UCRL-1410 Rev.
Unclassified-Physics Distribution

UNCLASSIFIED

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

THE SCATTERING OF PROTONS FROM CARBON

Peter A. Wolff

March 12, 1952

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

THE SCATTERING OF PROTONS FROM CARBON

Peter A. Wolff

Radiation Laboratory, Physics Department
University of California, Berkeley, California

ABSTRACT

Using the impulse approximation, a formula has been derived which gives the energy spectra of protons scattered from carbon in terms of the nuclear momentum distribution. Estimates of the errors involved in this formula give values of 5-10% for a 340 Mev bombarding energy. A comparison is made with the experimental data of Cladis and it is concluded that a gaussian momentum distribution gives a good fit to his results.

THE SCATTERING OF PROTONS FROM CARBON

Peter A. Wolff

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

February 28, 1952

-I-

Collisions of fast nucleons with nuclei can be understood on the basis of a model proposed by Serber[1] which treats such encounters as being made up of a series of nucleon-nucleon interactions. In particular, if the target nucleus is small and the bombarding energy high (long mean free path), this theory predicts that an incoming particle will ordinarily only scatter once in traversing the nucleus and, therefore, that the overall collision will have many of the features of a free nucleon-nucleon interaction. The differences between this so-called 'quasi-elastic scattering' and true nucleon-nucleon scattering are then determined by the binding of the struck particle and thus can be used to obtain information about nuclear structure, especially about the nuclear momentum distribution. Or, knowing the momentum distribution, one could use data on the interaction of nucleons with nuclei to study fundamental processes involving them.

In the present paper we treat the particular problem of the quasi-elastic scattering of protons from carbon. This reaction has

---

1 R. Serber, Phys. Rev. 72, 1114 (1947).

recently been studied experimentally by Cladis[2] who found that the scattering is, indeed, very much like that of free nucleons. Thus, the hypothesis of a two particle collision is verified and can be used as a basis for understanding high energy collisions. Our aim in this work will be to use this point of view to obtain formulas giving the shapes and widths of the energy spectra of scattered protons in terms of a single momentum distribution of particles in the carbon nucleus. Comparison of these formulas with Cladis' results will then enable us to obtain an expression for the momentum distribution and, finally, will permit a judgment as to the degree of correctness of this type of approach to problems involving energetic collisions.

-II-

The mathematical technique for handling a scattering problem of the type we are considering is the impulse approximation, first formulated by Fermi[3] and elaborated by Chew[4]. This method takes advantage of the briefness of nucleon-nucleon interactions as compared to nuclear periods, assuming that during the collision the nucleus is 'frozen' in the sense that no momentum is exchanged between the two interacting particles and the rest of the nuclear system. The collision, therefore, can be looked upon as an ordinary two body scattering with the exception that the target particle is moving at the time of impact. Thus, the

---

[2] John Cladis, Thesis, University of California (1952).

[3] E. Fermi, Ricerca Scient. VII - II, 13 (1936).

[4] G. F. Chew, Phys. Rev. 80, 196 (1950).

amplitude for scattering from the particle labelled r_1 in the nucleus is

$$A_1 = \sum_{\xi_1} \int \phi_o(k\xi_1, r_2, \ldots r_{12})\, a(p\xi_o, k\xi_1; q\xi_o', s\xi_1')\phi_f(r_2, \ldots r_{12})$$

$$\delta(\vec{p} + \vec{k} - \vec{q} - \vec{s})\, d\gamma\, dk$$

where ϕ_o is the ground state of carbon with the portion referring to particle one written in momentum space, $a(p\xi_o\, k\xi_1; q\xi_o', s\xi_1')$ the amplitude for scattering of free nucleons of momenta $\vec{p}$, $\vec{k}$ and spins (isotopic and real) ξ_o, ξ_1 into the state labelled by $\vec{q}$, $\vec{s}$ and ξ_o', ξ_1', and ϕ_f a wave function of the residual nucleus. Of course, the total amplitude involves a sum of scattered waves from all the particles within the nucleus; hence the cross-section contains interference terms between waves scattered from different nucleons. However, these are small for angles of scattering greater than about twenty degrees since they then involve high fourier components of the ground state wave function[5,6]. Thus, it is sufficient for our purposes to calculate A_i, square it, and then sum over the different particles within the nucleus. Furthermore, since we are principally interested in the energy spectrum of the outgoing particles, we also sum over all final states consistent with a momentum $\vec{q}$ for the scattered proton. The cross-section is then

---

5, 6 For further discussion of this point see J. Heidenam, Phys. Rev. 80, 171 (1950) and E. M. Henley, Thesis, University of California (1951).

$$\frac{d^2\sigma}{d\Omega dq} = \left(\frac{2\pi}{\hbar}\right)\left(\frac{M}{\hbar p}\right)\left(\frac{2M}{\hbar^2}\right) \sum_{1}^{12} \sum_{f} \int \frac{ds}{(2\pi)^3} \left\{ \delta\left[p^2 - q^2 - s^2 + \frac{2M}{\hbar^2}(E_o - E_f)\right] \right.$$

$$\sum_{\xi_o \xi_1} \int \left| \sum_{\xi_1} \phi_o(\overrightarrow{q+s-p}\,\xi_1, \vec{r}_2, \ldots \vec{r}_{12})\, a(\vec{p}\,\xi_o, \overrightarrow{q+s-p}\,\xi_1; q\xi_o', s\xi_1') \right.$$

$$\left. \left. \phi_f(r_2 \ldots, r_{12})\, d\tau \right|^2 \frac{q^2}{(2\pi)^3} \right\}$$

Because of the two particle nature of the interaction the final states, ϕ_f, that contribute appreciably to the cross-section are clustered in a fairly narrow energy band. This means we can replace E_f in the δ-function by some average value $\overline{E}_f$ and perform the sum over f explicitly by the closure principle[7]. The result is

$$\frac{d^2\sigma}{d\Omega dq} = \frac{2\pi}{\hbar} \frac{M}{\hbar p} \left(\frac{2M}{h^2}\right) \sum_{1}^{12} \int \frac{d\vec{s}}{(2\pi)^3} \left\{ \delta\left[p^2 - s^2 - q^2 - \frac{2M}{\hbar^2} B_{if}\right] \right.$$

$$\left. \sum_{\xi_o \xi_1} \int \left| \sum_{\xi_1} \phi_o(\overrightarrow{q+s-p}\,\xi_o, \vec{r}_2, \ldots \vec{r}_{12})\, a(p\xi_o, \overrightarrow{q+s-p}\,\xi_1; q\xi_o', s\xi_1') \right|^2 d\tau \right\} \frac{q^2}{(2\pi)^3}$$

where $B_{if} = -E_o + \overline{E}_f$. Neglecting the cross-terms arising from

---

7 This analysis closely parallels that given by G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 470 (1950).

different orientations of ξ_1,[8] we can rewrite this formula as

$$\frac{d^2\sigma}{d\Omega dq} = \frac{2\pi}{\hbar}\left(\frac{M}{\hbar p}\right)\frac{2M}{\hbar^2}\sum_1^{12}\int\frac{ds}{(2\pi)^3}\sum_{\substack{\xi_0\,\xi_1\\ \xi_0'\,\xi_1'}}|a|^2\int\left|\phi_0(\overrightarrow{q+s-p}\ r_2,\ldots r_{12})\right|^2 d\tau$$

$$\delta\left[p^2-q^2-s^2-\frac{2M}{\hbar^2}B_{if}\right]\frac{q^2}{(2\pi)^3}$$

The integral $\int\phi_0(\overrightarrow{q+s-p},\ r_2,\ldots r_{12})^2\,d\tau$ is the momentum distribution which henceforth will be designated by $N(\overrightarrow{q+s-p})$. Furthermore, it is reasonable to suppose that $|a|^2$ is dependent only upon the momentum transfer, $\overrightarrow{p-q}$, which permits us to take it outside integral. Therefore, letting

$\sum_{\xi_0\,\xi_1}|a|^2 = |V|^2$ and $\overrightarrow{q+s-p} = \vec{k}$ we find

$$\frac{d^2\sigma}{d\Omega dq} = \frac{2\pi}{\hbar}\frac{M}{\hbar p}\frac{2M}{\hbar^2}\sum_1^{12}|V|^2\int\frac{\vec{dk}}{(2\pi)^3}N(\vec{k})\,\delta\left(p^2-q^2-(\overrightarrow{p+k-q})^2-\frac{2M}{\hbar^2}B_{if}\right)\frac{q^2}{(2\pi)^3} \tag{5}$$

Since $N(\vec{k})$ is dependent only upon the magnitude of $\vec{k}$, and not upon its direction, the angular integrations can be performed without putting

---

8 Strictly speaking, this is only correct if there are no tensor or spin-orbit forces operating in the carbon nucleus.

in an explicit expression for the momentum distribution. To do them expand the δ-function to

$$\delta\left[2pq\cos\theta - 2q^2 - k^2 - \frac{2M}{\hbar^2}B_{if} - 2\vec{k}\cdot\overrightarrow{p-q}\right]. \qquad (5')$$

(θ is the angle of scattering) and measure θ_k, the polar angle of $\vec{k}$, from $\overrightarrow{p-q}$. The integrations can now be done immediately to give

$$\frac{d^2\sigma}{d\Omega\, dq} = \frac{2M}{\hbar^2}\left(\frac{2\pi}{\hbar}\right)\left(\frac{Mq^2}{\hbar p}\right)\sum_{1}^{12}\left|V\right|^2\int\frac{1}{(2\pi)^5}\,\frac{N(k)\,k^2\,dk}{2k\left|\overrightarrow{p-q}\right|} \qquad (6)$$

where the factor $2k\left|\overrightarrow{p-q}\right|$ is a jacobian which is introduced by the integration over the delta function. The limits on the final integration over k are given by the condition $\left|\cos\theta_k\right| \leq 1$ which, combined with eq.(5'), gives the following equation to determine them

$$\left(2pq\cos\theta - 2q^2 - \frac{2M}{\hbar^2}B_{if} - k^2\right)^2 = 4k^2\left|p-q\right|^2 \qquad (7)$$

The solutions are

$$k^2 = 2p^2 - 2pq\cos\theta + \frac{2M}{\hbar^2}B_{if}$$

$$\pm\, 2\sqrt{\left(p^2 - pq\cos\theta + \frac{M}{\hbar^2}B_{if}\right)^2 - \left(q^2 - pq\cos\theta + \frac{M}{\hbar^2}B_{if}\right)^2} \qquad (8)$$

or

$$k \sim (2p^2 - 2pq + \frac{2M}{\hbar^2} B_{if})$$

$$(2p^2 - 2pq \cos\theta + \frac{2M}{\hbar^2} B_{if}) \left[1 - \frac{(q^2 - pq \cos\theta + \frac{M}{\hbar^2} B_{if})^2}{2(p^2 - pq \cos\theta + \frac{M}{\hbar^2} B_{if})^2} \right]$$

where in the last line we have made use of the fact that $\frac{M}{\hbar^2} B_{if}$ and $q - p \cos\theta$ are small for quasi-elastic scatterings ($q - p \cos\theta$ would be zero for free nucleon scattering). The upper limit on k is very large compared to all nuclear momenta so it can be replaced by infinity. Denoting the lower limit by k_{min} where

$$k^2_{min} = \frac{(q^2 - pq \cos\theta + \frac{M}{\hbar^2} B_{if})^2}{(p^2 - pq \cos\theta + \frac{M}{\hbar^2} B_{if})} \tag{9}$$

The final expression for the cross-section is

$$\frac{d^2\sigma}{d\Omega dE} = \frac{2\pi}{\hbar} \frac{qM^2}{\hbar^3 p(2\pi)^5} \left(\frac{2M}{\hbar^2}\right) \sum_1^{12} |V|^2 \int_{k_{min}}^{\infty} \frac{N(k)\, d(k^2)}{4\, |\vec{p} - \vec{q}|} \tag{10}$$

Because of the similarity between quasi-elastic scattering and that of free nucleons, the values of q for which $\frac{d^2\sigma}{d\Omega dE}$ is large lie near $p \cos\theta$. Thus, in the slowly varying parts of the above expression it is a good approximation to replace q by $p \cos\theta$ for, as will be

seen later, the small errors made by this replacement are no larger than those inherent in the method of deriving formula (10). Thus k^2_{min} becomes simply

$$k^2_{min} = \frac{p \cos \theta \, (q - p \cos \theta) + \frac{MB_{if}}{\hbar^2}}{p^2 \sin^2 \theta + \frac{MB_{if}}{\hbar^2}} \tag{11}$$

which, if $\frac{MB_{if}}{\hbar^2}$ is negligible, reduces to the even simpler formula

$$k_{min} = (q - p \cos \theta) \operatorname{ctn} \theta. \tag{12}$$

Before these formulas can be used to obtain $N(k)$ we must decide what to substitute for $|V|^2$. If $q = p \cos \theta$ (the free scattering value) then $|V|^2$ is directly proportional to either the n-p or p-p cross-section, depending upon whether the particle scattered from is a neutron or a proton. In the applications, therefore, we will replace $|V|^2$ by the appropriate free nucleon cross-sections. This procedure is actually quite good because, at the angles and energies at which we will work, the n-p and p-p cross-sections are changing slowly and the spectra are fairly sharp. However, the formula is left in the form (10) to make explicit the fact that the formalism can equally well handle a case where the basic matrix element is changing rapidly.

-III-

To investigate the errors inherent in formula (10) two types of approach were used. The first consisted in solving a simple problem for which the matrix elements are known, and then comparing the result with that obtained by using equation (10). The problem chosen is that of the scattering of an energetic particle from another bound harmonically to a center of force (the incoming particle assumed not to interact with the oscillator potential). This is about the simplest system imaginable which represents, even crudely, the interaction of a fast nucleon with a neutron or proton in a nucleus. The results of this calculation are illuminating; they show that energy spectra obtained by the two methods are quite similar, differing only in terms of the order k^2/p^2, where k is the momentum of the harmonically bound particle at time of collision and p that of the bombarding particle (here the bombarding energy is high so that $k^2/p^2 \ll 1$). In particular, the positions of the maxima in the two spectra differ by $k_o^2/4p_o \cos\theta$, where θ is again the angle of scattering and $k_o^2 = \frac{2M\omega}{\hbar^2}$. This means a percentage error of $k_o^2/4p_o^2 \cos\theta$.

The other estimate of the error in (10) was found by re-deriving this equation in a way which, though more cumbersome and not as physically meaningful as the one we used previously, permits a rough evaluation of some of the factors neglected in obtaining it[9]. This new derivation is carried out by taking the current of scattered particles

[9] For an elaboration of this method see P. A. Wolff, Thesis, University of California (1951).

$$J = \left(\frac{M}{2\hbar^2}\right)^2 \sum_n \left\{ \left| \int \psi_o^*(r_1 \ldots r_{12})\, e^{-i\,\vec{p}\cdot\vec{r}_o}\, V_{int.}\, e^{i\,\vec{k}_n\cdot\vec{r}_o}\, \psi_n\, d\tau \right|^2 \frac{\hbar k_n}{M} \right\} \tag{13}$$

(here both ψ_o and ψ_f are wave functions of C^{12}, and k_n is the wave number of the outgoing proton given by $\frac{\hbar^2 k_n^2}{2M} = \frac{\hbar^2 p^2}{2M} + E_o - E_n$), replacing E_n by the Hamiltonian, H_c, of the carbon nucleus (since it operates on ψ_n), and then expanding $e^{i\,\vec{k}_n\cdot\vec{r}_o}$ into a power series in H_c. The first three terms in this expansion can be evaluated exactly by swinging H_c over to operate on $(V\psi_o)^*$, using the closure principle to do the sum over n, and then evaluating the resulting integrals by fourier analyzing both ψ_o and V. The result is the integral over q of a formula which is just the same as (10) except that in this case there is no term representing the excitation formerly lumped into B_{if}. Furthermore, the fourth term in the series, which is much too complicated to calculate exactly, can be roughly evaluated and gives an estimate on the error in formula (10) again of the order of k^2/p^2.

For the scattering of protons from carbon k^2 ranges up to values corresponding to energies over 30 Mev so that, with the bombarding energy of 340 Mev used by Cladis, we should expect the spectra we predict to be in error by as much as 10% in the wings of the curves where large k values are involved. Therefore, in picking a momentum distribution to fit the data it must always be remembered that the theory can only differentiate between those giving spectra that differ by 10% or more.

In connection with the estimates of error made here one final point is worth mentioning. As was stated, the first three terms in the series expansion described above lead directly to formula (10) after simple manipulations. On the other hand, the fourth term in this expansion cannot be calculated at all precisely, since it is of quite a different type from the previous three, requiring detailed knowledge of H_c and ψ_o for its evaluation. The conclusion which might be drawn from this fact is that to improve the impulse approximation one must have much more detailed information about nuclear structure than that given merely by a momentum distribution and, therefore, that a formula such as (10) is the best that can be obtained with the present knowledge about the constitution of nuclei.

-IV-

Before using equation (10) to determine the momentum distribution, there are two small points about this formula that must be considered. The first of these has to do with the fact that the nucleon-nucleon collision, instead of occurring in free space, actually takes place inside the nucleus which is a region of a negative potential. Therefore the wave numbers that go into equations (1) - (12) should be those appropriate to nuclear matter rather than vacuum. To illustrate how this effect works let us consider the interaction of a fast proton with a stationary nucleon in the nuclear well. Letting primed quantities denote wave numbers inside the well, unprimed those outside, and D the depth of the well we find the following equations relating

the final to the incident energy.

$$q' = p' \cos\theta \quad (\theta = \text{scattering angle})$$

$$E_{final} = \frac{\hbar^2 q^2}{2M} = \frac{\hbar^2 q'^2}{2M} - D = \frac{\hbar^2 p'^2 \cos^2\theta}{2M} - D \tag{14}$$

$$= \frac{\hbar^2 p^2 \cos^2\theta}{2M} - D \sin^2\theta = E_{incident} \cos^2\theta - D \sin^2\theta.$$

Thus, in first approximation, the effect of the nuclear well is to move all proton energy spectra to a lower energy by an amount $D \sin^2\theta$. This effect will be taken into account in the calculations by assumming that formula (10) applies to wave numbers inside the well and then correcting initial and final energies by the well depth. The values of D used were the slightly energy dependent ones calculated by Roberts and Jastrow[10]. However, for the angles at which we worked a constant D of 30 Mev gives almost the same results.

The second point concerns the quantity B_{if} which has tacitly been assumed to be a constant, independent of the scattering angle. In the case of pickup deuterons this assumption was correct for Chew and Goldberger[7] found that by subtracting a fixed 25 Mev from the energy of the outgoing deuteron they obtained good agreement with experiment. Actually, however, the formalism given in section two is inadequate for treating excitation effects and it is not surprising that in the present case that it turns out that we would have to choose

---

10 R. Jastrow and J. Roberts (unpublished).

B_{if} angularly dependent to make formula (10) fit Cladis' data. If we did this, though, we would be going at the problem just backwards for, as Chew and Goldberger emphasize, the experiments show that the excitation is constant and that we should modify (10) instead of making B_{if} a function of the scattering angle. Unfortunately, we have not found a way of carrying out this modification, nor has it been possible to construct a model, simple enough to solve, which would give quantitative insight as to how this excitation and binding energy is removed from that available to the scattered nucleons. Hence, in treating the data the B_{if} term will be dropped entirely from formula (10). All the curves will then be misplaced somewhat on the energy scale but we will still be able to compare their shapes with experiment since a correct calculation of the binding and excitation effects would probably shift the spectra without radically altering their shape.

-V-

In comparing (10) with experiment there are a number of empirically obtained expressions for $N(k)$ that we may use. For instance, Chew and Goldberger[7] in their paper on the formation of pickup deuterons in carbon use a distribution of the form

$$N(k) = \frac{8\pi\alpha_p}{(\alpha_p^2 + k^2)^2} \quad \text{with} \quad \frac{\hbar^2\alpha_p^2}{2M} = 18 \text{ Mev.}$$

Similarly, Henley and Huddlestone[11] in papers on meson production in carbon have used a gaussian distribution;

$$N(k) = \frac{8\pi^{3/2}}{\alpha^3} e^{-k^2/\alpha^2} \qquad \text{with } \frac{\hbar^2 \alpha^2}{2M} = 16 \text{ Mev.}$$

Finally, there is the theoretically obtained Fermi distribution of a degenerate nucleon gas. Figures (1) and (2) give the comparison of the spectra, obtained by substituting these three momentum distributions into formula (10), with Cladis' experimental data. In each case the theoretical curve has been shifted down in energy as explained in the previous section. Furthermore, there is included a correction for the multiple scattering (assumed to be mainly double) of the proton within the carbon nucleus. The form of these spectra is calculated using the Monte Carlo method[12] to evaluate a complicated multiple integral that arises and then the ratio of singly to multiply scattered protons estimated by using values of nuclear radius and mean free path given by Fernbach[13]. Typical shapes of these double collision spectra, which contribute about fifty percent of all scattered protons, are illustrated in figure (3). Fortunately, inclusion of these curves has a negligible

---

11 E. M. Henley and R. H. Huddlestone, Phys. Rev. 82, 754 (1951). Also see reference 6.

12 M. L. Goldberger, Phys. Rev. 74, 1269 (1948), gives details of the application of this method to nucleon-nucleus collisions.

13 S. Fernbach, Thesis, University of California (1951).

effect on the forms of the quasi-elastic curves provided we restrict ourselves to the high energy side of the spectra. Thus, by using the upper half of the energy distributions, we are able to draw reliable conclusions about the momentum distribution in spite of the fact that the multiple collision spectra cannot be calculated accurately. Finally, it should be mentioned that the theoretical curves in figures (1) and (2) have been normalized. This should not be taken to mean that the theory does not predict total cross-sections well for it does. However, because of the uncertainty in the multiple collision spectra it is very hard to estimate the relative contribution of the quasi-elastic peak as compared to the long tail of lower energy protons. From the point of view of the theory, this means that, although we do not know the height of the quasi-elastic peak too well, its shape is determined and permits a study of momentum distributions.

From figures (1) and (2) it is clear that the only one of the three momentum distributions that is suitable is the gaussian. This is in agreement with the work of Henley and Huddlestone who found, as we do, that the Chew-Goldberger and Fermi distributions have, respectively, too many and too few high momentum components. Moreover, further calculations done with values of α^2 corresponding to 12 and 20 Mev show that in neither of these cases can the theoretical spectra be reconciled with Cladis' data. Thus, the theory, if correct, seems to be quite sensitive to the type of distribution used. Of course, agreement at two angles does not verify the theory and it will take considerably more extensive data before a really critical test is

obtained. What is needed are accurate spectra for a number of angles. With these one could determine N(k), as was done here, from the spectrum at one angle and check it, and the whole theory as well, by examining the fit at other angles. However, until such data is available, it is interesting to see that the spectra at two angles can be understood in terms of a simple model and that this fit already fixes the momentum distribution to a very considerable extent.

In conclusion, the author would like to express his thanks to Dr. Cladis for many stimulating conversations; and to Professor Robert Serber, whose guidance and encouragement contributed materially to the writing of this paper. This work was performed under the auspices of the Atomic Energy Commission.

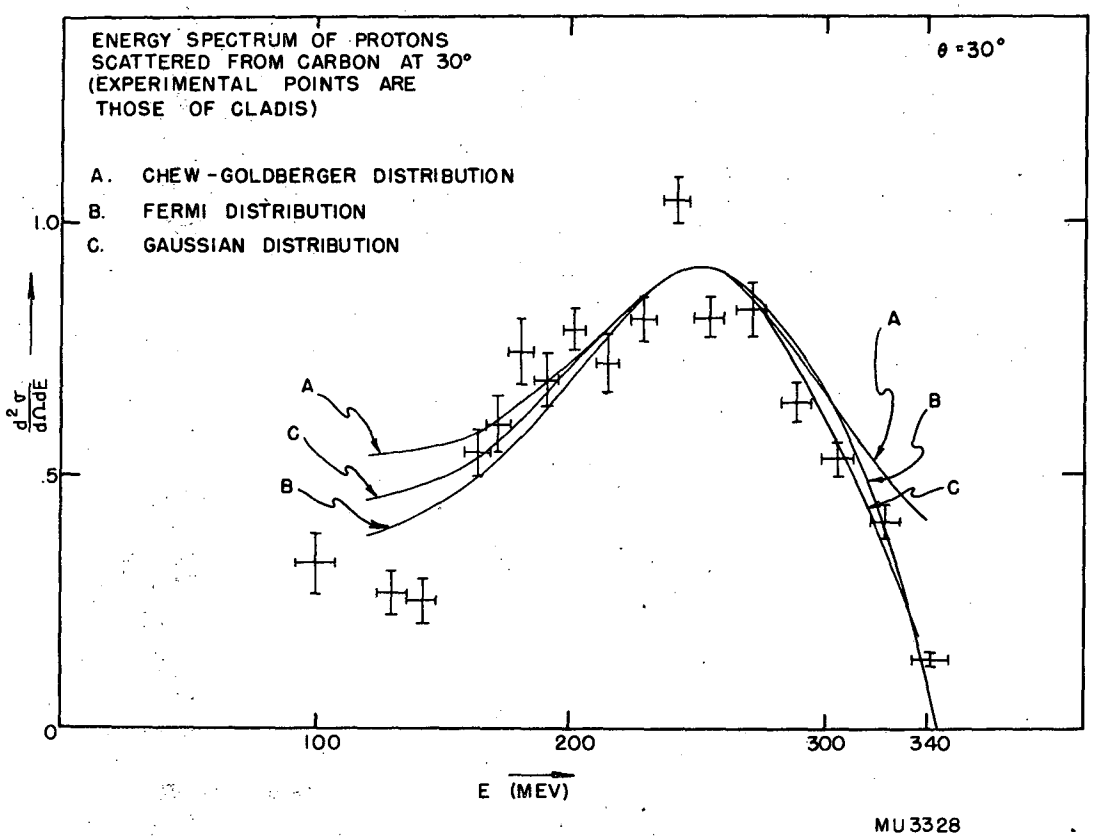
ENERGY SPECTRUM OF PROTONS
SCATTERED FROM CARBON AT 30°
(EXPERIMENTAL POINTS ARE
THOSE OF CLADIS)
θ = 30°
A. CHEW-GOLDBERGER DISTRIBUTION
B. FERMI DISTRIBUTION
C. GAUSSIAN DISTRIBUTION
1.0
.5
0
100
200
300
340
E (MEV)
A
B
C
MU3328

Fig. 1

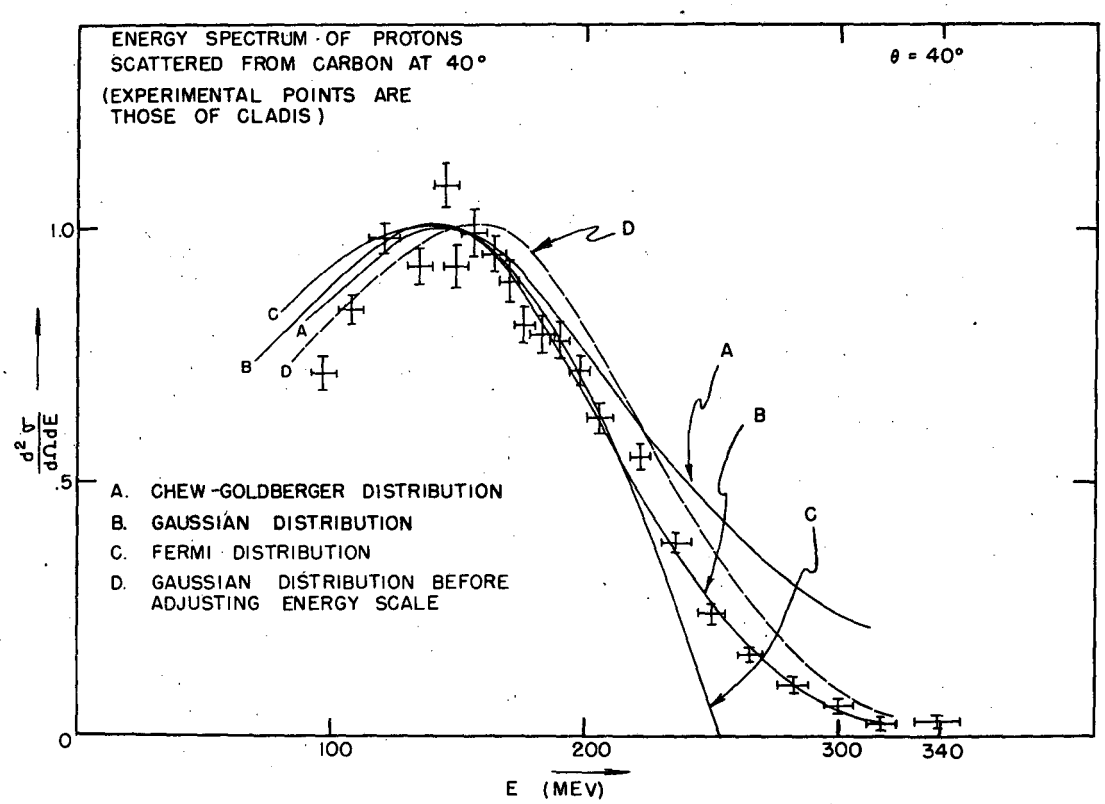
ENERGY SPECTRUM OF PROTONS
SCATTERED FROM CARBON AT 40°
(EXPERIMENTAL POINTS ARE
THOSE OF CLADIS)
θ = 40°
A. CHEW-GOLDBERGER DISTRIBUTION
B. GAUSSIAN DISTRIBUTION
C. FERMI DISTRIBUTION
D. GAUSSIAN DISTRIBUTION BEFORE
ADJUSTING ENERGY SCALE
A
B
C
D
1.0
.5
0
100
200
300
340
E (MEV)
d²σ/dΩdE

FIG. 2

MU3329

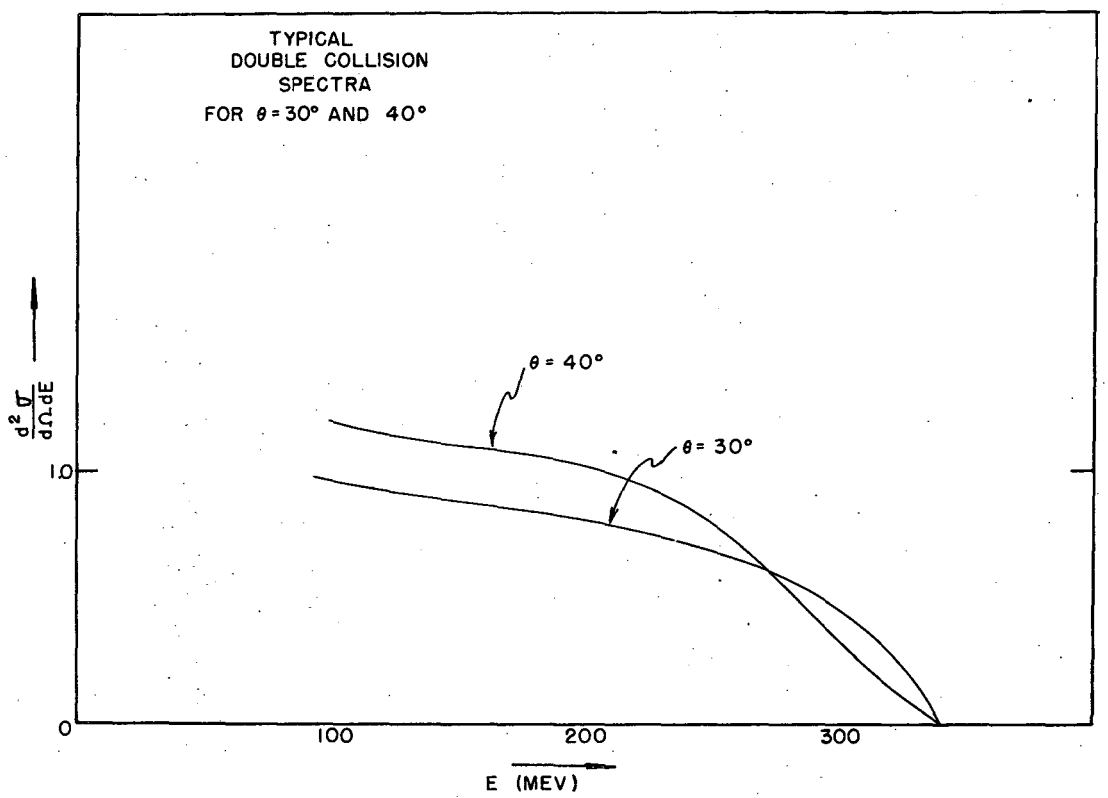
TYPICAL
DOUBLE COLLISION
SPECTRA
FOR θ = 30° AND 40°
θ = 40°
θ = 30°
$\frac{d^2\sigma}{d\Omega \cdot dE}$
1.0
0
100
200
300
E (MEV)

FIG. 3

MU3330