UCRL-16560

University of California

Ernest O. Lawrence Radiation Laboratory

PARAMETERS OF THE $K_0\bar{K}_0$ SYSTEM

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

UCRL-16560 c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Physical Review Letters UCRL-16560

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

PARAMETERS OF THE $K_0\bar{K}_0$ SYSTEM

Philippe H. Eberhard

December 6, 1965

Parameters of the $K_0 \bar{K}_0$ System*

Philippe H. Eberhard

Lawrence Radiation Laboratory
University of California
Berkeley, California

December 6, 1965

When no conservation law is assumed--P, C, CP, T, or CPT--R. Sachs[1] has shown that the time dependence of a K_0-$\bar{K}_0$ mixed state can still be described with the help of a short-lived state $|K_S\rangle$ and a long-lived state $|K_L\rangle$:

$$\left.\begin{aligned} |K_S\rangle &= p_S\,|K_0\rangle + q_S\,|\bar{K}_0\rangle \\ |K_L\rangle &= p_L\,|K_0\rangle - q_L\,|\bar{K}_0\rangle . \end{aligned}\right\} \quad (1)$$

Several authors[1-4] have set limits for the parameters p_S, q_S, p_L, and q_L under different assumptions concerning the decay processes--either CPT invariance, the $\Delta I = \frac{1}{2}$ rule, or excluding the possibility of accidental cancellation. To our present knowledge, it has not been pointed out yet how well all these parameters are known from already performed experiments, from the properties of unitarity, and from an assumption of negligible C violations in strong interactions only.

Of the eight real parameters that define the four complex parameters p_S, q_S, p_L, and q_L, five can be chosen by convention if no conservation law is assumed. We take the following conventions:

$$\left.\begin{aligned} p_S &= \cos\alpha_S\, e^{i\,\theta/2} & \qquad p_L &= \cos\alpha_L\, e^{-i\,\theta/2} \\ q_S &= \sin\alpha_S\, e^{-i\,\theta/2} & \qquad q_L &= \sin\alpha_L\, e^{i\,\theta/2} . \end{aligned}\right\} \quad (2)$$

The following restrictions can be imposed:

$$0 \leq \alpha_S \leq \pi/2$$
$$0 \leq \alpha_L \leq \pi/2 \qquad (3)$$
$$-\pi/2 \leq \theta \leq \pi/2 \, .$$

Without assuming any invariance in the decay process, we determine α_S, α_L, and θ from known experimental results.[5-11] Our result is given in Eq. 20.

If $|j\rangle$ is a particular state into which the K mesons can decay, we call Γ_j^S (Γ_j^L) the rate of decay of a pure $|K_S\rangle$ ($|K_L\rangle$) state into $|j\rangle$ and

$$\kappa = \langle K_S | K_L \rangle = p_S^* p_L - q_S^* q_L$$
$$= \cos(\alpha_S + \alpha_L)\cos\theta - i\cos(\alpha_S - \alpha_L)\sin\theta. \qquad (4)$$

Unitarity gives the relation[12]

$$|\kappa| \leq \frac{2\sum_j (\Gamma_j^S \Gamma_j^L)^{1/2}}{|\tau_S^{-1} + \tau_L^{-1} - 2i\,\Delta m|}, \qquad (5)$$

where[5]

$$\left.\begin{aligned} &\tau_S = |K_S\rangle \text{ lifetime} = (0.88 \pm 0.01)\, 10^{-10} \text{ sec}, \\ &\tau_L = |K_L\rangle \text{ lifetime} = (5.8 \pm 0.6)\, 10^{-8} \text{ sec}, \\ \text{and}\quad &\Delta m = \text{mass difference between } |K_S\rangle \text{ and } |K_L\rangle = (0.55 \pm 0.1)\, \tau_S^{-1}. \end{aligned}\right\} \qquad (6)$$

From the inequality (5) and using a Schwarz inequality and the relations $\sum_j \Gamma_j^S = \tau_S^{-1}$ and $\sum_j \Gamma_j^L = \tau_L^{-1}$, we get the relation[13]

$$\left.\begin{aligned} &|\kappa| \leq \frac{2(\tau_S/\tau_L)^{1/2}}{|1 + (\tau_S/\tau_L) - 2i\,\Delta m\,\tau_S|} \cong 5 \times 10^{-2} \\ &|\kappa|^2 \leq 3 \times 10^{-3} \, . \end{aligned}\right\} \qquad (7)$$

Three experiments have measured the short-life decay rate of K_0 into $\pi^+\pi^-$.[6] According to our description and convention, those results are a measurement of the quantity

$$\frac{\Gamma^S_{\pi^+\pi^-} \quad \sin^2\alpha_L}{1 - |\kappa|^2} = (0.36 \pm 0.01)\ \tau_S^{-1}. \tag{8}$$

Several experiments have measured the branching ratio of $|K_S\rangle$ into $2\pi^0$ to $|K_S\rangle$ into $\pi^+\pi^-$:[7, 8]

$$\Gamma^S_{2\pi^0}/\Gamma^S_{\pi^+\pi^-} = 0.46 \pm 0.02\ . \tag{9}$$

Two experiments have measured the ratio R of the reactions $\bar{p}p \to K_0$ + (system of particles with negative strangeness) and $\bar{p}p \to \bar{K}_0$ + (system of particles with positive strangeness) when the K_0 or the $\bar{K}_0$ decays into two charged pions.[9] We assume that C violations in strong interactions are small compared to the accuracy of the experiments; therefore equal amounts of K_0 and $\bar{K}_0$ are produced. Considering that the ratio R is about 1 and that the state $|K_L\rangle$ decays into $\pi^+\pi^-$ no more than 0.2% of the time,[10] while the state $|K_S\rangle$ decays into $\pi^+\pi^-$ with a branching ratio of at least 0.36 [from relations (7) and (8)], we conclude that only the short-lived state $|K_S\rangle$ contributed sensibly to the $\pi^+\pi^-$ decay, whenever a K_0 or a $\bar{K}_0$ was produced initially. Therefore R is a measurement of $\tan^2\alpha_L$. Combining both experiments, we get

$$\tan^2\alpha_L = 1.0 \pm 0.03. \tag{10}$$

Moreover, combining (7), (8), (9), (10) we get

$$\left.\begin{aligned} &\Gamma^S_{\pi^+\pi^-} = (0.72 \pm 0.023)\ \tau_S^{-1} \\ \text{and}\quad &\Gamma^S_{2\pi^0} = (0.33 \pm 0.018)\ \tau_S^{-1}\ . \end{aligned}\right\} \tag{11}$$

We also can get the total rate for decay into two pions, $\Gamma^S_{\pi^+\pi^-} + \Gamma^S_{2\pi^0}$, by using references 6 and 7, and combine it with the direct measurement of $\Gamma^S_{\pi^+\pi^-} + \Gamma^S_{2\pi^0}$ given in reference 8 [using also relations (7) and (10):

$$\Gamma^S_{\pi^+\pi^-} + \Gamma^S_{2\pi^0} = (1.034 \pm 0.03)\, \tau_S^{-1}. \tag{12}$$

Because the sum of all decay rates of the $|K_S\rangle$ state should add up to τ_S^{-1}, by using Eq. (12) one can set an upper limit to the decay rates into all final states that are not two pions:

$$\Gamma^S_{\text{not } 2\pi} < 0.03\, \tau_S^{-1}. \tag{13}$$

We can now turn to the $|K_L\rangle$ decay rates. Christenson et al.[10] have measured

$$\Gamma^L_{\pi^+\pi^-} \cong 2 \times 10^{-3}\, \tau_L^{-1}. \tag{14}$$

An upper limit for the $2\pi^0$ decay mode of the $|K_L\rangle$ state,

$$\Gamma^L_{2\pi^0} < 8 \times 10^{-3}\, \tau_L^{-1}, \tag{15}$$

has been set.[11] Therefore most of the $|K_L\rangle$ state decays into states that are not two pions:

$$\Gamma^L_{\text{not } 2\pi} \cong \tau_L^{-1}. \tag{16}$$

Substituting (11), (13), (14), (15) and (16) in Eq. (5), we get

$$|\kappa| \leq \frac{2\left[(\Gamma^S_{\pi^+\pi^-}\Gamma^L_{\pi^+\pi^-})^{1/2} + (\Gamma^S_{2\pi^0}\Gamma^L_{2\pi^0})^{1/2} + (\Gamma^S_{\text{not } 2\pi}\Gamma^L_{\text{not } 2\pi})^{1/2}\right]}{|\tau_S^{-1} + \tau_L^{-1} - 2i\,\Delta m|}$$

$$< (1.3 \pm 0.45) \times 10^{-2}. \tag{17}$$

Relation (4) implies that

$$|\kappa|^2 = \cos^2(\alpha_S + \alpha_L)\cos^2\theta + \cos^2(\alpha_S - \alpha_L)\sin^2\theta, \tag{18}$$

In (18) $\cos(\alpha_S - \alpha_L)$ cannot be close to zero because of relation (10) and the restrictions (3). Therefore (17) and (18) imply that $\alpha_S + \alpha_L \cong \pi/2$, and $\theta \cong 0$. More precisely, we have

$$[\alpha_S + \alpha_L - (\pi/2)]^2 + \theta^2 < 1.7 \times 10^{-4}. \tag{19}$$

The value given in Eq. (19) can be increased by a factor of 2 if we stretch the error by one more standard deviation.

Relation (19) represents a condition for both $\alpha_S + \alpha_L$ and θ. Using (19) and (10), we have

$$\left.\begin{aligned} \alpha_S &= \pi/4 \pm 0.015, \\ \alpha_L &= \pi/4 \pm 0.008, \\ \text{and} \qquad \theta &= 0 \pm 0.013. \end{aligned}\right\} \tag{20}$$

These values show that our parameters are determined without assumption of a conservation law in weak interactions. They are close to their theoretical value if CP were conserved, i.e.,

$$\left.\begin{aligned} \alpha_S &= \alpha_L = \pi/4 \\ \text{and} \qquad \theta &= 0. \end{aligned}\right\} \tag{21}$$

Relation (20) shows that the measurement of $|K_L\rangle$ decay rates[14] (made under the assumption that p_S, q_S, p_L, and q_L are close to $1/\sqrt{2}$) are valid. We could now split the $\Gamma_{\text{not } 2\pi}$ into several categories, but this would not improve our result (20) sensibly because of the uncertainties about the $|K_S\rangle$ decay rates in the already performed experiments[15] and about possible unknown modes of decay of $|K_L\rangle$.[16]

The errors quoted in (20) result from quadratic combination of experimental errors and the condition of unitarity (5). Because the latter is an absolute limit and should not be interpreted as a Gaussian error, the

probabilities of situations corresponding to several standard deviations in the errors in α_S and θ are somewhat less than those expected for a Gaussian distribution.

Since this work was performed, we have seen a preprint by Bell and Steinberger[17] in which our unitarity condition (5) is derived. They obtain a somewhat better constraint than our relation (17) for $|\kappa|$ under reasonable assumptions about a few decay rates.

I am indebted to Prof. L. W. Alvarez for his support, Prof. F. S. Crawford, Jr. for enlightening discussions, and Dr. A. Barbaro-Galtieri for making available some experimental data used in this paper.

FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U. S. Atomic Energy Commission.

1. R. Sachs, Ann. Phys. 22, 239 (1963).

2. T. D. Lee and L. W. Wolfenstein, Phys. Rev. 138B, 1490 (1965).

3. S. Weinberg, Phys. Rev. 110, 782 (1958).

4. T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

5. A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Lawrence Radiation Laboratory Report UCRL-8030, August 1965. The values of lifetimes have been determined only by experiments measuring exponential decay of the particle (A. Barbaro-Galtieri, private communication). The value of Δm used is an average given by M. L. Good at the International Conference on Weak Interactions, Argonne National Laboratory, October 1965.

6. F. S. Crawford, M. Cresti, R. Douglass, M. Good, G. Kalbfleisch, M. L. Stevenson, and H. Ticho, Phys. Rev. Letters 2, 266 (1959); M. Schwartz, in Proceedings of the Annual International Conference on High-Energy Physics at Rochester (Interscience, New York, 1960), p. 727; J. A. Anderson, F. S. Crawford, B. B. Crawford, R. L. Golden, L. J. Lloyd, G. W. Meisner, and L. Price, Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN (CERN Scientific Information Service, Geneva, 1962), p. 836.

7. C. Baglin et al., Nuovo Cimento 18, 1043 (1960); M. Chretien et al., Phys. Rev. 131, 2208 (1963).

8. J. L. Brown et al., Nuovo Cimento 19, 1155 (1961); J. L. Brown, J. A. Kadyk, G. H. Trilling, B. P. Roe, D. S. Sinclair, and J. C. Vander Velde, Phys. Rev. 130, 769 (1963).

9. R. Armenteros et al., Physics Letters 17, 170 (1965); C. Baltay et al., Phys. Rev. Letters 15, 591 (1965).

10. J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

11. L. Criegee, J. D. Fox, H. Frauenfelder, A. O. Hanson, G. Moscati, C. F. Perdrisat, and J. Todoroff, communication at the International Conference on Weak Interactions, Argonne National Laboratory, October 1965.

12. If all $|j\rangle$ represent a complete orthogonal set of final states and $\mathcal{M}$ is the transition matrix, then we have

$$\Gamma_j^S = |\langle j|\mathcal{M}|K_S\rangle|^2,$$

$$\Gamma_j^L = |\langle j|\mathcal{M}|K_L\rangle|^2,$$

and

$$(\tau_S^{-1}/2 + \tau_L^{-1}/2 - i\ \Delta m) \times (p_S^* p_L - q_S^* q_L) = \zeta$$
$$= \sum_j \langle j|\mathcal{M}|K_S\rangle^* \langle j|\mathcal{M}|K_L\rangle = \sum_j (\Gamma_j^S \Gamma_j^L)^{1/2} e^{i\phi_j},$$

where ϕ_j is the phase difference between $\langle j|\mathcal{M}|K_L\rangle$ and $\langle j|\mathcal{M}|K_S\rangle$. Therefore we can write

$$|\zeta| \leq \sum_j (\Gamma_j^S \Gamma_j^L)^{1/2}.$$

If j is used to describe several states of decay of the orthogonal set, the above inequality is still valid as a result of the Schwarz inequality.

13. Inequality (7) is weaker than inequality (5); it has already been reported in reference 2.

14. The data considered here for branching ratios is the world compilation quoted in reference 5.

15. J. A. Anderson, F. S. Crawford, R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters 14, 475 (1965); P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and R. J. Plano, Phys. Rev. 140B, 127 (1965). An upper limit of 10 $\Gamma^{L}_{3\pi 0}$ for $\Gamma^{S}_{3\pi 0}$ has been given us by the Paris-Milan-Padua collaboration (B. Aubert, Ecole Polytechnique, Paris, private communication).

16. When we sum up all known decay rates of $|K_L\rangle$, also taking into account measurements of some of the ratios of the decay rates, and compare the result to the value of τ_L^{-1} determined by exponential decay, the experimental accuracies allow us to write only the inequality

$$\Gamma^{L}_{\text{unknown}} < 0.08\ \tau_L^{-1}$$

(A. Barbaro-Galtieri, Lawrence Radiation Laboratory, private communication).

17. J. S. Bell and J. Steinberger, Weak Interactions of Kaons, CERN report 65/1524/5-TH. 605, October 13, 1965; lectures given at Oxford International Conference on Elementary Particles, September 1965.