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Berkeley, California

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Klaus Halbach and Ronald F. Holsinger

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Klaus Halbach and Ronald F. Holsinger

Lawrence Radiation Laboratory
University of California
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ABSTRACT

Several useful computer programs have been developed which, given the geometry and current configuration of a system of coils, calculate with great speed and accuracy the properties of the magnetic field produced by the given coils. Because one of our experimental programs requires a somewhat sophisticated magnetic-field shape, we have developed a method and appropriate computer programs that reverse this process. Rather than arrive at a solution for the geometry and current configuration by trial and error, the method we used systematically solves for the geometry and current configuration required to produce the desired magnetic-field shape.

The coil configuration that gives the desired field shape is determined by manipulating with matrix algebra appropriate variables of all individual coils in the coil system. The variables used in our calculations are currents, relative positions, and lengths of the coils.

* Work done under auspices of the U. S. Atomic Energy Commission.

I. INTRODUCTION

The design of magnet coil systems is a frequently encountered problem in the CTR program that becomes increasingly difficult as progress is made and more sophisticated equipment is needed. For a new Homopolar Gun we need a coil system that produces a magnetic field distribution such as the one depicted in Fig. 1, and it is, of course, very desirable to produce that field with a coil system that is practical to design, construct, and operate. The method that we used for the design, and that will be used to adjust the pertinent parameters in practical operation, is Newton's method. Although this procedure is well known and often used in numerical analysis work,[1] it seems to be seldom used by people who design equipment or work with it in the laboratory. Since Newton's method can be a very powerful tool for the analysis of systems involving many parameters, and particularly in designing complicated equipment and in performing experiments and evaluating their results, we use its application to our specific problem as an illustrative example. We hope that this may encourage more people to apply it directly in engineering design and in the laboratory.

II. BASIC DESIGN PROCEDURE

Our approach to the design of the coil system is the following. We try to design the coil system in such a way that its field matches exactly the desired field in as many points as is consistent with the number of parameters available for the design of the coils.

The features of the field distribution, given in Fig. 1, that are important for the experiment are the field strength of the mirror peaks and of region 1 relative to the field strength in region 2, the homogeneity of the field in region 2, and the uniformity of the field gradient in region 1. It is also very desirable

to be able to vary the relative field strengths mentioned above as well as the gradient in region 1. Not important for the experiment are the exact lengths of regions 1 and 2 and the transitions between regions 1, 2, and the mirror peaks, as long as the field does not under- or overshoot excessively.

The variables available to achieve the desired field distribution are coil geometry and current distribution. Some of the possible geometrical variables can be excluded from the beginning. For instance, the size of the vacuum vessel inside the coils is given, and one does not want to waste energy, therefore the radius of the coil system is essentially given. The thickness of the coils can also be eliminated, since it affects only weakly the desired characteristics of the field distribution, and it is essentially given by the magnitude of the fields and the allowable electrical losses in the coils. Because of manufacturing difficulties and the need to have a versatile coil system, we also exclude coils with varying pitch. The coil system therefore consists of several coils, and the current in the individual coils and the positions of the ends of the coils can be used as variables. We can see from Fig. 1 that one coil is needed for each of the mirror peaks, one for region 2, and one, two, or more for region 1, and it is desirable to minimize the number of coils required in region 1.

The first step in our procedure is described below. Although this step is not absolutely necessary, it is instructive to obtain an idea about the necessary number of coils in region 1 in the following way: We define the desired field distribution by specifying the value of the magnetic field on axis at 20 points, and also split the coil system up into 20 identical independent coils (Fig. 2). The fields b_n at the 20 points are obviously linear functions of the currents i_m in the 20 coils, and can therefore be expressed as

$$b_n = \sum_m A_{nm} i_m ,$$

or, using matrix notation,

$$b = A \cdot i , \tag{1}$$

where b and i are column vectors and A is the coefficient matrix. Although the coil system will be energized by a capacitor bank and there is a metal liner on the inside of the coil system, we assume vacuum fields throughout the design calculations. This is a reasonable simplification, since the effects of the liner will be small and can be taken into account experimentally, as is shown below. The vacuum-field assumption allows us to calculate the matrix elements with an existing computer program,[2] hereafter referred to as the "COILS" program, which gives B_z, B_r, B, A, rA, and $\partial B_z/\partial r$ as a function of r and z for any coaxial coil system. We use here only B_z at the points of interest.[3]

With the known matrix A, Eq. (1) can be solved for i:

$$i = A^{-1} \cdot b . \tag{2}$$

Figure 2 illustrates this procedure. The field distribution is specified by the circled points on the field plot. The coil system, with the indicated current distribution obtained from Eq. (2), gives a field distribution that goes exactly through the 20 specification points. From this graph it follows that two coils in region 1 are probably enough to obtain the required field distribution. The coil system in first approximation, then, consists of five coils with appropriate thicknesses and reasonable--but still arbitrarily chosen--positions of the end coordinates of the coils. The end points are selected so that there are practical gaps between adjacent coils, with no gap between the two coils in region 1. The currents in the individual coils are determined with the same method as used before, with one specified field point under each coil. This, in general, will

not give the desired field distribution with sufficient accuracy, since it has not been specified by a large enough number of points. However, the deviations from the desired field distribution will not be too large. To improve the field distribution, we now also utilize geometry variables, such as lengths and positions of the coils. This then allows us to specify the field at more points, so that the final field will very closely fit the desired field.

The field at the specification points is, of course, a nonlinear function of the geometrical variables. Since the field resulting from the starting geometry and currents is not too different from the desired field, the necessary changes in geometry will be small, so that a linear expansion in these variables will be a reasonable approximation. We denote our variables now by v_m, the first five being the currents and the remaining ones the geometry variables, such as lengths and positions. If we change these variables by small increments Δv_m, the resulting field changes Δb_n at the specified locations are then given in this linear approximation by

$$\Delta b_n = \sum_m \frac{\partial b_n}{\partial v_m} \cdot \Delta v_m ,$$

or, if we define the coefficient matrix elements

$$M_{nm} = \frac{\partial b_n}{\partial v_m} ,$$

then

$$\Delta b = M \cdot \Delta v . \qquad (3)$$

Equation (3) then is the matrix notation for the equations relating the change in the fields to the changes in the currents, lengths, and positions of the coils. The method used to calculate M is discussed below.

The Δb needed in Eq. (3) is the difference between (i) the fields produced by the starting geometry and the currents at the specified locations, and (ii) the desired fields there. To obtain the geometry and current changes necessary to produce the desired fields, we have to solve Eq. (3):

$$\Delta v = M^{-1} \cdot \Delta b. \tag{4}$$

Equation (4) would give the necessary changes exactly if the fields were really linear functions of all variables. Since this is not the case, Eq. (4) gives only a better approximation, but not the perfect solution, and this process has to be repeated until one arrives at a satisfactory solution. A block diagram of the iterative process is given in Fig. 3. This whole method of solving many simultaneous equations is, of course, nothing but Newton's method extended to many dimensions. The coil-system geometry and a plot of the axial components of the field obtained from such a computer run are represented in Fig. 4. Since in the planned experiment the field shape is important at a radius of 2 in., we fitted the field there at the axial positions indicated by the solid black circles. The field is plotted for r = 1 in., 2 in., and 3 in., and it can be seen that the actual field distribution comes very close to the desired one. The variables used for this run were the currents in the five coils, and the positions of the mirror coils and the coil producing the uniform field. The final position, radial thickness, and current densities of the coils are also indicated. Figure 5 is a plot of the flux tubes and field contour lines for this geometry.

III. COMPUTATION OF THE MATRIX M AND OPERATION OF THE COIL SYSTEM

The COILS program does not allow us to calculate directly $M_{nm} = \partial b_n / \partial v_m$ when v_m represents a geometrical variable. We therefore use, at least in

essence, difference quotients instead of the differential quotients. Our practical way of doing this is based on the following consideration.

According to Eq. (3), different vectors Δv lead to different vectors Δb. We now arrange these vectors to form matrices ΔV and ΔB such that the first column of Δv leads to the first column of ΔB, etc. Then, because of Eq. (3), we also have

$$\Delta B = M \Delta V. \tag{5}$$

To calculate M, we use the COILS program to calculate ΔB for a complete set of linearly independent Δv's, forming a square, nonsingular matrix ΔV. The columns of ΔV describe <u>finite</u> changes in geometry and currents, and the equivalent columns of ΔB are the resulting changes of the magnetic fields at the specification points. Since we are dealing here with finite changes, they represent in the case of geometry variables the addition of coils of finite length (chosen typically to be 1 in.), whose magnetic fields can be calculated by using the COILS program. Although in our particular case it was practical to make ΔV a diagonal matrix (i.e., calculating the effect of only one variable at a time), in cases with special symmetries it can be advantageous to use nondiagonal matrices ΔV. One advantage of this scheme for calculating M is that there is very little chance to get units and dimensions mixed up. From Eqs. (5) and (4) follows directly

$$\Delta v = \Delta V \Delta B^{-1} \cdot \Delta b. \tag{6}$$

When the coil system is constructed, its operation will be as follows. All the coils will be electrically in series with the energizing capacitor bank. Current control in the coils will be accomplished by firing crowbar ignitrons across individual coils at appropriate times to clamp the currents at the required values. Since there is the metallic liner inside the coils

and the actual fabricated coils will be different from the idealized ones used in the COILS program, we deem it inadvisable to rely on the calculations described above to determine the operating geometry and currents. These will be arrived at in the following way. There will be a number of pickup loops on the metal liner to allow an accurate measurement of the field at several axial positions. If a starting geometry and current distribution do not give the desired field configuration, the position and currents of the coils will be corrected according to Eq. (4). M in Eq. (4) will be obtained from Eq. (5), except that here ΔB will be determined by measuring the changes of the fields that result from changes of the positions of coils and the timing of the crowbar ignitrons. In other words, ΔB will be directly measured and the actual corrections will then, after ΔB is inverted, be directly obtained from Eq. (6). It is obvious that this procedure can be applied to any experiment or equipment that is controlled by many parameters. In practice we expect that the only real difficulty will be to obtain a uniform field gradient in region 1. This means that it will be necessary to invert only 3-by-3 or 4-by-4 matrices.

IV. PITFALLS

When one actually performs such calculations as the ones described above, there are, unfortunately, many ways to get into difficulties.

Although the linear approximation used to obtain the successive improvements is a good one, the real relationship between the geometry changes and the fields is nonlinear. This can lead to corrections that are too large, which in turn can result in divergence. We therefore reduced our original corrections by a relaxation factor 8, which was reduced atuomatically as the successive approximations became better.

A major source of trouble was that the matrices M were sometimes nearly singular. To detect the cause of such near signularities, we always printed out M^{-1}, as indicated in Eq. (4), although it would have been sufficient in principle to just solve Eq. (3) for Δv. Of the several observed causes for near singularity of M we mention only two here: (i) Unwise choice of the positions of the specification points relative to the coils, so that the field is overdetermined in one area and underdetermined in another. This, of course, can also happen as a result of successive approximations that move coils in an unanticipated manner. (ii) Loss of a variable because of successive geometry changes. This typically occurs when the end coordinates of coils are used individually as variables and the ends of two coils come very close to each other. The two columns of M describing the changes of b resulting from changes of these two end coordinates then become nearly identical, leading to a nearly singular matrix M.

V. PROGRAMMING DETAILS

The program used in these calculations (see Fig. 3) functions automatically through any number of iterative loops until the deviations from the desired field values at the specified points (Δb_n) are arbitrarily small, as predetermined by the programmer. Input data are read into the program, describing the number of coils, their starting positions, length, radius, and thickness; the radial and axial positions of the specification points; and the values of the field at these points which define the required field shape. Output from the program describes the final current and geometry configuration of the coil system and lists the deviations from the desired field at each specification point. Also, information related to each loop in the iteration sequence is printed so that the convergence (or in some cases, divergence) of the iteration process may be examined.

The program package itself consists of the COILS program with various modifications and additions. Subroutines were added to perform the following operations: (i) to internally define the geometry and current configuration of the coils and of short segments of the coils at appropriate positions for calculating the coefficient matrix elements [M_{nm} in Eq. (4)]; (ii) to redefine the current and geometry configuration of the coil system for each iteration as dictated by the solution to Eq. (4); and (iii) to perform the required matrix manipulation. If the reader wants a listing of the program package and usage instructions, which would be impractical to give here, he may communicate with the authors.

To examine in detail the fields produced by the idealized coil system, other programs (modifications and additions to the COILS program) were written. The output of these programs consists of graphical plots of various field properties such as shown in Fig. 4 and Fig. 5.

APPENDIX

Instead of using an equal number of specification points and variables and producing a field that fits the desired field exactly at the specification points, one can also use more specification points than variables and minimize the sum of the squares of the differences between the produced field and the desired field at the specification points.

If Δb_n are the field changes necessary to obtain an exact fit and $\sum_m M_{nm}\Delta v_m$ are the field changes produced by changing the variables v, one has then to minimize

$$\sum_n \left(\sum_m M_{nm}\Delta v_m - \Delta b_n \right)^2 W_n$$

with respect to all Δv_m. Here W_n is a weighting function that can be introduced if one wants to assign particular weights to individual field regions.

Differentiating the expression above leads to the set of equations that has to be fulfilled for all k:

$$\sum_{n,m} M_{nk} W_n M_{nm} \Delta v_m = \sum_n M_{nk} W_n \Delta b_n .$$

Introducing the transposed matrix $M^T (M_{nk} = M^T_{kn})$ and defining the diagonal matrix W by

$$W_{n,j} = W_n \cdot \delta_{nj}; \quad \delta_{n,j} \begin{array}{l} = 1 \text{ for } j = n \\ = 0 \text{ for } j \neq n \end{array} ,$$

one obtains the matrix equation

$$M^T W M \cdot \Delta v = M^T W \cdot \Delta b .$$

The solution of this equation,

$$\Delta v = (M^T W M)^{-1} M^T W \cdot \Delta b ,$$

gives the changes of the variables v necessary to obtain the least-squares fit as defined above.

It is of course also possible to make a least squares fit for some points, and, simultaneously, an exact fit for others. We do not describe the procedure in detail since, for all practical purposes, the same can be more easily obtained by using the least squares fit method with an appropriately chosen weight matrix.

Although we have not used this method extensively, we would like to mention that it can be advantageous for design work, and particularly for data analysis.

FOOTNOTES

1. F. B. Hildebrand, Introduction to Numerical Analysis (McGraw-Hill Book Company, New York, 1956). This reference is quoted only as one representative example of many books that describe Newton's method.

2. Mathematics and Computing Center, Lawrence Radiation Laboratory, Berkeley, California, Program TI EO COIL, April 29, 1963. This program is a modification of one developed by Prof. M. W. Garrett of Swarthmore College under Oak Ridge National Laboratory auspices. The majority of this write-up is abstracted from an unpublished manuscript of Prof. Garrett's entitled "Computer Program Using Elliptic Integrals for Magnetic Properties of Current Systems with Special Reference to the IBM 7090."

3. Instead of using only B_z to specify the field distribution, it probably would have been practical to use also $\partial B_z/\partial z$. However, $\partial B_z/\partial z$ was not available from the COILS program, and we did not invest the additional programming effort necessary to obtain $\partial B_z/\partial z$.

FIGURE CAPTIONS

Fig. 1. A typical desired field distribution.

Fig. 2. A 20-coil system for producing the desired field distribution.

Fig. 3. A block diagram of the iteration process.

Fig. 4. A representation of the coil system and plots of B_z at 1 in., 2 in., and 3 in. radii.

Fig. 5. Representation of the coil system (radial scale is twice axial scale), and plots of flux tubes and B_{tot} contour lines.

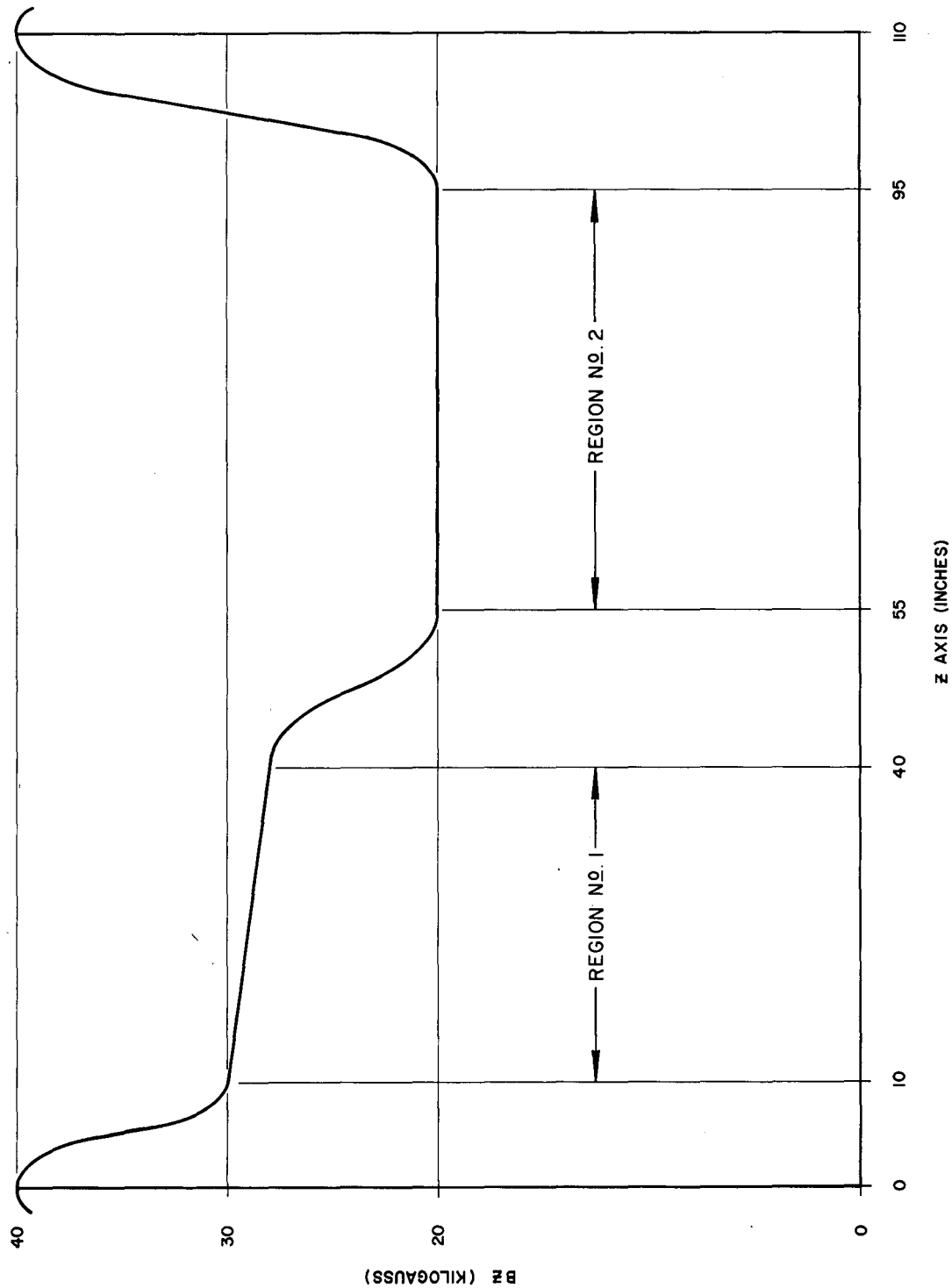

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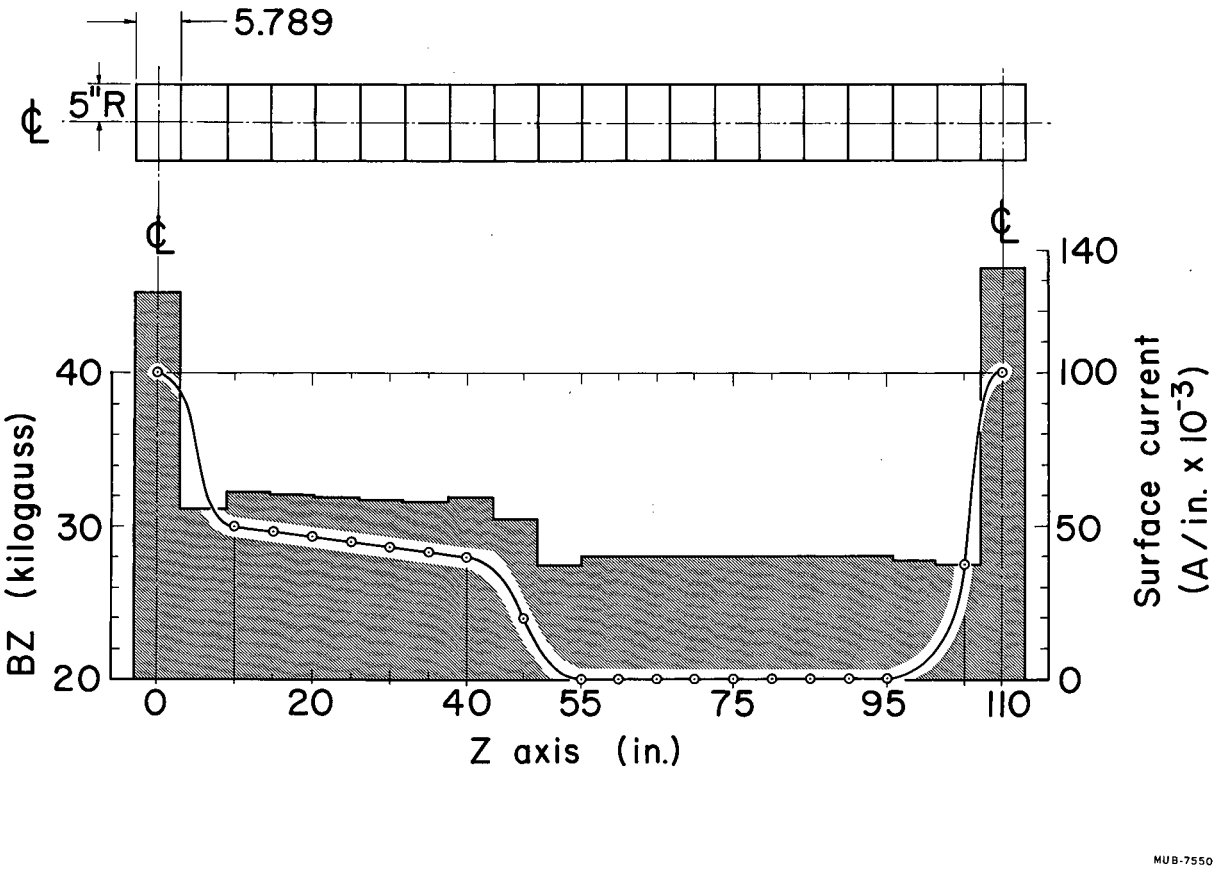
Fig. 1


5.789
5"R
℄
℄
℄
140
100
50
0
40
30
20
BZ (kilogauss)
Surface current
(A/in. x 10^-3)
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20
40
55
75
95
110
Z axis (in.)
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Fig. 2

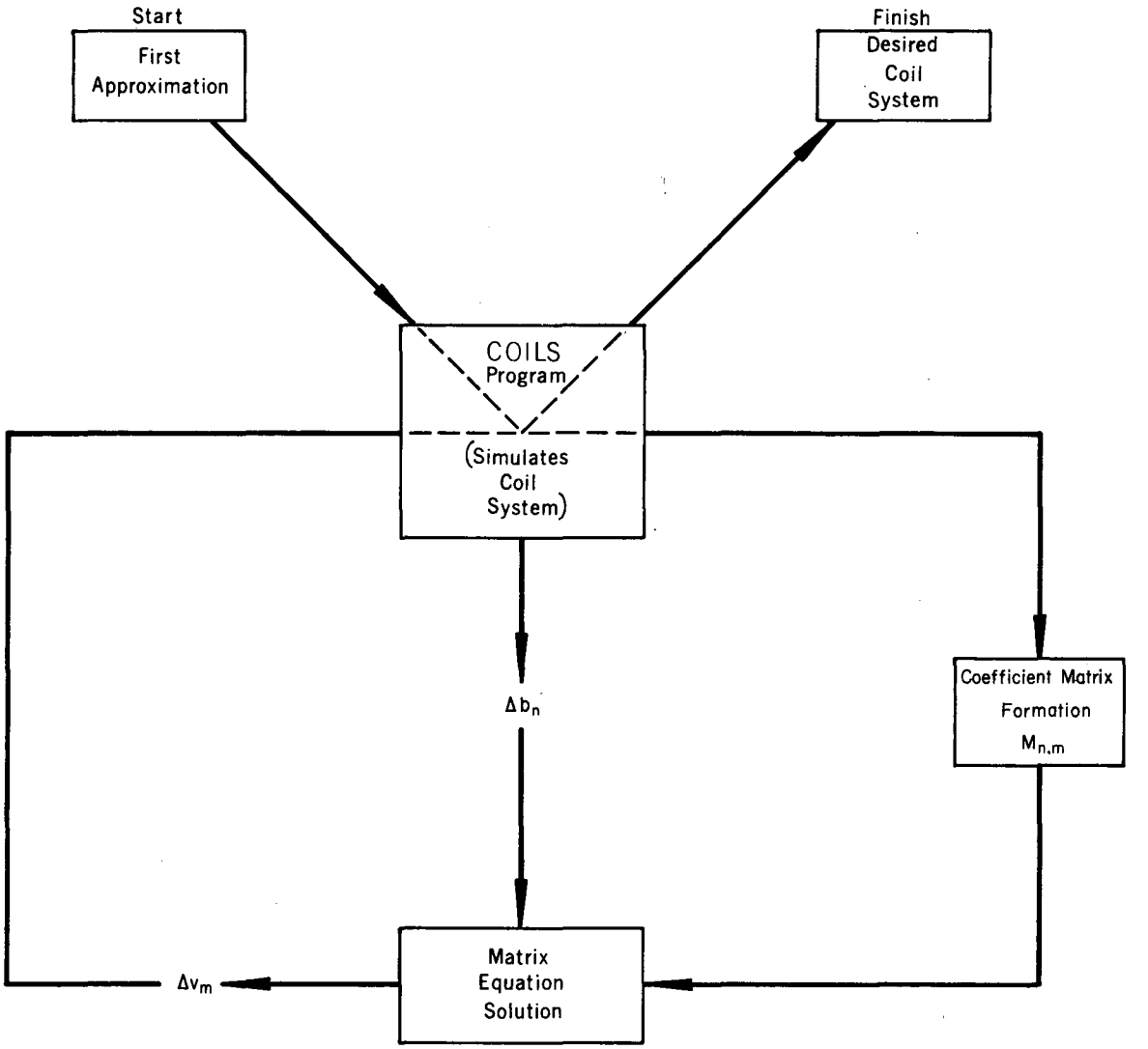

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Fig. 3

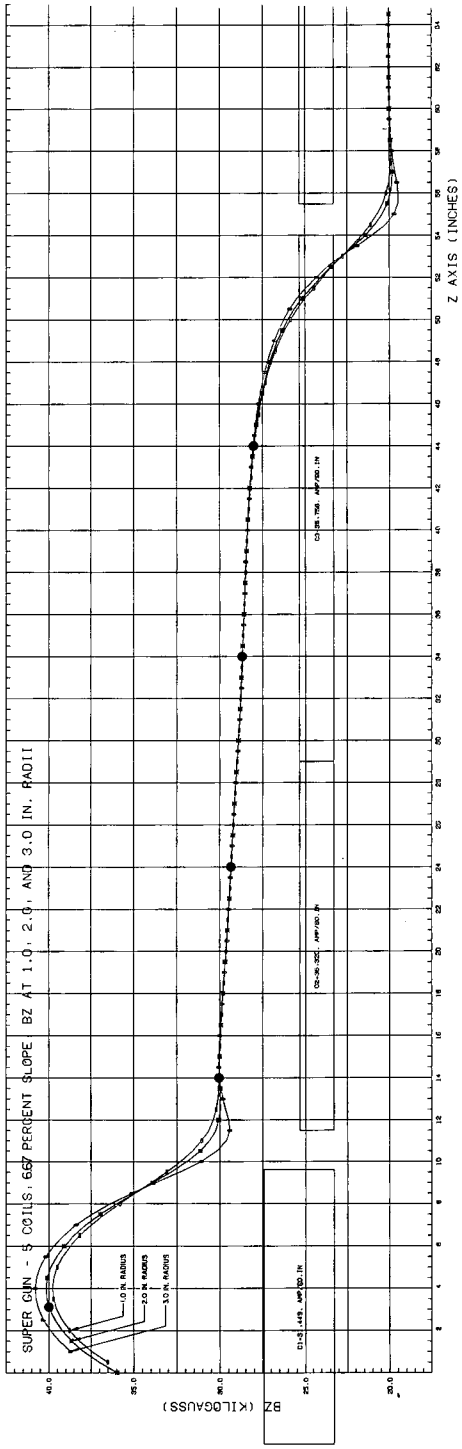

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Fig. 4

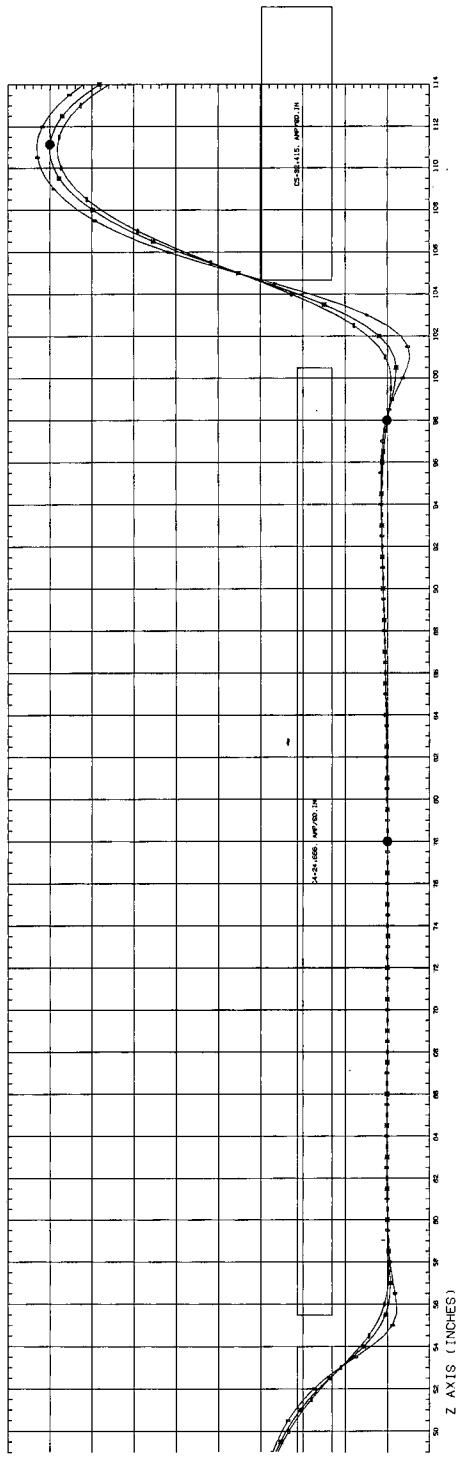

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Fig. 4 continued

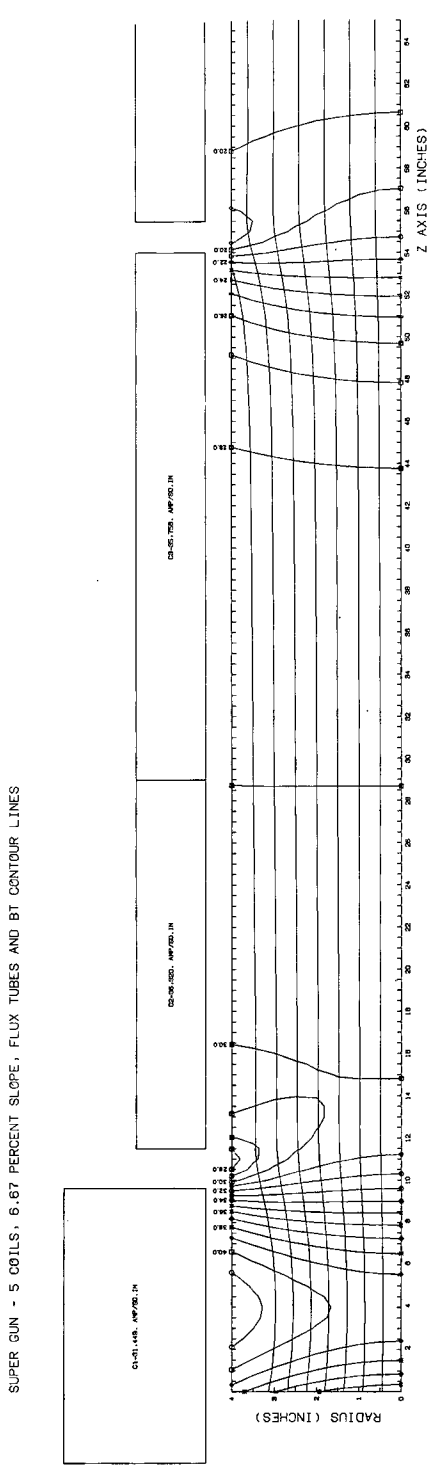

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Fig. 5

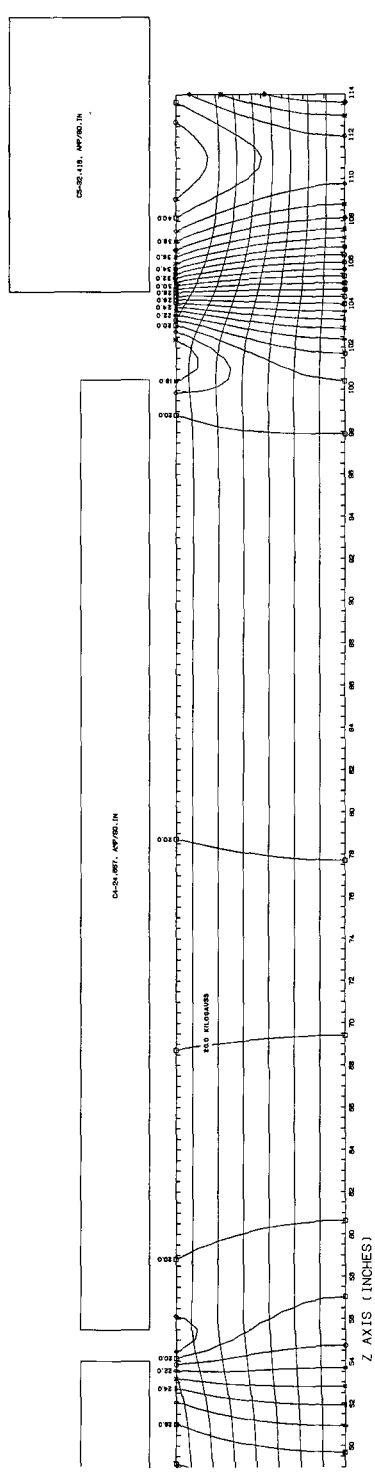

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Fig. 5 continued

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