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REMARKS ON THE SATURATION OF EQUAL-TIME COMMUTATORS AND PHYSICAL SUM RULES

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August 24, 1965

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1. The equal-time commutators of unrenormalized current operators have recently been the object of intensive investigation. Much of the discussion has been focused on the following aspect. Consider a theory which is invariant under a Lie group that is generated by charge operators constructed from a set of currents. Then the invariance requires that the expectation value of the equal-time commutator of two charge operators in a physical state belonging to some irreducible representation be saturated by a single intermediate state of the same irreducible representation. Conversely, if it is assumed that the expectation value of such a commutator is saturated by a single intermediate state, then results characteristic of the symmetry are obtained.[1,2,3] This latter circumstance has led to the interesting speculation that the saturation of equal-time charge commutators by judiciously selected intermediate states may be taken as a kind of dynamical mechanism for the induction of approximate symmetries.[2] Unfortunately, the phrase "dynamical mechanism" is difficult to define precisely in this context. Equal-time commutators, per se, are objects devoid of any special

dynamical significance. In particular, the mass of an intermediate state has no apparent bearing on its importance in the sum over states. Thus, the choice of an intermediate state cannot be predicated on any simple dynamical principle of the type underlying, say, the Goldberger-Treiman formulae.

2. The purpose of the present note is twofold. We first develop a field theoretical identity that relates equal-time current commutators to the off-mass-shell analytic continuation of physical scattering amplitudes. Here our work is identical in spirit to and largely motivated by that of Fubini, Furlan, and Rossetti,[4] but it differs in detail and suffers from no ambiguities. We then investigate the possibility of extracting useful information from this identity by approximating the physical scattering amplitude. Our approximation can be justified to some extent on dynamical grounds, for it is related to the familiar technique of pole approximation.

3. Let $J_\nu(x)$ be a current operator and $j(x)$ any Heisenberg operator. Gauss' theorem implies the trivial identity

$$\int d^4x\, \partial^\nu \left\{ e^{iq'x} \langle\, p' \mid i\, [\, J_\nu(x),\, j(0)\,]\, \theta(x_0) \mid p\, \rangle \right\} = 0 \,. \tag{1}$$

The states of momentum p' and p will, for the sake of definiteness, be taken as single baryon states. On evaluating the divergence we obtain

$$\int d^3x \; e^{-i\underset{\sim}{q}'\cdot\underset{\sim}{x}} \langle\, p' \mid i\,[\, J_0(\underset{\sim}{x}, 0),\, j(0)\,] \mid p \,\rangle$$

$$= q'^{\nu} \int d^4x \; e^{iq'x} \langle\, p' \mid [\, J_\nu(x),\, j(0)\,]\, \theta(x_0) \mid p \,\rangle \tag{2}$$

$$- c \int d^4x \; e^{iq'x} \langle\, p' \mid i\,[\, P(x),\, j(0)\,]\, \theta(x_0) \mid p \,\rangle \,,$$

where

$$\partial^{\nu} J_\nu(x) = c\, P(x) \,. \tag{3}$$

We take $j(x)$ to be a current density and denote by $\phi(x)$ the field generated by this current,

$$\left(\Box + m_\phi^2\right)\phi(x) = j(x) \,. \tag{4}$$

Then, by use of standard reduction techniques, Eq. (2) may be cast into the form

$$\int d^3x \; e^{-i\underset{\sim}{q}'\cdot\underset{\sim}{x}} \langle\, p' \mid i\,[\, J_0(\underset{\sim}{x}, 0),\, j(0)\,] \mid p \,\rangle$$

$$= q'^{\nu} \int d^4x \; e^{iq'x} \langle\, p' \mid [\, J_\nu(x),\, j(0)\,]\, \theta(x_0) \mid p \,\rangle \tag{5}$$

$$+ \frac{icT}{\mu^2 - q'^2} \,,$$

where μ is the mass of the P particle and T is the amplitude for ϕ + baryon $\rightarrow$ P + baryon scattering.

We defer a full investigation of Eq. (5) to a later publication. In the present note we consider only the limit in which q' vanishes,

$$\langle p' \mid i[Q, j(0)] \mid p \rangle$$

$$= \lim_{q' \to 0} q'^{\nu} \int d^4x \; e^{iq'x} \langle p' \mid [J_\nu(x), j(0)]\theta(x_0) \mid p \rangle$$

$$+ (ic/\mu^2) \lim_{q' \to 0} T \; , \tag{6}$$

where

$$Q = \int d^3x \; J_0(\underset{\sim}{x}, 0) \; . \tag{7}$$

The first term on the right-hand side of Eq. (6) vanishes unless an intermediate state contributes which is degenerate in mass with either of the baryon states. If this is the case, this contribution combines with a corresponding term in the Born-approximation part of the amplitude T to give a well-defined and unambiguous limit, although the limit of the separate terms is ill defined. It is in this respect that our method is superior to that of Fubini, Furlan, and Rossetti.[4]

Equation (6) forms the basis for the discussion of the remainder of this note. We identify $J_\nu(x)$ with one of the 8 components of the axial current density which transform as the generators of SU(3) or with an SU(3) singlet axial current. We may then assume a generalized

version of the partially conserved axial current hypothesis (PCAC) and take $P(x)$ to be the field operator of the corresponding member of the pseudoscalar octet with c a uniform constant for all members of the octet, or the field operator for an $SU(3)$ singlet state (η'). If $j(x)$ is identified with a nonet of polar or axial vector currents, the equal-time commutator appearing on the left-hand side of Eq. (6) can be computed by assuming that these currents are composed of bilinear combinations of Fermi fields that satisfy canonical commutation relations. If $j(x)$ is taken to be the source of the pseudoscalar meson octet or singlet, we obtain a generalization of a relation of Adler.[5] In this case it can be shown that the relevant commutator vanishes at the unphysical value of the momentum transfer $(p' - p)^2 = \mu^2$. This is adequate for our purposes.

4. On choosing $j(x)$ to be the pseudoscalar current we obtain a constraint on meson-baryon scattering. In order to get useful information from this constraint, we make the dynamical assumption that low-energy meson-baryon scattering is dominated by the exchange of a few systems with specific transformation properties under $SU(3)$. More precisely, we assume that systems of unit baryonic charge exchanged in the s and u channels transform only as an octet and decuplet, and systems of zero baryonic charge exchanged in the t channel transform only as singlets and octets.

While the above ansatz is somewhat ad hoc, it should be stressed that it is no more so than some of the assumptions that have gone into recent re-derivations of some $SU(6)$ results. Indeed it is a more

reasonable ansatz in the sense that one is imposing a well-defined condition on physical scattering amplitudes which has the virtue of being realizable in simple dynamical models such as the pole approximation with low-lying states.

We have investigated this constraint using routine manipulations with Clebsch-Gordan coefficients and also, as an algebraic check, using tensor methods. We find that a consistent solution is possible only if

$$(D/F)_{\text{Meson-Baryon Coupling}} = 3/2, \quad -1, \quad \text{or} \quad -3 \, . \tag{8}$$

If the ϕ field is distinct from P, we find

$$(D/F)_{\phi B\bar{B}} = (D/F)_{PB\bar{B}} \, . \tag{9}$$

5. The first solution in Eq. (8) agrees with the standard predictions of SU(6), SU(6,6), and, more importantly, with experiment.[6] The second solution is manifestly unphysical, for it gives a vanishing pion-nucleon coupling. It corresponds to invariance under a W(3) group.[7] We defer discussion of the third solution until the end of this note.

If the ansatz of Section 4 is to be understood in terms of a pole approximation, we require, in addition to the well-established baryon octet and decuplet, a low-lying nonet of mesons with even spin and positive parity which are normal under charge conjugation. There appears to be reasonable evidence for a 2^+ nonet[8] and some, albeit considerably

less convincing, evidence for a 0^+ nonet.[9] The existence of either or both is sufficient for our purpose.

Since the spin of the exchanged systems is essentially irrelevant in our model, its justification in terms of an ordinary pole approximation may be replaced by one involving the exchange of Regge trajectories of prescribed signature.

6. A similar calculation can be carried out for the case in which the Heisenberg operator $j(0)$ is identified with the electromagnetic current density $J_\mu(0)$. Here the relevant amplitude T refers to photoproduction processes. A straightforward application of Eq. (6), together with a dispersion analysis of the photoproduction amplitude, yields the sum rules for the isoscalar and isovector anomalous magnetic moments obtained by Fubini, Furlan, and Rossetti.[4] Using the same kind of pole approximation as that described above, we find that the (D/F) ratio for the magnetic moments is the same as the (D/F) ratio for the pseudoscalar coupling. Although the baryon octet contribution does yield the anomalous magnetic moments, the dispersion analysis determines the amplitude only up to subtraction constants and does not tell us whether we should use the full moments or the anomalous parts. In any static model calculation of low-energy photoproduction, the amplitude is proportional to the full magnetic moments.[10] If one uses the full moments as an "ansatz", the first solution for the $(D/F)_{PB\bar{B}}$ ratio yields the well-known SU(6) result

$$\mu(p)/\mu(n) = -3/2 . \tag{10}$$

The third solution in Eq. (8) gives the unacceptable result $\mu(p) = 0$.

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FOOTNOTES AND REFERENCES

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