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THE PASSAGE OF CHARGED PARTICLES THROUGH MATTER

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I. INTRODUCTION

The development of high energy charged particle accelerators has necessitated the extension of previous calculations on the penetration of charged particles through matter. In particular, calculations for solids were required, since only solid materials will give sizeable effects in equipment of reasonable dimensions.

The calculations presented here are concerned with the passage of protons through matter. No calculations for other particles have been made. The results for protons can easily be converted for other heavy singly charged particles. For multiply charged particles, the conversion is possible for approximate values but requires a small low energy correction for exact values.

In the range of energies considered, processes other than ionization and excitation of the atoms of the material seldom occur. Only the latter processes have been taken into account here. The rate of energy loss and the residual range for protons have been found as a function of energy.

Preliminary calculations were compared with experimental values. Out of this comparison, a better value of the mean excitation potential was obtained. The final results were calculated with these improved values.

II. THEORETICAL DEVELOPMENT

The rate of energy loss of heavy charged particles has been calculated by various authors[1,2,3,4,5]. At high energies it can be written in the form:

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2 N}{mv^2} B \qquad (1)$$

$$B = Z\left[\ln \frac{2mv^2}{I} - \ln(1-\beta^2) - \beta^2\right] \qquad (2)$$

where e is the electron charge, ze the charge of the incident particle, m the electron mass, v the magnitude of the incident particle's velocity, N the number of atoms of the stopping material per unit volume, and B the so-called "stopping number". In the formula for B, Z is the atomic number of the stopping material, $\beta = v/c$, c is the velocity of light and I is the mean excitation potential for an atom of the stopping material. These formulae apply to a single element. The generalization to mixtures and compounds will be discussed later.

At low energies, the probability of occurrence of some atomic transitions decreases or vanishes and the corresponding energy losses by the particle become impossible. Hence the rate of energy loss decreases and no longer has the simple form given

in (2). The correction for the K shell has been given by Bethe[6] for light elements.

$$B = Z\left[\ln \frac{2mv^2}{I} - \ln(1-\beta^2) - \beta^2\right] - C_K(\eta) \tag{3a}$$

or

$$B = (Z - 1.81)\left[\ln \frac{2mv^2}{I'} - \ln(1-\beta^2) - \beta^2\right] + B_K(\eta) \tag{3b}$$

where

$$\eta = \frac{2mv^2}{Z_{eff}^2\, Ry} \quad , \qquad Z_{eff} = Z - 0.3$$

$$Z \ln I = (Z - 1.81)\ln I' + 1.81 \ln I_K$$

$$I_K = 1.103\, Z_{eff}^2\, Ry \quad .$$

The form of the correction used in (3b) represents the total stopping power of an atom as the sum of the stopping powers due to each shell individually. The form of (3a) represents the correction for large values of η. The values of the correction factors are given graphically*.

For heavy elements the corrections have been made in another way. Here the K, L, and M shell corrections may all be involved. To calculate the form of B at low velocities,

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* L. Brown[7] recently has given the asymptotic form of the correction for a wider range of elements.

the original definition of $\ell n\, I$ was used. If f_j are the oscillator strengths corresponding to a transition with energy I_j,

$$Z\ \ell n\, I = \sum f_j\, \ell n\, I_j$$

subject to $Z = \sum f_j$. Then B can be written

$$B = \sum{}' f_j \left[\ell n \frac{2mv^2}{I_j} - \ell n\,(1 - \beta^2) - \beta^2 \right] \tag{4}$$

The use of $\sum'$ is to indicate that the sum extends only over values of j for which $\left[\ell n \frac{2mv^2}{I} - \ell n\,(1 - \beta^2) - \beta^2 \right]$ is positive. This method is equivalent to defining a new Z^* and I^* by

$$\left.\begin{aligned} Z^*\ \ell n\, I^* &= Z\, \ell n\, I - \sum{}'' f_j\ \ell n\, I_j \\ Z^* &= Z - \sum{}'' f_j \end{aligned}\right\} \tag{5a}$$

$\sum''$ indicates a sum over the values of j omitted in (4). Z^* and I^* are functions of the velocity. Then the stopping number takes the form

$$B = Z^* \left[\ell n \frac{2mv^2}{I^*} - \ell n\,(1 - \beta^2) - \beta^2 \right] \tag{5b}$$

This method corresponds roughly to the form of the K shell correction given in (3b).

The corrections as described are not sufficient to describe the behavior at very low velocities. In particular, no account is taken of the possibility of charge pick-up by the particle. For this reason the calculations were started at energies of the order of 1 Mev and the behavior of the particles below that energy derived from experiments.

The problem of determining I at high energies still remains. In principle one might hope to calculate it from its definition and from measured or calculated values of the oscillator strengths and energies. This is not feasible for most elements due to the complexity of the problem. However, Bloch[5] has shown, by a calculation based on the Fermi-Thomas model of the atom, that for elements of sufficiently large atomic number, I should be proportional to Z. The values of I and of the constant of proportionality have been measured experimentally[8,9,10,11]. The value of the constant used in the original calculations was that of R. R. Wilson. He measured the constant for aluminum and found a value

$$I = Z \times 11.5 \text{ ev.} \tag{6}$$

For hydrogen and helium, the evaluation of I has been done theoretically by E. J. Williams[12], yielding 17.5 ev. and 44 ev. respectively.

At high velocities ($v \simeq c$) equations (1) and (2) would indicate an increase in the rate of energy loss which is

logarithmic in the energy of the particle. It has been shown[13,14,15,16], however, that at these velocities the effect of the density of the material must be taken into account. For dense materials, the polarization of the material due to its dielectric constant decreases the effectiveness of the electric force and thus decreases the rate of energy loss. However, this effect is not important for the energies considered here.

Some work has been done on the passage of particles through mixtures and compounds. Such calculations are particularly useful in two fields. The use of photographic plates often requires knowledge of particle ranges, etc. The second field of application is in the use of hydrocarbons for the study of nuclear effects on hydrogen by the method of differences.

In making these calculations it was assumed that the stopping power of an atom was unaltered by the proximity of other atoms or by the atom being bound in a molecule. Thus, the stopping power of the material becomes the sum of the stopping powers of its constituents. This assumption of additivity has not been justified accurately. To do this would require a detailed study of the shifts in energy levels due to molecular binding. However, for most electrons the shifts in the levels should be small compared to the energies of the atomic levels so the effects should be small.

With this assumption equations (1) and (2) become

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{mv^2} \sum N_j B_j \tag{7}$$

$$B_j = Z_j \left[\ln \frac{2mv^2}{I_j} - \ln (1 - \beta^2) - \beta^2 \right] \tag{8}$$

where N_j is the number per unit volume, Z_j the atomic number and I_j the mean excitation potential of atoms of kind j. For some purposes of calculation it was convenient to define a mean atomic number and mean excitation potential by

$$Z_{mean} = \sum N_j Z_j / \sum N_j \tag{9a}$$

$$Z_{mean} \ln I_{mean} = \sum N_j Z_j \ln I_j / \sum N_j \tag{9b}$$

Using this, the relation for the rate of energy loss becomes the same as in equations (1) and (2) but with Z_{mean}, and I_{mean} being used for Z and I. Where shell corrections were required, they were made in the manner outlined for heavy elements (equations (4) and (5)). This scheme of correction causes no essential changes in the form of (8).

In addition to the rate of energy loss of particles, the mean range was desired. Its value follows directly from a knowledge of the rate of energy loss. The range, $R(E)$, for

a particle of kinetic energy, E, is

$$R(E) = \int_0^E \left(- \frac{dE}{dx} \right)^{-1} dE \tag{10}$$

Since the particles with which these calculations are concerned lose their energy in a large number of small steps, the distance traversed for a given energy loss fluctuates about a mean value. Similarly the energy loss for a given distance of penetration has a spread. The differential changes in the variance of these distributions have been calculated by Bethe[6] and take the forms

$$\frac{d}{dX} = \left[(E^2)_{Av} - (E_{Av})^2 \right] = 4\pi e^4 N$$

$$\times \left(Z^* + \sum K_j \frac{I_j Z_j}{mv^2} \ln \frac{2mv^2}{I_j} \right) \tag{11}$$

and

$$\frac{d}{dE} \left[(X^2)_{Av} - (X_{Av})^2 \right] = \frac{d}{dX} \left[(E^2)_{Av} - (E_{Av})^2 \right]$$

$$\times \left(dE/dX \right)^{-3} \tag{12}$$

The quantities here are the same as in (5) with $K_j \simeq 4/3$. These relations both apply for infinitesimal intervals. Making

use of the result that the variance of the distribution of the sum of a number of independent variables is the sum of their variances, (12) may be integrated to give the variance of the distance distribution for a finite energy change. The relation in (11) may not be integrated since the energy losses in successive intervals are not independent.

III. INTERPRETATION OF EXPERIMENTAL RESULTS

In checking the calculated curves against experimental data, two types of experiments can be used. One consists of measurements of the relative stopping power of various elements. The second consists of absolute range measurements. The analysis of either type involves certain corrections which will be discussed first.

The first consideration concerns the straggling of particles. In interpreting experiments, the straggling generally must be combined with the energy dependence of the recording instrument. Thus a counter has a simple energy dependence, giving a single count for a particle with any positive residual range. On the other hand, an ionization chamber gives a response depending on the particle energy. In general, for an incident monoenergetic beam of particles, the response, $I(R)$, at a distance R from the end of range is

$$I(R) = n \int_{-\infty}^{\infty} P(r, R)\, i(2R - r)\, dr \quad . \tag{13}$$

n is the number of particles, $P(r, R)\, dr$ is the probability that a particle has, in an interval $(r, r + dr)$, the energy corresponding to a residual range R and $i(R)$ is the response of the instrument to a particle of this energy. $i(R)$ is zero for R negative. The distances R and r are measured from the

end of the range of the particle, the positive direction being toward the source of the beam. It is assumed that in the interval between R and r the particle has the average behavior.

For a counter, $i(R) = 1$, $R > 0$. Then (13) becomes

$$I(R) = n \int_{-\infty}^{2R} P(r, R)\, dr \quad . \tag{14}$$

This is just the number of particles having a positive residual range at the point R. At the end of the range, (14) gives the result

$$I(0) = n \int_{-\infty}^{0} P(r, 0)\, dr \quad . \tag{15a}$$

If $P(r, 0)$ is symmetric about the end of the range,

$$I(0) = \frac{n}{2} \tag{15b}$$

that is, half of the incident particles have stopped at this point and half have a positive residual range.

In applying (13) to the interpretation of the range measurements it is necessary to determine the form of $P(r, R)$ and $i(R)$. The quantity actually measured, R^* , is the range parallel to the incident beam. It has a distribution which arises from three sources. The total ranges of particles, measured along their paths, fluctuate about a mean value, R , which is the mean range the theoretical calculations yield. The

distribution of this straggling is quite closely a normal distribution. The second source of fluctuation is the variation of particle energies in the incident beam. The distribution for this is not known.

The third source of the R^* distribution arises from the scattering of the particles. The value of the range for a particle, measured parallel to the incident beam is obviously never greater than the value along the particle's actual path. Hence, the distribution from this cause is asymmetric. If R is the mean range along the particle paths and R^* the mean measured range, it can be shown that

$$R - R^* = \frac{1}{2} \int_0^R \langle \theta^2 \rangle_{Av} \, dx \tag{16a}$$

where $\langle \theta^2 \rangle_{Av}$ is the mean squared scattering angle at depth x. For 340 Mev protons, an approximate calculation gives

$$\frac{R - R^*}{R} = Z/6400 \tag{16b}$$

or about one percent for the heaviest elements.

The standard deviation of the straggling, calculated theoretically, varies from 0.85% to 1.1%. The energy spread of the beam used in the experiment also amounts to about 1%.

In the experiments considered here, it was found that the instrument response was proportional to $R^{-0.46}$. This

neglects the decrease in ionization at the end of the range. The distribution function $P(r, R)$ was assumed to be a normal distribution with an arbitrary half width, which was adjusted to fit the data. It was believed that the asymmetric scattering distribution was narrow and hence should not make $P(r, R)$ very asymmetric. This point should be examined more critically. The contribution to the distribution of the initial energy spread was unknown. The simplest distribution was a symmetric one and there seemed to be no particularly preferable one.

Using $P(r, R)$ and $i(R)$ as

$$P(r, R) = \frac{1}{\alpha\sqrt{2\pi}} e^{-\frac{(R-r)^2}{2\alpha^2}} \tag{17a}$$

$$\alpha^2 = \left\langle (R - r)^2 \right\rangle_{Av} \tag{17b}$$

$$i(R) = \text{const.} \times R^{-0.46} \tag{18}$$

the value of $I(R)$ becomes

$$I(R) = \text{const.} \times \int_{-\infty}^{2R} \frac{e^{-\frac{(R-r)^2}{2\alpha^2}}}{\alpha\sqrt{2\pi}} (2R - r)^{-0.46} dr \tag{19a}$$

$$I(R) = \text{const.} \times \int_{0}^{\infty} \frac{e^{-\frac{(r'-R)^2}{2\alpha^2}}}{\alpha\sqrt{2\pi}} (r')^{-0.46} dr' \tag{19b}$$

Introducing α as a unit of length, and letting $x = -R/\alpha$, $t = r'/\alpha$.

$$I(x) = \text{const.}\ x \int_0^\infty e^{-\frac{(x+t)^2}{2}}\, t^{-0.46}\, dt \,. \tag{20}$$

In analyzing the data, the ratio of $I(0)$ to the maximum value of $I(x)$ was used to locate the point corresponding to the mean energy. This ratio has been calculated numerically and is equal to 0.82. Neither this number nor the shape of the curve is critically dependent on the value of the exponent in $i(R)$ which is known to about three percent. A ten percent change in the exponent changes $I(0)/I_{max.}$ by only two percent and the fitting of the steep part of the ionization curve is unchanged.

When all the corrections have been applied, the comparison with theory can be accomplished. We shall discuss the relative stopping power measurements first.

The stopping power per electron is $(NZ)^{-1}(dE/dx)$. It is conventional to define a quantity, q , which is the stopping power per electron for an element relative to the stopping power per electron of aluminum. From equations (1) and (2), for similar particles

$$q = B_Z / B_{A\ell} \tag{21}$$

If the velocities are high enough that shell corrections vanish, q may be written explicitly.

$$q = \frac{-\ln I + \left[\ln 2mv^2 - \ln(1-\beta^2) - \beta^2\right]}{-\ln I_{A\ell} + \left[\ln 2mv^2 - \ln(1-\beta^2) - \beta^2\right]} \tag{22}$$

If q , $I_{A\ell}$ and v are known, I can easily be calculated.

$$\ln I = \left[\ln 2mv^2 - \ln(1-\beta^2) - \beta^2\right](1-q) + q\ln I_{A\ell} \tag{23}$$

In the relative stopping power experiments, q is measured by finding the thickness, $\Delta x_{A\ell}$, of aluminum required for the same energy loss as a given thickness, Δx_{sample} of the element. Then,

$$q_{exp.} = \frac{\Delta x_{A\ell}}{(NZ)_{A\ell}} \frac{(N\,Z)_{sample}}{\Delta x_{sample}} \tag{24}$$

In using this q in equation (23), it is necessary to choose an average velocity. Since the energy losses were small compared to the energy and the velocity dependence of (23) small, an arithmetic mean value was satisfactory. The values of I obtained by Bakker and Segre[10] are given in Table I.

The total range measurements are not as easily interpretable but the results for I are independent of any assumption about the value of I for aluminum. To find the

change in I required by a difference between theoretical and experimental ranges the following procedure was used. Let $R_1(E)$ and $R_2(E)$ be the ranges calculated for two values of the mean excitation potential, I_1, and I_2 respectively. Using the form of equations (1) and (10)

$$R_1 - R_2 = \frac{m}{4\pi e^4 N} \left\{ \int_0^E v^2 \, dE \left[\frac{1}{B_1(v)} - \frac{1}{B_2(v)} \right] \right\} . \tag{25}$$

Using any of the forms for putting in the shell corrections

$$\frac{1}{B_1(v)} - \frac{1}{B_2(v)} = \frac{1}{B_1(v)} \left\{ 1 - \left[1 - \frac{Z \ \ln(I_2/I_1)}{B_1(v)} \right]^{-1} \right\} \tag{26}$$

For small values of $Z \ln (I_2/I_1)$, this can be expanded as a power series. Then (25) becomes

$$R_1 - R_2 = - \frac{mc^2}{4\pi e^4 N} \sum_{\gamma = 1} \vartheta_\gamma (E) \left[Z \ln (I_2/I_1) \right] \tag{27a}$$

where

$$\vartheta_\gamma(E) = \int_0^E \beta^2 \left[B_1(v) \right]^{-\gamma - 1} dE \tag{27b}$$

Using the relation (27a) and the difference between the experimental range and the calculated range, it is possible to calculate the value of I which would give the experimental range. The same relation can also be used to correct the calculated ranges to correspond to the new value of I. The values of I obtained in this way from the experiments of Mather and Segre[11] are given in Table I.

IV. METHOD OF CALCULATION

The quantities desired were both the rate of energy loss and the mean range of the particle as a function of its energy. Since the form of the stopping number B did not allow any convenient explicit integration for the value of the range, the reciprocals of the values of $(-dE/dx)$ were taken and the range found by numerical integration.

The calculations were first performed using Wilson's value for I, $Z \times 11.5$ ev, for $Z > 2$, and using Williams' values for hydrogen and helium. Using these, the values of B, dE/dx and the range were calculated. The range was found by integrating numerically.

$$R(E) = \int_{E_o}^{E} (-dE/dx)^{-1}\, dE + R(E_o) \qquad (28)$$

E_o is a low energy below which the calculations were not attempted. The range for this energy was obtained empirically.

For light elements (lithium to argon), the K shell corrections were used as given by Bethe[6]. $R(E_o)$ was determined from the experiments of R. R. Wilson[8] and Mano[17] who measured the relative stopping power for particles of low velocities of various elements relative to air. Using these relative stopping powers and the range in air, $R(E_o)$ could be

obtained. This result assumes that the relative stopping power is constant at low velocities. The errors introduced by this estimation are negligible at high energies since $R(E_o)$ is only a negligible part of the high energy range. The numbers used are given in Table II.

For heavy elements the corrections for the various shells were made as outlined in equations (4) and (5). The values used for f_j were those given by Hönl[18] and Bethe[6]. The values of I_j used were the ionization potentials for the various shells[19]. These are only approximations to the correct values but the errors are not serious. The initial ranges, $R(E_o)$, were determined in the same way as for the light elements. The constants used are given in Table III.

Having calculated the range energy relation, it became possible to compare the theoretical calculation with experiment. This was done for beryllium, carbon, aluminum, copper and lead as outlined in section three. To do this required calculating the $\mathcal{S}_\nu(E)$. This could be done easily, since B had already been calculated. The integrals were again evaluated numerically. Only $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$ were calculated. Using these, new values of the I's were calculated, corresponding to the experimental ranges. This gives better values for I for these elements.

In addition, one may use the relative stopping power experiments to determine I. These should give satisfactory

results, since the range measurement indicated that the value of I for aluminum was correct.

Using the values of I determined from both types of experiment, a curve, Figure 1, was drawn of I/Z as a function of Z and used to determine new values of I for substances not measured in the experiments. Using the new values of I, the stopping power and range calculations were corrected.

For the elements for which range measurements were made this value of I was used. The range can be corrected by the series (27a). The stopping power can be corrected by the relation

$$(-dE/dx)_2 - (-dE/dx)_1 = \frac{4\pi e^4 N}{mv^2} Z \ln (I_2/I_1) \quad . \tag{29}$$

For elements for which no range measurements were made, the correction is made by using (29) to correct dE/dx and recalculating the integral for the range. Hydrogen and helium were not changed, the theoretical values being used.

The values of dE/dx and range given in the tables are the corrected values and correspond to the values of I given. The shell corrections remain unaltered by changes in I.

Thus far we have discussed only the calculations for elements. For the calculation of photographic emulsion ranges, a different method was used. Lattes, Fowler, and Cuer[20] had determined a range-energy relation for protons up to 12 Mev energy

experimentally. An extrapolation to 36 Mev was desired. It seemed better to avoid calculating I_{mean} from the assumption of additivity and a value of $I = Z \times 11.5$ ev. since an alternate approach was available. Using the composition of the emulsion as given by the manufacturer, Z_{mean} was calculated. The rate of energy loss at several energies near 10 Mev was calculated from the experimental data. Using the shell corrections as given in (4) and (5) for each element, a value of I_{mean} was calculated. This value was then used to extend the rate of energy loss and range calculations above 10 Mev. The value of range at and below 10 Mev were taken from the experimental data.

It should be noted that this method tends to compensate an error in the composition by an error in the calculated value of I_{mean}. An analysis of the value of I_{mean} has been made, assuming additivity. It yielded avalue of about $Z \times 16$ ev. for the excitation potentials of atoms other than hydrogen. This is much larger than any of the experimental values of I. It does not seem probable that the assumption of additivity is sufficiently in error to account for these differences.

Finally, the variance of the distribution of particles in space was calculated. This was done for copper and aluminum only, these being most frequently used. The calculation consisted merely of a numerical integration of equation (12). The values of I used were the uncorrected ones. It does not seem profitable to correct the copper values, since the errors are small. The aluminum calculation, of course, needs no correction since its I is not changed by the new experiments.

V. CONCLUSION

The rate of energy loss and the range of protons has been calculated as a function of energy, attempting to get a good fit to new experimental data. It is believed that above 10 Mev the ranges are good to one-half percent. Below 10 Mev the range is increasingly more sensitive to errors in the initial ranges, $R(E_o)$. The rate of energy loss is believed to be good to one-half percent down to about 5 Mev. The values are given in Tables IV - XVI.

I should like to express my appreciation to Professor R. L. Thornton and Professor E. Segre for many interesting discussions concerning the experimental work and especially to Professor Robert Serber who has been most encouraging throughout these calculations.

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TABLE I.

Element	Z	I/Z (Bakker & Segre)	I/Z (Mather & Segre)
Li	3	11.3	
Be	4	15.1	14.75
C	6	12.7	12.91
Al	13	11.5*	11.56 11.19
Fe	26	9.3	
Cu	29	9.6	10.77 10.48 10.81
Ag	47	9.1	
Sn	50	9.5	
W	74	9.4	
Pb	82	9.2	10.11 9.67
U	92	9.6	

* The aluminum value is taken as the reference value

TABLE II. Light elements.

Element	E_o(Mev)	$R(E_o)$	I(ev)	Source of I
H	1	10.025 cm	17.5	Theory
He	1	13.03 cm	44	Theory
Li	1	2.87 mg cm^{-2}	34.5	Relative stopping power.
Be	1	2.91 mg cm^{-2}	59.0	Range
C	1	2.76 mg cm^{-2}	74.4	Range
Al	.4	34.19 mg cm^{-2}	150	Range and relative stopping power
A	1	7.27 cm	194	Figure 1

TABLE III

Element	E_o	$R(E_o)$	I(ev)	Source of I	Oscillator strengths K	L	M	Ionization potentials K	L	M
Cu	4	46.66 mg cm^{-2}	310	Range	1.315	8		8.86kv	1.10kv	
Ag	4	62.60 mg cm^{-2}	470	Figure I	1.221	6.135		25.5 kv	3.79kv	
Pb	4	81.68 mg cm^{-2}	811	Range	1.14	4.72	11.7	87.6 kv	15.8 kv	3.85kv

TABLE IV: Rate of energy loss and range of protons in hydrogen at 15° C., 760 mm. Hg, I = 17.5 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev cm^{-1}	R cm
1	5.8848×10^{-2}	10.025
3	2.4149	69.061
5	1.5782	1.7410×10^{2}
6	1.3543	2.4270
8	1.06288	4.1076
10	8.8024×10^{-3}	6.1855
12	7.5440	8.6480
14	6.6211	1.1485×10^{3}
16	5.9136	1.4687
18	5.3530	1.8246
20	4.8972	2.2157
30	3.4823	4.6776
40	2.7408	7.9416
50	2.2813	1.1961×10^{4}
60	1.9676	1.6697
70	1.7391	2.2115
80	1.5650	2.8188
90	1.4279	3.4886
100	1.3170	4.2186
150	9.7692×10^{-4}	8.6958
200	8.0240	1.4386×10^{5}
250	6.9626	2.1104
300	6.2513	2.8704
350	5.7431	3.7065
400	5.3635	4.6086
450	5.0703	5.5683
500	4.8380	6.5787
600	4.4959	8.7275
700	4.2591	1.1016×10^{6}
800	4.0884	1.3415
900	3.9613	1.5901
1000	3.8647	1.8458
2000	3.5436	4.5892
3000	3.5362	7.4209
4000	3.5826	1.0232×10^{7}
5000	3.6401	1.3001
6000	3.6977	1.5727
7000	3.7525	1.8411
8000	3.8036	2.1058
9000	3.8511	2.3670
10,000	3.8953	2.6252

TABLE V: Rate of energy loss and range of protons in helium at 15° C., 760 mm. Hg, I = 44 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev cm^{-1}	R cm
1	4.7599×10^{-2}	1.303×10
3	2.0387	8.389
5	1.3518	2.0725×10^{2}
6	1.1654	2.8714
8	9.2070×10^{-3}	4.8174
10	7.6614	7.2100
12	6.5902	1.0034×10^{3}
14	5.8010	1.3276
16	5.1938	1.6926
18	4.7111	2.0975
20	4.3176	2.5414
30	3.0899	5.3236
40	2.4420	8.9938
50	2.0386	1.3498×10^{4}
60	1.7622	1.8791
70	1.5605	2.4835
80	1.4064	3.1597
90	1.2849	3.9046
100	1.1864	4.7154
150	8.8366×10^{-4}	9.6737
200	7.2770	1.5955×10^{5}
250	6.3268	2.3356
300	5.6892	3.1712
350	5.2334	4.0893
400	4.8927	5.0787
450	4.6296	6.1303
500	4.4213	7.2363
600	4.1147	9.5859
700	3.9029	1.2085×10^{6}
800	3.7506	1.4701
900	3.6376	1.7410
1000	3.5521	2.0194
2000	3.2771	4.9940
3000	3.2824	8.0499
4000	3.3343	1.1074×10^{7}
5000	3.3946	1.4046
6000	3.4539	1.6967
7000	3.5097	1.9839
8000	3.5616	2.2666
9000	3.6096	2.5456
10,000	3.6541	2.8209

TABLE VI: Rate of energy loss and range of protons in lithium, I = 34.5 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R gm. cm^{-2}
1	258.71	2.8738×10^{-3}
2	151.18	8.1376
4	86.055	2.6379×10^{-2}
6	62.174	5.4049
8	49.024	9.0563
10	40.737	1.3553×10^{-1}
12	35.004	1.8867
14	30.786	2.4973
16	27.544	3.1854
18	24.969	3.9490
20	22.872	4.7869
30	16.338	1.0044×10^{0}
40	12.897	1.6990
50	10.758	2.5522
60	9.2929	3.5556
70	8.2247	4.7021
80	7.4097	5.9853
90	6.7667	7.3994
100	6.2462	8.9392
150	4.6469	1.8363×10
200	3.8238	3.0313
250	3.3228	4.4401
300	2.9866	6.0316
350	2.7463	7.7808
400	2.5668	9.6665
450	2.4281	1.1671×10^{2}
500	2.3183	1.3780
600	2.1566	1.8262
700	2.0449	2.3031
800	1.9645	2.8025
900	1.9048	3.3198
1000	1.8595	3.8515
2000	1.7126	9.5391
3000	1.7136	1.5390×10^{3}
4000	1.7394	2.1185
5000	1.7699	2.6884
6000	1.8000	3.2487
7000	1.8284	3.7998
8000	1.8548	4.3428
9000	1.8793	4.8783
10,000	1.9021	5.4072

TABLE VII: Rate of energy loss and range of protons in beryllium, I = 59.0 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev $gm^{-1}cm^{2}$	R $gm.\ cm^{-2}$
1	231.05	2.91×10^{-3}
2	137.89	8.73
4	80.267	2.853×10^{-2}
6	58.011	5.828
8	45.950	9.732
10	38.306	1.4521×10^{-1}
12	32.996	2.0165
14	29.078	2.6636
16	26.058	3.3914
18	23.655	4.1981
20	21.694	5.0819
30	15.562	1.0612×10^{0}
40	12.318	1.7892
50	10.295	2.6816
60	8.9066	3.7293
70	7.8920	4.9248
80	7.1171	6.2614
90	6.5050	7.7330
100	6.0090	9.3342
150	4.4822	1.9116×10
200	3.6948	3.1493
250	3.2145	4.6063
300	2.8921	6.2506
350	2.6616	8.0561
400	2.4894	1.0001×10^{2}
450	2.3563	1.2068
500	2.2509	1.4240
600	2.0960	1.8854
700	1.9890	2.3759
800	1.9121	2.8891
900	1.8552	3.4204
1000	1.8121	3.9661
2000	1.6755	9.7898
3000	1.6804	1.5763×10^{3}
4000	1.7086	2.1667
5000	1.7407	2.7466
6000	1.7721	3.3159
7000	1.8015	3.8755
8000	1.8289	4.4264
9000	1.8542	4.9693
10,000	1.8776	5.5052

TABLE VIII: Rate of energy loss and range of protons in carbon, I = 74.44 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev $gm^{-1}cm^{2}$	R gm. cm^{-2}
1	236.29	2.76×10^{-3}
2	146.94	8.29
4	86.209	2.678×10^{-2}
6	62.533	5.442
8	49.644	9.058
10	41.452	1.3488×10^{-1}
12	35.749	1.8699
14	31.534	2.4669
16	28.282	3.1378
18	25.692	3.8807
20	23.575	4.6943
30	16.946	9.7768
40	13.431	1.6458×10^{0}
50	11.235	2.4638
60	9.7274	3.4234
70	8.6244	4.5177
80	7.7812	5.7405
90	7.1149	7.0863
100	6.5747	8.5499
150	4.9103	1.7483×10
200	4.0508	2.8777
250	3.5265	4.2062
300	3.1743	5.7046
350	2.9224	7.3493
400	2.7342	9.1205
450	2.5888	1.1002×10^{2}
500	2.4736	1.2979
600	2.3044	1.7176
700	2.1876	2.1637
800	2.1037	2.6303
900	2.0417	3.1131
1000	1.9948	3.6089
2000	1.8477	8.8938
3000	1.8552	1.4307×10^{3}
4000	1.8877	1.9652
5000	1.9243	2.4899
6000	1.9600	3.0048
7000	1.9933	3.5107
8000	2.0242	4.0084
9000	2.0528	4.4990
10,000	2.0793	4.9830

TABLE IX: Rate of energy loss and range of protons in aluminum, I = 150 ev. (These are essentially the values of J. H. Smith .)

E(Mev)	$-\frac{dE}{dx}$ Mev $gm^{-1}cm^{2}$	R gm. cm^{-2}
1		3.45×10^{-3}
2	115.0	1.08×10^{-2}
4	69.6	3.45
6	51.2	6.91
8	41.0	1.132×10^{-1}
10	34.5	1.667
12	29.9	2.290
14	26.58	3.006
15	25.18	3.393
17	22.81	4.228
19	20.89	5.146
21	19.30	6.143
23	17.96	7.218
25	16.82	8.369
30	14.56	1.157×10^{0}
35	12.89	1.523
40	11.60	1.933
45	10.58	2.385
50	9.743	2.878
60	8.458	3.983
70	7.516	5.240
80	6.794	6.642
90	6.222	8.182
100	5.757	9.854
120	5.047	1.358×10
140	4.530	1.777
160	4.136	2.240
180	3.826	2.743
200	3.576	3.284
250	3.120	4.787
300	2.813	6.480
350	2.593	8.334
400	2.428	1.033×10^{2}
500	2.201	1.467
600	2.054	1.938
700	1.952	2.438
800	1.879	2.961
900	1.826	3.501
1000	1.785	4.055

(Continued)

TABLE IX: (Continued)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^{2}	R gm. cm^{-2}
1250	1.721	5.484
1500	1.688	6.952
1750	1.671	8.441
2000	1.664	9.941
2250	1.663	1.144×10^{3}
2500	1.665	1.295
2750	1.670	1.445
3000	1.677	1.594
4000	1.710	2.185
5000	1.747	2.763
6000	1.782	3.330
7000	1.815	3.886
8000	1.845	4.432
9000	1.873	4.970
10,000	1.898	5.501

TABLE X: Rate of energy loss and range of protons in argon at 15° C., 760 mm. Hg, I = 198 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev cm^{-1}	R cm
1	2.6311×10^{-1}	7.2727
2	1.6984	1.2135×10
4	1.0423	2.7693
6	7.7131×10^{-2}	5.0293
8	6.1982	7.9419
10	5.2195	1.1473×10^{2}
12	4.5303	1.5598
14	4.0164	2.0297
16	3.6171	2.5552
18	3.2972	3.1351
20	3.0348	3.7680
30	2.2046	7.6924
40	1.7589	1.2810×10^{3}
50	1.4783	1.9040
60	1.2845	2.6319
70	1.1421	3.4594
80	1.0329	4.3816
90	9.4636×10^{-3}	5.3943
100	8.7603	6.4937
150	6.5832	1.3175×10^{4}
200	5.4525	2.1580
250	4.7604	3.1435
300	4.2947	4.2522
350	3.9613	5.4667
400	3.7120	6.7723
450	3.5194	8.1570
500	3.3671	9.6105
600	3.1433	1.2691×10^{5}
700	2.9895	1.5957
800	2.8794	1.9369
900	2.7984	2.2894
1000	2.7377	2.6509
2000	2.5578	6.4832
3000	2.5814	1.0383×10^{6}
4000	2.6361	1.4217
5000	2.6945	1.7969
6000	2.7503	2.1642
7000	2.8022	2.5244
8000	2.8499	2.8782
9000	2.8939	3.2264
10,000	2.9345	3.5695

TABLE XI: Rate of energy loss and range of protons in copper, I = 309.91 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R gm. cm^{-2}
4		4.666×10^{-2}
5	47.05	6.803
6	41.27	9.078
8	33.42	1.4499×10^{-1}
10	28.29	2.1031
12	24.65	2.8626
14	21.92	3.7248
16	19.79	4.6865
18	18.08	5.7452
20	16.67	6.8985
22	15.49	8.1444
24	14.48	9.4811
26	13.61	1.0907×10^{0}
28	12.85	1.2420
30	12.19	1.4019
35	10.82	1.8384
40	9.757	2.3259
45	8.913	2.8628
50	8.223	3.4474
55	7.647	4.0785
60	7.160	4.7547
65	6.741	5.4749
70	6.377	6.2380
75	6.057	7.0430
80	5.774	7.8888
85	5.523	8.7746
90	5.296	9.6994
95	5.093	1.0662×10
100	4.908	1.1663
110	4.585	1.3773
120	4.312	1.6023
130	4.079	1.8409
140	3.877	2.0925
150	3.701	2.3566
160	3.545	2.6328
170	3.407	2.9206
180	3.284	3.2197
190	3.172	3.5296
200	3.072	3.8500

(Continued)

TABLE XI: (Continued - Page 2 of 3)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^{2}	R gm. cm^{-2}
225	2.858	4.6948
250	2.686	5.5979
275	2.545	6.5548
300	2.426	7.5615
325	2.326	8.6143
350	2.240	9.7099
375	2.166	1.0845×10^{2}
400	2.101	1.2018
425	2.045	1.3224
450	1.994	1.4463
475	1.948	1.5731
500	1.909	1.7028
550	1.840	1.9698
600	1.784	2.2459
650	1.737	2.5300
700	1.698	2.8212
750	1.665	3.1186
800	1.638	3.4215
850	1.613	3.7294
900	1.592	4.0414
950	1.575	4.3572
1000	1.558	4.6764
1100	1.533	5.3235
1200	1.514	5.9800
1300	1.499	6.6439
1400	1.488	7.3134
1500	1.480	7.9874
1600	1.473	8.6647
1700	1.469	9.3445
1800	1.466	1.0026×10^{3}
1900	1.464	1.0709
2000	1.463	1.1393
2250	1.464	1.3102
2500	1.467	1.4808
2750	1.473	1.6508
3000	1.481	1.8201
3250	1.489	1.9885
3500	1.497	2.1560
3750	1.506	2.3226
4000	1.515	2.4881
4250	1.524	2.6526
4500	1.533	2.8163

(Continued)

TABLE XI: (Continued - Page 3 of 3)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R gm. cm^{-2}
4750	1.542	2.9789
5000	1.551	3.1407
5500	1.568	3.4613
6000	1.584	3.7786
6500	1.600	4.0926
7000	1.616	4.4036
7500	1.630	4.7116
8000	1.644	5.0170
8500	1.658	5.3198
9000	1.671	5.6202
9500	1.683	5.9182
10,000	1.695	6.2142

TABLE XII: Rate of energy loss and range of protons in silver, I = 470 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R gm. cm^{-2}
4	46.436	6.260×10^{-2}
6	35.194	1.1263×10^{-1}
8	28.682	1.7597
10	24.385	2.5189
12	21.320	3.3985
14	19.024	4.3935
16	17.224	5.5000
18	15.770	6.7149
20	14.569	8.0356
22	13.559	9.4598
26	11.950	1.2610×10^{0}
30	10.722	1.6150
34	9.7514	2.0067
38	8.9638	2.4350
42	8.3106	2.8989
46	7.7596	3.3974
50	7.2880	3.9296
60	6.3594	5.4028
70	5.6737	7.0711
80	5.1454	8.9248
90	4.7254	1.0955×10
100	4.3829	1.3155
110	4.0982	1.5516
120	3.8577	1.8033
130	3.6517	2.0699
140	3.4732	2.3508
150	3.3171	2.6455
160	3.1794	2.9536
170	3.0570	3.2744
180	2.9474	3.6077
190	2.8489	3.9528
200	2.7598	4.3095
225	2.5702	5.2493
250	2.4173	6.2532
275	2.2915	7.3161
300	2.1864	8.4337
325	2.0972	9.6017
350	2.0208	1.0817×10^{2}
375	1.9547	1.2075
400	1.8970	1.3374

(Continued)

TABLE XII: (Continued)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^{2}	R gm. cm^{-2}
425	1.8462	1.4710
450	1.8014	1.6081
475	1.7614	1.7485
500	1.7257	1.8919
600	1.6148	2.4921
700	1.5388	3.1274
800	1.4847	3.7895
900	1.4452	4.4727
1000	1.4158	5.1721
1200	1.3769	6.6063
1400	1.3548	8.0718
1600	1.3426	9.5554
1800	1.3368	1.1049×10^{3}
2000	1.3351	1.2546
2500	1.3413	1.6285
3000	1.3548	1.9995
3500	1.3712	2.3664
4000	1.3887	2.7287
4500	1.4063	3.0865
5000	1.4235	3.4399
6000	1.4563	4.1343
7000	1.4864	4.8139
8000	1.5141	5.4804
9000	1.5395	6.1353
10,000	1.5630	6.7799

TABLE XIII: Rate of energy loss and range of protons in lead,

I = 810.79 ev.

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^{2}	R gm. cm^{-2}
1	71.435	7.90×10^{-3}
2	51.304	2.505×10^{-2}
3	41.418	4.98
4	34.923	8.1668
6	26.940	1.4752×10^{-1}
8	22.176	2.2981
10	19.082	3.2755
12	16.808	4.3949
14	15.073	5.6537
16	13.700	7.0474
18	12.585	8.5722
20	11.659	1.0225×10^{0}
22	10.877	1.2002
24	10.206	1.3901
26	9.6238	1.5921
28	9.1136	1.8057
30	8.6622	2.0309
35	7.7317	2.6431
40	7.0065	3.3234
45	6.4252	4.0696
50	5.9480	4.8791
55	5.5483	5.7502
60	5.2081	6.6809
65	4.9148	7.6697
70	4.6592	8.7151
75	4.4343	9.8156
80	4.2348	1.0970×10
85	4.0566	1.2177
90	3.8963	1.3435
95	3.7514	1.4743
100	3.6198	1.6100
110	3.3894	1.8957
120	3.1944	2.1998
130	3.0271	2.5216
140	2.8820	2.8603
150	2.7549	3.2153
160	2.6427	3.5861
170	2.5428	3.9719
180	2.4534	4.3724
190	2.3729	4.7870

(Continued)

TABLE XIII: (Continued - Page 2 of 3)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R $gm. cm^{-2}$
200	2.3001	5.2151
225	2.1450	6.3419
250	2.0197	7.5440
275	1.9166	8.8155
300	1.8304	1.0151×10^2
325	1.7572	1.1546
350	1.6945	1.2995
375	1.6402	1.4495
400	1.5928	1.6042
425	1.5511	1.7633
450	1.5143	1.9265
475	1.4815	2.0934
500	1.4522	2.2639
550	1.4022	2.6145
600	1.3613	2.9766
650	1.3274	3.3487
700	1.2992	3.7295
750	1.2753	4.1180
800	1.2551	4.5133
850	1.2379	4.9145
900	1.2231	5.3209
950	1.2104	5.7319
1000	1.1994	6.1469
1100	1.1818	6.9871
1200	1.1686	7.8383
1300	1.1588	8.6978
1400	1.1516	9.5635
1500	1.1464	1.0434×10^3
1600	1.1428	1.1308
1700	1.1405	1.2184
1800	1.1392	1.3061
1900	1.1387	1.3939
2000	1.1390	1.4817
2250	1.1417	1.7010
2500	1.1467	1.9195
2750	1.1531	2.1369
3000	1.1604	2.3531
3250	1.1681	2.5678
3500	1.1762	2.7811
3750	1.1844	2.9929
4000	1.1927	3.2033

(Continued)

TABLE XIII: (Continued - Page 3 of 3)

E(Mev)	$-\frac{dE}{dx}$ Mev gm^{-1} cm^2	R gm. cm^{-2}
4250	1.2009	3.4122
4500	1.2091	3.6196
4750	1.2171	3.8257
5000	1.2251	4.0305
5500	1.2405	4.4360
6000	1.2553	4.8367
6500	1.2694	5.2328
7000	1.2830	5.6245
7500	1.2959	6.0123
8000	1.3083	6.3963
8500	1.3202	6.7767
9000	1.3315	7.1538
9500	1.3425	7.5278
10,000	1.3529	7.8988

TABLE XIV: Range of protons in Ilford emulsion.

E(Mev)	R(microns)
2	0.292×10^2
4	1.126
6	2.322
8	3.824
10	5.640
12	7.741
14	1.010×10^3
16	1.272
18	1.561
20	1.874
22	2.212
24	2.574
26	2.960
28	3.368
30	3.800
32	4.254
34	4.728
36	5.226

TABLE XV: Straggling of deuterons in Al, I = 150 ev.

E(Mev)	$(R - R_{Av})^2_{Av}$ $(mg\ cm^{-2})^2$
200	0
180	$.7170 \times 10^4$
160	1.2779
140	1.7018
120	2.0085
100	2.2179
80	2.3497
60	2.4234
40	2.4571
30	2.4643
20	2.4677
10	2.4689
6	2.4690
4	2.4690

TABLE XVI: Straggling of protons in copper, I = 333.5 ev.

E(Mev)	$(R - R_{Av})^2_{Av}$ (gm cm^{-2})^2
400	0
375	.1795
350	.3428
325	.4898
300	.6202
275	.7345
250	.8327
225	.9154
200	.9833
180	1.0274
160	1.0632
140	1.0911
120	1.1121
100	1.1270
80	1.1367
60	1.1424
40	1.1450
20	1.1459
10	1.1460
4	1.1460

TABLE XVI: Straggling of protons in copper, I = 333.5 ev.

E(Mev)	$(R - R_{Av})^2_{Av}$ (gm cm^{-2})^2
400	0
375	.1795
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275	.7345
250	.8327
225	.9154
200	.9833
180	1.0274
160	1.0632
140	1.0911
120	1.1121
100	1.1270
80	1.1367
60	1.1424
40	1.1450
20	1.1459
10	1.1460
4	1.1460

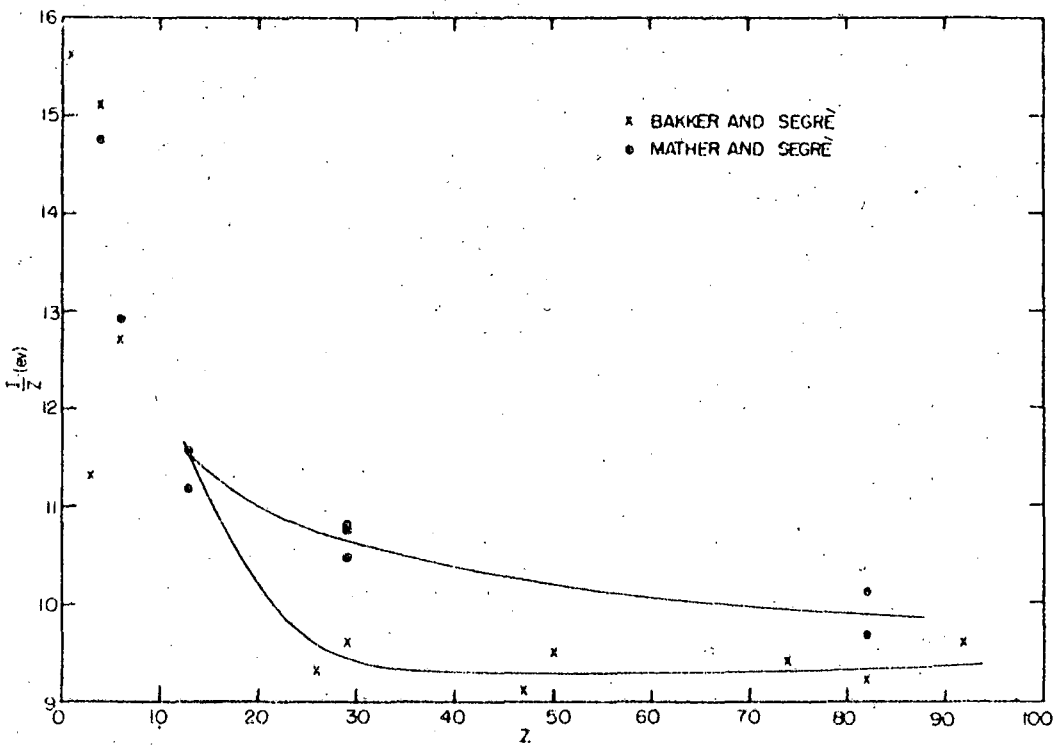

FIG. 1 - MEAN EXCITATION POTENTIAL, I, vs. ATOMIC NUMBER, Z

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