UCRL-16821

University of California

Ernest O. Lawrence Radiation Laboratory

AN ANALYSIS OF THE METHODS USED TO CALCULATE THE GRAIN BOUNDARY SLIDING CONTRIBUTION DURING CREEP

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Trans. AIME

UCRL-16821
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

AN ANALYSIS OF THE METHODS USED TO CALCULATE THE GRAIN BOUNDARY SLIDING CONTRIBUTION DURING CREEP

T. G. Langdon

September 21, 1966

AN ANALYSIS OF THE METHODS USED TO CALCULATE THE GRAIN BOUNDARY SLIDING CONTRIBUTION DURING CREEP*

T. G. Langdon

Inorganic Materials Research Division, Lawrence Radiation Laboratory, and Department of Mineral Technology, College of Engineering, University of California, Berkeley, California

Much interest has been centered recently on the phenomenon of grain boundary sliding, particularly in studies of the formation of cavities during high-temperature creep.[1] A large number of papers have been devoted to this subject but unfortunately many inconsistencies exist in the treatment of the experimental data. The purpose of this note is to draw attention to these inconsistencies and to present unambiguous formulae for use in future studies.

If a specimen is subjected to creep testing, the total strain, E_{tot}, consists of two component parts, such that

$$E_{tot} = E_s + E_{gb} \qquad (1)$$

where E_s is the strain due to trans-granular processes occurring within the grains, and E_{gb} is the strain due to grain boundary sliding. A number of workers have calculated the percentage contribution made by sliding to the overall extension in the form of the ratio E_{gb}/E_{tot} $(= \gamma)$, or, in some cases, have obtained sufficient readings that a calculation can be made from their results. A comprehensive review is given in Table 1, in which a classification has been made according to the material used

---

* Originally published as part of a Ph.D. thesis submitted to the University of London, June 1965.

and the method of measurement. The numbers in the Table cite references given in the bibliography. The u, v, and w offsets refer to a resolution of the sliding vector into three mutually perpendicular components: u is the sliding component measured parallel to the stress axis, v is the component perpendicular both to the stress axis and to the specimen surface, and w is the component perpendicular to the stress axis and in the plane of the surface. With only two exceptions (references 13 and 33), all determinations of γ from measurements of u, v or w refer specifically to surface observations. The method of grain shape measurements, originally proposed by Rachinger,[8] has the additional advantage of also allowing a determination of γ in the interior of specimens, as indicated in the Table.

However, although this large volume of work exists, it is usually not possible to compare results directly since different workers have often utilised different methods of calculation. This arises due to the complication of converting from measurements of the offsets either perpendicular to, or in the plane of, the surface to the resultant longitudinal extension of the specimen. Gifkins[44] has pointed out that a general equation to cover this conversion is given by

$$E_{gb} = k\, n\, \bar{x} \qquad (2)$$

where n = no. of grains/cm

$\bar{x}$ = average displacement at the boundary ($\bar{u}$, $\bar{v}$ or $\bar{w}$)

k = a geometrical averaging factor

In practice, the value of k in Eq. 2 is dependent on both the parameter chosen for $\bar{x}$ and the method of measurement. For example, for readings of u the value obtained for $\bar{u}$ will depend on whether measurements

are recorded along a longitudinal or transverse traverse of the specimen, since each traverse will intersect boundaries having a different angular distribution with respect to the stress axis. This fact has been overlooked by the majority of previous workers, and the values utilised for k have often been based on incorrect geometrical assumptions. A comparison between the results of different workers is thus only possible if allowance is made for these discrepancies.

If the longitudinal offset, u, is measured at every boundary intersected by a longitudinal line, then it follows directly that

$$E_{gb} = n\,\bar{u}_{long} \qquad (3)$$

where $\bar{u}_{long}$ refers to the average value of u obtained from a longitudinal traverse. In practice, measurements of u are more easily made from a transverse traverse, and Brunner and Grant[45] have shown that E_{gb} is then given by

$$E_{gb} = n\,(\overline{u_{trans} \cdot \tan\theta}) \qquad (4)$$

where the subscript "trans" refers to measurements taken along a transverse traverse, and θ, the angle between the boundary and the stress axis in the plane of the surface, is measured at every boundary intersected by the traverse. Equation 4 is correct at low values of E_{tot}.

Measurements of the w parameter are usually taken along a longitudinal traverse, and it has been shown[32] that E_{gb} is then given by

$$E_{gb} = 2\,n\,(\overline{w_{long}/\tan\theta}) \qquad (5)$$

where the average bar refers to the average of the quotient.

Difficulties arise in the determination of γ from the offsets perpendicular to the surface, since the values of v are directly affected

by the presence of a free surface, and by the change of internal angle which occurs as the boundaries migrate to become perpendicular to the surface during creep.[32] It has been pointed out elsewhere[32] that in this case E_{gb} is best calculated from the equation

$$E_{gb} = k\ n\ \bar{v}_{ran} \qquad (6)$$

where k is determined experimentally by the direct measurement of E_s, and $\bar{v}_{ran}$ refers to readings of v taken randomly rather than along a particular linear traverse (as, for example, by the use of interferometry). Experiments with magnox, a Mg - 0.78% Al alloy, have yielded values of k of the order of 1.07 when the total strain is about 2%.[32]

A compelte summary of the various equations used to calculate E_{gb} is given in Table II, and these will now be examined in detail.

One of the earliest techniques, put forward by McLean,[2] involved measuring, at the same point, both the vertical step height v and the component of sliding along the grain boundary in the plane of the surface. This was then used to calculate the total sliding displacement, but, since there was a large inaccuracy in the latter measurement, a method was later developed[46] whereby E_{gb} could be calculated directly from v measurements alone, and this was made use of in subsequent work.[3,4,18] To obtain a relationship between E_{gb} and $\bar{v}_{ran}$, McLean assumed that the angle between the sliding vector and the stress axis could take all values between 0° and 90° with equal frequency. This is correct, but McLean overlooked the fact that his v measurements were made on a planar surface parallel to the stress axis, where the angles of intersection follow a sinusoidal relation. Also, no allowance was made for the

internal change of angle which occurs due to the tendency for boundaries to migrate to an equilibrium position perpendicular to the surface. A number of other workers have computed γ using the McLean relationship, which includes a correction factor of $(1 + E_{tot})^{1/2}$ to compensate for the reduction in angle between the boundary and the stress axis as the specimen elongates. Some workers have included this correction[21,23,28,34,35,37,40], but others have not[14,38,39], and, furthermore, some workers have taken measurements of v along a longitudinal traverse instead of randomly.[21,23,37]

In an attempt to calculate γ for Al and Mg specimens, Dorn and co-workers[10,30] measured $\bar{u}_{trans}$. They then calculated E_{gb} by assuming a k factor of 1, but this is incorrect since the value of $\bar{u}$ obtained from a transverse traverse is not equal to that obtained from a longitudinal traverse. In later work,[11] it was assumed, without explanation, that the value of k was $\pi/2$, but there is no justification for this since a transverse traverse intersects, on average, boundaries more nearly parallel to the stress axis, and $\bar{u}_{trans}$ is greater than $\bar{u}_{long}$.[32] From Eq. 3 it therefore follows that the correct value of k must be less, and not greater, than unity. The same workers[7] also assumed a k factor of unity for use with measurements of $\bar{v}_{trans}$, but again this was unsubstantiated. Ishida, Mullendore and Grant[13] distinguished between longitudinal and transverse traverses, but they overlooked the fact[47] that measurements of w, unlike u, included no contribution from the sliding component perpendicular to the surface. The equation they used is incorrect since it underestimates γ by a factor of two. This can be seen by reference to Eq. 5.

Rachinger[8] attempted to determine γ on the surface by measuring both the displacement of grid lines and the offsets perpendicular to the surface. He showed theoretically that the average gap at a grain boundary ($\bar{u}$), measured parallel to the stress axis, between the ends of a longitudinal grid line was given by

$$(E_{tot} - E_s) = n\,\bar{u} \qquad (7)$$

This is correct provided $\bar{u}$ is determined from a longitudinal traverse. However, Rachinger incorrectly assumed that the value obtained for $\bar{u}$ was independent of the direction of traverse, and measurements were taken to determine $\bar{u}_{trans}$ from a transverse traverse.

Similarly, Rachinger theoretically showed that the average displacement $\bar{w}$, measured perpendicular to the stress axis between the ends of a transverse grid line, was given by

$$(1 + E_s)^{-1/2} - (1 + E_{tot})^{-1/2} = n\,\bar{w} \qquad (8)$$

Gifkins[39] has pointed out that, when E_s and E_{tot} are both small compared with unity, this latter expression reduces to

$$E_{gb} = 2\,n\,\bar{w} \qquad (9)$$

Rachinger incorrectly measured $\bar{w}_{long}$ for use in Eq. 8, and again made no distinction for $\bar{w}$ obtained from a particular traverse.

To utilise measurements of the offsets perpendicular to the surface, Rachinger assumed that $\bar{v}$ and $\bar{w}$ were equal, so that Eq. 8 reduces to

$$E_{gb} = 2\,n\,\bar{v} \qquad (10)$$

Results presented elsewhere[32] show that this assumption is invalid. Furthermore, Rachinger measured $\bar{v}$ using a Talysurf surface analyser along a longitudinal traverse, and thus in reality obtained $\bar{v}_{long}$.

To summarize, in each case Rachinger used an invalid formula for his surface measurements, and the agreement obtained for the different methods was therefore probably fortuitous. The value of $k = 2$, which is a direct consequence of Rachinger's formula,[8] merely results from the fact that u consists of both a v and a w component. Later calculations of γ from $\bar{v}_{ran}$[38,39] or $\bar{v}_{long}$[9] measurements using $k = 2$ are therefore not justified.

Couling and Roberts[29] attempted an experimental determination of k for use with $\bar{u}_{trans}$, and fully appreciated that E_{gb} was best calculated from Eq. 3. To convert from the transverse to the longitudinal component, they measured the average value of $\bar{u}$ in each 10° increment of θ, and determined the frequency of occurrence of each increment on a longitudinal traverse. This method is essentially correct, but their result must be accepted with reserve since their measured distribution of u with θ is unlike that obtained by other investigators.[11,14,32]

More recently, Gittins[14] attempted to determine γ by showing that

$$u = (v/\tan\psi) + (w/\tan\theta) \qquad (11)$$

where ψ is the angle, measured on a transverse section, between the boundary and the plane of the surface. He further assumed that it was possible to take the independent average of two variables appearing in a fraction, and since, in the interior, $\bar{w}_{long} = \bar{v}_{long}$, he obtained

$$E_{gb} = (2\ n/\tan\bar{\psi})\ \bar{v}_{long} = (2\ n/\tan\ \bar{\theta})\ \bar{w}_{long} \qquad (12)$$

Since $\bar{\theta} = \bar{\psi} = 57.3°$ initially (both theoretically and by direct measurement[48]), and by taking $\tan\bar{\psi}$ in place of $\overline{\tan\psi}$, he deduced a geometrical factor for the interior of $k = 2/\tan 57.3° = 1.28$. Because

of the change of angle occurring at a polished surface, Gittins measured the ratio $\overline{v}_{long}/\overline{w}_{long}$ for an annealed surface, and obtained a value of 1.5. Thus the value of k for use with $\overline{v}_{long}$ on this type of surface was k = 1.28/1.5 = 0.85. In fact, this analysis is unacceptable, since it is not always valid to take the average of the two variables w_{long} and θ independently. This is shown by some typical results obtained on magnox. Measurements of w_{long} and θ were recorded on the polished surface of a specimen deformed to 2.5% total strain, and, after a total of 300 readings, the values obtained were as follows:

$$\overline{w}_{long} = 0.872\ \mu m;\ \overline{\theta} = 58.0^\circ$$

$$(\overline{w_{long}/\tan\theta}) = 0.628$$

$$\therefore \quad E_{gb} = 2\ n\ (\overline{w_{long}}/\overline{\tan\theta}) \qquad (5)$$

$$= 2 \times 0.628 \times 10^{-4}\ n.$$

If the variables are taken separately, and $\tan\overline{\theta}$ used in place of $\overline{\tan\theta}$, then, by the Gittins analysis,

$$E_{gb} = (2\ n/\tan\overline{\theta})\ \overline{w}_{long} \qquad (12)$$

$$= (2\ n/\tan 58^\circ)\ 0.872$$

$$= 2 \times 0.545 \times 10^{-4}\ n.$$

The true value of k to use with individual values of $\overline{w}_{long}$ in this case is therefore k = (2 x 0.628)/0.872 = 1.44, which is even higher than the value of 1.28 calculated by Gittins for the annealed surface. Results on other specimens also indicate that the value of k derived in this manner is substantially greater than 1.28, although the average

intercept angle, $\bar{\theta}$, is always of the order of 57° - 58° and therefore agrees well with the theoretical value of 57.3°.

The method of determining γ from grain shape measurements, first put forward by Rachinger,[8] involved measuring the number of grains per unit length in directions both parallel and perpendicular to the stress axis, and this technique has been utilised in a number of publications[8,13,15,17,24,41]. A better procedure is to measure the maximum length and breadth of individual grains, in the manner suggested by Hensler and Gifkins,[49] and measurements of this type have been carried out by a number of workers[9,14,16,25,26,27,31,32,33,42,43] (although it should be noted that only the length of the grains was measured in reference 25). Even with this technique, however, difficulties may arise in the handling of the experimental data, as pointed out by Sellars.[50] Furthermore, results presented elsewhere[32] indicate that no reliable values of γ can be obtained from grain shape measurements, owing to the tendency for boundary migration to maintain the grains in an equi-axed configuration.

The conclusions to be drawn from this analysis are that all workers, with the exception of Brunner and Grant,[12,44] have used an incorrect method to calculate γ. These methods have been examined in detail, and it has been shown that the discrepancies can all be resolved in terms of invalid assumptions. Table II indicates that the range of magnitude of these discrepancies is often quite large. The method used to determine γ from measurements of $\bar{v}_{long}$ by Gittins,[14] for example, would yield a value less than half that obtained by the Rachinger[8] analysis from the

same readings. The methods used to calculate γ from $\bar{u}_{trans}$ measurements range from the use of k values of 1.00 to 1.57 in Eq. 2, although it was pointed out that the correct value of k should be less than unity. This suggests that the values of γ obtained by Dorn et al.,[11] for which $k = \pi/2 = 1.57$, and Couling and Roberts,[29] for which $k = 1.45$, may overestimate the true value by almost a factor of two. In this respect it is interesting to note that the values reported by Couling and Roberts were at all times very large, ranging from a minimum of 67% to a maximum of 93%.

I am grateful to Professor R. L. Bell and Dr. R. C. Gifkins for many helpful discussions. This work was carried out at Imperial College, University of London, under a contract furnished by the United Kingdom Atomic Energy Authority. Financial support was provided by the United States Atomic Energy Commission during the preparation of this paper.

REFERENCES

1. J. E. Harris, Trans. Am. Inst. Min. Metall. Engrs., 233, 1509-1516 (1965).

2. D. McLean, J. Inst. Metals, 80, 507-519, (1951-52).

3. D. McLean and M. H. Farmer, ibid., 85, 41-50 (1956-57).

4. D. McLean, ibid., 81, 293-300 (1952-53).

5. D. McLean, N.P.L. Symposium on Creep and Fracture, pp. 73-87, H.M.S.O., London (1956).

6. D. McLean, Rev. Mét., 53, 139-146 (1956).

7. J. Harper and J. E. Dorn, Acta Met., 5, 654-665 (1957).

8. W. A. Rachinger, J. Inst. Metals, 81, 33-41 (1952-53).

9. P. W. Davies, R. N. Stevens and B. Wilshire, ibid., 94, 49-55 (1966).

10. B. Fazan, O. D. Sherby and J. E. Dorn, Trans. Am. Inst. Min. Metall. Engrs., 200, 919-922, (1954).

11. J. G. Harper, L. A. Shepard and J. E. Dorn, Acta Met., 6, 509-518 (1958).

12. H. Brunner and N. J. Grant, Trans. Am. Inst. Min. Metall. Engrs., 218, 122-127 (1960).

13. Y. Ishida, A. W. Mullendore and N. J. Grant, ibid., 233, 204-212 (1965).

14. A. Gittins, Ph.D. Thesis, Univ. of Melbourne, (1964).

15. W. A. Rachinger, Acta Met., 7, 374-375 (1959).

16. P. W. Davies, R. N. Stevens and B. Wilshire, Nature, 206, 924 (1965).

17. H. J. Busboom, DMS Report No. 63-1, Stanford Univ., (1963).

18. D. McLean and M. H. Farmer, J. Inst. Metals, 83, 1-10 (1954-55).

19. A. W. Mullendore and N. J. Grant, Trans. Am. Inst. Min. Metall. Engrs., 227, 219-330 (1963).

20. Y. Ishida, A. W. Mullendore and N. J. Grant, ibid., 230, 1454-1459, (1964).

21. P. W. Davies, J. P. Dennison and R. W. Evans, J. Inst. Metals, 92, 409-412, (1963-64).

22. J. A. Martin, M. Herman and N. Brown, Trans. Am. Inst. Min. Metall. Engrs., 209, 78-81, (1957).

23. D. C. Dodds and P. W. Davies, J. Inst. Metals, 93, 483-485 (1964-65).

24. N. N. Davidenkov, E. I. Brainin and D. M. Vasil'yev, Fiz. Met. i Metalloved., 11, (3), 451-460, (1961).

25. V. I. Shabalin, Doklady Akad. Nauk SSSR, 144, 551-553 (1962).

26. F. Garofalo, Special Tech. Publ. No. 283, A.S.T.M., (1960).

27. F. Garofalo, L. Zwell, A. S. Keh and S. Weissmann, Acta Met., 9, 721-729, (1961).

28. V. M. Rozenberg, Izvest. Akad. Nauk SSSR, Met. i Topl., (1), 105-110, (1960).

29. S. L. Couling and C. S. Roberts, Trans. Am. Inst. Min. Metall. Engrs., 209, 1252-1256, (1957).

30. F. E. Hauser, C. D. Starr, L. Tietz and J. E. Dorn, Trans. Am. Soc. Metals, 47, 102-130, (1955).

31. J. H. Hensler and G. V. Cullen, J. Australian Inst. Metals, 9, 38-45, (1964).

32. T. G. Langdon and R. L. Bell, J. Materials Science, to be published.

33. C. Graeme-Barber, (Univ. of Southampton), unpublished work.

34. V. M. Rozenberg, Fiz. Met. i Metalloved., 15, (3), 397-404, (1963).

35. V. M. Rozenberg, ibid., 11, (6), 899-909, (1961).

36. V. M. Rozenberg, Zavodskaya Laboratoriya, 25, (1), 53-59, (1959).

37. P. W. Davies, J. P. Dennison and R. W. Evans, J. Inst. Metals, 93, 25-56, (1964-65).

38. R. C. Gifkins, J. Australian Inst. Metals, 8, 148-157, (1963).

39. R. C. Gifkins, Proc. Berkeley Nuclear Conf., (D. J. Littler, ed.), pp. 335-342, Butterworths, London, (1961).

40. M. G. Lozinsky and V. Ya. Ferenets, Fiz. Met. i Metalloved., 16, (3), 409-415 (1963).

41. N. N. Davidenkov and E. I. Brainin, Fiz. Tverd. Tela, 1, (9), 1479-1483, (1959).

42. W. R. Martin and J. R. Weir, ORNL-TM-1043, Oak Ridge Nat. Lab., (1965).

43. F. Garofalo, C. Richmond, W. F. Domis and F. von Gemmingen, Proc. Joint Int. Conf. on Creep, paper 30, I. Mech. Eng., London, (1963).

44. R. C. Gifkins, Fracture (B. L. Averbach et al., eds.), pp. 579-623, M.I.T. and John Wiley, New York, (1959).

45. H. Brunner and N. J. Grant, Trans. Am. Inst. Min. Metall. Engrs., 215, 48-56, (1959).

46. D. McLean, J. Inst. Metals, 81, 133-144, (1952-53).

47. R. L. Bell, C. Graeme-Barber and T. G. Langdon, Trans. Am.Inst. Min. Metall. Engrs., 233, 1648-1649, (1965).

48. A. Gittins and R. C. Gifkins, J. Inst. Metals, to be published.

49. J. H. Hensler and R. C. Gifkins, ibid., 92, 340, (1963-64).

50. C. M. Sellars, ibid., 93, 365-366, (1964-65).

Table I

Previous Determinations of γ

Material	γ from "vertical" (v) offsets	γ from "horizontal" (u,w) offsets	γ from grain shape		
			(i)*	(ii)	(iii)
Al	2,3,4,5,6,7,8,9	2,8,10,11,12,13	8,13,14,15		9,16,17
Al alloy	3,5,18	12,19,20			
Au	21				
β-brass	14,22	14	14		
Cd	3				
Cu	3				
Cu-0.15% Al	23				
Cu-0.37% Be	3				
Fe	3		24,25		26,27
Fe-30% Ni	28				
Mg	14	29,30			31
Mg-0.78% Al	32,33	32,33	32,33		
Ni	34,35,36,37				
Pb	38				
Pb-2.2% Tl	38,39				
Pt	40				
Sn	3				
Zn	3				
Steel				41	42,43

* (i) = Surface and interior.
(ii) = Surface only.
(iii) = Interior only.

Table II

Formulae Used by Previous Workers to Calculate E_{gb}

Workers	Reference	Component Measured	Formula Used
McLean	46	$\bar{v}_{ran}$	$E_{gb} = (\sqrt{5}/\sqrt{2})\ n\ \bar{v}_{ran}(1 + E_{tot})^{1/2}$
Fazan, Sherby, and Dorn	10	$\bar{u}_{trans}$	$E_{gb} = n\ \bar{u}_{trans}$
Harper, Shepard, and Dorn	11	$\bar{u}_{trans}$	$E_{gb} = (\pi/2)\ n\ \bar{u}_{trans}$
Harper and Dorn	7	$\bar{v}_{trans}$	$E_{gb} = n\ \bar{v}_{trans}$
Brunner and Grant	45	$\bar{u}_{trans}$	$E_{gb} = n\ (\overline{u_{trans} \cdot \tan\theta})$
Ishida, Mullendore, and Grant	13	$\bar{w}_{long}$	$E_{gb} = n\ (\overline{w_{long}/\tan\theta})$
Rachinger	8	$\bar{u}_{trans}$	$E_{gb} = n\ \bar{u}_{trans}$
"	"	$\bar{w}_{long}$	$E_{gb} = 2\ n\ \bar{w}_{long}$
"	"	$\bar{v}_{long}$	$E_{gb} = 2\ n\ \bar{v}_{long}$
Couling and Roberts	29	$\bar{u}_{trans}$	$E_{gb} = 1.45\ n\ \bar{u}_{trans}$
Gittins	14	$\bar{v}_{long}$*	$E_{gb} = 0.85\ n\ \bar{v}_{long}$
Langdon and Bell	32	$\bar{v}_{ran}$	$E_{gb} = 1.07\ n\ \bar{v}_{ran}$

* Measured on an annealed surface.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.