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DERIVATION OF CORRECTIONS TO CONTROL THE CLOSED ORBIT IN AN ALTERNATING-GRADIENT SYNCHROTRON

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DERIVATION OF CORRECTIONS TO CONTROL THE CLOSED ORBIT IN AN ALTERNATING-GRADIENT SYNCHROTRON*

G. R. Lambertson and L. Jackson Laslett
(Paper Presented by Dr. A. A. Garren)

I. INTRODUCTION

The position of the closed orbit in the magnet aperture of a synchrotron is affected by changes in magnetic guide field and by displacements of the magnet structure. Either effect may be used, with minor limitations, to adjust the closed-orbit position and to correct for disturbances from either source. With limited knowledge about the causes of closed-orbit disturbances and a limited number of observations of beam position, one cannot generally expect to detect and uniquely correct specific misalignments or field errors. A closed-orbit control system employing whatever adjustments are provided will at best position the closed orbit at specific points around the synchrotron; the designer or user must select the type and degree of adjustment to be used and judge the acceptability of the effects in regions between the controlled points.

This paper reports progress on a study of how to control the closed-orbit position by displacement of the synchrotron structure.[1] We consider a system in which the closed-orbit position is detectable at k points around the accelerator and the user may select a set of desired changes, Δb_o, in the beam position at these k points. A computer program will then be used to determine the structure displacements required to approximate the desired changes, making provision to avoid excessive, unproductive movements and undesirable excursions of the beam between detector stations. The user may then apply the adjustments and monitor the effect at the detector stations. This system employs feedback to the extent that desired changes are based upon the detected beam position, but this feature is not prominent although it may become important in repeated application of the system. Also, it may be noted that this system does not necessarily result in a direct cancellation of geometric misalignments of the magnet structure.

II. METHOD

We have taken as a model for study the 200 GeV proton synchrotron of the LRL Design Study.[2] This accelerator has a betatron frequency, ν, of 16-3/4; there are 504 alternating-gradient magnets and 12 Collins-type straight sections with two quadrupoles each. This structure is represented in Fig. 1. The structure is considered as divided into rigid segments joined at 276 hinge points, and radial or vertical displacements may be introduced at these hinge points indicated in Fig. 1. In these displacements, the vacuum tank and beam detectors move with the local magnet structure. Beam-position detectors which give the closed-orbit position with respect to the local structure are located at 72 places. This number k of detectors corresponds to 4.3 per betatron wavelength, and was judged to be near the minimum needed for a meaningful study and the maximum convenient for computational work in the initial investigation. More frequent detectors may be useful in practice. For judging the suitability of an adjustment, the resultant orbit position has been calculated at the 288 points indicated in Fig. 1, which include the 72 detector stations.

The matrix T that relates the 276 hinge-point movements, h, with the resulting 288 closed-orbit changes Δb, according to

$$\Delta b = Th, \qquad (1)$$

has been calculated for this synchrotron. When changes of closed-orbit positions Δb_o at only 72 points are specified, it has been convenient to displace the hinge points in 72 groups. A number of grouping schemes has been tried and a favorable example is shown in Fig. 2. In this case, movement of a single group produces a smooth hump involving 10 or 11 hinge points, with each hump centered at one of the beam-detector locations. The equal movement of all groups produces a uniform displacement of the magnet structure. Smoothness of the group shape and overlap between adjacent groups serve to reduce the introduction of short-wavelength displacements.[3] The amplitude of a group movement may be expressed by a single parameter and the displacement of the structure is then given by the 72 components of a vector h_c, composed of the amplitudes of all of the group movements. With a particular group pattern, the closed orbit changes at the 72 detector stations may now be written as

$$\Delta b_c = T_c h_c, \qquad (2)$$

where T_c is a 72 x 72 matrix operating on the group amplitudes h_c.

To find values of h_c that will exactly produce the desired changes Δb_o, it would appear that these are given, from Eq. (2) with $\Delta b_c = \Delta b_o$, by

$$h_c = T_c^{-1}(\Delta b_o).$$

There are, however, difficulties in this direct procedure. First, the matrix T_c has at least one eigenvalue equal to zero, or $|T_c| = 0$, so that the matrix has no inverse. This arises from the fact that uniform displacement of all magnets produces no relative displacement of the beam. Nevertheless, if the eigenvectors having zero eigenvalue were not complex, they could be deleted from Δb_o and real values of the h_c found for the correctable portion of Δb_o. A second difficulty appears if this procedure requires excessive displacements to produce exactly a desired set of Δb_o, which may include errors in beam-detector response and operator judgment. A more flexible approach has been adopted to deal with these difficulties.

We shall seek a modified solution of the least-squares problem

$$\chi^2 = \Sigma[(\Delta b_c)_i - (\Delta b_o)_i]^2 = \text{minimum}. \quad (3)$$

This minimization leads directly to the equations

$$\tilde{T}_c T_c h_c = \tilde{T}_c \Delta b_o, \quad (4)$$

where $\tilde{T}_c$ is the transpose of T_c and the product $\tilde{T}_c T_c$ is a symmetric matrix having all eigenvalues real and either zero or positive.

It is of interest to note that any particular eigenvalue λ of $\tilde{T}_c T_c$ is a measure of the effectiveness of the associated normalized eigenvector for influencing the closed orbit. The sum of the squares of the orbit changes produced by h_c is

$$\tilde{\Delta b}_c \Delta b_c = \tilde{h}_c \tilde{T}_c T_c h_c ,$$

from which one may see that the contribution of the jth eigenvector to $\Sigma(\Delta b_c)_i^2$ is $\lambda_j a_j^2$ where a_j is the amplitude of the jth normalized eigenvector in h_c.

This interpretation of the eigenvalue has suggested that in the minimization of Eq. (3), the eigenvectors with low eigenvalue could be deleted from the solution h_c with only minor effects on Δb_c, while substantially reducing the root-mean-square hinge-point movements required. Eigenvectors with eigenvalue zero are an extreme example and at least these must be deleted to obtain a set of finite displacements.[4]

To calculate a set of h_c with certain eigenvectors excluded, first the eigenvalues and 72 orthonormal eigenvectors of the matrix $\tilde{T}_c T_c$ have been obtained. Using these, an inverse matrix with selected eigenvectors deleted has been formed and combined with $\tilde{T}_c$ to produce a single matrix K that operates on any given Δb_c to generate the recommended hinge-point movements

$$h_c = K \Delta b_c . \tag{5}$$

The eigenvalues obtained with the groups of Fig. 2 are plotted in Fig. 3. Arranged in order of their dominant frequency, these eigenvalues demonstrate a resonant sensitivity to displacements with frequencies near the ν-value. Lowest eigenvalues are associated with low harmonic frequencies. The effects of excluding various groups of eigenvectors have been studied.

III. RESULTS

Realistic examples of closed-orbit disturbances for use in evaluating the correction system have been generated by introducing displacements h at the hinge points and calculating the resultant relative beam displacements from Eq. (1). The 72 beam displacements at the detector stations are then used in Eq. (5) to give the recommended gang amplitudes for correction of this disturbance. The effect of applying these displacements is calculated and the resulting corrected closed-orbit deviations evaluated at 288 points.

A long smooth bulge involving 17 hinge points has been one example used as a test case. This distortion of the structure and the resultant absolute beam position are plotted in Fig. 4. This figure also shows the result of correcting the closed-orbit deviation exactly at the 72 detector stations by excluding only the one eigenvector having zero eigenvalue. The presence of relatively unproductive low-frequency harmonics in the correction is apparent. The same case is shown in more detail over 1/6 of the circumference in Fig. 5. Here the closed-orbit position relative to the magnet structure is also given. For comparison with the "exact" correction, Fig. 6 shows the correction obtained with this same initial disturbance when 19 eigenvectors having lowest eigenvalues

are excluded. The corrected closed orbit no longer passes through zero at each detector station, but the maximum beam excursion from the aperture center is essentially unchanged from the exact solution while the magnet structure has assumed a much more acceptable form. The effect of excluding various eigenvectors is illustrated in Figs. 7 and 8, where the final maximum closed-orbit deviations and root-mean-square hinge movements are plotted.

A second type of test disturbance is the displacement of a single hinge point. Such an abrupt excursion cannot be approximated--nor removed--by the smooth groups. The result of correcting exactly the closed-orbit distortion at 72 stations is shown in Fig. 9; the correction with 19 vectors excluded is shown in Fig. 10. These illustrations and Figs. 7 and 8 show the advantage of excluding eigenvectors for this type of disturbance.

A preliminary indication of the effect of errors in the beam-detector response or of requesting abrupt closed-orbit changes has been given by assuming one detector station to be in error by one unit and seeking a compensating closed-orbit position. The results summarized in Figs. 11 and 12 point out the importance of removing unproductive movements to limit the response to errors of this type.[5]

IV. CONCLUSIONS

This study has revealed some of the characteristics of one type of closed-orbit control system. At this point we feel that the usefulness of such a system is limited to the mid-range of frequencies near the ν-value of the synchrotron. Fortunately it is in this range that large closed-orbit disturbances are most probable. Increased numbers of beam-detector stations will improve performance of the system and it seems reasonable to expect to obtain substantial reductions of the final closed-orbit deviations. The limited ability of the control system to realign specific misalignments of the structure indicates that closed-orbit control will only complement, but not entirely replace, the geometrical survey of an alternating-gradient synchrotron structure.

In the 200 GeV synchrotron, closed-orbit control would be useful for control of the location of unavoidable beam loss, so that radioactivation of accelerator components and of the enclosure may be limited in azimuthal extent, and could implement changes desired in the mode of operation to meet the varying needs of the experimental program. Computer analysis of beam-position detector data also would be useful as a diagnostic tool in determining the probable

location and cause of disturbances that may arise during accelerator operation.

In the past, prominent factors in determining synchrotron aperture requirements were the injected beam emittance and allowance for misalignments and errors. Experience with closed-orbit control and increased beam intensities may be expected to result in the replacement of these factors by the requirements of collective effects, beam use, and extraction.

It is a pleasure to acknowledge the invaluable assistance of Mrs. Harold (Barbara) Levine and of Miss Penelope A. Collom in the computational work.

FOOTNOTES

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[1]Extensive previous discussion of orbit control has been given by a number of Soviet authors--see, for example, E. L. Burshtein, A. A. Vasil'ev, A. L. Mints, V. A. Petukhov, and S. M. Rubchinskii, Atomnaya Energiya 12, 111 (1962) and papers in the Proceedings of the International Conference on High Energy Accelerators, Dubna, 1963 (Atomizdat, Moscow, 1964).

[2]Lawrence Radiation Laboratory Report UCRL-16000 (University of California Lawrence Radiation Laboratory unpublished report, June 1965.)

[3]Amongst other group configurations that appear satisfactory is that which employs movements of the hinge points in various long-wavelength sinusoidal combinations. The more localized groups shown in Fig. 2 lead, however, to equally good performance, and their relatively short range may be of convenience in the practical application of the technique.

[4]As an alternative to the suppression of selected eigenvectors, early work included the insertion of additional terms, with adjustable weights, into Eq. (3) for the purpose of reducing the total magnitude and curvature of the gang movements. This procedure led to corrections similar to those obtained by eigenvector suppression, but suffers from the feature that repeated applications of the method will eliminate, in effect, the influence of these additional terms.

[5]With 19 eigenvectors excluded, the maximum orbit response that results from a unit error in an individual detector signal ranges in magnitude, for various detector locations, between 0.682 and 0.836. The corresponding root-mean-square response is such that, for 72 uncorrelated signal errors of individual standard deviation ϵ, one expects a root-mean-square orbit deviation of 0.828ϵ. To achieve a 95% correction of tractable closed-orbit distortions in a single iteration, it would be necessary, therefore, that the detector signals have a relative probable error (0.6745ϵ) less than 4%. A second source of error that remains to be investigated concerns the effect of employing, in a single iteration, a matrix, T_c, that is based on the design specifications of the accelerator and does not describe exactly the performance of the actual machine--it may be presumed desirable, in this case, to adjust T_c to be consistent with the observed ν value, since this parameter has a significant long-range influence on the closed-orbit response.

FIGURE CAPTIONS

Fig. 1 Magnet configuration, monitor locations, and betatron function in a portion of the synchrotron. One superperiod covers 23 hinge points.

Fig. 2 Pattern of smooth hinge-point groups used for adjusting the magnet structure.

Fig. 3 Eigenvalues arranged in order of increasing dominant frequency of eigenvector formed from amplitudes of hinge-point groups. Eigenvalue levels below which eigenvectors were excluded in various tests are also indicated.

Fig. 4 Polar plot showing expanded absolute displacements of magnet structure and closed orbit, before and after corrections, in an example with only one eigenvector excluded. The initial magnet distortion is a long smooth bulge involving 17 hinge points.

Fig. 5 Detailed plot for a portion of the circumference of structure and orbit displacements of Fig. 4. The lower diagram shows the corrected closed-orbit position relative to the magnet structure.

Fig. 6 Orbits and structure as in Fig. 5, except with 19 eigenvectors excluded in making the correction.

Fig. 7 The maximum displacement of the corrected closed orbit, as a function of the number of eigenvectors excluded, for two types of initial disturbance.

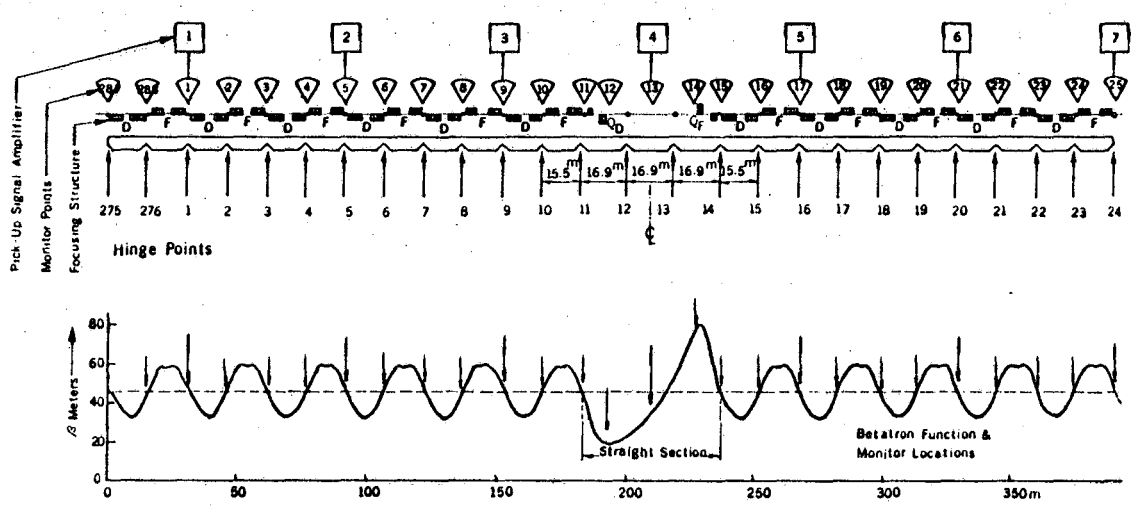
Fig. 8 Root-mean-square hinge-point movement employed in the cases depicted in Fig. 7.

Fig. 9 Magnet displacement and corrected orbit position, similar to Fig. 5, but for an initial displacement of a single hinge point. In making the correction, only one eigenvector was excluded.

Fig. 10 Orbits and structure as in Fig. 9, except with 19 eigenvectors excluded in making the correction.

Fig. 11 Effect of an adjustment of the structure made on the basis of unit error in the signal from one beam detector station. The graph shows the resultant maximum closed orbit deviation as a function of the number of eigenvectors excluded.

Fig. 12 Effect of unit error in one detector signal on the root-mean-square hinge-point movements.



Magnet Configuration & Monitor Locations
Within One Superperiod

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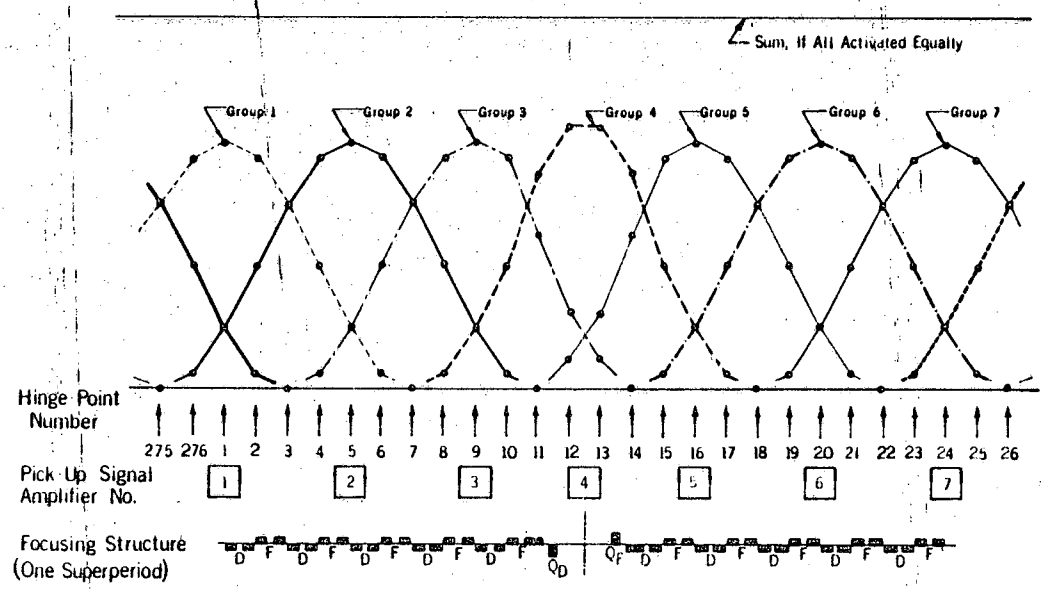
Fig. 1



Form of Hinge-Point Movements
For Successive Hinge-Point Groups

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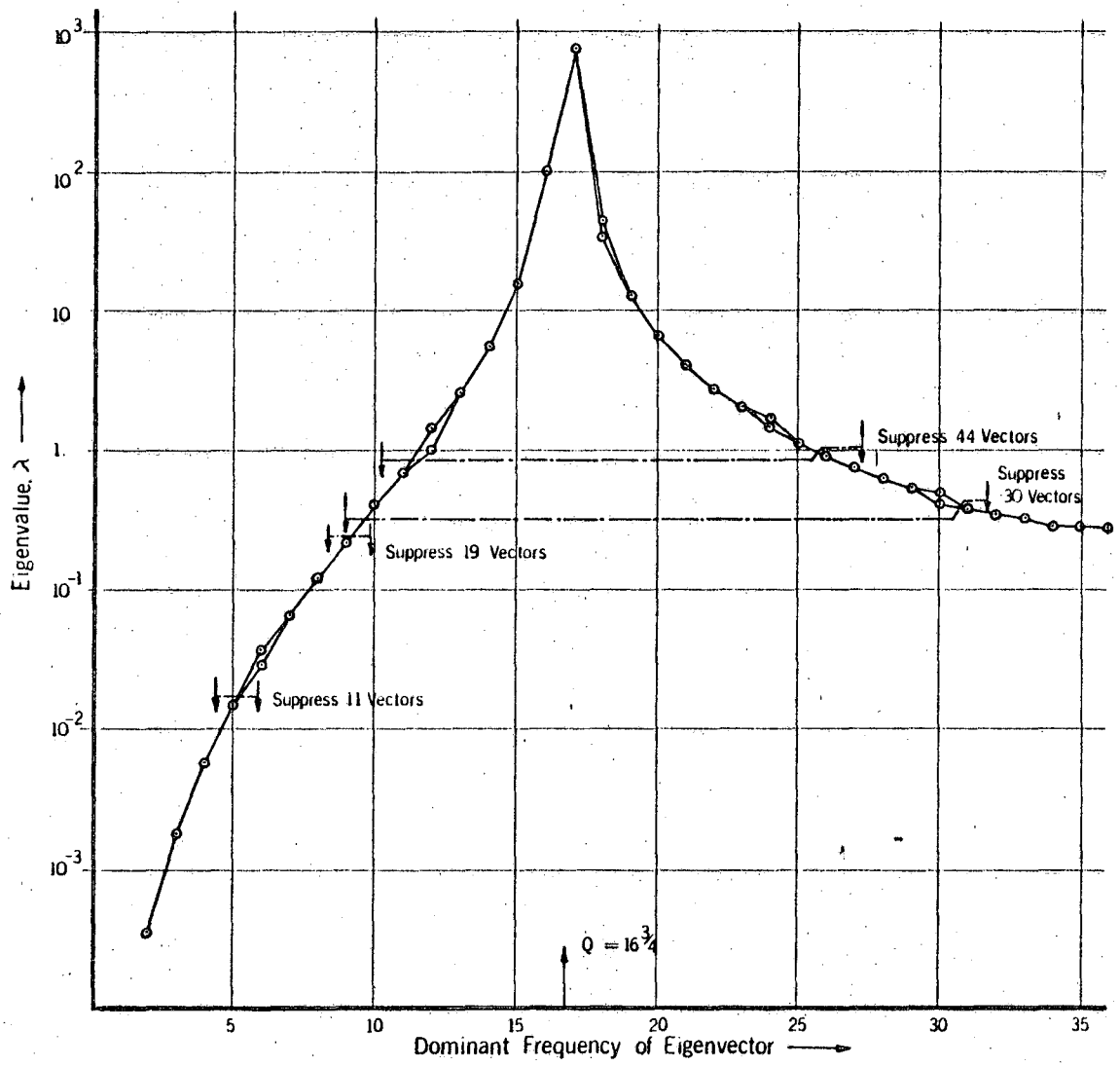
Fig. 2



Eigenvalue, vs. Dominant Frequency, of Eigenvector Formed From Amplitudes of Hinge Point Groups

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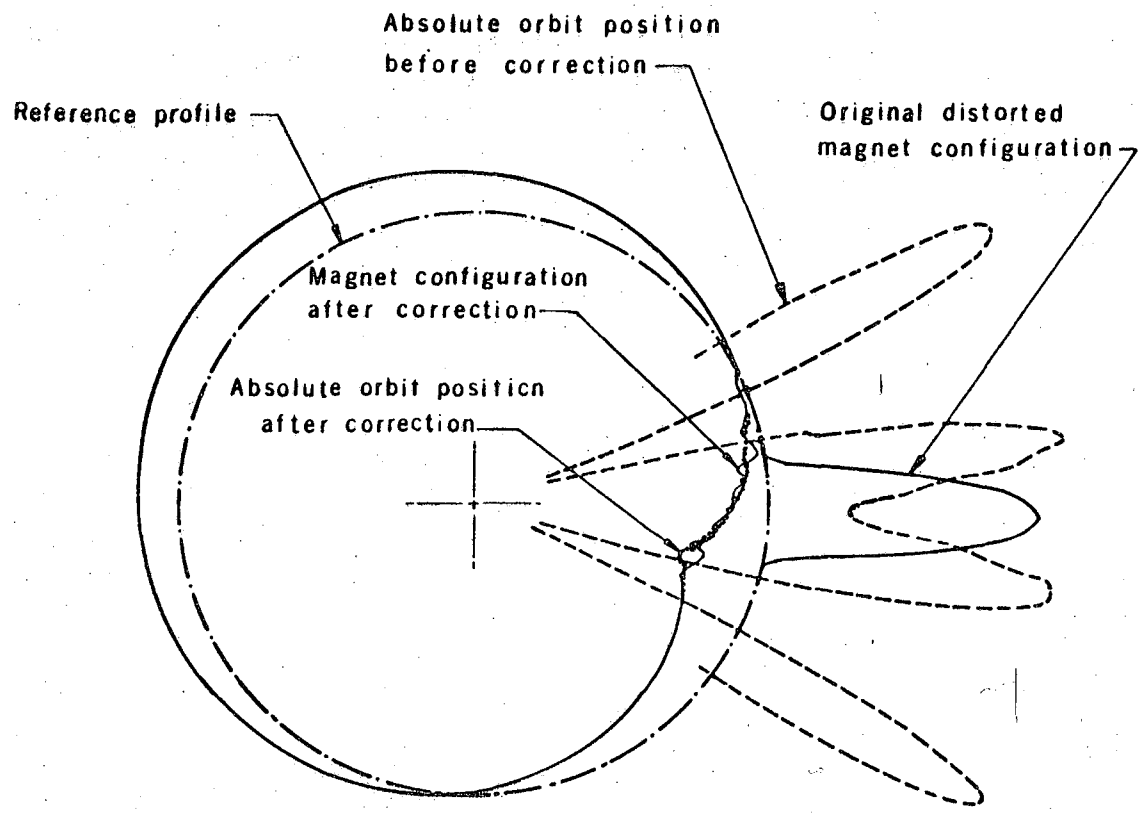
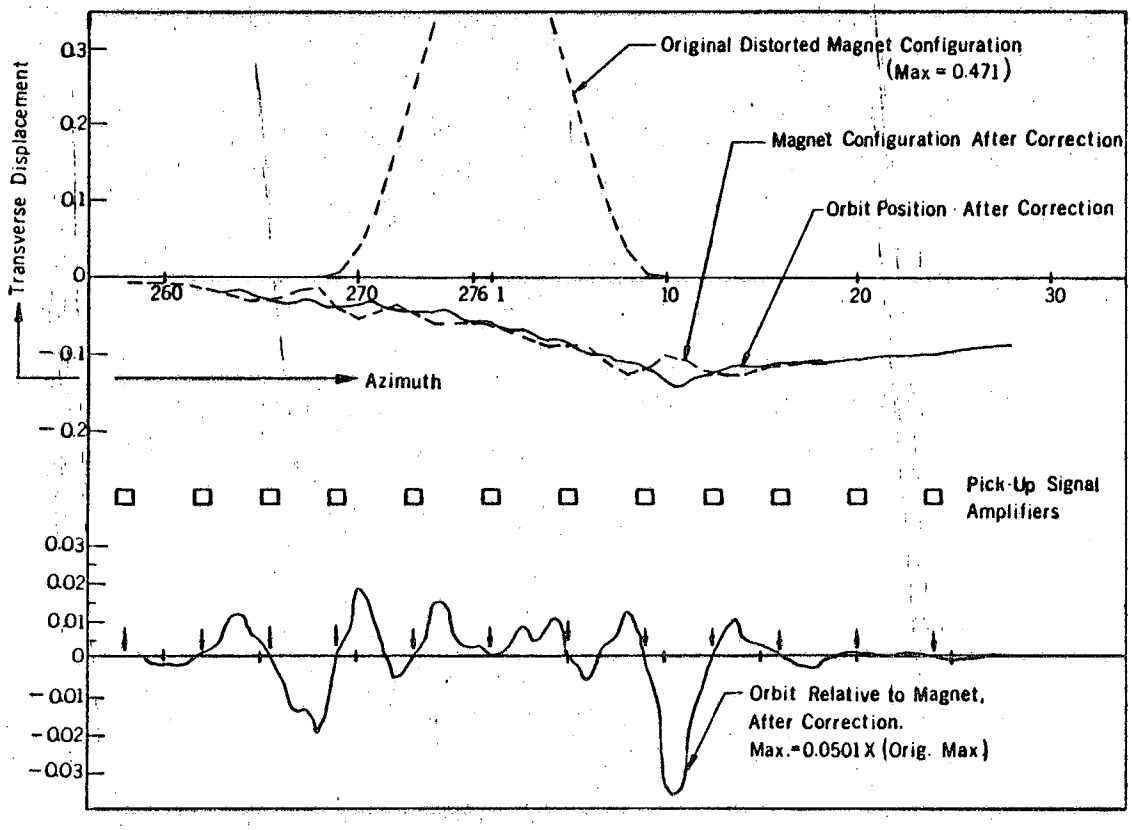
Fig. 3



Absolute Positions of Magnet Structure and Closed Orbit Before and After Corrections With Only One Eigenvector Excluded

MUB-7702

Fig. 4



Response With One Eigenvector Excluded

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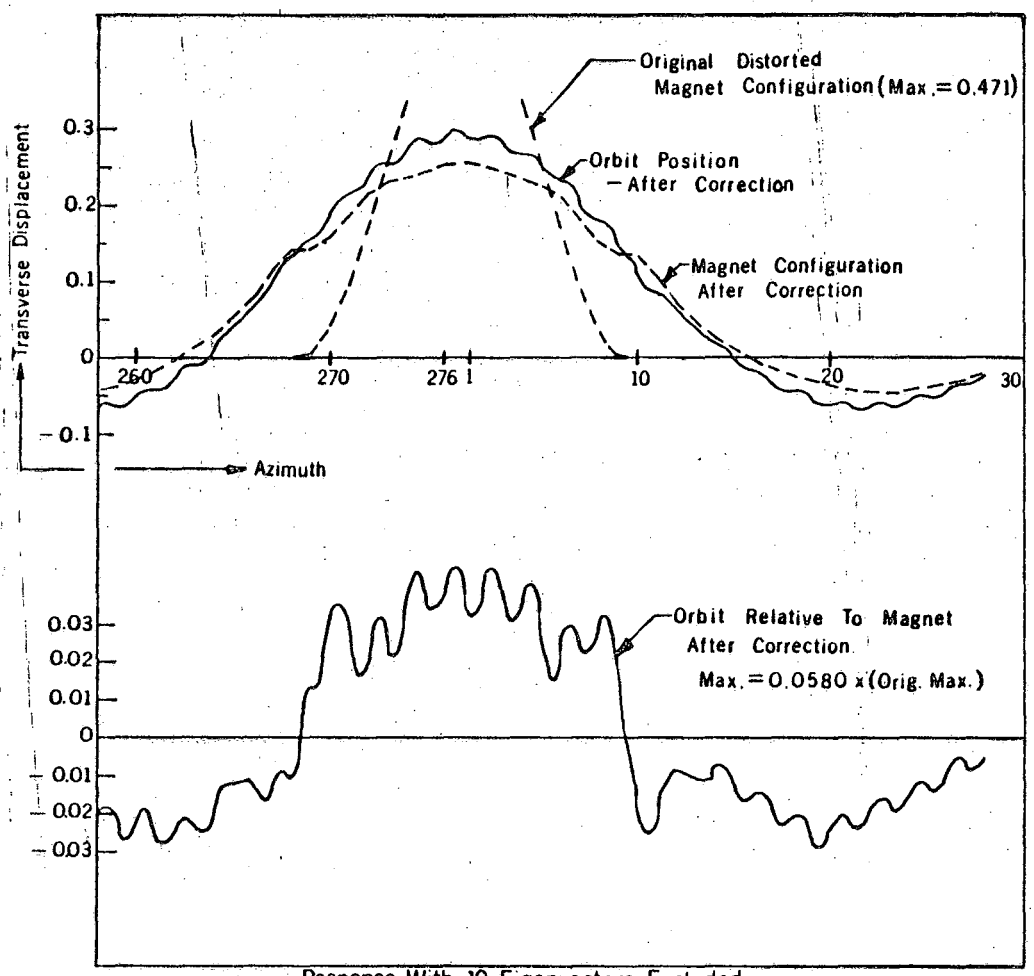
Fig. 5



Response With 19 Eigenvectors Excluded

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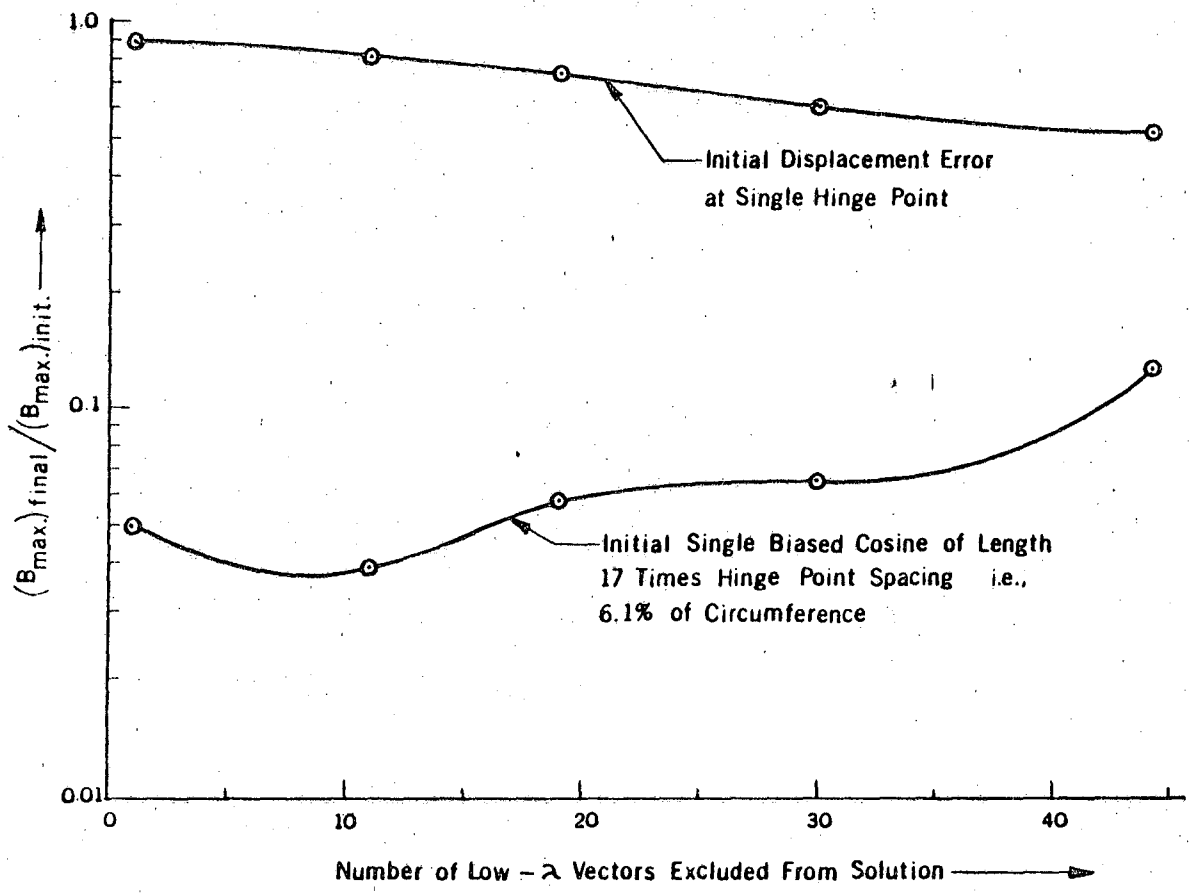
Fig. 6



Dependence of Residual Error
Upon Number of Vectors Excluded

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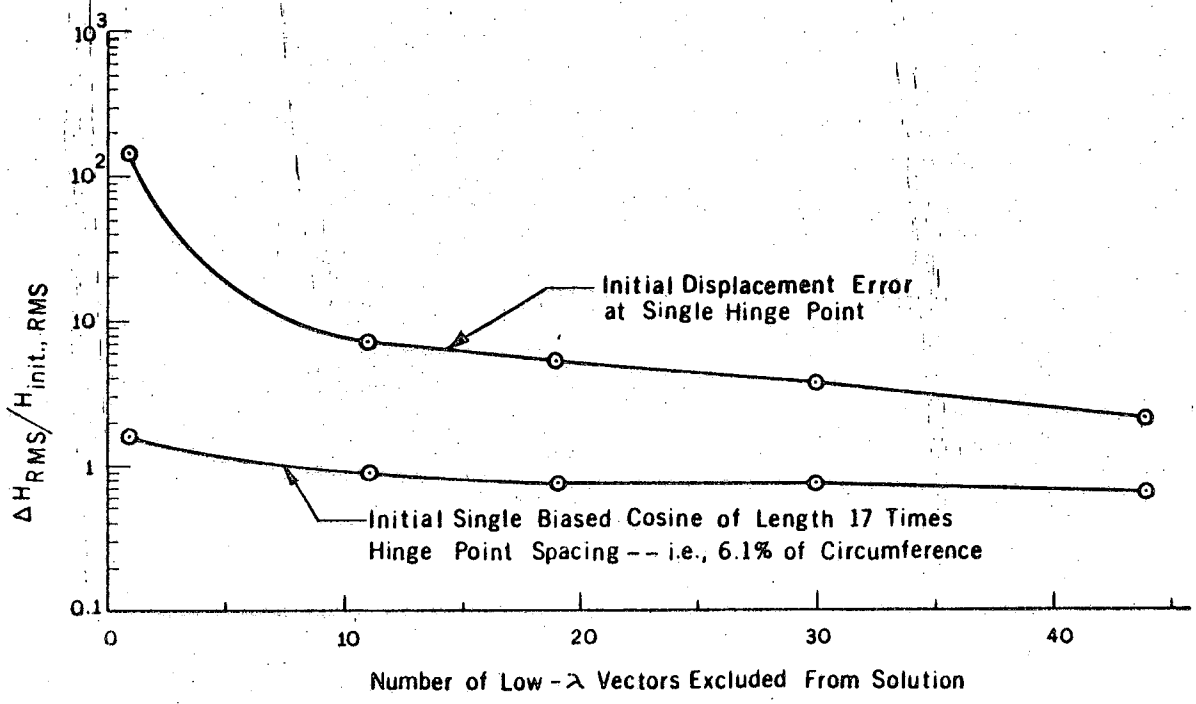
Fig. 7



Dependence of R.M.S. Hinge-Point Movement Upon Number of Vectors Excluded

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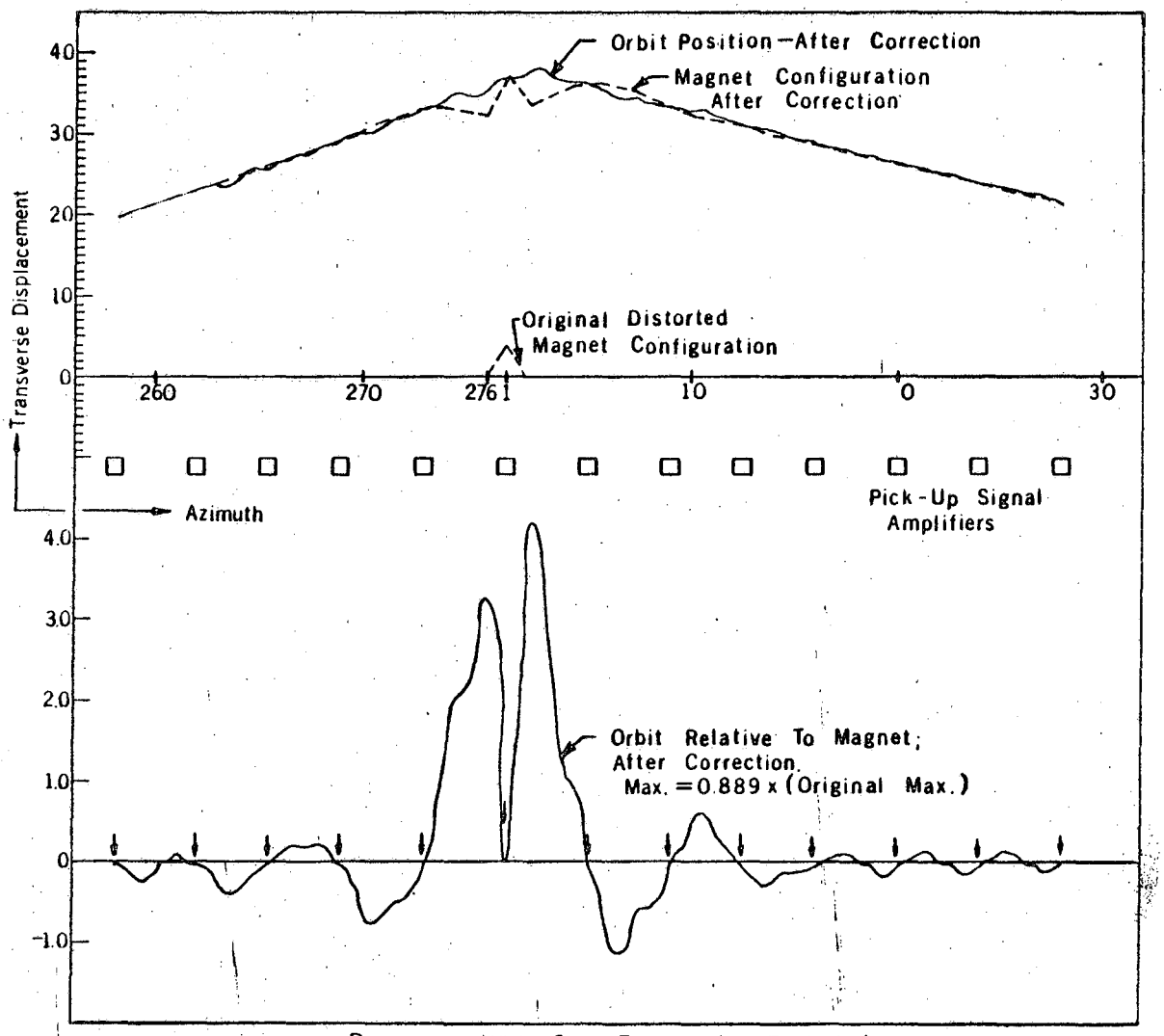
Fig. 8



Response With One Eigenvector Excluded

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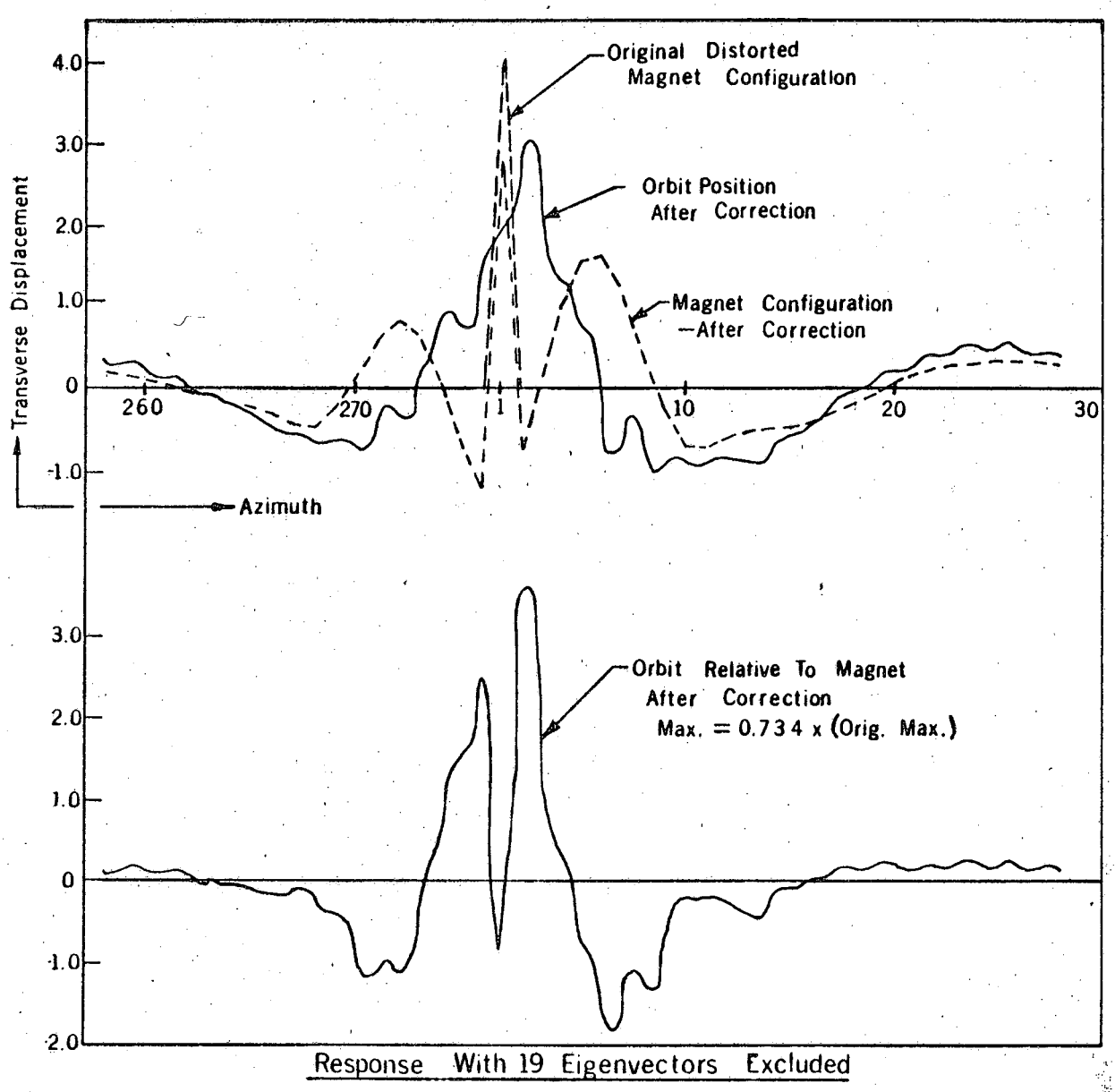
Fig. 9



Response With 19 Eigenvectors Excluded

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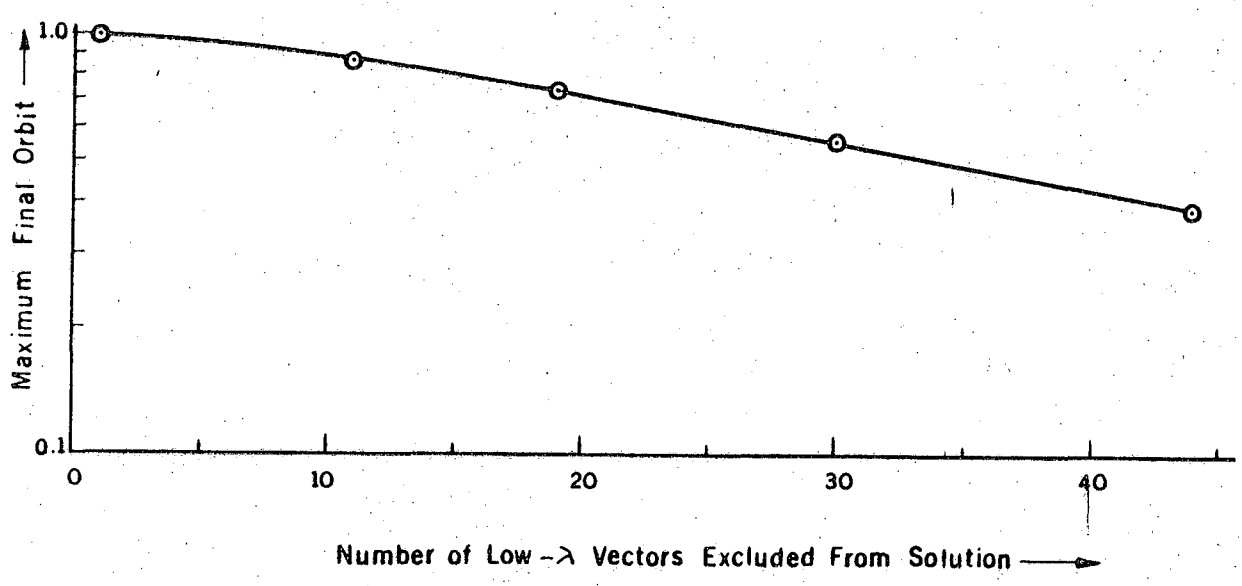
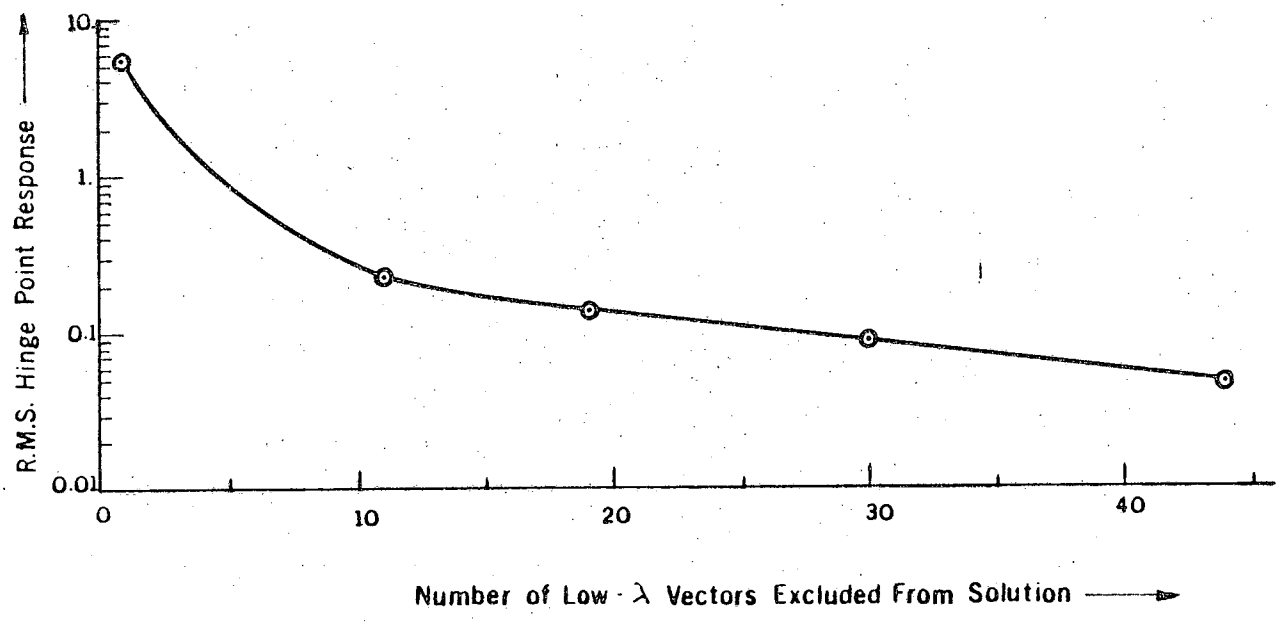
Fig. 10



Response to Unit Error
In the Signal From One Pick-Up Amplifier (#1)

MUB-7598

Fig. 11



Response to Unit Error
In the Signal From One Pick-Up Amplifier (#1)

MUB-7599

Fig. 12

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