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STUDY OF THE REACTION $\eta \to \pi^+\pi^-\gamma$

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UNIVERSITY OF CALIFORNIA

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STUDY OF THE REACTION $\eta \rightarrow \pi^+\pi^-\gamma$

LeRoy Raymond Price

(Ph. D. Thesis)

June 8, 1966

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ABSTRACT

The results of an analysis of 33 decays of the type $\eta \rightarrow \pi^+\pi^-\gamma$ are presented. It is found that the rho-dominant model fits the data quite well. No conclusions can be drawn regarding the possible existance of a C-nonconserving interaction.

I. INTRODUCTION

In early 1961, Gell-Mann[1] developed his now-famous "8-fold way". One of the predictions of this new symmetry scheme was that there should exist an $I = 0$, $J^P = 0^-$ meson (which he called χ^0). With the mass formula that he developed in that same paper,[2] the mass of the χ^0 was later predicted to be about 550 MeV.

Also in 1961, Pevsner et al.[3] in a Johns Hopkins-Northwestern collaboration were examining the mass distribution of the three pions in the reaction

$$\pi^+ d \rightarrow pp\pi^+\pi^-\pi^0 .$$

In addition to the ω^0 (which had been discovered just a short time before), this group saw a bump at about 550 MeV — and called it "eta." However, other than its width, which they found to be less than 25 MeV, they were unable to determine any of the quantum numbers of this new particle.

Soon Carmony, Rosenfeld, and Van de Walle[4] were able to show that the isotopic spin of the eta was equal to zero. And then Bastien et al.[5] showed that the most probable assignments for the quantum numbers of the eta were exactly the same as those Gell-Mann had predicted for the χ^0--$J^P = 0^-$ and G parity = +.

Thus the eta meson was the first particle to be predicted by a unitary symmetry scheme and subsequently found.

Independent and perhaps more reliable evidence that the spin of the eta is zero was obtained by Chretien et al.[6] They observed the decay

$$\eta \rightarrow 2\gamma.$$

By using the method of Chinowsky and Steinberger[7] for measuring the mass of the π^0, Chretien et al. were able to establish that the two gammas actually came from an eta (i.e., they observed the opening angle between the two gammas in the lab system).

An interesting aspect of eta decay is that <u>all</u> of the observed modes are electromagnetic. This can be easily seen by noting that the decay $\eta \to 2\pi$ is forbidden since $J^P = 0^-$ for the eta. But for two pions, P equals $(-1)^J$.

The decay

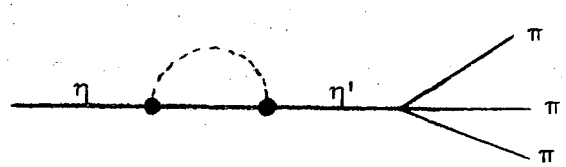
$$\eta \to 4\pi^0$$

is not forbidden by any quantum numbers, but the Q for this reaction is only about 8 MeV and the decay must surmount rather high angular momentum barriers.

The decay

$$\eta \to 3\pi$$

is of course forbidden by G parity (since G = +). But G-nonconserving decays are allowed to go electromagnetically, and this is exactly what does happen in the case of the eta. Thus one pictures the three-pion decay to be as follows:



That is, the eta first emits and then absorbs a virtual gamma ray before decaying into the three pions.

Table I. All of the known decay modes of the eta meson. Sources are:

a A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Phys. Rev. Letters (to be published).

b Frank S. Crawford, Jr., L. J. Lloyd, and Earle C. Fowler, Phys. Rev. Letters (to be published).

c G. Di Giugno, R. Querzoli, G. Troise, F. Vanoli, M. Giorgi, P. Schiavon, and V. Silvestrini, Phys. Rev. Letters 16, 767 (1966).

d R. Grossman, L. R. Price, F. S. Crawford, Jr., Conversion of gamma rays from the decay $\eta^0 \rightarrow \pi^+ + \pi^- + \gamma$, UCRL-16684 (1966), and Phys. Rev. (to be published).

e Frank S. Crawford, Jr., L. J. Lloyd and Earle C. Fowler, Phys. Rev. Letters, 10, 546 (1963).

Decay mode	Approximate percentage	Source
$\pi^+\pi^-\pi^0$	25	a
$\pi^0\pi^0\pi^0$	10	b
$\gamma\gamma$	39	a
$\pi^+\pi^-\gamma$	6	a
$\pi^0 e^+ e^+$	<1	a
$\pi^0\gamma\gamma$	18	c
$\pi^+\pi^- e^+ e^-$	0.1	d
$\gamma e^+ e^-$	0.6	e

In addition to the three-pion modes, the eta has other modes of decay. All of the known decay modes, along with their approximate rates, are given in Table I.

Another interesting feature of the eta is that it is in an eigenstate of C (charge conjugation). Thus the decays of the eta are an ideal place to look for violations of C-invariance, particularly for violations that might have a strength comparable to that of the electromagnetic interactions.[8]

Such a test has been recently performed on a sample of 1300 decays[9] of the type

$$\eta \rightarrow \pi^+\pi^-\pi^0 .$$

Because an effect of only about one or two standard deviations was found, it was concluded that it would take a much larger experiment (at least 7,000 decays) to get a definitive result.

The present paper describes the first attempt to find a C-nonconserving interaction in the decay

$$\eta \rightarrow \pi^+\pi^-\gamma.$$

Although the eta was discovered in 1961, it was not until 1963 that the mode

$$\eta \rightarrow \pi^+\pi^-\gamma \qquad (1)$$

was first reported (by Fowler et al.[10] with 12 events). Since then it has been reported by only three other groups: M. Foster[11] (26 events), Foelsche and Kraybill[12] (9 events), and Pauli and Muller (9 "class a" events[13] and 15 "class b" events[14]). In each of these experiments, the branching ratio

$$R \equiv \Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$$

was measured. The results are $R = 0.26 \pm 0.08$ (Ref. 10), $R = 0.20 \pm 0.04$ (Ref. 11), $R = 0.14 \pm 0.08$ (Ref. 12), $R = 0.27 \pm 0.10$ (Ref. 13), and $R = 0.73 \pm 0.25$ (Ref. 13). This last value reflects the fact that Pauli and Muller saw many events with low gamma-ray energies in the eta c.m. These "low-energy" events were not seen in any of the other three experiments.

In none of the above experiments was the final state analyzed for either the spin of the dipion or a parameterization of the gamma-ray energy spectrum. In addition, possible nonconservation of C in decay (1) was not tested in any of these experiments.[15]

The primary purposes of this experiment[16] were:

(a) To look for a low-energy gamma-ray enhancement,

(b) To determine the spin of the dipion,

(c) To fit the energy spectrum of the gamma to various hypotheses,

and

(d) To look for a possible nonconservation of C in the decay $\eta \to \pi^+\pi^-\gamma$.

II. THE BEAM

Our sample of events came from 60 rolls of film exposed to 1170-MeV/c pi-plus mesons in the Alvarez 72-in. hydrogen bubble chamber. The beam-transport system was designed and built by Professor Frank S. Crawford, Jr. As this beam has been well described in the literature,[17] we include here only a schematic diagram of the beam optics (Fig. 1).

The most important feature of this beam (as far as this experiment is concerned) is its unusually small spread in momentum (the half-width at half-maximum is ±3 MeV/c).[18] The smallness of this quantity enables us to successfully separate from one another the reactions:

a) $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^-$

b) $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \gamma$

and

c) $\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \pi^0$.

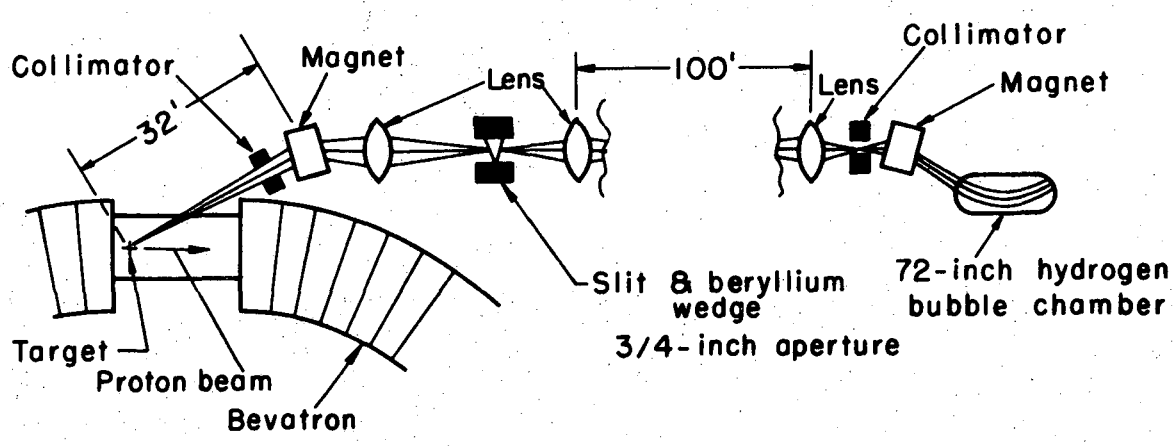


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Fig. 1. Schematic diagram of beam optics.

III. SEPARATING OUT THE $\eta \to \pi^+\pi^-\gamma$ EVENTS

Our initial sample contained about 4000 four-pronged events. Protons were identified on the scanning table on the basis of their bubble density. All events were fit to a number of hypotheses,[19] and then a series of cutoffs was applied (Sec. III. B). The effects of the cutoffs were estimated by means of the Monte Carlo program FAKE.[20]

A. The Use of FAKE for Calculating Corrections

In the past few years we have come to rely increasingly upon the Monte-Carlo program FAKE for calculating the effects of various cutoffs.

This program generates "bubble chamber events" by a Monte-Carlo method. These events are then sent thru the same programs used for the real events. In this manner one can easily calculate the effects of any set of correlated or uncorrelated cutoffs.

Our faith in the program stems from the fact that whenever we have compared such items as the χ^2 distributions or energy spectra obtained from a properly set-up FAKE run with the same quantities from known real events, the two sets have always agreed within the statistics.

As examples, in Fig. 2(a) we compare the distributions in $m^2(\gamma)$ (i.e., the square of the missing mass recoiling against the four charged particles) from a sample of 330 (arbitrarily selected) FAKE events[21] and our final sample of 33 events. Similarly in 2(b) we compare the (1C) Coulomb χ^2's. Figures 2(c) and 2(d) are comparisons of the mass (squared) of the $\pi^+\pi^-\gamma$ (unfitted data) and the (1c) gamma-ray χ^2, respectively. All agree quite well.

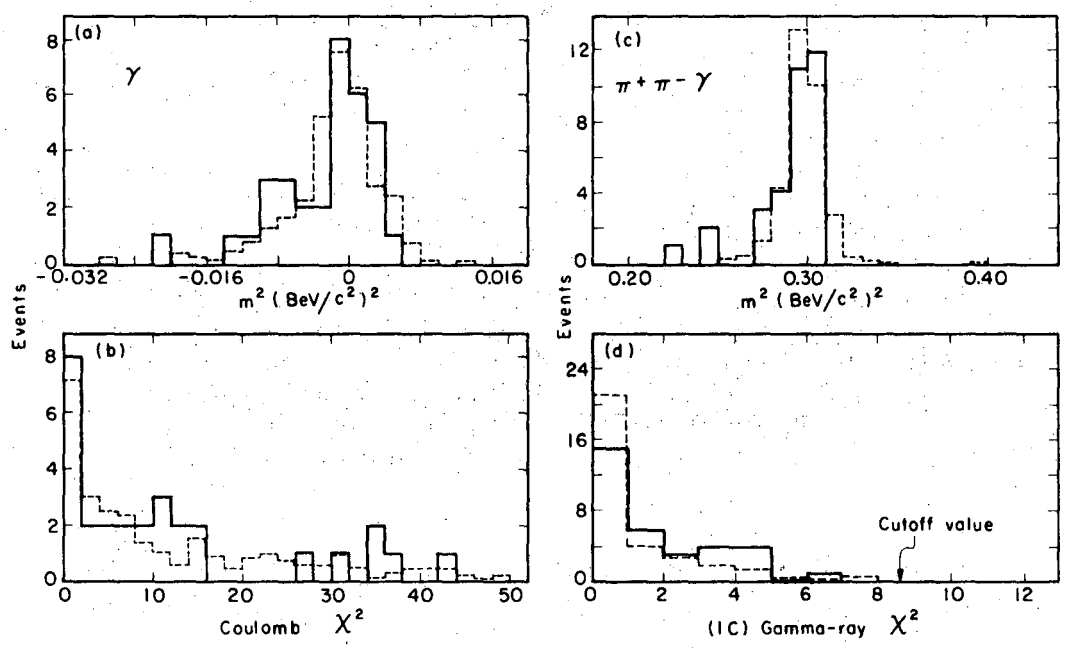

Fig. 2. Comparison of FAKE quantities with those of our final 33 events. There are 330 FAKE events. The areas under the FAKE histograms have been normalized to 33 counts. (a) The distribution in m^2 of the gamma ray in $\pi^+p \rightarrow \pi^+p\pi^+\pi^-\gamma$. (b) The (1C) Coulomb χ^2 distributions. (c) The distributions in m^2 of $\pi^+\pi^-\gamma$ [that π^+ is chosen which gives $m(\pi^+\pi^-\gamma)$ closest to the eta mass 548 MeV.] (d) The (1C) gamma-ray χ^2 distributions.

Using FAKE, we find that each of our cutoffs removes some of our events. The approximate percentages of the $\eta \to \pi^+\pi^-\gamma$ removed are listed in Table II: our detection efficiency for $\eta \to \pi^+\pi^-\gamma$ is about 80%.

Table II. The approximate percentages of $\eta \to \pi^+\pi^-\gamma$ events which are removed by each of our cutoffs (defined in Sec. III. B). These values were determined by using FAKE.

Criterion	Approx. % removed
1. 4C fit	4
2. $\pi^+\pi^-\pi^0$ production	7
3. $\pi^+\pi^-\gamma$ production	7
4. Coulomb scatter	2
5. (2C) gamma-ray fit	0

A plot of our detection efficiency vs the energy of the gamma ray in the laboratory system appears in Fig. 3(a). This particular parameter was not actually used in our analysis.

In Fig. 8 is plotted the detection efficiency vs the energy of the gamma ray in the eta frame. We used this parameter in comparing our data with the theoretical curves.

Figure 3(b) shows our detection efficiency vs $\cos\theta$, where θ is the angle between the π^+ and γ in the dipion c.m.

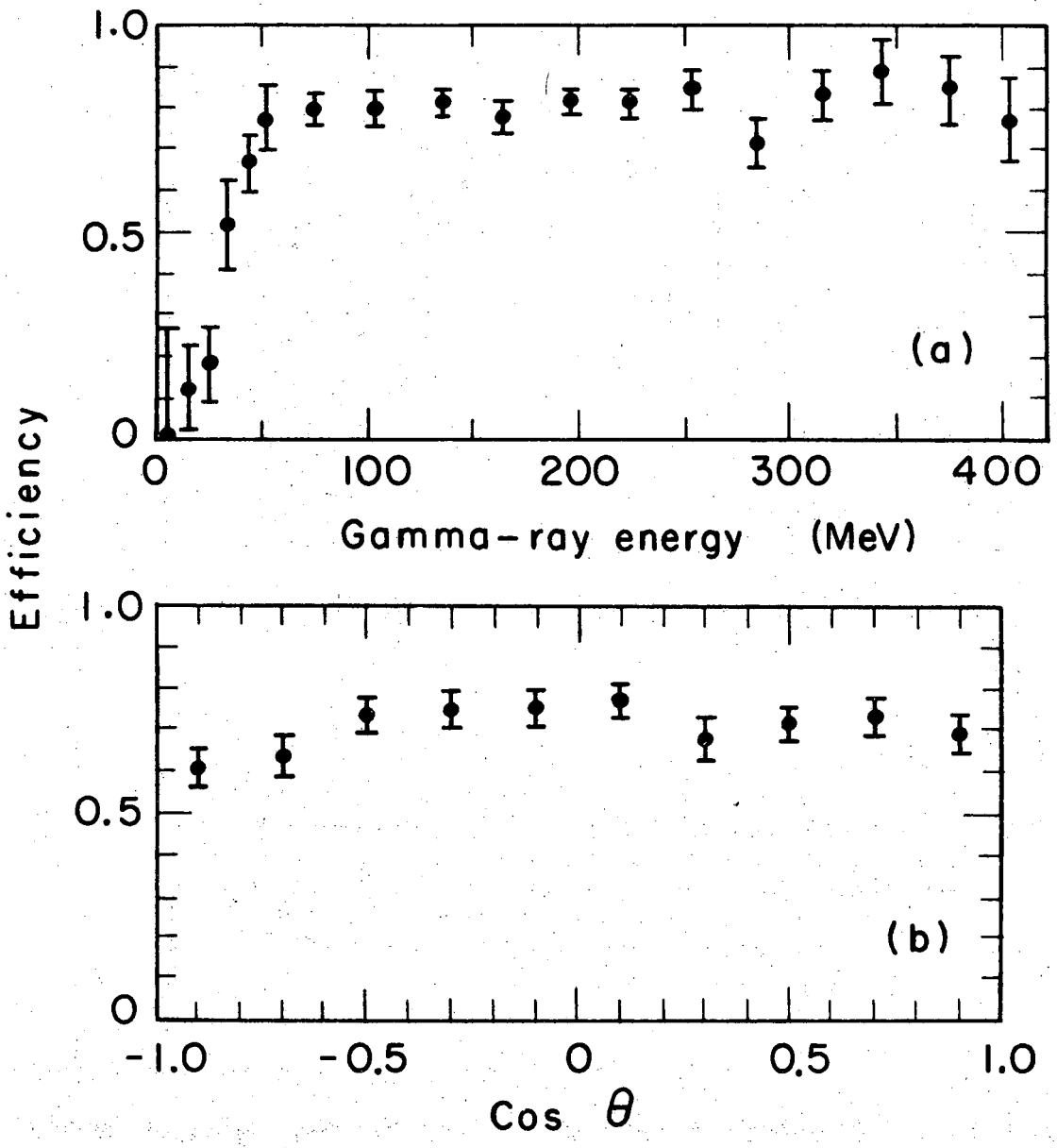

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Fig. 3. Detection efficiencies as determined from FAKE. (a) As a function of the gamma-ray energy in the laboratory system. The six data points at the left are for 10-MeV intervals, the remaining for 30-MeV intervals. (b) As a function of cos θ in the dipion c.m. All errors are purely statistical and depend only upon the number of FAKE events used.

B. The Cutoff Criteria

We now describe the cutoffs imposed on each of the initial 4,000 four-pronged events:

1. Four-Constraint (4C) Fit

If χ^2 for the reaction

$$\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \tag{2}$$

was less than 35, we rejected the event.[22]

2. $\pi^+\pi^-\pi^0$ Production

The events were fit (1C) to the reaction

$$\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \pi^0 \tag{3}$$

and removed if χ^2 was less than 7. Setting this cut at $\chi^2 = 7$ was somewhat arbitrary. This value was chosen in an effort to minimize the number of π^0 events retained, while at the same time not removing too many gamma-ray events. With this value of 7 we removed all but an estimated (according to FAKE) 2.2 events of type (3), while losing only 7% of our desired events of the type $\eta \rightarrow \pi^+\pi^-\gamma$.

3. $\pi^+\pi^-\gamma$ Production

If χ^2 for the fit (1C) to

$$\pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \gamma \tag{4}$$

was greater than 8.6 (Ref. 23), the event was discarded. We also demanded that this χ^2 be less than that for reaction (3).

4. Coulomb Scatter

It sometimes happens that one of the four charged tracks in the reaction (2) will undergo a small-angle Coulomb scatter. If these Coulomb scatters never fit reaction (4) we would not have to worry about

them. However, since we know (from FAKE) that the decays $\eta \to \pi^+\pi^-\gamma$ frequently have low χ^2's for Coulomb scatters, we also believe the reverse to be true.

In addition, it is reasonable that such an ambiguity should exist [i.e., that an event of the type shown in reaction (2) with a small-angle Coulomb scatter should fit reaction (4)], since a Coulomb scatter causes a small "kink" in a track. This kink then causes the momentum and energy to be slightly unbalanced at the vertex; if $|p|$ is about equal to the energy, then the event will surely fit gamma-ray production (since the "missing mass" at the vertex will then be about zero).

To get an idea of roughly how many Coulomb scatters there might be in our sample of 4000 four-prong events, we looked at the well-known Rutherford formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2mv^2}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} ,$$

which can be rewritten for small angles of scattering

$$\sigma \simeq 2\pi \left(\frac{e^2}{2p\beta}\right)^2 \int_{\theta_{min}}^{\theta_{max}} \frac{\theta d\theta}{\theta^4} = 16\pi \left(\frac{e^2}{p\beta}\right)^2 \left(\frac{1}{\theta_{max}} - \frac{1}{\theta_{min}}\right) .$$

Now we estimate that if a track Coulomb scatters less than 2°, it would probably not spoil the 4C fit (i.e., make the 4C $\chi^2 > 35.$). We further <u>estimate</u> that if a track scatters more than 8°, it would be noticed by the measurer and measured accordingly.

Thus we have

$$\sigma(\theta = 2°\text{-}8°) = 16\pi \left(\frac{e^4}{p\beta}\right)^2 (7.7\times10^2) .$$

Now the average $p\beta$ for the tracks from our four-pronged events is roughly 300 MeV/c, which gives

$$\sigma \approx 0.915 \times 10^{-26}\ \mathrm{cm}^2 .$$

This corresponds to a mean free path of

$$\lambda = \frac{1}{N\sigma} = \frac{1}{0.05 \times 6.22 \times 10^{23} \times 0.915 \times 10^{-26}} \approx 3 \times 10^{3}\ \mathrm{cm}.$$

Since the average length of a track in a typical four-pronged event is about 30 cm, the probability of one track Coulomb scattering between 2 and 8 degrees is

$$\frac{n_{scatt}}{n_0} = (1 - e^{-x/\lambda}) = 1 - e^{-0.01} = 0.01 .$$

In our 4000 events we have $4000 \times 4 = 16\,000$ tracks. Thus there could be of the order of a hundred four-prong events which are really good 4C's [i.e., reaction (2)] but have Coulomb scatters between 2 and 8 degrees. As these could completely swamp the desired gamma-ray events, they have to be removed.

The removal is done by deleting one of the four final tracks and fitting the remaining tracks (1C) to reaction (2). Events for which this χ^2 is less than the χ^2 for reaction (4) are removed, provided that $\psi p \beta$ is less than 35 rad-MeV/c, where ψ is the space angle between the fitted and measured momentum vectors of the deleted track,[15] and p is the fitted momentum of this track.

The reason for invoking this $\psi p \beta$ criterion is that FAKE tells us that we would lose about 20% (rather than only 2%) of our good decays $\eta \rightarrow \pi^+\pi^-\gamma$ if we were to cut off on the basis of the χ^2 criterion alone.

Now if $\psi p\beta = 35$, then for a typical track $\psi = \frac{35 \text{ rad.}}{300} \approx 7°$. Then from the formulas above,[24] we estimate that less than 0.1 real Coulomb scatter could have a $\psi p\beta$ as large as 35.

It is unfortunate that FAKE is unable to simulate Coulomb scatter events. Thus we are unable to process known Coulomb scatters thru our standard programs and to calculate the number of events remaining after the above criteria are imposed. Lacking such a technique, we estimate that there are no Coulomb scatter events in our final sample of 33 $\eta \to \pi^+\pi^-\gamma$ decays. This estimate is based on examining the Coulomb χ^2 and $\psi p\beta$ for each of our final events individually.

5. Scanning-Table Examination

The remaining events were examined on the scanning table for possible electrons misfit as pions. Two events involved Dalitz-pair electrons and were removed.

Finally, all remaining events were remeasured at least twice in order to remove any events that had survived merely due to measurement errors.

After the above steps, 38 good events of type (4) remained. These were then fit (2C) to

$$\pi^+ p \to \pi^+ p \eta, \quad \eta \to \pi^+\pi^-\gamma, \tag{5}$$

with an eta mass of 548 MeV. Their χ^2 distribution (Fig. 4) agreed well with the theoretical χ^2 (2C) distribution[27] to a χ^2 of about 20. Five events had large χ^2 (2C) and are believed to be type (4), but not from eta decay type (5). We took the 33 events with χ^2 (2C) less than 20 as our final sample. In Table III we give details of these 33 events.[25]

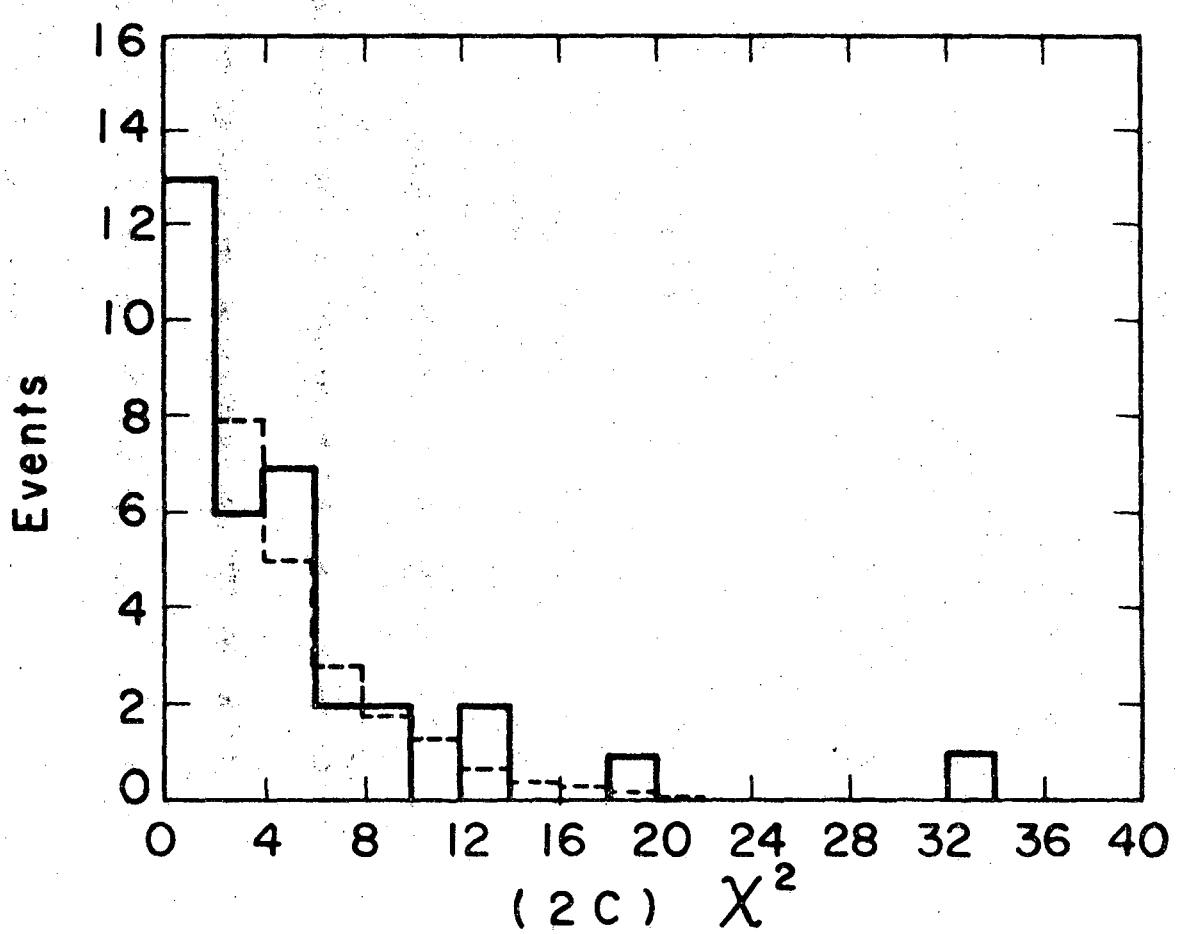

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Fig. 4. The (2C) χ^2 distribution of our $33_2 + 5$ events for reaction (7). Four events have χ^2's above 70 and are not plotted. The dashed lines represent the "theoretical" 2C χ^2 distribution normalized to 33 counts. The theoretical χ^2's have been multiplied by two before plotting (see footnote 22).

Table III. Details of 33 events. θ is the angle between the π^+ and γ in the dipion rest frame; p is the γ energy in the eta frame.

Event	Cos θ	p(MeV)	Event	Cos θ	p(MeV)
2153458	0.521	146.3	2180249	0.035	116.7
2159233	0.570	80.5	2183369	0.765	140.5
2159366	0.670	62.3	2184040	0.937	132.9
2162397	0.322	79.7	2195383	-0.434	123.1
2163095	-0.492	144.3	2196202	-0.149	107.8
2163288	0.288	117.4	2197247	0.176	193.5
2163466	-0.564	123.1	2197352	-0.444	176.0
2169380	-0.187	185.4	2198452	0.236	76.7
2172460	-0.387	63.3	2199275	-0.739	84.2
2175201	-0.750	99.1	2200242	-0.739	167.8
2175317	0.656	92.3	2202342	-0.167	103.4
2175441	0.229	69.5	2202417	0.251	77.0
2176531	-0.662	155.7	2202520	-0.269	147.1
2177176	0.197	112.7	2205066	-0.453	123.4
2177572	0.500	145.3	2208477	0.670	140.9
2179400	0.548	113.4	2211063	-0.273	166.8
2180190	-0.578	97.6			

From the same sample of 4000 four-pronged events, using a similar method of analysis,[26] we found 113 good events of the type

$$\pi^+ p \rightarrow \pi^+ p \quad , \quad \eta \rightarrow \pi^+ \pi^- \pi^0 \, . \tag{6}$$

To illustrate the clean separation between $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\gamma$ production, we show in Fig. 5(a) a plot of the unfit missing neutral mass (squared) recoiling against the $\pi^+ p \pi^+ \pi^-$ for our 33 + 5 + 113 events. To illustrate the lack of non-eta background for $\pi^+\pi^-\gamma$ production, we show in Fig. 5(b) a plot of $m^2(\pi^+\pi^-\gamma)$, using the final π^+ that gives a mass closest to the eta, for our 38 events of type (4). Figure 6 shows the same 38 events in a two-dimensional display. We see that our selection criteria based on χ^2 give essentially the same sample that we would obtain if we selected on the basis of missing neutral mass and on $m(\pi^+\pi^-\gamma)$. The χ^2 method carries less visual appeal than the mass plots, but has the advantages that it takes the measurement errors into account systematically, and that it is easier for one to calculate (using FAKE) the effects of cutoffs based on χ^2 than of cutoffs based on calculated errors in missing mass.[10]

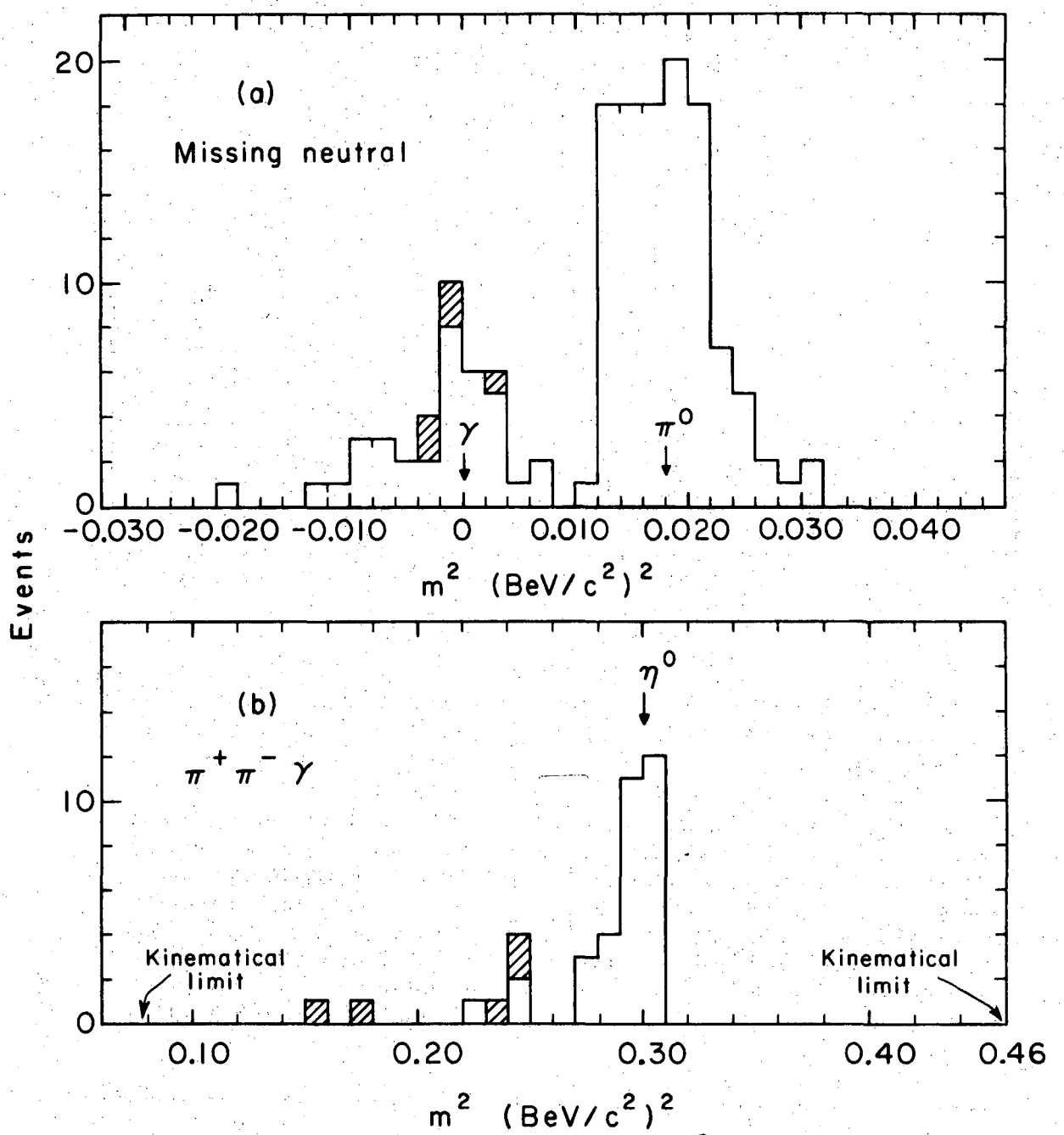

Fig. 5. Mass distributions. (a) Distribution in m^2 (mass-squared) of the missing neutral in $\pi^+p \to \pi^+p\pi^+\pi^- +$neutral. All events with m^2 less than 0.006 $(BeV/c^2)^2$ also happen to satisfy our $\chi^2(1C)$ criteria for selecting gamma rays. The five shaded gamma rays do not come from eta decay. (b) Distribution in m^2 of $\pi^+\pi^-\gamma$ for $\pi^+p \to \pi^+p\pi^+\pi^-\gamma$. (That π^+ is chosen which gives $m(\pi^+\pi^-\gamma)$ closest to the eta mass 548.) The five shaded gamma rays do not satisfy our $\chi^2(2C)$ criteria for $\eta \to \pi^+\pi^-\gamma$. The three "good-eta gamma rays" that lie outside the main eta peak do satisfy our χ^2 criteria and are used. (According to our FAKE calculation, the sample contains an estimated 2.2 spurious gammas arising from neutral pions with large measurement errors.)

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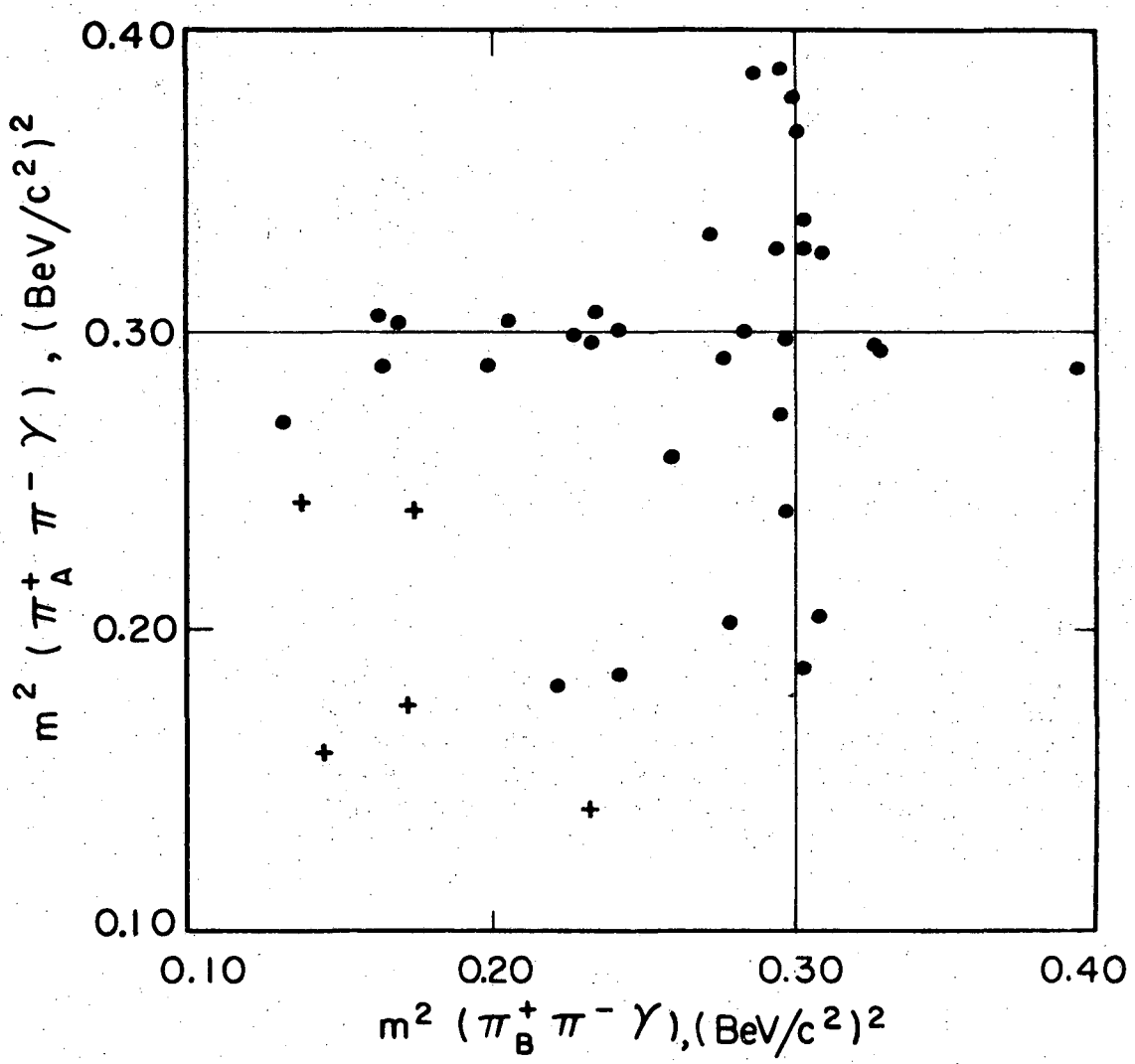

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Fig. 6. An eta "cross diagram." For each event two "eta" masses can be calculated (each corresponding to one of the two π^+'s in the final state). The thin lines are at the eta mass. Our 33 final events are indicated by dots. The five events of type (6), but not corresponding to eta decays, are represented by crosses.

IV. GAMMA-RAY CONVERSIONS

We now wish to establish that these gamma rays (which we have identified by a purely kinematical analysis) are perfectly ordinary, in the sense that they exhibit both "external" and "internal" electromagnetic conversion into e^+e^-, with roughly the expected probability. (We find one example of each.)

A. "External" Conversion

We re-examined each of our $\eta \to \pi^+\pi^-\gamma$ events on the scanning table. We looked along the predicted line of flight of the gamma ray for electron pairs (or triplets) produced in the liquid hydrogen via the external-conversion reactions

$$\gamma p \to p e^+ e^- \tag{7}$$

and

$$\gamma e^- \to e^- e^+ e^- . \tag{8}$$

We found one electron pair; it had excellent self-consistent kinematic fits to all stages of the sequence given by

$$\pi^+ p \to \pi^+ p \eta, \quad \eta \to \pi^+\pi^-\gamma . \tag{9}$$

and Eq. (7).[27]

For our events, the average potential gamma-ray path was about 30 cm, and the average laboratory gamma-ray energy was 144 MeV. The energy-weighted average cross section for materialization of pairs or triplets is 11.3 mb (Ref. 28). The average conversion efficiency is then 0.012, and the expected number of conversions is $32 \times 0.012 = 0.38$; the probability of getting at least one conversion is $1 - \exp(-0.38) = 0.32$.

B. "Internal" Conversion

We also systematically scanned our same sample (60 rolls) of film for six-pronged events. One such event gave an excellent fit[29] to the sequence

$$\pi^+ p \rightarrow \pi^+ p \eta \ , \qquad \eta \rightarrow \pi^+ \pi^- e^+ e^- \ . \tag{10}$$

The invariant mass of the electron pair was small (5 MeV). Thus the virtual gamma ray γ_V in $\eta \rightarrow \pi^+ \pi^- \gamma_V \rightarrow \pi^+ \pi^- e^+ e^-$ was "almost real." We expected the internal-conversion probability to be of order α, and since we had 38.0 corrected events of type (9) (as we shall see later), we expected to see $38 \times \alpha = 0.30$ events of type (10). The probability of getting at least one conversion was then $1 - \exp(-0.30) = 0.26$.

V. EXPERIMENTAL RESULTS

A. Angular Distribution

In Fig. 7 we plotted the folded angular distribution in $|\cos\theta|$, where θ is the angle between the π^+ and γ in the dipion c.m. system. From FAKE we found that our detection efficiency was essentially independent of $\cos\theta$ (footnote 30). Angular-momentum conservation and zero eta spin demand that the dipion have $J_z = \pm 1$ for z along the gamma-ray direction in the dipion frame. Thus for a pure dipion state J, the distribution in $\cos\theta$ is given by $|Y_J^{\pm 1}|^2$. Normalizing to 33 counts and calculating[31] χ^2 for curves corresponding to J = 1, 2, and 3, we find $\chi^2 = 4.5$ for J = 1, 49.4 for J = 2, and 102.0 for J = 3; in each case the "expected" χ^2 is 4. We conclude that the dipion is dominated by J = 1 and thus has the spin and isotopic spin of the rho meson.

B. Charge Asymmetry

It has recently been proposed that the decay $\eta \rightarrow \pi^+\pi^-\gamma$ might exhibit a "charge asymmetry" due to a possible C-noninvariant interaction; further, the magnitude of such an asymmetry has been estimated to be as large as 10%.[15]

Since the eta and gamma have C = +1 and -1, respectively, C conservation demands that $C(\pi^+\pi^-)$ equal -1. But $C(\pi^+\pi^-)$ equals $(-1)^J$. Thus C conservation demands the spin of the dipion to be odd. Likewise, C nonconservation would demand J to be even. Thus if there is a small amount of C-nonconserving amplitude, we may have some J = 2 (or other even value) amplitude present. The interference between the dominant J = 1 and any even J leads to odd powers of $\cos\theta$ in the angular distribution, or, equivalently, to a charge asymmetry $f_+ - f_- \neq 0$ in the pion-energy distribution in the eta rest frame. Since we have 17

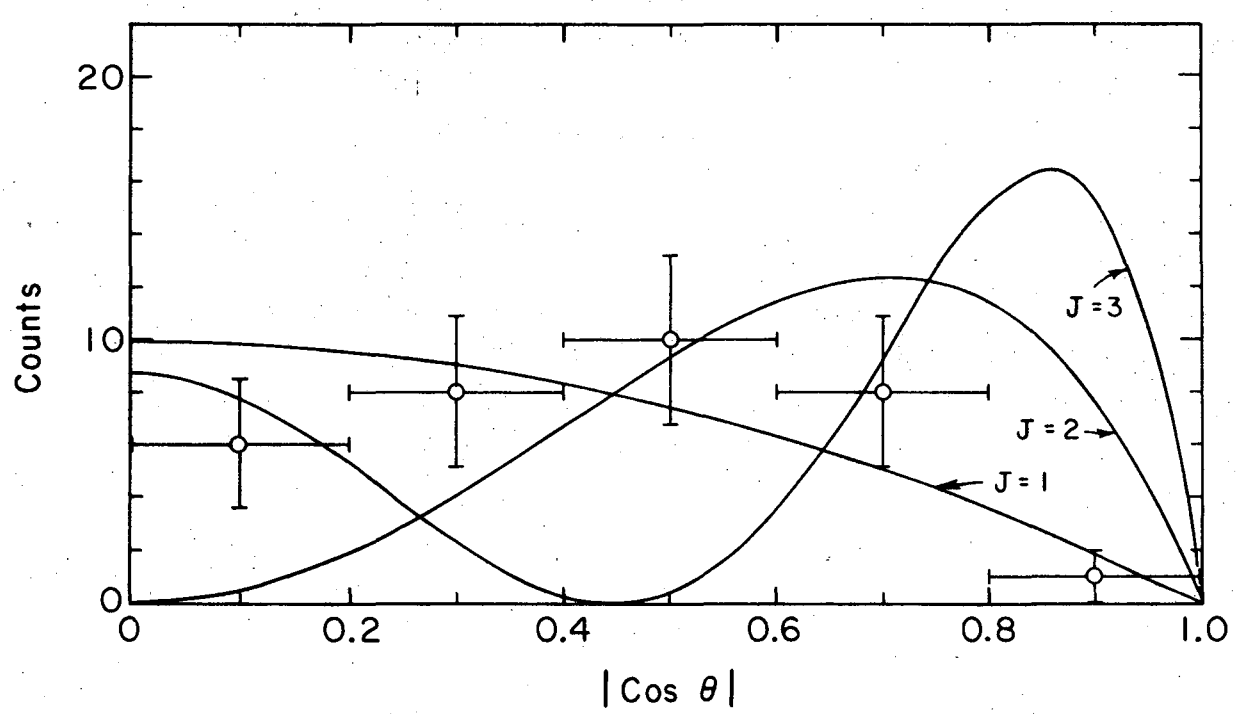

Fig. 7. Angular distribution for $\eta \rightarrow \pi^+\pi^-\gamma$. Here θ is the angle between the π^+ and the γ in the dipion c.m. rest frame. The three smooth curves correspond to J = 1, 2, and 3 for the dipion. (J = 0 is forbidden by angular-momentum conservation, since the eta spin is zero.) We see that J = 1 fits well, and J = 2 and 3 fit poorly.

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events with $\cos\theta > 0$ and 16 with $\cos\theta < 0$, the raw data give $f_+ - f_- = -(1/33) \pm 0.17$. Using FAKE, we find that the choice of the wrong π^+ in a small fraction of the events leads to a small spurious asymmetry.[32] Correcting for this, we find the result

$$R = 0.02 \pm 0.17.$$

Thus no conclusions concerning a charge asymmetry may be drawn from this experiment (except that it is probably not greater than 0.5).

We note that <u>if</u> there is a real asymmetry in the decay $\eta \to \pi^+\pi^-\gamma$, and <u>if</u> it is as large as the largest estimates (10%), then it would take an experiment of about 1600 $\eta \to \pi^+\pi^-\gamma$ events to establish it to four standard deviations.

C. Energy Spectrum of the Gamma Ray

Since the eta has spin 0 and the gamma has spin 1, the dipion in the decay

$$\eta \to \pi^+\pi^-\gamma$$

must also have spin.

Further, since $C_\eta = +$ and $C_\gamma = -$, C on the dipion must be negative, assuming that C is conserved in electromagnetic decays. Thus the dipion must have odd J.

1. Simplest Matrix Element

To construct the matrix elements that correspond to this state, we first list all the possible vectors in the problem: $\underset{\sim}{p}$, $\underset{\sim}{q}$, and $\underset{\sim}{\epsilon}$, where $\underset{\sim}{p}$ is the momentum of the gamma in the eta frame, $\underset{\sim}{q}$ the momentum of the π^+ in the dipion frame, and $\underset{\sim}{\epsilon}$ the "polarization vector" of the gamma.

Now if a 0^- (eta) goes to two pions, the matrix element must be a pseudoscalar. The simplest pseudoscalar that can be formed from three vectors is

$$M = \underset{\sim}{\epsilon} \cdot (\underset{\sim}{p} \times \underset{\sim}{q})$$
$$= pq \sin\theta \, (\underset{\sim}{\epsilon} \cdot \hat{n}) ,$$

where θ is the angle between the π^+ and the gamma in the dipion system, and $\hat{n}$ is a unit vector normal to both $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

Then $\sum_{pol} |M|^2 = p^2 q^2 \sin^2\theta \sum_{pol} |(\underset{\sim}{\epsilon} \cdot \hat{n})|.^2$

But $\sum_{pol} |\bar{\epsilon} \cdot \hat{n}|^2 = \pi$, since $\hat{n}$ always lies in the plane of $\underset{\sim}{\epsilon}$

Thus the "simplest" matrix element for the decay $\eta \to \pi^+\pi^-\gamma$ is (omitting the constant factor):

$$|M|^2 = p^2 q^2 \sin^2\theta . \tag{11}$$

2. <u>"Rho Dominant" Matrix Element</u>

If we now wish to say that the I = 1, J = 1 dipion phase shift in our region of interest (i.e., $m_{\pi\pi}$ from 380 to 550 MeV) is completely dominated by the rho meson, then (according to Jackson[33]) we replace the q^2 in Eq. (13) by the resonance factor

$$q^2 \to \frac{m_{\pi\pi}}{q} \frac{\Gamma}{(m_p^2 - m_{\pi\pi}^2) + m_p^2 \Gamma^2} , \tag{12}$$

where $\Gamma = (q^3/q_0^3)\gamma$, $m_{\pi\pi}$ is the dipion mass, m_p is the rho mass (765 MeV), q_0 is the value of q at resonance (357 MeV), and γ is the "reduced width" of the rho (124 MeV).

In Fig. 8 we plotted the distribution of the energy p of the gamma ray in the eta rest frame, for the 33 events. We have no explicit cutoff on gamma-ray energy. The detection efficiency $\epsilon(p)$ depends on the gamma energy and on our χ^2 cutoffs; it is calculated using FAKE and plotted in Fig. 8. We multiplied all theoretical curves by $\epsilon(p)$ before fitting them to the data.

Using the simplest matrix element, Eq. (11) and integrating over $\cos\theta$, we obtain

$$dN = C\,\epsilon(p)\,p^2 q^2\,\frac{pq}{m_{\pi\pi}}\,dp,$$

where C is a normalization constant, and dN is the number of counts expected in the interval dp, taking into account the detection efficiency. This nonresonant $J = 1$ curve is normalized to 33 events in Fig. 8 and gives $\chi^2 = 13.2$ with 6 degrees of freedom for a χ^2 probability of 4%, a rather poor fit.

Next we assume that the $I = 1$, $J = 1$ dipion phase shifts are dominated by the rho meson.[34] Using Eq. (12) we get the "rho-dominant" curve of Fig. 8. This curve is also normalized to 33 events and gives $\chi^2 = 5.9$ for a χ^2 probability of 40%, a good fit. Thus we lend some support to the rho-dominant model. (However, it is apparent from Fig. 8 that any other model that shifts the spectrum towards lower gamma-ray energies would also fit.)

The fact that we find evidence for final-state interactions in the dipion system lends encouragement to the possibility of (eventually) detecting the charge asymmetry, if a small amount of C-nonconserving amplitude is actually present; if there were no final-state interactions,

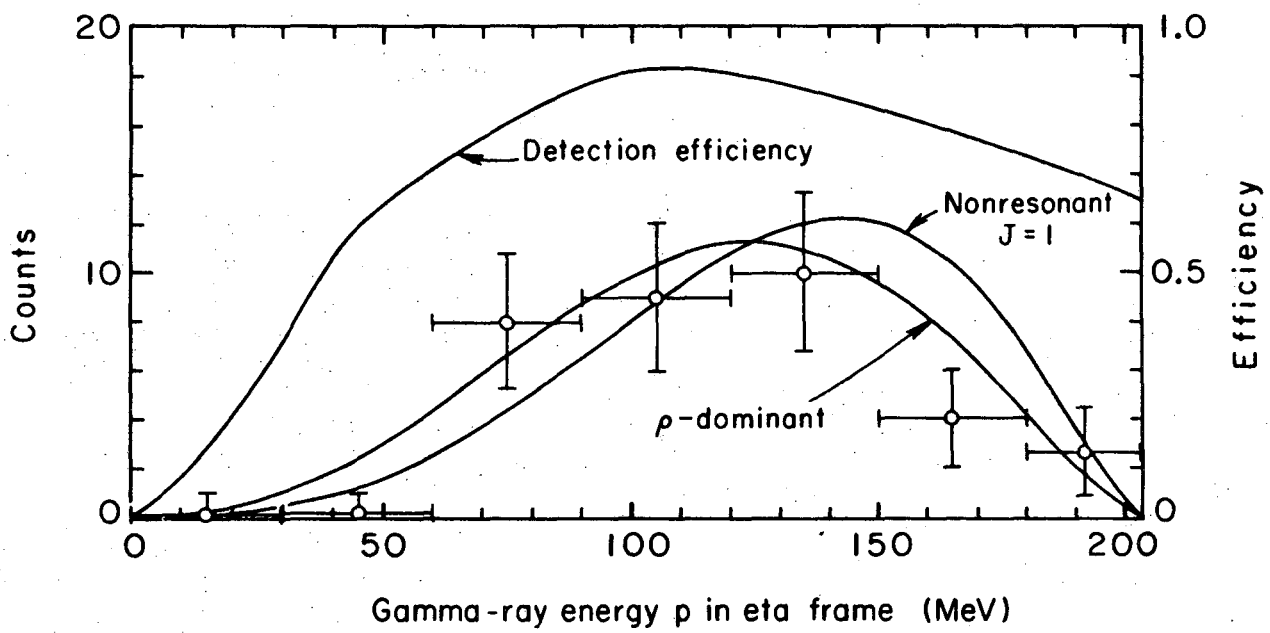

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Fig. 8. Energy distribution. The detection efficiency $\epsilon(p)$ is calculated using FAKE. The two theoretical curves correspond to a nonresonant J = 1 and to a rho-dominant J = 1 dipion. Neither curve has any free parameter except for a normalization constant. The curves are multiplied by $\epsilon(p)$ before plotting and comparing with the data. The rho-dominant model fits very well, the nonresonant model not so well.

the interference term would necessarily vanish (by CPT invariance) and there could then be no charge asymmetry even if C were not conserved.[35]

D. Branching Ratio

We next divide our 33 events by the average detection efficiency $\bar{\epsilon}$, where $\bar{\epsilon} \equiv [\ \epsilon(P)\ (dN/dp)dp]/[\ (dN/dp)dp]$ and the integral extends over the entire range p=0 to 203 MeV. If dN/dp is given by the rho-dominant curve, we find ϵ=0.81. The nonresonant spectrum (11) gives ϵ=0.820, and thus the same branching ratio R. Thus we calculate that our total corrected number of decays $\eta \to \pi^+\pi^-\gamma$ is 40.2±7.1. Using FAKE, we find that this includes 2.2±0.5 events of type (6) that were not removed by the cutoffs. All other corrections are negligible [for example, we estimate that our 33 events include less than 0.1 event of type (2)]. We also calculate from our 113 good events of type (6) a corrected number 128.0±11.3. We thus find R = (40.2 - 2.2)/128 = 0.30±0.06.

Now one of the mysteries of eta decay is why this ratio

$$R \equiv \frac{\Gamma(\pi^+\pi^-\gamma)}{\Gamma(\pi^+\pi^-\pi^0)}$$

is so small. From first principles one might expect $\pi^+\pi^-\gamma$ to be the dominant mode of decay since it goes as α rather than as α^2. Also the Q of the reaction is twice as large as it is in the three-pion mode, so that phase space is about 4 times as large, since for a three-body state, phase space is approximately proportional to Q^2.

Thus one might expect

$$R \approx \frac{4}{\alpha} \approx 500.$$

If, however, we look at the decay a little more closely, we see that the dipion must have spin, and in particular the simplest matrix element is

$$M = pq \sin\theta .$$

Now if we evaluate numerically the quantity

$$K = \frac{\iint p^2 q^2 \sin^2\theta \, \frac{pq}{m_{12}} \, dp \, d\cos\theta}{\iint \frac{pq}{m_{12}} \, dp \, d\cos\theta} ,$$

where the integrals run over the entire $\eta \to \pi^+\pi^-\gamma$ phase space, we find $K \approx 0.48$. Then in general

$$R \approx [K(L_{int} \cdot m_\pi)^4] \times 500,$$

where L_{int} is the "radius of interaction." Thus the effect of the matrix element is to reduce the expected ratio by about 0.5 (if we assume $L_{int} \sim \lambda\!\!\!^{-}_\pi$).

Thus we would now "expect"

$$R \sim 250.$$

But in reality (as we have seen) it is about 0.3!

By assuming the existence of an $I = 0$, $J = 0$ dipion (the σ^0), Brown and Singer[36] have been able to reduce the "expected" R to about the experimentally observed value. This is possible because the σ^0 cannot participate in the decay $\eta \to \pi^+\pi^-\gamma$ (since the σ^0 has $J = 0$), but it can contribute to the three-pion mode. In fact, it changes the three-pion mode from a three-body process to a two-body process (thus increasing the rate about a hundred fold).

According to a model recently proposed by Stevenson,[37] if the true radius of interaction were $L_{int} \approx \frac{1}{5m_\pi}$ (the length which corresponds to a typical vector particle), then we would have

$$R \approx 0.4,$$

which agrees rather well with the experimental value. This suggestion is reasonable in the light of current developments in Unitary Symmetry schemes.[38]

E. Low-Energy Gamma Rays

Pauli and Muller[14] found about as many $\eta \to \pi^+\pi^-\gamma$ decays with gamma energy between 10 and 60 MeV in the eta frame as they did between 60 and the maximum allowed value of 203 MeV; namely, they found about 12 events above estimated background in each of the two regions. If the "true" spectrum (corresponding to 100% detection efficiency) gave equal numbers of counts in these two regions, then, taking into account our detection efficiency $\epsilon(p)$ as plotted in Fig. 8, we would expect to find 20 counts below 60 MeV. But we found none. (For our best-fit curve in Fig. 8, we expected two counts below 60 MeV and found none; this is an entirely reasonable statistical fluctuation.) We conclude that there are no anomalous low-energy gamma rays.[39]

VI. SUMMARY

The results of this experiment can be summarized as follows:

1. The "rho-dominant" model fits both the dipion angular distribution and the gamma-ray energy distribution quite well.

2. The branching ratio

$$R = \Gamma(\eta \rightarrow \pi^+\pi^-\gamma)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$$

is found to be 0.30±0.06, which is consistent with previously reported values.

3. We find no evidence for an enhancement of gamma rays at low energy, as reported by Pauli and Muller.

4. No conclusions can be drawn from this experiment regarding the possible existence of a small-C nonconserving interaction.

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14. E. Pauli and A. Muller (Ref. 13) and E. Pauli, [University of Strasbourg, Ph. D. Thesis, 1965 (unpublished)] find an enhancement at gamma energies between 10 and 60 MeV in the eta frame. The enhancement arises from their "Class b" events, which are kinematically ambiguous with a zero-energy gamma ray, i. e., with no gamma ray at all. Their Class b peak at the eta-mass band amounts to about 2.7 standard deviations above their non-eta background.

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21. We start with about 1000 simulated events of type (5). The momentum distribution of the etas and of the other final-state particles is taken to be that observed in our larger sample of etas that decay via mode (6). The simulated events are analyzed by the same kinematics programs and subjected to the same cutoffs as those used for the real candidates.

22. Experience has shown that, for a correct hypothesis, the χ^2 distributions for 72-in. bubble chamber events agree with the theoretical distributions for the appropriate constraint class, provided the theoretical values of χ^2 are multiplied by 2. Thus the 4C cutoff at $\chi^2 = 35$ corresponds to a theoretical χ^2 of 17.5 (and thus a probability of about 0.3%).

23. The value 8.6 was chosen (arbitrarily) because it corresponded to a probability of about 4%.

24. Notice that ψ is a "fictitious" scattering angle. It is equal to the actual angle of scattering (θ_{true}) <u>only</u> if the scatter occurred at the vertex. For a given event

$$\psi \approx (1 - x/\ell)\,\theta_{true} ,$$

where ℓ is the length of the scattered track and x is the distance from the four-pronged vertex to the scatter.

25. For eight events, it happens that each of the two π^+'s in the final state had a (2C) gamma-ray χ^2 of less than 20. Such events are called "ambiguous pion" events, and the ambiguity is always "resolved" by choosing the interpretation which gives the lowest χ^2. We always neglect any biases which may be introduced by such a technique, except when looking at a possible charge asymmetry (see footnote 32). In addition, one event (2176531 Table III) is actually a six prong (see footnote 29).

26. We reject events with $\chi^2(4C) < 35$, and then demand $\chi^2(1C) < 8.6$ for reaction (3), $\chi^2(2C) < 6$ for reaction (6), and $\chi^2(2C)$ for (6) less than that for (5).

27. The fit to reaction (9) gives $\chi^2(2C) = 2.7$. The fit to the subsequent conversion (7) gives $\chi^2(1C) = 2.0$, with a transverse momentum transfer to the proton of 9 MeV/c.

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29. The proton, e^+, and e^- are unambiguously identified on the scanning table. The fit to $\pi^+ p \rightarrow \pi^+ p \eta$ gives $\chi^2(4C) = 3.2$. This event also gives a good fit to the external-conversion sequence given by Eqs. (9) and (7). However, the corresponding gamma-ray path has zero length (with an experimental upper limit of about 2 mm); we therefore discount the possibility of

external conversion. Further, since we consider the decay $\eta \rightarrow \pi^+\pi^-e^+e^-$ to be basically the same reaction as $\eta \rightarrow \pi^+\pi^-\gamma$, this event has been included in our final sample of 33 good events. Perhaps it should not have been used in the analyses of Section V. However, it does fit reaction (9) very well, and we thus feel that the internal conversion of the gamma ray probably does not materially affect the dipion quantities.

30. From Fig. 3(b) we see that the detection efficiency vs $|\cos\theta|$ would vary only about 12% from $|\cos\theta| = 0$ to $|\cos\theta| = 1$, and since this is only the order of the statistical fluctuations in the real data (Fig. 7), we neglect this small $|\cos\theta|$ dependence.

31. We define χ^2 as $\Sigma[(N_i - \overline{N}_i)^2]/\overline{N}_i$, where N_i is the observed number of counts in the ith bin, and $\overline{N}_i$ is the number expected from the smooth theoretical curve. The smooth curve is normalized to the observed 33 counts, and there are then no free parameters.

32. From FAKE we find that this asymmetry is $R = +0.01 \pm 0.02$, where the error is completely statistical and depends upon the number of FAKE events we generated.

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