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SEMIMICROSCOPIC DESCRIPTION OF Hg^{198}_I

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UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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G. Alaga and G. Ialongo

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SUMMARY

In the present note we offer a qualitative explanation of the low lying collective states of mercury isotopes as well as the known transition probabilities between them.¹ In addition we reaffirm the prediction of new² states of angular momentum 1^+ and 0^+ in the region between 1.0-1.5 MeV and predict positive quadrupole moments for the first excited 2^+ , 4^+ and 6^+ states of the order of magnitude of the transition moments. The quadrupole moments of the second and third 2^+ states are of the same order of magnitude and negative.

The model used in the calculation is that of two holes coupled to a vibrator.²

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INTRODUCTION

The purpose of the present investigation was to reconsider and supplement the calculations² of two proton holes distributed in the single hole states $s_{1/2}$, $d_{3/2}$, $h_{11/2}$, and $d_{5/2}$ coupled to a five dimensional vibrator including the states of the vibrator up to two and three phonons. Furthermore the investigation was stimulated by new experimental results on the single proton hole states³ in Tl^{207} and new measurements of the spectra of mercury isotopes (Cf. Fig. 2[‡]).

The work was made possible by the help of the program written by R. Goldstein.

The actual calculation was divided into four steps. First we investigated the simplest case where we took only $s_{1/2}$ and $d_{3/2}$ single hole states and only states up to two phonons of the vibrator. In case 2 we studied the effect of the third phonon state on case 1. Case 3 consisted in taking additional single hole states $h_{11/2}$ and $d_{5/2}$ and only two phonons ($d_{5/2}^{-2}$ was omitted). Finally in case 4 we took all the mentioned single particle configurations and vibrator states up to three phonons. In that case we have so far succeeded to calculate only the energy levels and the wave functions but not the transition probabilities.

The model we used is²

$$\begin{aligned}
 H \psi = & \left[H_{SH} + H_p + \frac{1}{2} C_2 \sum_{\mu=-2}^2 |\alpha_2^\mu|^2 + \frac{1}{2} B_2 \sum_{\mu=-2}^2 |\alpha_2^{\mu*}|^2 \right. \\
 & \left. + K \sum_{i\mu=-2}^2 \alpha_2^\mu Y_2^{\mu*}(\theta_i, \phi_i) \right] \psi \quad (1)
 \end{aligned}$$

[‡] Figure 2 shows the similarity between the low lying spectra of mercury isotopes as reported by Sakai et al. However, recently B.P. Maier et al. have classified the 1029.33 keV

The term H_{SH} is the standard shell model hamiltonian describing the single hole states. H_p is the short range pairing force acting between proton holes. C_2 and B_2 are parameters of the vibrator and α_2^μ is a set of collective coordinates.⁴ The last term represents the coupling between the motion of the holes and the vibrator. K measures its coupling strength at the surface. $Y_2(\theta_i, \phi_i)$ is the spherical harmonics and is a function of the angles of the "i" hole. "i" is the index of summation running over all hole states.

The energies of the single hole states used are³

$d_{3/2}^{-s} s_{1/3} = 0.350$ MeV $h_{11/2}^{-s} s_{1/2} = 1.343$ MeV
 $d_{5/2}^{-s} s_{1/2} = 1.673$ MeV. The energy of the first excited state of the vibrator was taken as 1.027 MeV and the pairing coupling constant $G = 0.1$ MeV.⁵

RESULTS AND DISCUSSION

The results of our calculations are given in figures and tables.

Figure 1 represents the energy levels of the coupled system for cases 1, 2, 3 and 4. The value of the parameter

$$"a" = \sqrt{\frac{\hbar\omega}{2C_2}} \frac{\langle K \rangle}{\sqrt{4\pi}} = 0.6 \text{ MeV}$$

seems to give the spectrum which mostly resembles the experimental spectrum. Cf.

Fig. 2. The characteristic of the calculated spectra is

level in Hg^{200} as 0^+ . The two methods of measurements are quite different. In the first case the conversion lines following the beta decay and electron capture of Tl^{200} were measured, while in the second case the gamma ray energies resulting from the neutron capture in Hg^{199} were found. This indicates that there are still ambiguities in the assignment of the low lying states of mercury isotopes.

that they contain the experimentally observed 2_{1+} , 2_{2+} , (0_+) , 2_{3+} and 4_{1+} states and then we have a gap between those and higher excited states as observed experimentally. The changes in the spectra going from case 1 to other cases as indicated in Fig. 1, are not too dramatic. In case 2 the collective states 4_1 and 0_3 are appreciably depressed as expected. In case 3 we see the effect of the additional configurations $h_{11/2}$ and $d_{5/2}$ creating new states of angular momentum 0, 1, 2, 3 etc. and slightly depressing the low lying 0_{2+} and 1_{1+} states. In case 4 we probably have to use a smaller value of "a" to get closer to the experimental situation.

Tables 1 and 4 contain the information on the electric and magnetic transition probabilities. The $B(E2)_{\text{exp}} = 0.903$ of the $2_{1+} \rightarrow 0_{1+}$ transition and $\mu_{\text{exp}}^6 = 0.76 \frac{eh}{2Mc}$ of the 2_{1+} state for Hg^{198} has been used to give extra conditions to determine the parameters C_2 , $\langle K \rangle$ and g_R . Those parameters are presented in Table 2 and have been used to calculate the values of the $B(E2)$ and $B(M1)$ of all the states involved.

Tables 1 and 4 indicate considerable changes in the electromagnetic properties of some states in going from case 1 to cases 2 and 3. These changes are less dramatic in changing the effective charge and the effective gyromagnetic ratios of protons. They do not exceed the factor of 3. To understand the changes of $B(E2)$ in the transitions $(2_{2+} \rightarrow 0_{1+})$, $(2_{3+} \rightarrow 0_{1+})$, $(2_{3+} \rightarrow 0_{2+})$ etc. and of $B(M1)$ in the transitions $(2_{2+} \rightarrow 1_{1+})$, $(2_{2+} \rightarrow 2_{1+})$ etc., let us make the following general comment.

The transition probabilities of the coupled system are built up of the coherent contributions of the transition probabilities of proton holes and the collective contribution of the vibrator. We can distinguish the following situations:

- a) Contributions of the holes and the vibrator are in phase.
- b) Contributions of the holes and the vibrator are out of phase.

In each of these two cases we can talk about a large or small participation of the vibrator or the holes in building up the transition probabilities. This might be due to the lack of coherence in the contribution of different hole configurations or due to the contribution of only smaller components in the hole wave function. The same argument holds for the vibrator. As an illustration of a practically complete coherence let us quote the $(2_1 \rightarrow 0_1)$, $(4_1 \rightarrow 2_1)$ electric quadrupole and $(1_1 \rightarrow 2_1)$ magnetic dipole transitions. For the electric quadrupole transition $(2_1 \rightarrow 0_1)$ with the effective charge $e_{\text{eff}}^P = e_p$, the contribution of the collective and the hole part to the matrix element is on an average 4:1, while for $e_{\text{eff}}^P = 2e_p$ it becomes 2:1. For the $(4_1 \rightarrow 2_1)$ quadrupole transition we again have 6:1 and 2:1 respectively. This indicates that in these states the contributions of the holes and the collective motion for $e_{\text{eff}}^P = 2e_p$ are in phase and comparable. For the magnetic dipole transition $(1_1 \rightarrow 2_1)$ the contribution of the collective part to the matrix element is negligible. Generally speaking the participation of the collective motion in the magnetic properties of the coupled system is as small as expected.

Let us now turn to explain the fluctuation of the $B(E2)$ in the $(2_2 \rightarrow 0_1)$, $(2_3 \rightarrow 0_1)$ and $(2_3 \rightarrow 0_2)$ transitions. The best way of doing that is perhaps to quote the values of "A"'s and "B"'s for the mentioned transitions (Cf. Appendix 1).

Transition	A			B		
	1	2	3	1	2	3
$(2_2 \rightarrow 0_1)$	0.020	-0.068	-0.036	-0.042	0.238	0.054
$(2_3 \rightarrow 0_1)$	-0.115	0.075	0.053	0.323	-0.206	-0.294
$(2_3 \rightarrow 0_2)$	0.000	0.133	-0.097	0.069	0.189	-0.176
$(2_1 \rightarrow 0_1)$	0.380	0.360	0.470	0.496	0.580	0.500

In the first two of these transitions the contribution of the hole part "A" and the contribution of the vibrator part "B" are out of phase. Besides, in the $(2_2 \rightarrow 0_1)$ transition, except when three phonon states are included, all the contributions are small compared to the contributions of the $(2_1 \rightarrow 0_1)$ transition. Contrary to the case $(2_2 \rightarrow 0_1)$ where the collective part has been changed by including three phonon states, in case $(2_3 \rightarrow 0_2)$ the hole part is the one which has appreciably been changed. This is the reason for the fluctuations of the calculated transition probabilities.

In the same way we can go on arguing from transition to transition and from state to state. Instead we might formulate a tentative conclusion like this: We expect that only for those transitions in which the large components of the wave function do overlap and the two parts do add up, the description of the transition probabilities is expected to be fair. In all other cases we prefer to draw only qualitative conclusions about the slowness of those transitions.

The same arguments, of course, hold for the magnetic transitions.

The fact that $\begin{Bmatrix} 3/2 & 3/2 & 2 \\ 2 & 2 & 3/2 \end{Bmatrix} = 0$ and the inclusion

of the spin orbit partner to the $d_{3/2}$ states explain the relative importance of the three phonon states and the change in the magnetic properties respectively.

Table 2 contains the values of the parameters C_2 , $\langle K \rangle$ and g_R for cases 1, 2 and 3. The parameters change continuously except for the parameter g_R in case 2. Besides the mentioned parameters, Table 2 also contains the predicted $B(E2)$ values for the $2_1 \rightarrow 0_1$ transition in Pb^{200} .[‡]

Comparison with the experiment has so far been done only on the first four excited states. Electromagnetic transitions have been observed between the states $2_3 \rightarrow 2_1$, $2_3 \rightarrow 0_1$, $2_2 \rightarrow 2_1$, $2_2 \rightarrow 0_1$, $4_1 \rightarrow 2_1$, $2_1 \rightarrow 0_1$. The main characteristics of these transitions are that the transition $2_2 \rightarrow 2_1$ is a mixed electric quadrupole and magnetic dipole transition with the mixing ratio $\delta \approx 1$, that the ratio of the intensities $W_y(2_2 \rightarrow 2_1)/W_y(2_2 \rightarrow 0_1) \approx 25$ and $W_y(2_3 \rightarrow 2_1)/W_y(2_3 \rightarrow 0_1)$ is also larger than unity.

Let us make a table of the relative intensities resulting from the model calculations.

Relative intensities	$\frac{W_y(a \rightarrow b)}{W_y(a' \rightarrow b')}$ $e_{eff}^P = 2e_P$ $g_{eff}^P = 0.7 g_P$		
	1	2	3
$(2_2 \rightarrow 2_1)$			
$(2_2 \rightarrow 0_1)$	94.0	0.815	38.3
$(2_3 \rightarrow 2_1)$			
$(2_3 \rightarrow 0_1)$	52.7	10.5	7.5

[‡] We assume, of course, that the interaction of the closed shell protons with the neutron holes induces collective vibrations of lead isotopes. Antisymmetrization of such states and of the two proton hole state is neglected.

From this table and Table 4 for the values of δ we can conclude that our results are in qualitative agreement with experiments. The argument runs as follows.

The magnetic component in the $2_2 \rightarrow 2_1$ transition is slowed down at least ^a100 times compared to the normal transition as indicated by the interpolated value from Table 1 between case 2 and case 3. The interpolated value for δ is close to unity. The interpolated value of the ratio

$\frac{2_2 \rightarrow 2_1}{2_2 \rightarrow 0_1}$ is around 20, in close agreement with the experiment.

The ratio $\frac{2_3 \rightarrow 2_1}{2_3 \rightarrow 0_1}$ is not much different from the

ratio given by the experiment. In addition we predict a small admixture of electric quadrupole transitions, if any, in the $2_3 \rightarrow 2_1$ and $2_3 \rightarrow 2_2$ transitions. Also, the intensity of the $2_3 \rightarrow 2_2$ transition is rather small.

The $4_1 \rightarrow 2_1$ transition seems to be fairly strong.

Now we turn to the hitherto not observed 0_2^+ and 1_1^+ states in Hg^{198} . If these states are in the region as indicated in Fig. 1 the smallness of the energy difference indicates that they could be only populated from higher excited states. If these states appear to be considerably higher than indicated in the figure due to the neglected monopole modes and exchange part of the interaction between the proton holes, [‡] their character will also appreciably change and one needs additional consideration.

There exist extensive data on higher excited states in Hg^{198} and other mercury isotopes. The main characteristics of mercury spectra are the similarity between the spectra of difference isotopes and an apparent gap between the first four excited states and high density states above.

[‡] This point has been investigated by A. Covello and G. Sartoris (private communication).

This might indicate that minor changes in the Hg^{198} parameters will possibly account for small differences in mercury spectra.

We have so far been discussing only the even parity states in mercury isotopes. We are also expecting the negative parity states with high angular momentum of the type $(h_{11/2} s_{1/2})_{5-,6-}$ coupled to the vibrator ground state to occur approximately between 1.5 - 2.0 MeV. Such a state of angular momentum 5- has been observed in Hg^{198} at 1.635 MeV.

In Table 3 we list the static quadrupole moments for six excited states. The first excited state 2_1+ has a positive quadrupole moment as well as the 4_1+ and 6_1+ states. These quadrupole moments are of the order of magnitude of the transition moments. Such a large quadrupole moment for the first excited state has experimentally been found in Cd^{114} .⁷

The large static quadrupole moment finds a natural explanation in our treatment where the particle vibrational coupling relaxes the strict conditions of the vibrational model by admixing to the one phonon states no phonon, two and three phonon states. These admixtures produce anharmonicities and together with the holes contribute to the static quadrupole moments. The approximate equality of the quadrupole moments of the 2_1+ , 4_1+ and 6_1+ states might indicate that the states $0+$, $2+$, 4_1+ and 6_1+ are the states of a distorted rotational band. (That is even more probable in lighter mercury isotopes.) The position of the $6+$ level is not experimentally known. For Hg^{198} we expect the lowest $6+$ state in the region of 2 MeV. The quadrupole moments of the 2_2+ , 2_3+ and 4_2+ states are negative and somewhat smaller. The 2_2+ state has a somewhat larger $B(E2)$ for the

transition to the 2_1+ state. That offers support to the interpretation of the 2_2+ state as a two phonon vibrational state. Anyhow the appearance of the 2_3+ state which has been observed and is apparently not a three phonon state but rather a particle phonon mixture, indicates that the vibrator gets much distorted, and new states start to appear at the energies of the two phonon states. Such a situation is rather similar to that in the region of strongly deformed nuclei.⁸

Characteristics similar to those in mercury spectra have experimentally been observed in the case of even Cd isotopes (and some others), which is not too surprising because Cd has two proton holes in the closed shell of 50 protons. Preliminary results obtained with the same method for Cd¹¹⁴ seem encouraging.⁹

As mentioned at the beginning we used $G = 0.1$ MeV and $\hbar\omega = 1.027$ in the reported calculation. In addition, we also looked at the results for $G = 0$ and $G = 0.2$ MeV as well as $\hbar\omega = 0.700$ and 1.5 MeV. The results are not too sensitive to the variation of G but for $\hbar\omega$ outside the region 850-1.200 MeV the calculated spectrum gets more distorted and the agreement with experiment much poorer. Hence we are tempted to conclude that the parameter $\hbar\omega$ can approximately be taken from the corresponding lead isotopes. In that case the neutrons coupled to the closed proton shells seem to provide the basic vibrational structure in lead isotopes and the proton holes coupled to these vibrations create a new situation.

The new wave functions offer also a new possibility of describing inelastic scattering relaxing the strict conditions of the vibrational model. The investigation of inelastic scattering will be presented elsewhere.

CONCLUSION

From the qualitative agreement of our calculations for Hg^{198} with the experiment we are tempted to conclude that the method used in this treatment is more suitable than the linearization methods for the treatment of the nuclei close to the closed shells. In that case there is practically no difference between the holes and the quasiparticles. The reasons for that might be in the low spin single hole configurations and the rather high value of $\hbar\omega$.

The short range proton neutron force (except in the creation of the vibrator) is in the present treatment neglected and the spectra are given by the aligned proton neutron force, contrary to the short proton neutron force in the linearization methods.

Which part of the nuclear dynamics one has to use in the treatment of nuclei apart from the closed shells will be decided by future experimental and theoretical investigations.

ACKNOWLEDGMENTS

We would like to acknowledge the hospitality and excellent working conditions provided by the Chemistry Department of the Lawrence Radiation Laboratory and stimulating discussions with members and visitors of the theoretical group. Special thanks are due to N.K. Glendenning, L. Kisslinger, K. Kumar, A. Lande, R. Lemmer, J. Rasmussen, and M. Veneroni. The experimental part was extensively discussed with M. Sakai and T. Yamazaki,

which we gratefully acknowledge. We also thank Dr. O.W.B. Schult for helpful correspondence.

It is great pleasure for one of the authors (G.A.) to acknowledge stimulating discussions with A. Bohr and B. Mottelson during his visit to Copenhagen.

The help in computations by Chin W. ^{Ma} is gratefully acknowledged. The computation was carried out on the computing facilities at the Lawrence Radiation Laboratory with great help and understanding. In particular we would like to thank R. Goldstein for writing the program.

One of us (G.I.) is indebted to the Physics Department of the New York University for the financial support which made his stay in Berkeley possible.

APPENDIX 1

For the sake of definiteness we write our definition of the reduced matrix element $B(E2)$.

$$B(E2) = 2.722 \left(\frac{197}{E_\gamma} \right)^5 \frac{10^{-21}}{\tau r_0^4 A^{4/3}} \text{ sec} \quad (1a)$$

E_γ is the energy of the γ ray in MeV and τ is the lifetime of the initial state in seconds. r_0 is the constant factor in the definition of the nuclear radius $R_0 = r_0 A^{1/3} \times 10^{-13}$ cm. A is the atomic number. For $A = 198$, $r_0 = 1.2$, $\tau = 3.16 \times 10^{-11}$ sec and $E_\gamma = 0.412$ MeV we obtain $B(E2) = 0.903$ as a dimensionless quantity. [The conversion contribution is negligible ($\alpha_2 = 0.02$).]

The intensities of the lines are calculated from the standard formula

$$W_\gamma(E2) = 7.60 \left(\frac{E_\gamma}{197} \right)^5 A^{4/3} B(E2) \times 10^{20} \frac{1}{\text{sec}} \quad (1b)$$

It is easy to connect the definition of $B(E2)$ from (1a) to its usual definition $B(E2) = \frac{B(E2)_{st}}{e^2 R_0^4}$ or

$B(E2)_{st} = B(E2) e^2 R_0^4 = 0.217 e^2 10^{-48} \text{ cm}^4$. In the same way for the magnetic transitions we have

$$W_\gamma(M1) = 1.35 \left(\frac{E_\gamma}{197} \right)^3 B(M1) \times 10^{20} \frac{1}{\text{sec}} \quad (1c)$$

The operators for the electric quadrupole and magnetic dipole transitions are chosen in the form

$$M(E2) = -e_{\text{eff}}^P \sum_i r_i^2 Y_2^{\mu*}(\theta_i, \phi_i) + \frac{3}{4\pi} Ze R_0^2 \alpha_2^{\mu*} \quad (1d)$$

$$M(M1) = \frac{e \hbar}{2Mc} \sqrt{\frac{3}{4\pi}} \left\{ \sum_i \left(g_s s_1^{\mu*} + g_l l_1^{\mu*} \right)_i + g_R R_1^{\mu*} \right\} \quad (1e)$$

The summation runs over all holes. The negative sign in the first term is chosen because we are dealing with hole states. Z is the charge of the nucleus. R_0 its radius. s_1^μ , l_1^μ and R_1^μ are the spherical components of the spin, orbital and surface angular momentum operators. g_s , g_l and g_R are the corresponding giromagnetic ratios. e_{eff}^P is the effective charge of proton holes.

The additional quantity we need is the ratio of the intensities of the electric and magnetic transitions. From (1b) and (1c) we obtain

$$\delta^2 = \frac{7.60}{1.35} \left(\frac{E_\gamma}{197} \right)^2 A^{4/3} \frac{B(E2)}{B(M1)} = \frac{W(E2)}{W(M1)} \quad (1f)$$

It should be mentioned that in calculating the transition probabilities we set all radial integrals equal to $\frac{3}{5} R_0^2$. For Hg^{198} we have $\frac{3}{5} R_0^2 = 29.4 \times 10^{-26} \text{ cm}^2$. The calculated radial integrals for a Woods-Saxon well for the holes in the lead region are $\langle d_{3/2} | r^2 | d_{3/2} \rangle = 28.5$, $\langle d_{5/2} | r^2 | d_{5/2} \rangle = 29.4$, $\langle h_{11/2} | r^2 | g_{11/2} \rangle = 38.5$, $\langle d_{3/2} | r^2 | d_{5/2} \rangle = 28.8$. All these values are in Fermis.

We see that our estimate is extremely good except for the

$h_{11/2}$ state. The pairing force certainly overestimates the influence of the configurations with high values of the angular momenta, so the approximation used is satisfactory.

After determining $B(E2)_{\text{exp}}$ for the $2_1 \rightarrow 0$ transition we can use the relation $\sqrt{B(E2)_{\text{exp}} (2I_i + 1)} =$

$$= (A e_{\text{eff}} + Bx) = \langle I_f || M(E2) || I_i \rangle \quad \text{to obtain } x,$$

$$C_2 = \frac{\hbar\omega}{2} \left(\frac{Z}{x} \right)^2, \quad \langle K \rangle = a \sqrt{4\pi} \frac{Z}{x} \quad \text{and } B(E2) = \left(\frac{3}{4\pi} \right)^2 x^2 e^2 R_0^4$$

for the vibrator. A and B are quantities calculated from the model (3f).

The spectroscopic quadrupole moment is defined as

$$\langle I I | Q_2^0 | I I \rangle = \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \sqrt{\frac{16\pi}{5}} \langle I || M(E2) || I \rangle \quad (1g)$$

$\begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}$ is the three-j coefficient. For calculating the quadrupole moments we have to use the same effective charge as for the transition probabilities.

The magnetic moment is given by

$$\mu = \sqrt{\frac{4\pi}{3}} \begin{pmatrix} I & 1 & I \\ -I & 0 & I \end{pmatrix} \langle I || M(M1) || I \rangle = \quad (1h)$$

$$= \sqrt{\frac{4\pi}{3}} \begin{pmatrix} I & 1 & I \\ -I & 0 & I \end{pmatrix} [D'g_R + E'(g_1 - g_R) + F'(g_s - g_1)]$$

Choosing appropriate values for g_1 and g_s and knowing the magnetic moment of the first excited state we can determine the value of g_R and use it to calculate the magnetic transition probabilities.

If the magnetic moment is not known we can obtain g_R from the known $B(Ml)$ using the formula

$$\sqrt{B(Ml)_{\text{exp}}(2I_i+1)} = [D g_R + E(g_1 - g_R) + F(g_s - g_1)] = \\ = \langle I_f || M(Ml) || I_i \rangle$$

D, D', E, E', F and F' are quantities calculated from the model (3g).

APPENDIX 2

In this section we would like to describe the calculation of the reduced matrix elements of the vibrator and give all the phase conventions. Let us first introduce a new set of variables instead of α_λ^μ and $\dot{\alpha}_\lambda^\mu$, by means of the transformation

$$\alpha_2^\mu = \sqrt{\frac{\hbar}{2B\omega}} \left(b_2^\mu + (-1)^\mu b_2^{-\mu\dagger} \right) \quad (2a)$$

$$\dot{\alpha}_2^\mu = i \sqrt{\frac{\hbar B\omega}{2}} \left(b_2^{\mu\dagger} - (-1)^\mu b_2^{-\mu} \right) \quad (2b)$$

In addition the new variables should satisfy the following relations

$$b_2^{\mu'} b_2^{\mu\dagger} - b_2^{\mu\dagger} b_2^{\mu'} = \delta_{\mu\mu'}; \quad b_2^\mu b_2^{\mu\dagger} = n_\mu + 1 \quad \text{and} \quad b_2^{\mu\dagger} b_2^\mu = n_\mu \quad (2c)$$

Hence we see that $b_2^{\mu\dagger}$ and b_2^μ are the creation and destruction operators of the phonons, respectively. n_μ is the number operator.

The symmetric states for one, two and three phonons can be written as follows

$$|12\mu\rangle = b_2^{\mu\dagger} |0\rangle; |2RR_z\rangle = \frac{1}{\sqrt{2}} \sum_{\mu\mu'} \langle 2\mu 2\mu'; RR_z \rangle b_2^{\mu\dagger} b_2^{\mu'\dagger} |0\rangle$$

$$|3RR_z\rangle = |(22)R'_2; RR_z\rangle = N \sum_{R'_0} [\delta_{R'_0 R'} + 2 \sqrt{(2R'+1)(2R'_0+1)} \left\{ \begin{matrix} 2 & 2 & R' \\ & 2 & R & R' \end{matrix} \right\}] | (22)R'_0 2; RR_z \rangle \quad (2d)$$

with

$$|(22)R'_0 2; RR_z\rangle = \sum_{\mu\mu'\nu R'_0} \langle 2\mu 2\mu'; R'_0 R'_{0z} \rangle \langle R'_0 R'_{0z} 2\nu; RR_z \rangle \times b_2^{\mu\dagger}(1) b_2^{\mu'\dagger}(2) b_2^{\nu\dagger}(3) |00\rangle$$

or equivalently

$$|3RR_z\rangle = |22R'_2; RR_z\rangle = \frac{1}{\sqrt{2[1 + 2(2R'+1) \left\{ \begin{matrix} 2 & 2 & R' \\ & 2 & R & R' \end{matrix} \right\}]}}$$

$$\sum_{\mu\mu'\nu R'_z} \langle 2\mu 2\mu'; R'R'_z \rangle \langle R'R'_z 2\nu; RR_z \rangle b_2^{\mu\dagger} b_2^{\mu'\dagger} b_2^{\nu\dagger} |0\rangle$$

$\langle ; \rangle$ are the vector coupling coefficients as defined in TAS. $\{ \}$ is the six-j coefficient as given by Edmonds.¹¹ $|0\rangle$ is the phonon vacuum. The normalization factor is given by

$$N = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{1 + 2(2R'+1) \left\{ \begin{matrix} 2 & 2 & R' \\ & 2 & R & R' \end{matrix} \right\}}} \quad (2e)$$

From (2d) and (2e) it is evident that we use the representation where we do and do not label the phonons explicitly.

The three phonon wave function can conveniently be written as

$$| 2^3 R'_z ; RR_z \rangle = \sum_{R'_0} | 2^2 R'_0 2 ; RR_z \rangle \langle 2^2 R'_0 2R | 2^3 R'R \rangle$$

where $\langle 2^2 R'_0 2R | 2^3 R'R \rangle$ is the fractional parantage coefficient which is explicitly given by the help of (2d) and (2e). If there is no degeneracy in R as in our case, the wave function should be independent of the choice of R' provided it satisfies the triangular relations.

To set up the interaction matrix and to calculate the transition probabilities according to (1) and (1d), we have to compute the matrix elements of α_λ^μ and $\alpha_\lambda^{\mu\dagger}$. Using the wave functions (2d) and the definition (2a) for the reduced matrix elements b and b^\dagger we obtain

$$\langle NRR_z | \alpha_2^\mu | N'R'_z \rangle = \sqrt{\frac{\hbar}{2B\omega}} (-1)^{R-R_z} \begin{pmatrix} R & 2 & R' \\ -R_z & \mu & R'_z \end{pmatrix} \times \quad (2f)$$

$$[\langle NR || b_2^\dagger || N'R' \rangle \delta_{NN'+1} + (-1)^{R-R'} \langle N'R' || b_2^\dagger || NR \rangle \delta_{N'N+1}]$$

$$\text{with } \langle NR || b_2^\dagger || N'R' \rangle = (-1)^{R-R'} \langle N'R' || b_2 || NR \rangle \quad \text{and}$$

$$\langle 1 R || b_2^\dagger || 00 \rangle = \sqrt{2R+1} \quad R = 2 \quad (2g)$$

$$\langle 2 R || b_2^\dagger || 12 \rangle = \sqrt{2(2R+1)} \quad R = 0, 2, 4$$

$$\begin{aligned}
 \langle 3 R || b_2^\dagger || 2R' \rangle &= \sqrt{3(2R+1)} & \langle 2^3 R_0 R | 2^2 R' R \rangle &= \\
 &= \frac{\sqrt{(2R+1)}}{\sqrt{1 + 2(2R_0+1) \begin{Bmatrix} 2 & 2 & R_0 \\ 2 & R & R_0 \end{Bmatrix}}} \left[\delta_{R_0 R'} + 2 \sqrt{(2R_0+1)(2R'+1)} \begin{Bmatrix} 2 & 2 & R' \\ 2 & R & R_0 \end{Bmatrix} \right]
 \end{aligned}$$

N and N' are the numbers of phonons in the final and initial states respectively. R' and R₀ are even. As mentioned earlier, R₀ is chosen arbitrarily and its choice gives the sign of the reduced matrix elements. However the physical results do not depend on that sign so we choose R₀ = 4 and R₀ = 0 to get the phases used in the literature.

Let us list the reduced matrix elements used in the present calculation:

$$\begin{aligned}
 \langle 30 || b_2^\dagger || 22 \rangle &= \sqrt{3} & \langle 33 || b_2^\dagger || 22 \rangle &= -\sqrt{3 \cdot 5} \\
 \langle 32 || b_2^\dagger || 20 \rangle &= \sqrt{7} & \langle 33 || b_2^\dagger || 24 \rangle &= \sqrt{2 \cdot 3} \\
 \langle 32 || b_2^\dagger || 22 \rangle &= 2\sqrt{\frac{5}{7}} & \langle 34 || b_2^\dagger || 22 \rangle &= 3\sqrt{\frac{4}{7}} \\
 \langle 32 || b_2^\dagger || 24 \rangle &= 2 \cdot 3 \frac{1}{\sqrt{7}} & \langle 34 || b_2^\dagger || 24 \rangle &= 3 \sqrt{\frac{10}{7}} \\
 & & \langle 36 || b_2^\dagger || 24 \rangle &= \sqrt{3 \cdot 13}
 \end{aligned}$$

For the choice of R₀ = 0 and R₀ = 2 the only difference is the change in the relative sign between the matrix elements $\langle 33 || b_2^\dagger || 22 \rangle$ and $\langle 33 || b_2^\dagger || 24 \rangle$.

APPENDIX 3

The actual calculation was performed using the expression for the interaction matrix (1)

$$\begin{aligned}
 & \langle J, NR; I | H_P + K \sum_{i\mu=-2}^2 \alpha_2^\mu Y_2^{\mu*}(\theta_i, \phi_i) | J', N'R'; I \rangle = \\
 & = -\frac{G}{2} \sqrt{(2j_1+1)(2j_1'+1)} (-1)^{l_1+l_1'} \delta_{1_1 1_2} \delta_{1_1' 1_2'} \delta_{j_1 j_2} \delta_{j_1' j_2'} \delta_{RR'} \delta_{NN'} + \\
 & + \langle K \rangle \sqrt{\frac{\hbar\omega}{2C_2}} \left\{ \begin{matrix} R' J' I \\ J R 2 \end{matrix} \right\} (-1)^{J'+R+I} \langle (n_1 j_1, n_2 j_2) J || \sum_{i=1}^2 Y_2(i) || (n_1' j_1', n_2' j_2') J' \rangle \times \\
 & \times [\langle NR || b_2^\dagger || N'R' \rangle \delta_{NN'+1} + \langle NR || b_2 || N'R' \rangle \delta_{N'N+1}] \quad (3a)
 \end{aligned}$$

where

$$\begin{aligned}
 & \langle (n_1 j_1, n_2 j_2) J || \sum_{i=1}^2 Y_2(i) || (n_1' j_1', n_2' j_2') J' \rangle = \sqrt{\frac{(2J+1)(2J'+1)}{(1+\delta_{j_1 j_2})(1+\delta_{j_1' j_2'})}} \times \\
 & \times [(-1)^{j_1+j_2+J'+n_1+n_1'+l_1+l_1'} \langle j_1 || Y_2 || j_1' \rangle \delta_{j_2 j_2'} \delta_{1_2 1_2'} \left\{ \begin{matrix} j_1 & j_1' & 2 \\ J' & J & j_2 \end{matrix} \right\} + \\
 & + (-1)^{j_1+j_2'+n_1+n_2'+l_1+l_1'} \langle j_1 || Y_2 || j_2' \rangle \delta_{j_2 j_1'} \delta_{1_2 1_1'} \left\{ \begin{matrix} j_1 & j_2' & 2 \\ J' & J & j_2 \end{matrix} \right\} + \\
 & + (-1)^{J+J'+1+n_2+n_2'+l_2+l_2'} \langle j_2 || Y_2 || j_1' \rangle \delta_{j_1 j_2'} \delta_{1_1 1_2'} \left\{ \begin{matrix} j_2 & j_1' & 2 \\ J' & J & j_1 \end{matrix} \right\} + \\
 & + (-1)^{J+j_1+j_2'+n_2+n_2'+l_2+l_2'} \langle j_2 || Y_2 || j_2' \rangle \delta_{j_1 j_1'} \delta_{1_1 1_1'} \left\{ \begin{matrix} j_2 & j_2' & 2 \\ J' & J & j_1 \end{matrix} \right\}]
 \end{aligned}$$

and

(3b)

$$\langle (1_1 \ 1/2)j_1 || Y_2 || (1'_1 \ 1/2)j'_1 \rangle = \frac{1}{2} (-1)^{j'_1 - 1/2} [1 + (-1)^{l_1 + l'_1}] \times$$

$$\times \sqrt{\frac{(2j_1 + 1)(2j'_1 + 1)5}{4\pi}} \begin{pmatrix} j_1 & j'_1 & 2 \\ 1/2 & -1/2 & 0 \end{pmatrix} \quad (3a)$$

All the symbols have already been defined.

Setting up and solving the matrix for a given I_i we obtain the energy eigenvalues E_{I_i} and the components of the eigen vectors $a_{n_1(1_1 \ 1/2)j_1 \ n_2(1_2 \ 1/2)j_2; JNR}^{I_i}$ so that we can write the solution of the coupled system in the form

$$|I_i^M \rangle = \sum_{\substack{n_1 l_1 j_1 n_2 l_2 j_2 \\ JNR}} a_{n_1(1_1 \ 1/2)j_1 \ n_2(1_2 \ 1/2)j_2; JNR}^{I_i} \quad (3e)$$

$$[[n_1(1_1 \ 1/2)j_1 \ n_2(1_2 \ 1/2)j_2]_{JNR} I_i^M \rangle$$

The summation runs over all base vectors of the free system.

With the help of (3d), (1d) and (1e) we can calculate the reduced matrix elements of the electric quadrupole and magnetic dipole transitions. The result is

$$\langle I_i || M(E2) || I'_s \rangle = \sqrt{(2I_i + 1)(2I'_s + 1)} \sum_{\substack{n_1 l_1 j_1 n_2 l_2 j_2 \\ JNR}} \sum_{\substack{n'_1 l'_1 j'_1 n'_2 l'_2 j'_2 \\ J'N'R'}} a_{n_1(1_1 \ 1/2)j_1 \ n_2(1_2 \ 1/2)j_2; JNR}^{I_i} a_{n'_1(1'_1 \ 1/2)j'_1 \ n'_2(1'_2 \ 1/2)j'_2; J'N'R'}^{I'_s}$$

$$[-e_{\text{eff}}^P \frac{3}{5} R_o^2 (-1)^{J+I'_s+R'} \delta_{NN'} \delta_{RR'} \begin{Bmatrix} J' & J & 2 \\ I_i & I'_s & R' \end{Bmatrix}] \times$$

$$\times \langle n_1(1_1 \ 1/2)j_1 \ n_2(1_2 \ 1/2)j_2 \ J || \sum_{i=1}^2 Y_2(i) || n'_1(1'_1 \ 1/2)j'_1 \ n'_2(1'_2 \ 1/2)j'_2 \ J' \rangle +$$

$$\begin{aligned}
 & + (-1)^{I_1+R'+J'} \delta_{n_1 n'_1} \delta_{l_1 l'_1} \delta_{j_1 j'_1} \delta_{n_2 n'_2} \delta_{l_2 l'_2} \delta_{j_2 j'_2} \delta_{JJ'} \sqrt{\frac{\hbar\omega}{2G}} \frac{3}{4\pi} Z e R_0^2 \times \\
 & \times \left\{ \begin{matrix} I'_s & I_1 & 2 \\ R & R' & J' \end{matrix} \right\} [\langle NR || b_2 || N'R' \rangle \delta_{N+1 N'} + \langle NR || b_2^\dagger || N'R' \rangle \delta_{N N'+1}] \quad (3f)
 \end{aligned}$$

$$\langle I_1 || M(M1) || I'_s \rangle = \frac{e \hbar}{2Mc} \sqrt{\frac{3}{4\pi}} \sum_{JNR} \sum_{J'N'R'} \begin{matrix} I_1 \\ a_{n_1} (l_1 \ 1/2) j_1 \ n_2 (l_2 \ 1/2) j_2, JNR \\ n_1 l_1 j_1 n_2 l_2 j_2 \quad n'_1 l'_1 j'_1 n'_2 l'_2 j'_2 \end{matrix}$$

$$a_{n_1}^{I'_s} (l_1 \ 1/2) j'_1 \ n_2 (l_2 \ 1/2) j'_2, J'N'R' \left[\sqrt{(2I'_s+1)(I'_s+1)I'_s} \ g_R \delta_{I_1 I'_s} \delta_{NN'} \delta_{RR'} \delta_{JJ'} \times \right.$$

$$\times \delta_{j_1 j'_1} \delta_{j_2 j'_2} \delta_{n_1 n'_1} \delta_{n_2 n'_2} \delta_{l_1 l'_1} \delta_{l_2 l'_2} + \sqrt{(2I'_s+1)(2I_1+1)} \left[(-1)^{J+I'_s+1+R'} \delta_{NN'} \delta_{RR'} \delta_{JJ'} \right.$$

$$\left. \delta_{n_1 n'_1} \delta_{l_1 l'_1} \delta_{j_1 j'_1} \delta_{n_2 n'_2} \delta_{l_2 l'_2} \delta_{j_2 j'_2} \sqrt{(2J+1)(J+1)J} \left\{ \begin{matrix} J & J & 1 \\ I_1 & I'_s & R' \end{matrix} \right\} (g_1 - g_R) + \right.$$

$$\left. + (-1)^{J+I'_s+1+R'} \delta_{NN'} \delta_{RR'} \left\{ \begin{matrix} J' & J & 1 \\ I_1 & I'_s & R' \end{matrix} \right\} \times \right.$$

$$\left. \times \langle n_1 (l_1 \ 1/2) j_1 \ n_2 (l_2 \ 1/2) j_2 \ J || S_1 || n'_1 (l'_1 \ 1/2) j'_1 \ n'_2 (l'_2 \ 1/2) j'_2 \ J' \rangle (g_s - g_1) \right] \quad (3g)$$

with

$$\langle n_1 (l_1 \ 1/2) j_1 \ n_2 (l_2 \ 1/2) j_2 \ J || S_1 || n'_1 (l'_1 \ 1/2) j'_1 \ n'_2 (l'_2 \ 1/2) j'_2 \ J' \rangle =$$

$$= \sqrt{\frac{(2J+1)(2J'+1)}{(1+\delta_{j_1 j_2})(1+\delta_{j'_1 j'_2})}} \left[(-1)^{j_1+j_2+J'+1} \left\{ \begin{matrix} j_1 & j'_1 & 1 \\ J & J & j_2 \end{matrix} \right\} \delta_{j_2 j'_2} \delta_{l_2 l'_2} \times \right.$$

$$\begin{aligned}
 & \langle (1_1 \ 1/2)j_1 || S_1 || (1'_1 \ 1/2)j'_1 \rangle + (-1)^{j_1+j'_2+1} \left\{ \begin{matrix} j_1 & j'_2 & 1 \\ j' & j & j_2 \end{matrix} \right\} \delta_{j_2 j'_1} \delta_{1_2 1'_1} \times \\
 & \times \langle (1_1 \ 1/2)j_1 || S_1 || (1'_2 \ 1/2)j'_2 \rangle + (-1)^{j+j'} \left\{ \begin{matrix} j_2 & j'_1 & 1 \\ j' & j & j_1 \end{matrix} \right\} \delta_{j_1 j'_2} \delta_{1_1 1'_2} \times \\
 & \times \langle (1_2 \ 1/2)j_2 || S_1 || (1'_1 \ 1/2)j'_1 \rangle + (-1)^{j+j_1+j'_2+1} \left\{ \begin{matrix} j_2 & j'_2 & 1 \\ j' & j & j_1 \end{matrix} \right\} \delta_{j_1 j'_1} \delta_{1_1 1'_1} \times \\
 & \times \langle (1_2 \ 1/2)j_2 || S_1 || (1'_2 \ 1/2)j'_2 \rangle] \quad (3h)
 \end{aligned}$$

and

$$\langle (1 \ 1/2)j || S_1 || (1' \ 1/2)j' \rangle = (-1)^{j+1/2+1'+1} \sqrt{(2j+1)(2j'+1)} \sqrt{\frac{3}{2}} \left\{ \begin{matrix} j' & j & 1 \\ 1/2 & 1/2 & 1' \end{matrix} \right\} \delta_{11'} \quad (3i)$$

The line intensities and the static moments can easily be calculated from (lb), (lg) and (lf).

The relative magnitude and phase between the electric quadrupole and magnetic dipole transitions are obtained from¹²⁾

$$\delta = \sqrt{\frac{7.60}{1.35}} \left(\frac{E_V}{197} \right) A^{2/3} \frac{\langle I_1 || M(E2) || I'_s \rangle}{\langle I_1 || M(M1) || I'_s \rangle} \quad (3j)$$

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FIGURE CAPTIONS

Fig. 1. In Fig. 1 we plot the lowest energy levels of the coupled system. The spacing of the single hole levels was taken from the experiment to be $d_{3/2}^{-1} - s_{1/2}^{-1} = 0.350$ MeV, $h_{11/2}^{-1} - s_{1/2}^{-1} = 1.343$ MeV and $d_{5/2}^{-1} - s_{1/2}^{-1} = 1.673$ MeV. For the vibrator energy we took $\hbar\omega = 1.027$ MeV. The pairing coupling constant

$$G = 0.1 \text{ MeV was used. } a = \frac{\langle K \rangle}{\sqrt{4\pi}} \sqrt{\frac{\hbar\omega}{2C_2}} = 0.6.$$

Cases 1, 2, 3 and 4 represent the following situations. In case 1 we use two phonons of the vibrator and $s_{1/2}^{-1}$ and $d_{3/2}^{-1}$ hole states. Case 2 uses the same number of hole states but includes the vibrator states up to three phonons. In case 3 only two phonon states were used but the additional $h_{11/2}^{-1}$ and $d_{5/2}^{-1}$ states were included. However the state $d_{5/2}^{-2}$ was omitted. Case 4 contains all the hole states as in case 3, but also the vibrator states up to three phonons.

(Fig. 1 MUB-7694)

Fig. 2. contains the experimentally known lowest lying states in mercury isotopes.

(Fig. 2 MUB-7693)

TABLE CAPTIONS

Table 1 contains data on electromagnetic transition probabilities of low lying states. The reduced matrix elements $B(E2)$ and $B(M1)$ are given in units $e^2 R_0^4$ and $(\frac{e\hbar}{2Mc})^2$ respectively. The transition probabilities are given in units of $10^{10} \frac{1}{\text{sec}}$. In order to calculate the reduced matrix elements, we used the values of $\hbar\omega = 1.027$ MeV and the parameters C_2 and g_R from table 2. The reduced matrix elements are listed for different values of the effective charge and the effective giromagnetic ratio of the proton. Those transitions which are forbidden by the general selection rules following from the conservation of the angular momentum are left out. All other parameters are the same as in Fig. 1. The energies adopted to calculate the transition probabilities are those from Fig. 2.

Table 2. In Table 2 we list the values of the average coupling constant $\langle K \rangle$ in MeV. The constant of the restoring force C_2 in MeV, the corresponding values of the reduced matrix element of the vibrator in $e^2 R_0^4$ and the collective giromagnetic factor g_R respectively. Besides the parameters used in Fig. 1 we employed $Z = 80$, $A = 198$, $B(E2)(2_1 \rightarrow 0_1)_{\text{exp}} = 0.900 e^2 R_0^4$, and $\mu(2)_{\text{exp}} = 0.76 \frac{e\hbar}{2Mc}$. Different sets of values are given for different effective charges and different giromagnetic ratios of the proton.

Table 3 gives the static quadrupole moment of a few lowest excited states. The parameters are again those from Fig. 1 and Table 2. Two sets of values are given corresponding to different values of the effective charge.

Table 4 contains the calculated ratios of the electric quadrupole and magnetic dipole transitions.

Table 1

Transition	B(E2), $e_{eff}^P = e_p$			B(E2), $e_{eff}^P = 2e_p$			B(M1), $g_s^P = 5.586$			B(M1), $g_s^P_{eff} = 0.7g_s^P$			$W_{\beta}(E2), e_{eff}^P = 2e_p$			$W_{\beta}(M1), g_s^P_{eff} = 0.7g_s^P$		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
$2_1 \rightarrow 0_1$	0.900	0.900	0.900	0.900	0.900	0.900	-	-	-	-	-	-	3.16	3.18	3.18			
$2_2 \rightarrow 0_1$	0.003	0.086	0.004	0.001	0.038	0.001	-	-	-	-	-	-	0.457	17.4	0.457			
$2_3 \rightarrow 0_1$	0.206	0.061	0.168	0.086	0.024	0.069	-	-	-	-	-	-	149.0	41.7	120.0			
$2_2 \rightarrow 0_2$	0.250	0.101	0.232	0.236	0.098	0.210	-	-	-	-	-	-						
$2_3 \rightarrow 0_2$	0.012	0.100	0.092	0.007	0.103	0.074	-	-	-	-	-	-	0.008	0.122	0.087			
$1_1 \rightarrow 2_1$	0.006	0.002	0.014	0.005	0.017	0.011	1.580	1.176	1.230	0.640	0.476	0.595						
$2_2 \rightarrow 1_1$	0.490	0.102	0.449	0.464	0.104	0.400	0.182	0.007	0.211	0.042	0.009	0.056						
$2_3 \rightarrow 1_1$	0.097	0.464	0.189	0.088	0.310	0.167	0.712	0.875	1.700	0.281	0.307	0.700						
$4_1 \rightarrow 2_1$	1.270	1.420	1.341	1.150	1.302	1.210	-	-	-	-	-	-	35.2	39.9	37.0			
$2_2 \rightarrow 4_1$	0.020	0.056	0.006	0.014	0.063	0.003	-	-	-	-	-	-						
$2_3 \rightarrow 4_1$	0.009	0.057	0.000	0.052	0.075	0.007	-	-	-	-	-	-	0.114	0.165	0.015			
$2_2 \rightarrow 2_1$	0.095	0.372	0.128	0.089	0.291	0.112	0.169	0.060	0.061	0.071	0.003	0.023	3.82	12.5	4.8	39.2	1.66	12.7
$2_3 \rightarrow 2_1$	0.284	0.050	0.184	0.216	0.035	0.134	0.271	0.400	0.660	0.395	0.236	0.470	65.5	10.9	42.0	720	430	860
$2_3 \rightarrow 2_2$	0.000	0.004	0.004	0.000	0.009	0.001	0.071	0.025	0.047	0.044	0.000	0.036	0.000	0.011	0.001	2.83	0.000	2.32
$0_2 \rightarrow 2_1$	0.033	0.083	0.027	0.044	0.082	0.007	-	-	-	-	-	-	1.87	3.50	0.298			
$1_1 \rightarrow 0_1$	-	-	-	-	-	-	0.000	0.000	0.001	0.000	0.000	0.002						
$1_1 \rightarrow 0_2$	-	-	-	-	-	-	0.002	0.000	0.011	0.007	0.003	0.008						

Case	$e_{\text{eff}}^P = e_p$			$e_{\text{eff}}^P = 2e_p$			$\epsilon_S^P = 5.586$	$\epsilon_S^P \text{ eff} = 0.7 \epsilon_S^P$
	$\langle K \rangle$	C_2	$B(E2)_{\text{Pb}^{200}}$	$\langle K \rangle$	C_2	$B(E2)_{\text{Pb}^{200}}$	ϵ_R	ϵ_R
1	47.5	262	0.695	62.0	425	0.427	-0.27	-0.20
2	56.0	346	0.530	70.5	545	0.334	0.00	0.036
3	51.5	295	0.620	72.2	575	0.316	-0.43	-0.34

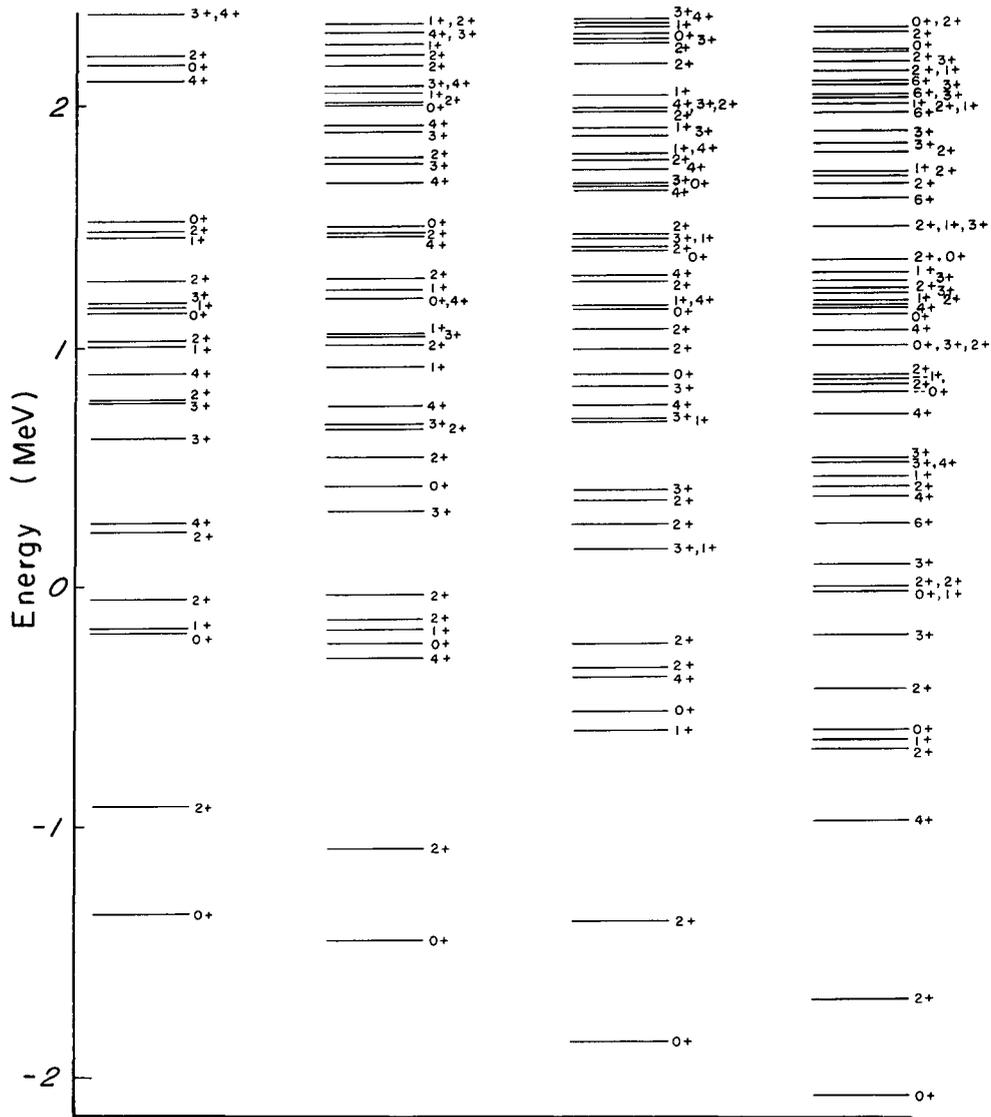
Table 2

Spin	Static quadrupole moments in $e_P R_0^2$					
	$e_{\text{eff}}^P = e_P$			$e_{\text{eff}}^P = 2e_P$		
	1	2	3	1	2	3
2_1	1.65	1.70	1.57	1.68	1.75	1.66
2_2	-1.01	-1.22	-0.922	-0.984	-1.28	-0.844
2_3	-0.456	-0.892	-0.121	-0.642	-0.930	-0.284
4_1	1.07	1.47	1.05	1.39	1.64	1.28
4_2	-0.651	-1.02	-0.667	-0.721	-1.03	-0.645
6_1	?	0.99	1.11	?	1.29	1.46

Table 3

Table 4

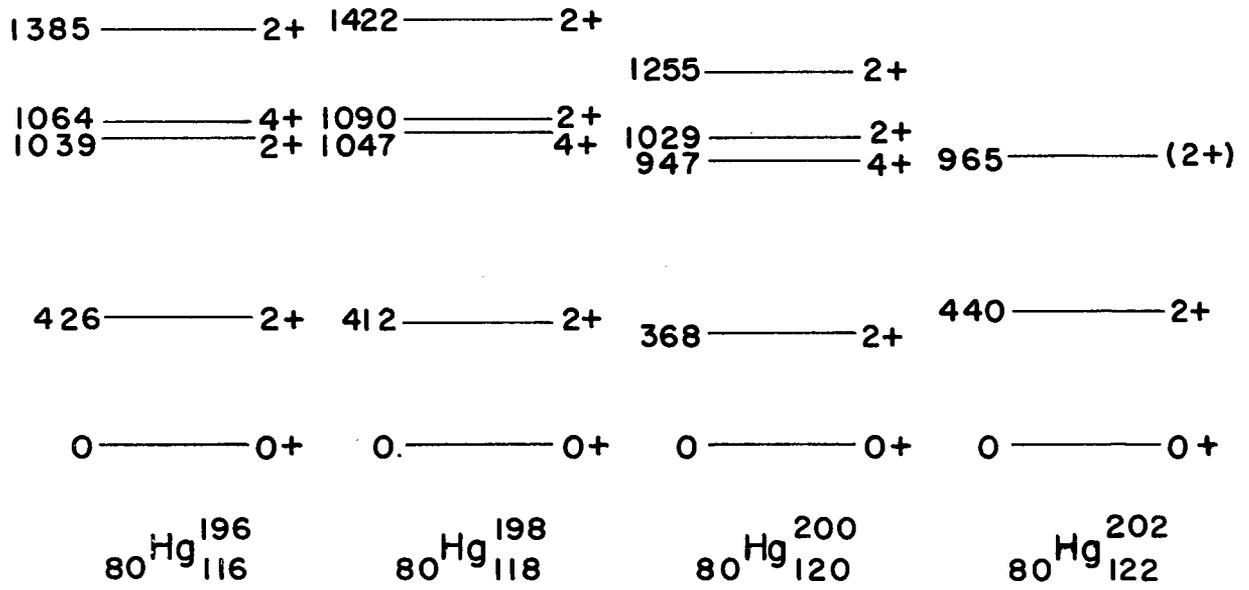
Transition	$\sigma, e_{\text{eff}} = e_p \quad g_{\text{eff}}^P = 0.7g_p$		
	1	2	3
$2_2 \rightarrow 2_1$	-0.310	2.54	-0.612
$2_3 \rightarrow 2_1$	0.305	0.158	0.201
$2_3 \rightarrow 2_2$	0.008	-1.25	-0.018



Calculated energy levels of ^{198}Hg for cases 1,2,3,4
($a = 0.6$)

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Fig. 1



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Fig. 2

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