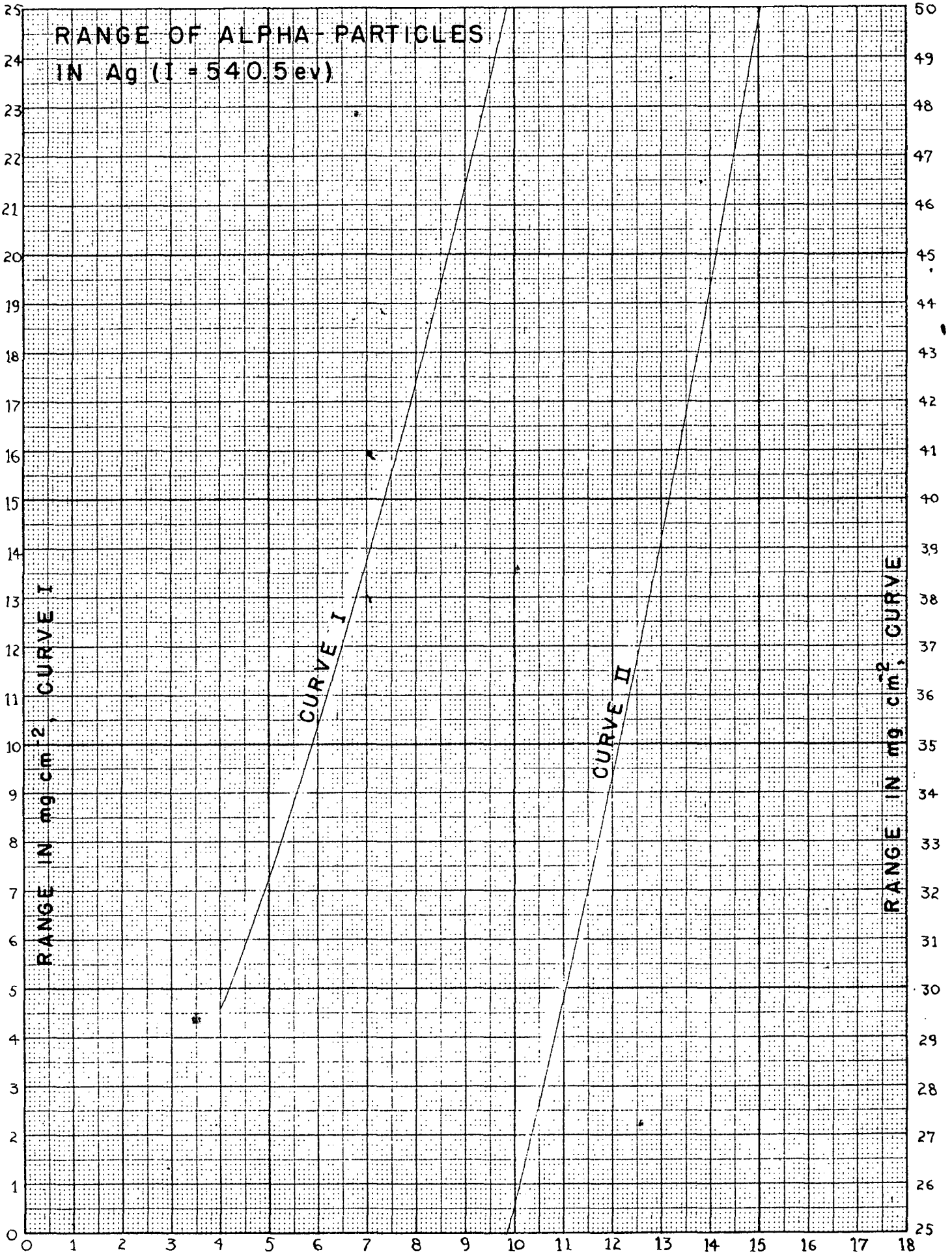
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH

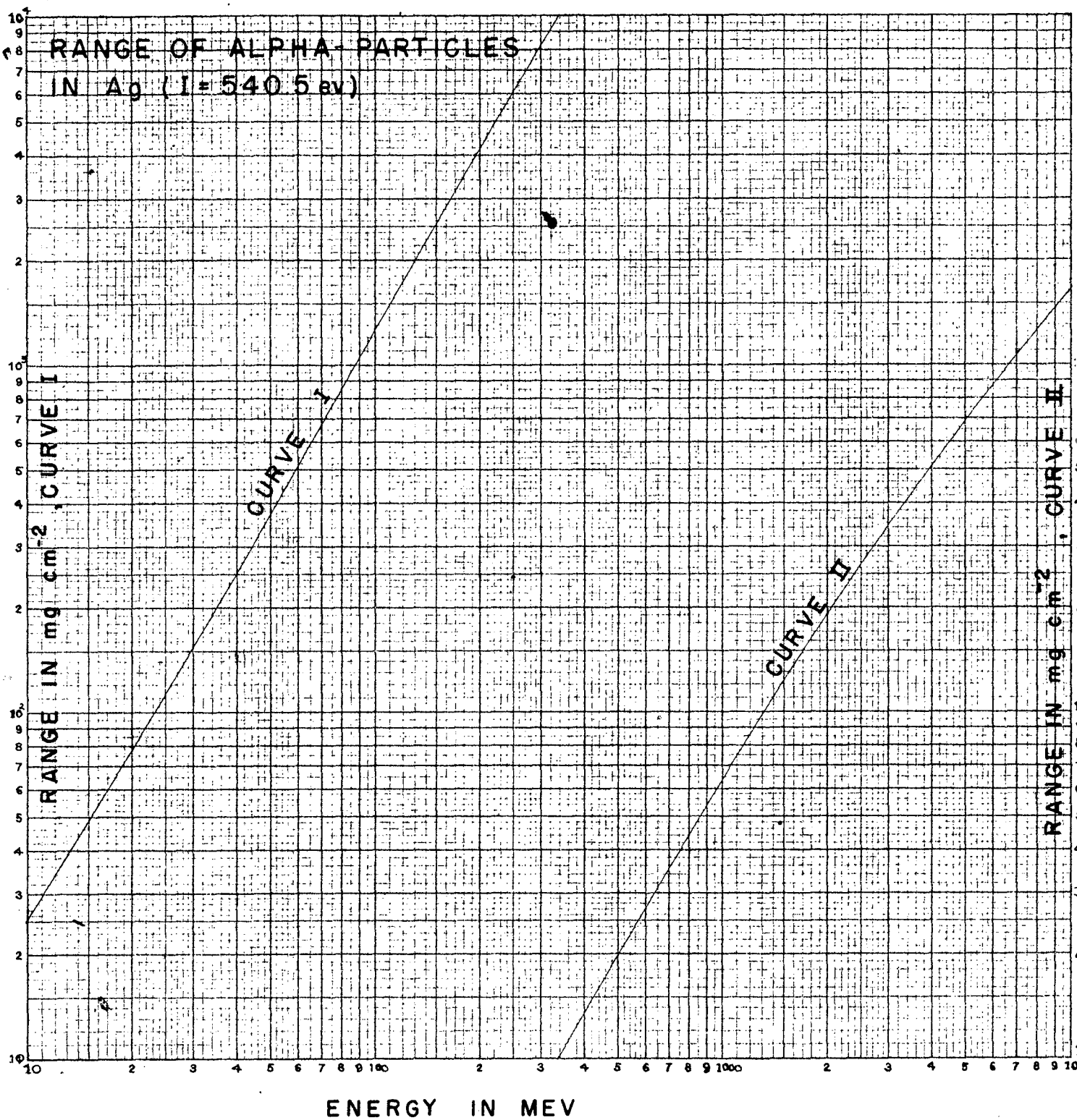

RANGE OF ALPHA-PARTICLES
IN Cu (I = 333.5 ev)
CURVE I
CURVE II
RANGE IN mg cm⁻², CURVE I
RANGE IN mg cm⁻², CURVE II
ENERGY IN MEV
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50

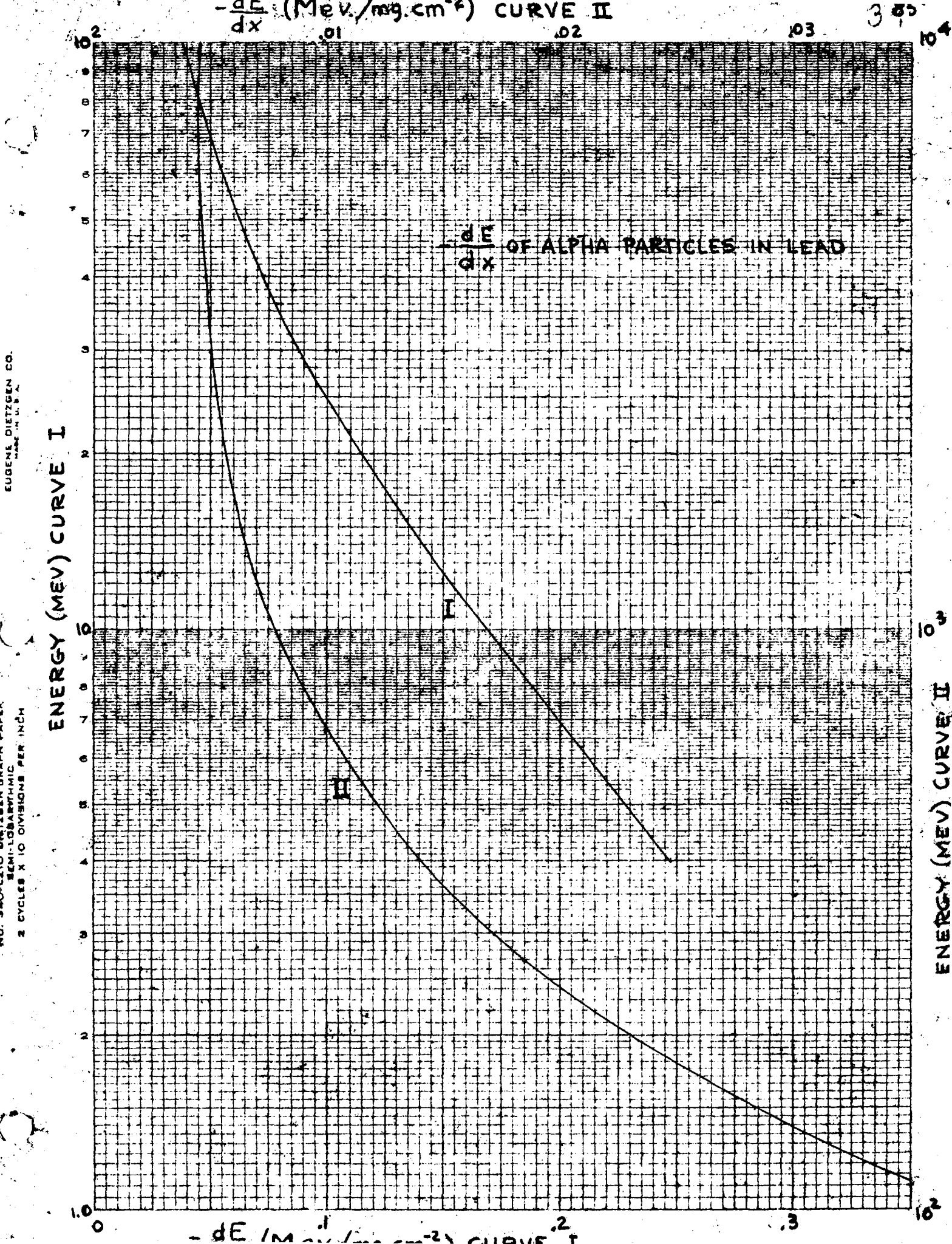


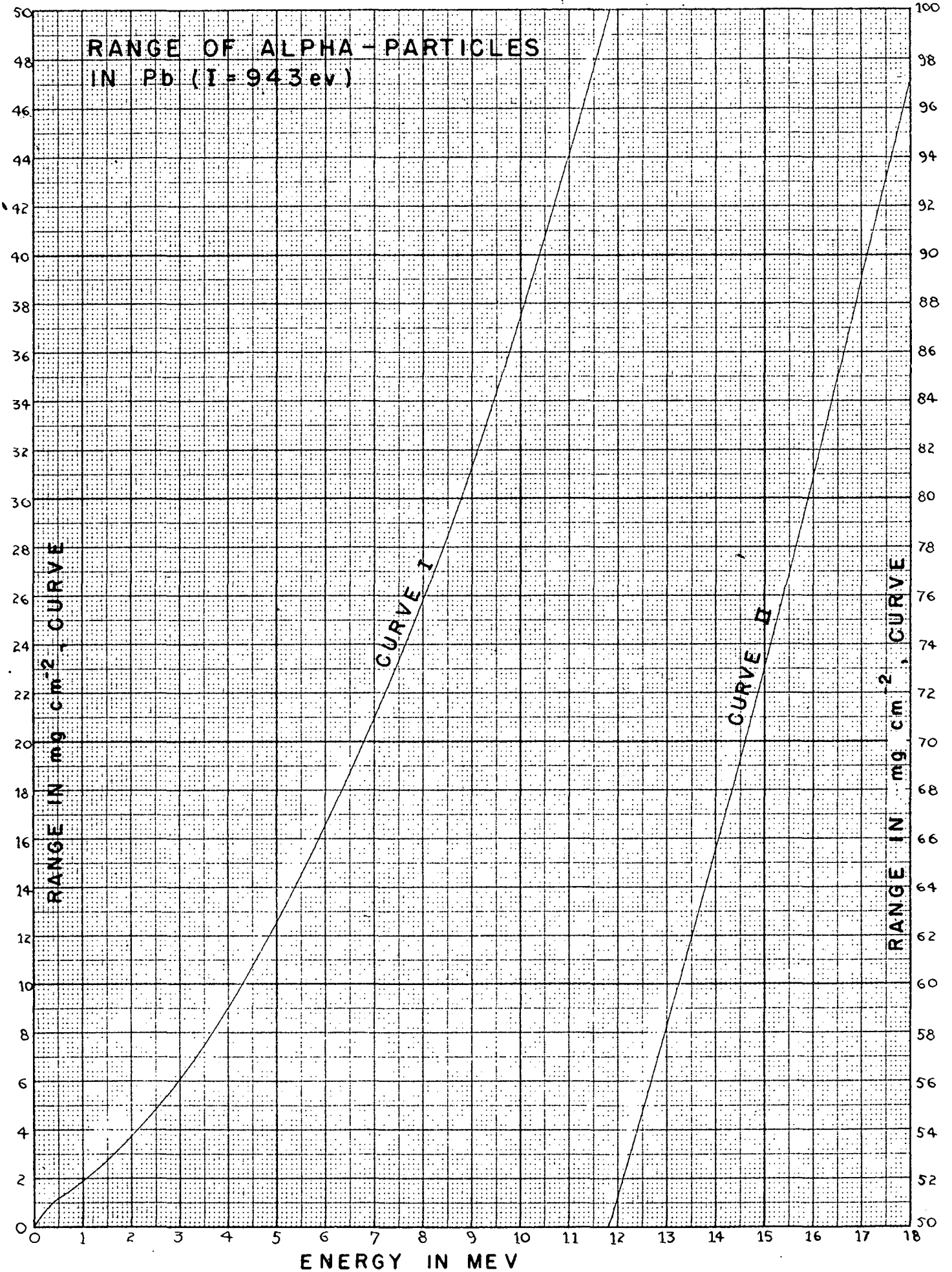
KEUFFEL & ESSER CO., N. Y. NO. 359-120G
Logarithmic, 3 X 3 Cycles.
MADE IN U. S. A.


$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.01
.02
.03
.04
.05
.06
$-\frac{dE}{dx}$ OF ALPHA PARTICLES IN SILVER
I
II
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
10^2
10^3
10^4
.1
.2
.3
$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE I
EUGENE DIETZGEN CO.
MADE IN U.S.A.
NO. 340-L210 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF ALPHA-PARTICLES
IN Ag (I = 540.5 ev)
RANGE IN mg cm⁻², CURVE I
RANGE IN mg cm⁻², CURVE
CURVE I
CURVE II
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50


RANGE OF ALPHA-PARTICLES
IN Ag (I = 540.5 ev)
RANGE IN mg cm-2, CURVE I
CURVE I
CURVE II
RANGE IN mg cm-2, CURVE II
ENERGY IN MEV
KEUFFEL & ESSER CO., N. Y. NO. 359-129G
Logarithmic, 3 X 3 Cycles
MADE IN U.S.A.


$-\frac{dE}{dx}$ (Mev./mg.cm^{-2}) CURVE II
.01
.02
.03
10^4
10^3
10^2
$-\frac{dE}{dx}$ OF ALPHA PARTICLES IN LEAD
ENERGY (MEV) CURVE I
ENERGY (MEV) CURVE II
I
II
10
1.0
0
.1
.2
.3
$-dE$ (Mev./mg.cm^{-2}) CURVE I
EUGENE DIETZGEN CO.
MADE IN U.S.A.
NO. 359-L10 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
2 CYCLES X 10 DIVISIONS PER INCH


RANGE OF ALPHA-PARTICLES
IN Pb (I = 943 ev)
CURVE I
CURVE II
RANGE IN mg cm⁻² CURVE I
RANGE IN mg cm⁻² CURVE II
ENERGY IN MEV
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
0
2
4
6
8
10
12
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16
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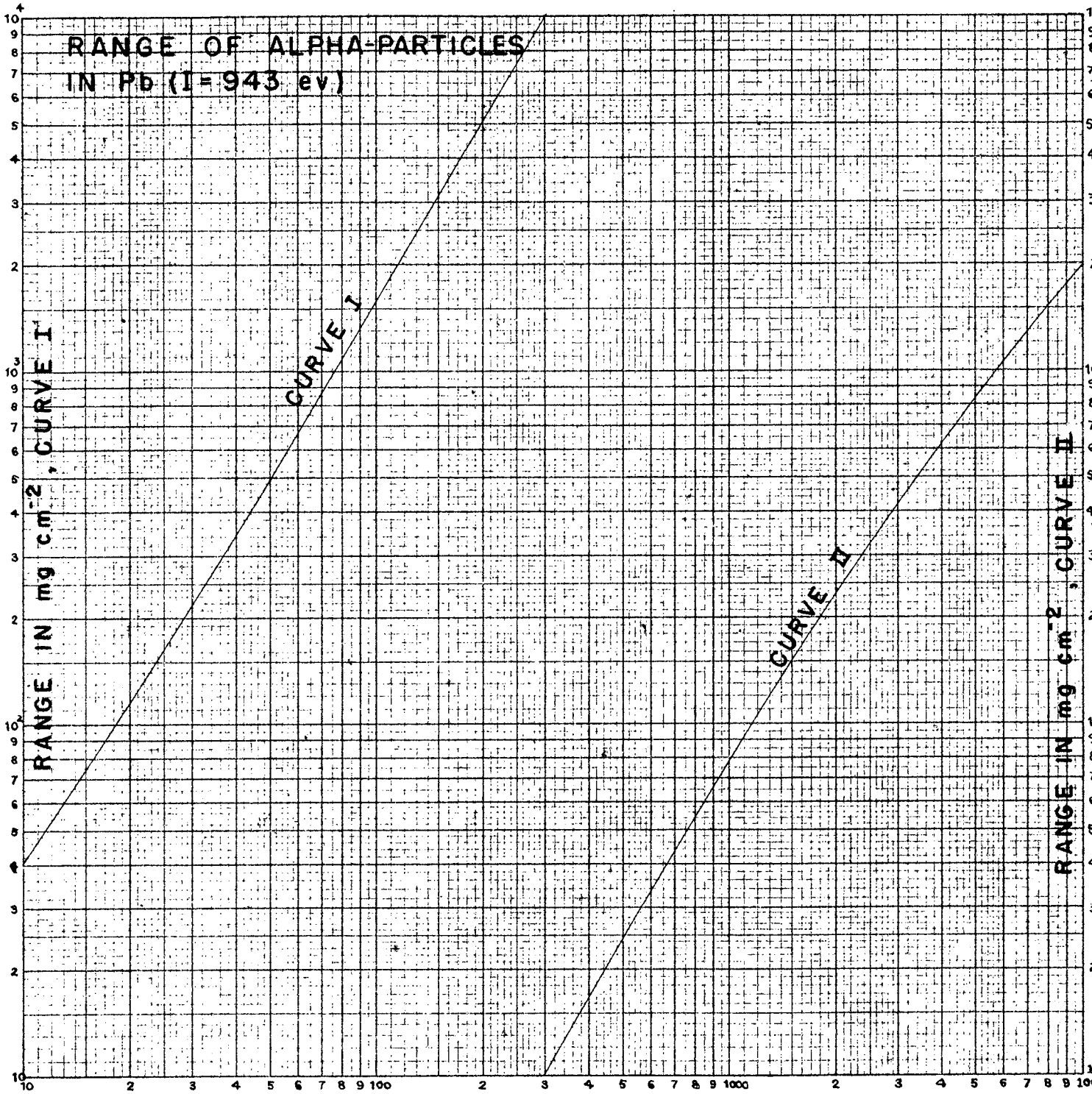
94
96
98
100



~~RANGE~~ IN MEV

Energy

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ELECTRON RANGES

Electron Ranges

The expression for the energy loss of electrons in a stopping medium has been discussed and developed by Bethe[1] and Serber[2] for the case in which energy loss by collisions only is considered, and is

$$-\frac{dT}{dx} = \frac{2\pi N Z e^4}{m v^2}\left[\log_e \frac{(mc^2)^2}{2I^2} + \log_e \beta^2\gamma^2 \frac{(E_r - mc^2)}{mc^2} - \left(\frac{2}{\gamma} - 1 + \beta\right)\log_e 2 + 1 - \beta^2\right] \qquad (1)$$

where

e = electronic charge,

m = electronic mass

NZ = number of electrons per unit volume of stopping material,

β = v/c; where v is the velocity of the electron, and c is the velocity of light,

$$\gamma = \frac{1}{\sqrt{1 - \quad ^2}}$$

$$E_r = mc^2(\gamma - 1) = mc^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right),$$

I = the mean excitation potential.

Making the substitution $u = \frac{T}{mc^2}$, where T is the kinetic energy, $(E_r = T - mc^2)$, equation (1) takes on the form

$$-\frac{dT}{dx} = \frac{2\pi N Z e^4}{mc^2 \frac{u^2+2u}{(1+u)^2}}\left[\log_e \frac{1}{2}\left(\frac{mc^2}{I}\right)^2 + \log_e\left[u(u^2+2u)\right] - \left[1 - \left(\frac{u}{1+u}\right)^2\right]\log_e 2 + \frac{1}{(1+u)^2}\right] \qquad (2)$$

The variable components of this equation have been tabulated for various electron energies (see table IX).

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[1] Bethe, H.: Quantenmechanik der Ein - und Zwei - Electronenprobleme; Handbuch der Physik, Vol. 24, pt. 1, Quantentheorie, pp. 519 - 523, 1933

[2] Serber, R.: Serber Says; University of California Radiation Laboratory, BP-96, 1947. pp. 14-20

Table IX. The relativistic equation for the energy loss of an electron going through matter and due to excitation and ionization by collision is given by

$$-\frac{dT}{dx} = \frac{2\pi N z e^4}{mc^2 \frac{u^2+2u}{(1+u)^2}} \left[\log_e \frac{1}{2}\left(\frac{mc^2}{I}\right)^2 + \log_e\left[u(u^2+2u)\right] - \left[1-\left(\frac{u}{1+u}\right)^2\right]\log_e 2 + \frac{1}{(1+u)^2} \right]$$

where $u = T/mc^2$, where m is the rest mass of the electron and T is the relativistic kinetic energy.

E in Mev	u	$\frac{u^2+2u}{(1+u)^2}$	$\log_e[u(u^2+2u)] - [1-(\frac{u}{1+u})^2]\log_e 2 + \frac{1}{(1+u)^2} + \log_e \frac{1}{2}(mc^2)^2$
.001	1.95760×10^{-3}	3.90373×10^{-3}	9.5142
.002	3.91520	7.78465	10.8976
.003	5.87280	1.16429×10^{-2}	11.7057
.004	7.83039	1.54787	12.2782
.005	9.78799	1.92923	12.7217
.006	1.17456×10^{-2}	2.30837	13.0835
.007	1.37032	2.68532	13.3891
.008	1.56608	3.06009	13.6534
.009	1.76184	3.43270	13.8862
.010	1.95760	3.80316	14.0943
.011	2.15336	4.17150	14.2822
.012	2.34912	4.53772	14.4536
.013	2.54488	4.90185	14.6111
.014	2.74064	5.26390	14.7567
.015	2.93640	5.62389	14.8921
.016	3.13216	5.98183	15.0187
.017	3.32792	6.33774	15.1374
.018	3.52368	6.69163	15.2492
.019	3.71944	7.04351	15.3549
.020	3.91520	7.39342	15.4550
.025	4.89399	9.11363	15.8894
.030	5.87280	1.07864×10^{-1}	16.2427
.035	6.85159	1.24133	16.5402
.040	7.83039	1.39962	16.7970
.045	8.80919	1.55366	17.0227
.050	9.78799	1.70359	17.2241
.055	1.07668×10^{-1}	1.84956	17.4058
.060	1.17456	1.99172	17.5714
.065	1.27244	2.13019	17.7234
.070	1.37032	2.26510	17.8639
.075	1.46820	2.39657	17.9946
.080	1.56608	2.52472	18.1168
.085	1.66396	2.64965	18.2315
.090	1.76184	2.77148	18.3397
.095	1.85972	2.89030	18.4418
.100	1.95760	3.00622	18.5388

E in Mev	u	$\frac{u^2 + 2u}{(1+u)^2}$	$\log_e[u(u+2u)] - [1-(\frac{u}{1+u})^2]\log_e 2 + \frac{1}{(1+u)^2} + \log_e \frac{1}{2}(mc^2)^2$
.150	2.93640	4.02452	19.3087
.200	3.91520	4.83558	19.8639
.250	4.89400	5.49207	20.3046
.300	5.87280	6.03089	20.6739
.350	6.85159	6.47858	20.9943
.400	7.83039	6.85458	21.2787
.450	8.80919	7.17343	21.5353
.500	9.78799	7.44614	21.7697
.550	1.07668×10^{0}	7.68121	21.9859
.600	1.17456	7.88526	22.1867
.650	1.27244	8.06351	22.3744
.700	1.37032	8.22014	22.5508
.750	1.46820	8.35851	22.7172
.800	1.56608	8.48134	22.8747
.850	1.66396	8.59089	23.0243
.900	1.76184	8.68899	23.1668
.950	1.85972	8.77721	23.3029
1.000	1.95760	8.85680	23.4331
1.50	2.93640	9.35464	24.4973
2.00	3.91520	9.58608	25.2845
2.50	4.89400	9.71214	25.9094
3.00	5.87280	9.78829	26.4274
3.50	6.85159	9.83779	26.8697
4.00	7.83039	9.87176	27.2554
4.50	8.80919	9.89607	27.5975
5.00	9.78799	9.91408	27.9046
5.50	1.07668×10^{1}	9.92778	28.1834
6.00	1.17456	9.93844	28.4385
6.50	1.27244	9.94691	28.6753
7.00	1.37032	9.95374	28.8919
7.50	1.46820	9.95934	29.0953
8.00	1.56608	9.96397	29.2858
8.50	1.66396	9.96786	29.4677
9.00	1.76184	9.97115	29.6340
9.50	1.85972	9.97396	29.7941
10.00	1.95760	9.97638	29.9460
15.0	2.93640	9.98915	31.1484
20.0	3.91520	9.99380	32.0067
25.0	4.89400	9.99599	32.6732
30.0	5.87280	9.99720	33.2187
35.0	6.85159	9.99793	33.6796
40.0	7.83039	9.99841	34.0790
45.0	8.80919	9.99874	34.4316
50.0	9.78799	9.99898	34.7468
55.0	1.07668×10^{2}	9.99915	35.0321
60.0	1.17456	9.99929	35.2927
65.0	1.27244	9.99939	35.5324
70.0	1.37032	9.99948	35.7548
75.0	1.46820	9.99956	35.9587
80.0	1.56608	9.99960	36.1544

E in Mev	u	$\frac{u^2 + 2u}{(1+u)^2}$	$\log_e[u(u^2+2u)] - \left[1-\left(\frac{u}{1+u}\right)^2\right]\log_e 2 + \frac{1}{(1+u)^2} + \log_e \frac{1}{2}(mc^2)^2$
85	1.66396	9.99964	36.3360
90	1.76184	9.99968	36.5073
95	1.85972	9.99971	36.6693
100	1.95760	9.99974	36.8230
110	2.15336	9.99979	37.1087
120	2.34912	9.99982	37.3695
130	2.54488	9.99985	37.6094
140	2.74064	9.99987	37.8316
150	2.93640	9.99989	38.0384
160	3.13216	9.99990	38.2319
170	3.32792	9.99991	38.4136
180	3.52368	9.99992	38.5850
190	3.71944	9.99993	38.7471
200	3.91520	9.99994	38.9009
210	4.11096	9.99994	39.0472
220	4.30672	9.99995	39.1867
230	4.50248	9.99995	39.3200
240	4.69824	9.99995	39.4476
250	4.89400	9.99996	39.5700
260	5.08976	9.99996	39.6876
270	5.28552	9.99996	39.8008
280	5.48128	9.99997	39.9099
290	5.67704	9.99997	40.0151
300	5.87280	9.99997	40.1168
310	6.06856	9.99997	40.2151
320	6.26431	9.99997	40.3103
330	6.46007	9.99998	40.4026
340	6.65583	9.99998	40.4921
350	6.85159	9.99998	40.5791
360	7.04735	9.99998	40.6635
370	7.24311	9.99998	40.7457
380	7.43888	9.99998	40.8257
390	7.63463	9.99998	40.9036
400	7.83039	9.99998	40.9795

$mc^2 = 0.51083$ Mev

$\log_e \frac{1}{2}(mc^2)^2 = 20.9892$

I to be entered in ev.

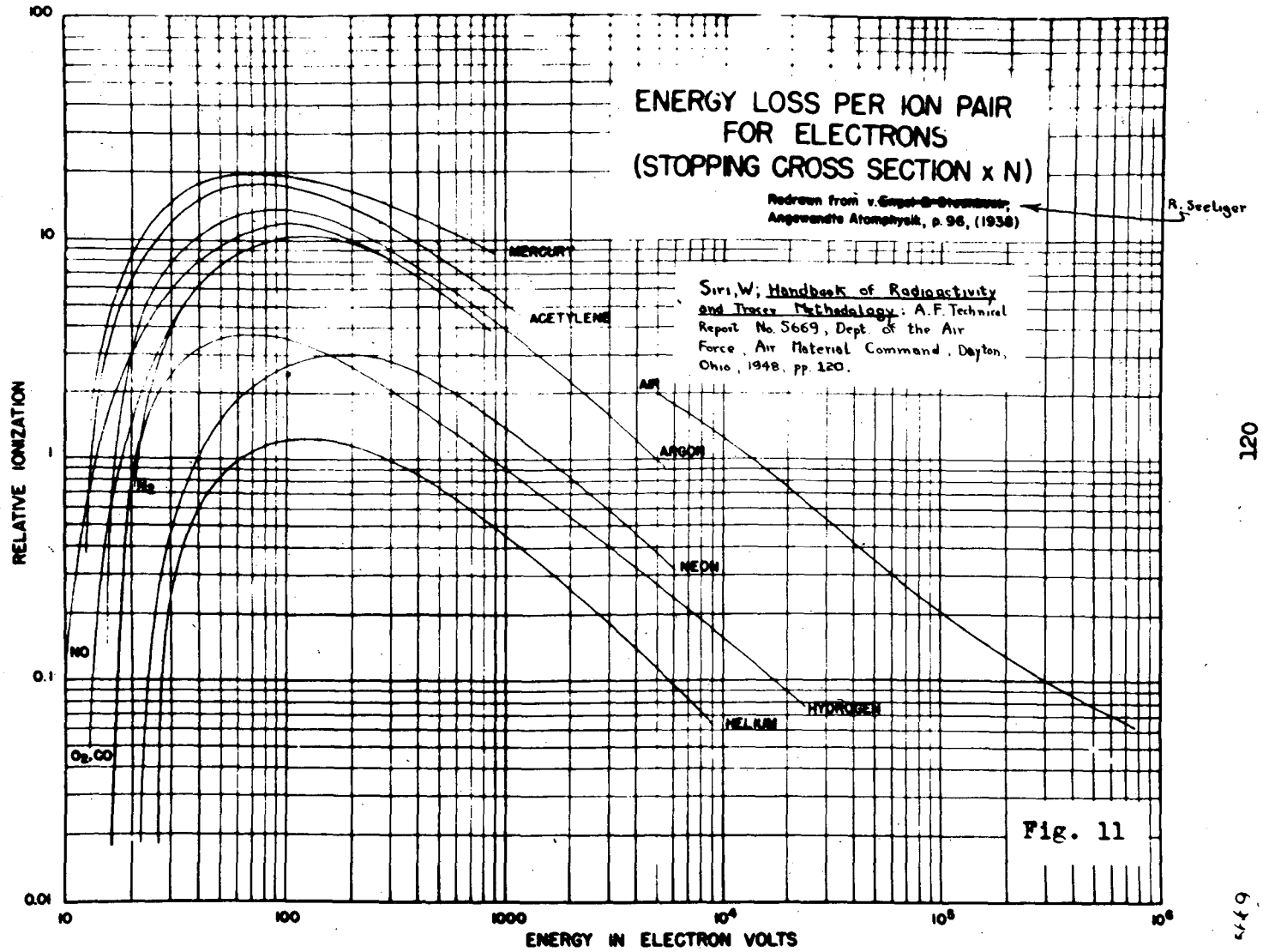

Fig. 11

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