UCRL-17063

University of California

Ernest O. Lawrence Radiation Laboratory

APPLICATION OF THE STARS PRODUCED IN A NUCLEAR EMULSION TO THE DETERMINATION OF A HIGH NEUTRON ENERGY SPECTRUM

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

Berkeley, California

UCRL-17063
UC-34 Physics
TID-4500 (49th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

APPLICATION OF THE STARS PRODUCED IN A NUCLEAR EMULSION TO THE DETERMINATION OF A HIGH NEUTRON ENERGY SPECTRUM

Ronald P. Omberg and H. Wade Patterson

February 1967

APPLICATION OF THE STARS PRODUCED IN A NUCLEAR EMULSION TO THE DETERMINATION OF A HIGH NEUTRON ENERGY SPECTRUM

Ronald P. Omberg and H. Wade Patterson

Lawrence Radiation Laboratory
University of California
Berkeley, California

February 1967

ABSTRACT

A method is presented for determining the relative shape of a high-energy neutron spectrum by utilizing the gray prongs produced in a nuclear emulsion. The method consists of assuming a flux shape that contains a single arbitrary parameter; this parameter is then determined by requiring the calculated number of gray prongs per star to agree with the value measured in the emulsion. The advantage of this method is that its accuracy is relatively independent of the magnitude of the energy range considered; this is because the product of the flux and the star production rate, if the cross section is assumed to be independent of energy, is relatively constant over a large energy range for fluxes that vary approximately as 1/E.

INTRODUCTION

The mean number of gray prongs per star produced at a given energy was assumed to be given by an extension to the relationship found experimentally by Remy.[1] The probability of producing a star per unit track length was assumed to be independent of energy. In particular, for these calculations, a flux shape of the form $1/E^n$ was assumed up to an arbitrarily specified junction energy, and a polynomial of the second degree was assumed from this junction energy to the maximum energy of the spectrum.

The calculated flux shapes were all physically reasonable, and the applicable ones agreed well with those presented by Smith,[2] who used the threshold detector method.

CALCULATIONAL MODEL

Given a nuclear emulsion containing N elements capable of producing stars, the number of stars produced per unit volume R will be

$$R = \sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \phi(E) \; \Sigma_i(E)\, dE,$$

where E_{min} is the minimum energy required to produce a star and $\Sigma_i(E)$ is the probability of producing a star per unit track length in element i, or the cross section for star production. If $A_i(E)$ is the mean number of gray prongs per star produced at energy E in element i, then the number of gray prongs produced per unit volume S will be

$$S = \sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \phi(E)\, A_i(E)\, \Sigma_i(E)\, dE.$$

The mean number of gray prongs per star $\bar{A}$ in the emulsion is then given by

$$\bar{A} = S/R,$$

$$\bar{A} = \frac{\sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \phi(E)\, A_i(E)\, \Sigma_i(E)\, dE}{\sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \phi(E)\, \Sigma_i(E)\, dE} \quad . \qquad (1)$$

Remy,[1] who investigated experimentally the mean number of gray prongs per star produced in a nuclear emulsion, used, in essence, a δ-function source, and hence measured

$$S = \sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \delta(E'-E)\, A_i(E')\, \Sigma_i(E')\, dE'$$

$$= \sum_{i=1}^{N} A_i(E)\, \Sigma_i(E)$$

and

$$R = \sum_{i=1}^{N} \int_{E_{min}}^{E_{max}} \delta(E'-E)\, \Sigma_i(E')\, dE'$$

$$= \sum_{i=1}^{N} \Sigma_i(E) \ ,$$

giving

$$\overline{A} = \frac{\sum_{i=1}^{N} A_1(E)\, \Sigma_i(E)}{\sum_{i=1}^{N} \Sigma_i(E)} \equiv A(E) \ , \tag{2}$$

where Eq. 2 is considered to be a definition of A(E). Now assuming that A(E) as a function of E is known for a δ-function flux, the problem is to find $\phi(E)$ for an arbitrary flux, if the mean number of gray prongs per star $\overline{A}$ produced by this flux is known. From Eq. 2 we have

$$\sum_{i=1}^{N} A_i(E)\, \Sigma_i(E) = A(E) \sum_{i=1}^{N} \Sigma_i(E),$$

and hence putting this into Eq. (1), after interchanging summation and integration, gives

$$\overline{A} = \frac{\int_{E_{min}}^{E_{max}} \phi(E)\, A(E) \left(\sum_{i=1}^{N} \Sigma_i(E) \right) dE}{\int_{E_{min}}^{E_{max}} \phi(E) \sum_{i=1}^{N} \left(\Sigma_i(E) \right) dE} \ .$$

If $\Sigma_i(E)$ can be considered independent of E for all i, then the cross sections can be taken out of the integral and canceled, and hence

$$\bar{A} = \frac{\int_{E_{min}}^{E_{max}} A(E)\,\phi(E)\,dE}{\int_{E_{min}}^{E_{max}} \phi(E)\,dE} \quad . \tag{3}$$

The problem now is to solve Eq. 3 for $\phi(E)$ with E_{min}, E_{max}, $A(E)$, and $\bar{A}$ known. This was done by assuming the flux to be a function of a single parameter only and then solving Eq. 3 for this parameter. In particular, it was assumed

$$\phi(E) = \frac{1}{E^n} \quad , \quad \text{for } E_{min} \leq E \leq E_c, \tag{4}$$

$$\phi(E) = \alpha + \beta E + \gamma E^2, \text{ for } E_c \leq E \leq E_{max}, \tag{5}$$

with E_c arbitrary and with the conditions

$$\phi(E_c) \text{ continuous}, \tag{6}$$

$$\frac{d\phi}{dE}(E_c) \text{ continuous}, \tag{7}$$

$$\phi(E_{max}) = 0. \tag{8}$$

Conditions 6, 7, and 8 determine the coefficients α, β, and γ; the parameter n was then determined by solving Eq. 3 with Eqs. 4 and 5. The computation of n was carried out on a machine, since $A(E)$ is not easily described analytically. The machine program is shown in the Appendix.

A plot of $A(E)$ as a function of E is shown in Fig. 1. The values from 20 to 300 MeV are due to Remy; above 300 MeV no experimental data were available and consequently the curve was extrapolated. This was done by making a rather rough estimate; the number of gray prongs per star at 1.8 BeV was assumed to be equal to the total number of cascade protons and π^+ and π^- mesons--i.e., the total number of charged particles, predicted by Metropolis et al.[3] for proton bombardment of copper. This number is 4.0. At 10 BeV, the number of gray prongs per star was assumed equal to the predicted number of cascade protons;[4] this number is 7 to 25. Below 20 MeV gray prong production was arbitrarily taken to be zero. Numerical values for the solid line drawn in Fig. 1 are shown in Table I. This line was used in the calculations; the dashed line in Fig. 1 is discussed later.

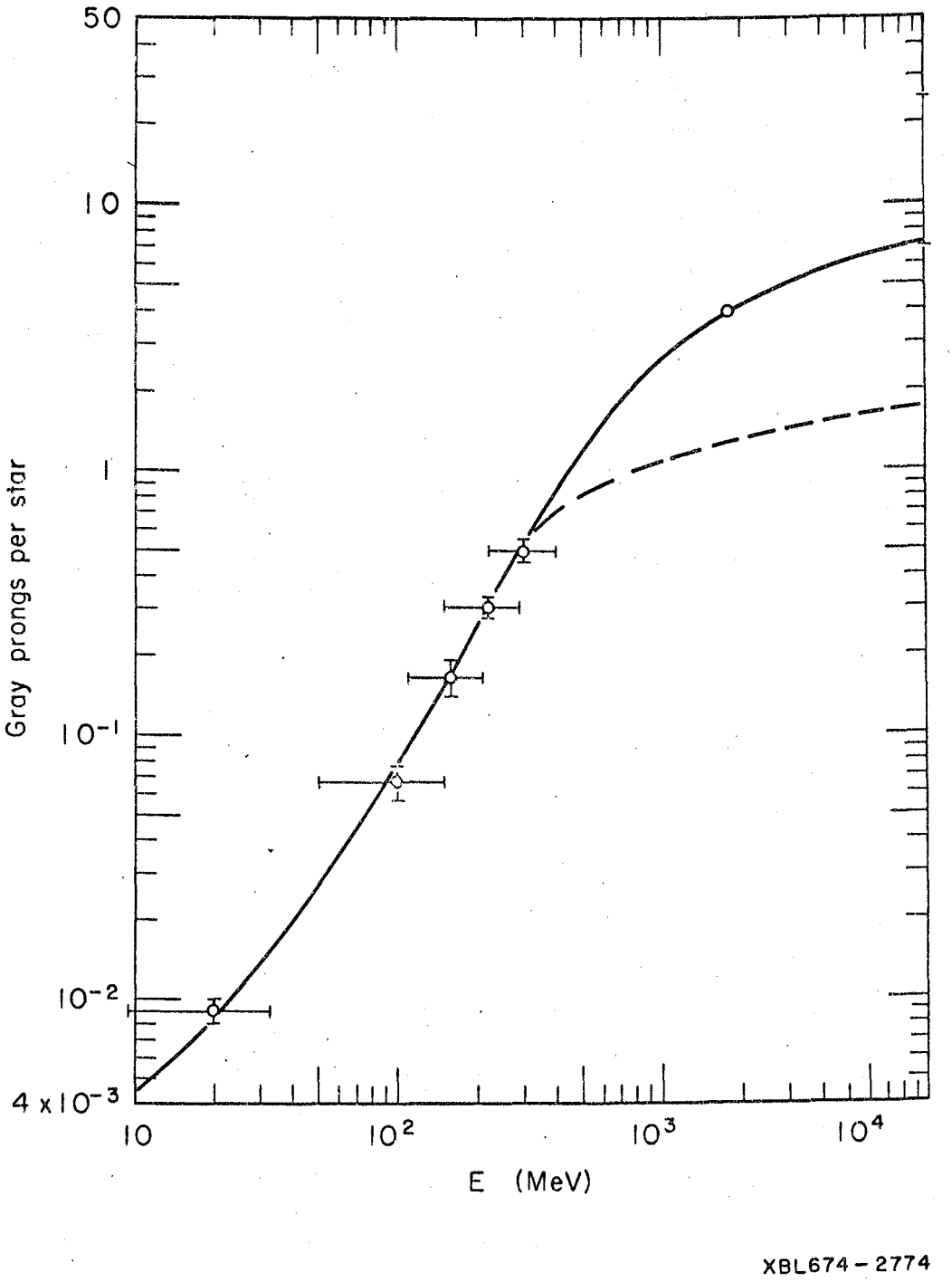


Fig. 1. Gray prongs per star vs neutron energy.

Table I. Gray prongs per star as a function of neutron energy.

E	A(E)
20	0.0085
40	0.020
70	0.044
100	0.075
200	0.25
500	1.15
700	1.85
1 000	2.6
1 500	3.5
2 000	4.1
3 000	5.0
5 000	6.0
10 000	7.2

RESULTS

Prior to this work nuclear emulsions were irradiated in six different neutron fields by the Health Physics Department. The results are shown in Table II. Since this calculation requires a known maximum energy, the maximum energy of the cosmix-ray neutron spectrum was arbitrarily taken to be 10 000 MeV. In addition, the computation requires a value for E_{min} and a value

Table II. Nuclear emulsion results.

Location	Number of gray prongs per star	E_{max} (MeV)
Bevatron WTT shielding wall	0.500	6 200
Bevatron Col. 7, main floor	0.321	6 200
Bevatron mezzanine	0.272	6 200
184-inch cyclotron between Bay 10 and 11	0.442	730
White Mountain, 12 000 ft	1.071	10 000
White Mountain, 14 000 ft	1.038	10 000

for the junction energy E_c. E_{min} was arbitrarily taken to be 20 MeV and E_c was arbitrarily chosen as 2/3 E_{max} for the accelerators. For the cosmic ray spectrum E_c was taken equal to E_{max}, i.e., the flux was considered to vary as $1/E^n$ right up to the assumed maximum energy.

The results as computed by the machine program, which solves Eqs. and conditions 3 through 8, are shown in Tables III and IV. Important to note is that this type of calculation cannot compute the absolute magnitude of the flux, but only the relative shape. Hence, the flux values in Table IV are meaningful only relative to other values in the same column. The spectra are also shown graphically in Fig. 2. The Bevatron spectrum from Ref. 2 was obtained by the threshold detector method at the WTT shielding wall. The agreement between this spectrum and the WTT shielding wall spectrum obtained by the star method is relatively good. The difference in the slopes at any point is small and the slope of the star spectrum appears to approximate the mean slope of the threshold detector spectrum quite closely. The other two Bevatron spectra, for which no measurements by an independent method were available, do not radically depart from the WTT shielding wall spectra and hence do not appear to be physically unreasonable.

Table III. Computed flux shape parameters.

Location	n	α^a	β^a	γ^a	E_c (MeV)
Bevatron WTT shielding wall	1.56	3.8×10^{-6}	0.24	-1.24	4140
Bevatron Col. 7, main floor	1.73	1.2×10^{-6}	-0.42	-0.58	4140
Bevatron mezzanine	1.80	7.5×10^{-7}	-0.59	-0.41	4140
184-inch cyclotron between Bay 10 and 11	0.64	-0.02	-6.60	5.60	487
White Mountain, 12 000 ft	1.32	-	-	-	-
White Mountain, 14 000 ft	1.33	-	-	-	-

a - The coefficients α, β, and γ are not identical to the ones in Eq. 5; this equation was modified to $\phi(E) = \alpha(1 + \beta(E/E_{max}) + \gamma(E/E_{max})^2)$ in the program. The coefficients in the table are for the latter equation. The reason for the modification is that the numbers in the latter equation are easier to handle.

The cosmic ray spectrum was obtained by measurements on White Mountain and on high-altitude flights, using the threshold detector method. It can be seen that the two White Mountain spectra obtained by the star method agree relatively well with this spectrum, i.e., the relative slopes do not differ to a large extent.

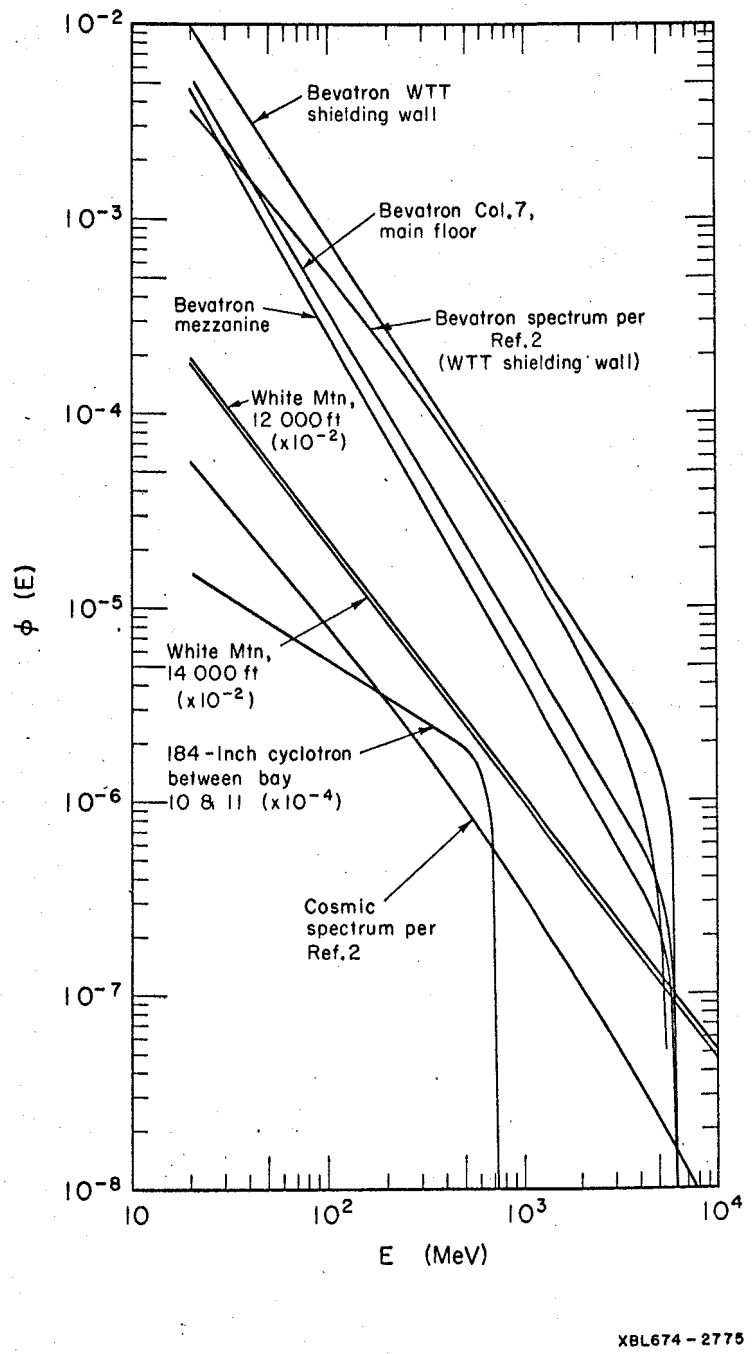


Fig. 2. Neutron flux shapes.

Table IV. Computed flux values.

Bevatron WTT shielding wall		Bevatron Col. 7 main floor	Bevatron mezzanine	White Mountain		184-inch cyclotron between Bay 10 and 11	
				12 000 ft	14 000 ft		
E	ϕ	ϕ	ϕ	ϕ		E	ϕ
20	9.4×10^{-3}	5.6×10^{-3}	4.6×10^{-3}	1.9×10^{-2}	1.8×10^{-2}	20	1.5×10^{-1}
850	2.8×10^{-5}	8.6×10^{-6}	5.5×10^{-6}			115	4.9×10^{-2}
1 650	9.6×10^{-6}	2.6×10^{-6}	1.6×10^{-6}			210	3.3×10^{-2}
2 000				4.3×10^{-5}	3.9×10^{-5}	300	2.6×10^{-2}
2 500	5.1×10^{-6}	1.3×10^{-6}	7.9×10^{-7}			400	2.2×10^{-2}
3 300	3.3×10^{-6}	8.0×10^{-7}	4.7×10^{-7}			490	1.9×10^{-2}
4 000				1.7×10^{-5}	1.5×10^{-5}	540	1.7×10^{-2}
4 140	2.3×10^{-6}	5.5×10^{-7}	3.2×10^{-7}			590	1.5×10^{-2}
4 550	1.9×10^{-6}	4.5×10^{-7}	2.6×10^{-7}			630	1.1×10^{-2}
5 000	1.5×10^{-6}	3.5×10^{-7}	2.0×10^{-7}			680	5.9×10^{-3}
5 400	1.1×10^{-6}	2.4×10^{-7}	1.4×10^{-7}			730	0
5 800	5.5×10^{-7}	1.2×10^{-7}	6.9×10^{-8}				
6 000				1.0×10^{-5}	9.0×10^{-6}		
6 200	0	0	0				
8 000				6.9×10^{-6}	6.2×10^{-6}		
10 000				5.1×10^{-6}	4.6×10^{-6}		

Finally, the spectrum at the 184-inch cyclotron was computed; no data were available, however, for a comparison. In any case, the shape does not appear unreasonable.

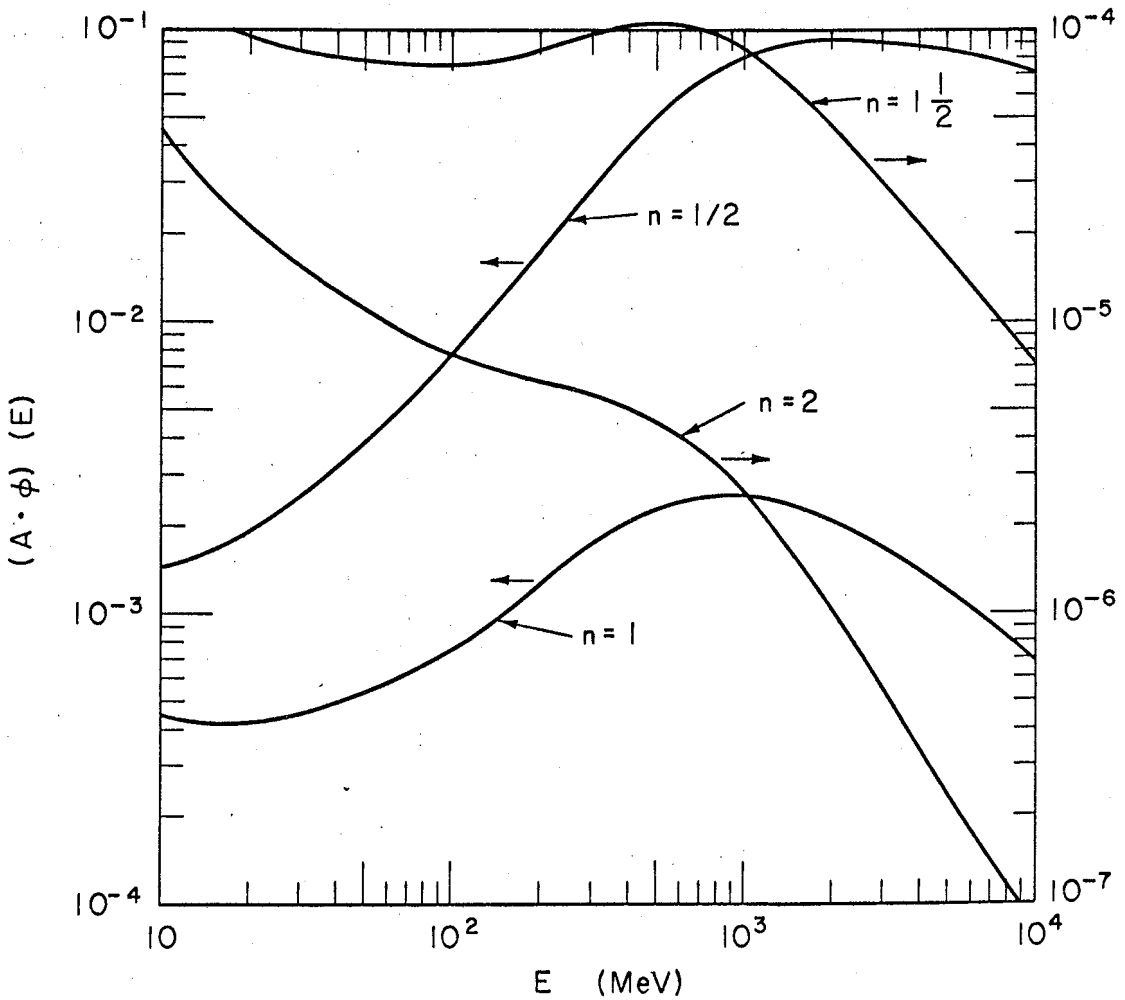
Figure 3 shows the product of $\phi(E)$ and $A(E)$ as a function of energy with $\phi(E)$ varying as $1/E^n$ for several n. On the assumption, as before, that the cross section for star production is independent of energy, the significance of this figure is that the star production rate per unit energy interval does not vary more than an order of magnitude for fluxes with n in the vicinity of 1. For n less than 1, the production rate increases with increasing energy. Only for n approaching 2 does the production rate decrease by significantly more than an order of magnitude. This independence of the star production rate on energy should make this method relatively insensitive to the energy range considered.

The dashed line in Fig. 1 indicates a later change made in the gray-prong-per-star relationship, so that the curve fits an experimental data point indicating 1.1 gray prongs per star at 1.1 MeV. Further investigation showed, however, that the actual energy was uncertain and hence the point was not useful in this analysis. Nevertheless, the results of perturbing the gray-prong-per-star relationship were interesting. No agreement was obtained between these perturbed results and either the previous ones from the gray-prong method or the threshold detector method. The results of this perturbation are shown in Table V.

Table V. Computed flux shape parameters for the perturbed gray-prong-per-star curve.

Location	n	α	β	γ	E_c (MeV)
Bevatron WTT shielding wall	1.16	2.9×10^{-5}	7.5	-8.4	4140
Bevatron Col. 7, main floor	1.40	9.7×10^{-6}	1.2	-2.3	4140
Bevatron mezzanine	1.49	5.8×10^{-6}	0.6	-1.6	4140
184-inch cyclotron, between Bay 10 and 11	0.29	-0.04	-4.6	3.6	487
White Mountain, 12 000 ft	0.67	-	-	-	-
White Mountain, 14 000 ft	0.71	-	-	-	-

Since the parameter α is in fact the slope of the linear portion in Fig. 2, the data in Table V represent an average change of 21% in the slope of the Bevatron spectra and 47% in the White Mountain spectra. A change of this much would cause a noticeable discrepancy between the star method and the threshold detector method for the Bevatron and a considerable difference for



XBL674 – 2776

Fig. 3. Star production rate per unit energy interval (constant cross section assumed).

White Mountain. In fact for White Mountain there would be no agreement at all. Given the assumptions basic to this work, this indicates that the extension of the gray-prong-per-star curve is a reasonable estimate of the actual curve shape.

CONCLUSIONS

Gray-prong production in a nuclear emulsion was used to reconstruct spectra which can be reasonably well described by a single parameter, and showed good agreement with the flux shapes obtained independently by the threshold detector method. For those cases in which no comparisons were available, the computed flux shapes do not appear physically unreasonable.

ACKNOWLEDGMENTS

We would like to thank Dr. Roger Wallace for suggesting this problem and for his assistance.

Work done under the auspices of the U. S. Atomic Energy Commission.

APPENDIX

Description of the machine program
(The basic structure of this program was written by Kenneth G. Wiley.)

Card No.	Input Required	
	Data	Format
1	MAXFUN, X, XSTEP, ABSACC, RELAC	I10, 2F10.5, 2F20.5
2	TITLE	8A10
3	AN, EMAX, EMIN	3F20.5
4	EJCT	F20.5
5	NUTBLE	L5
6	N	I10
7	AZ(I), BZ(I)	2F20.5
.	.	.
.	.	.
.	.	.
6 + N	.	.
6 + N + 1	ENDATA	F20.5

NOMENCLATURE

MAXFUN: The maximum number of times the program is allowed to iterate in attempting to find the exponent of E.

X: The arbitrary parameter in the flux shape, i.e., the exponent of E.

XSTEP: The amount by which the program should vary X on the first iteration; after the first iteration, the program determines its own variation.

ABSACC: The allowable difference between the measured gray-prong-per-star value and the computed value.

RELAC: The relative allowable difference, i.e., ABSACC divided by the measured gray-prong-per-star value.

TITLE: Title of the individual case being run.

AN: Measured number of gray prongs per star.

EMAX: The maximum energy of the spectrum.

EMIN: The minimum energy of the spectrum.

EJCT: The junction energy between the $1/E^X$ representation of the flux and the polynomial representation.

NUTBLE: A logical variable which must be true so that the table for gray prongs per star versus energy can be read in; it must be false otherwise.

N: The number of ordered pairs in the table of gray prongs per star versus energy.

AZ(I): Energy in the table of gray prongs per star versus energy.

BZ(I): Gray prongs per star in the table of gray prongs per star versus energy.

ENDATA: If ENDATA is zero, the program reads another case; if it is greater than zero, the program makes a normal exit.

Note that if NUTBLE is false, then cards 6 through 6 + N must not be inserted.

THE MACHINE PROGRAM

```
      PROGRAM STAR(INPUT,OUTPUT)
      COMMON /MD/ EMAX, EMIN, E(100), EN(100), N, Y, EJCT
      COMMON /PQ/ ALPHA, BETA, GAMMA
      DIMENSION U(100), TITLE(8), TS(8), AZ(100), BZ(100)
      LOGICAL SWPR, NUTBLE
      DOUBLE PRECISION TS,HPRINT
      READ 1000,MAXFUN,X,XSTEP,ABSACC,RELAC
5000  READ 5001,TITLE
5001  FORMAT(8A10)
      PRINT 1015
      PRINT 5002,TITLE
5002  FORMAT(1H ,8A10)
1015  FORMAT(1H1///////////)
      IFLAG=0
C
C     IFLAG DENOTES WHEN CONVERGENCE HAS BEEN ATTAINED AND THEN BEGINS
C     PRINTOUT
C
      NCALL=0
C
C     NCALL = NUMBER OF TIMES LINMIN HAS BEEN CALLED
C
C
C     MAXFUN = MAXIMUM NUMBER OF ITERATIONS MADE IN FINDING THE
C     EXPONENT OF E
C     X = INITIAL GUESS AT THE EXPONENT OF E
C     XSTEP = STEP VALUE FROM INITIAL GUESS POINT TO BE MADE IN
C     DETERMINING THE SLOPE
C     ABSACC = ABSOLUTE CONVERGENCE CRITERIA
C     RELAC = RELATIVE CONVERGENCE CRITERIA
C
1000  FORMAT(I10,2F10.5,2F20.5)
      READ 1001,AN,EMAX,EMIN
C
C     AN = MEASURED NUMBER OF GRAY PRONGS PER STAR
C     EMAX = MAXIMUM ENERGY OF SPECTRUM GIVING MEASURED VALUE
C     EMIN = MINIMUM ENERGY OF SPECTRUM GIVING MEASURED VALUE
C
1001  FORMAT(3F20.5)
      READ 1001, EJCT
      R = EJCT / EMAX
      READ 1003, NUTBLE
1003  FORMAT(L5)
      IF ( .NOT. NUTBLE) GO TO 1025
      READ 1002, N
      NSTOR = N
C
C     N = NUMBER OF PIECEWISE ANALYTICAL EXPRESSIONS OR NUMBER OF
C     ORDERED PAIRS IN TABLE
C
1002  FORMAT(2I10)
      DO 1010 I=1,N
1010  READ 1001, AZ(I), BZ(I)
1025  CONTINUE
```

```
      READ 1001, ENDATA
      PRINT 1030,X,XSTEP,ABSACC,RELAC,MAXFUN
1030  FORMAT(15H0INITIAL VALUE=,F10.5/14H INITIAL STEP=,F10.5/
     *20H ABS CONV CRITERION=,F15.13/
     $20H REL CONV CRITERION=,F15.13/8H MAXFUN=,I10)
      PRINT 1031,AN,EMAX,EMIN
1031  FORMAT(27H0MEASURED GRAY PRONGS/STAR=,F15.9,
     1//6H EMAX=,F15.9,
     2//6H EMIN=,F15.9)
      N = NSTOR
1041  FORMAT(4F15.5)
1060  CONTINUE
1005  FORMAT(1H0, 5HN1 = , I3, 5X, 5HN2 = , I3, 5X, 5HN3 = , I3)
      PRINT 1009, EJCT
1009  FORMAT(8H0EJCT = , F15.9)
      CALL SORT(N1, N2, N3, AZ)
      PRINT 1005, N1, N2, N3
      PRINT 1015
      PRINT 1061
1061  FORMAT(31H      ENERGY     GRAY PRONGS/STAR)
      DO 1065 I=1,N
1065  PRINT 1066, AZ(I), BZ(I)
C
C     AZ(I)= ENERGY IN THE TABLE OF ORDERED PAIRS
C     BZ(I) = GRAY PRONGS PER STAR IN THE TABLE OF ORDERED PAIRS
C
      CALL POINT(N1, N2, N3, AZ, BZ)
1066  FORMAT(2F13.6)
1095  CONTINUE
      PRINT 5002
      DO 1069 I = 1,N
      PRINT 1066, E(I), EN(I)
      E(I) = ALOG(EMAX / E(I))
1069  EN(I) = ALOG(EN(I))
C
C     E(I) IS NOW LETHARGY AND EN(I) IS ON A LOG SCALE
C
1070  CONTINUE
      ITEST=2
C
C     IF CONVERGENCE CRITERIA IS NOT SATISFIED LINMIN SETS ITEST = 1,
C     IF IT IS SATISFIED IT SETS ITEST = 2, IF CONVERGENCE IS NOT BEING
C     OBTAINED IT SETS ITEST = 3, AND IF MAXIMUN NUMBER OF CALLS IS
C     EXCEEDED IT SETS ITEST = 4
C
500   CALL LINMIN(ITEST,X,F,MAXFUN,ABSACC,RELAC,XSTEP)
      NCALL=NCALL+1
      GO TO (1,2,3,4),ITEST
1     Y=1.-X
      IF (EJCT .EQ. EMAX) GO TO 3750
      CALL COMPT(X, R, EJCT)
3750  CONTINUE
C
C     X = CALCULATED VALUE OF THE EXPONENT OF E
C     CALCULATE THE DENOMINATOR B
C
      IF(ABS(Y).LT..001) GO TO 5
C
C     IF X IS CLOSE TO UNITY GO TO 5
```

```
C       CALCULATE B DIRECTLY FOR X NOT NEAR UNITY
C
        B = (EJCT**Y - EMIN**Y) / Y
        GO TO 10
C
C       USE A SERIES EXPANSION TO OBTAIN B FOR X NEAR UNITY
C
5       G = ALOG(EJCT)
        H=ALOG(EMIN)
        B=1.+Y*(G+H)*.5+Y*Y*(G*G+G*H+H*H)/6.
        B=B+Y**3*(G**3+G*G*H+G*H*H+H**3)/24.
        B=B+Y**4*(G**4+G**3*H+G*G*H*H+G*H**3+H**4)/120.
        B=(G-H)*B
10      CONTINUE
C
C       ADD CONTRIBUTION FROM PARABOLA
C
        IF (EJCT .EQ. EMAX) GO TO 4010
        TOP = ALPHA * EMAX * (1.0 + BETA / 2.0 + GAMMA / 3.0)
        BTM = ALPHA * EJCT * (1.0 + BETA * R / 2.0 + GAMMA * R**2 / 3.0)
        B = B + TOP - BTM
4010    CONTINUE
200     AA=0.
C
C       NUMERATOR CALCULATION FOR A TABLE OF ORDERED PAIRS
C       N = NUMBER OF ORDERED PAIRS IN GRAY PRONGS PER STAR VERSUS
C       ENERGY TABLE
C
        DO 230 I=2,N
        J=N-I+1
C
C       J RUNS BACKWARD FROM N-1 TO 1
C
        T=E(J+1)
C
C       T TAKES ON EACH LETHARGY VALUE IN TURN WITH THE EXCEPTION OF E(1)
C       CORRESPONDING TO THE HIGHEST ENERGY OR THE LOWEST LETHARGY
C
        A=0.
        HPRINT=E(J)-E(J+1)
C
C       HPRINT IS THE LETHARGY DIFFERENCE BETWEEN THE LETHARGY VALUE
C       BEING CONSIDERED AND ITS NEXT HIGHEST VALUE, HPRINT COVERS EACH
C       INTERVAL BETWEEN E(1) AND E(N)
C       CALL INTO GIVING IT A LETHARGY VALUE AND A LETHARGY DIFFERENCE,
C       THE LETHARGY VALUE T IS AT THE BOTTOM OF THE INTERVAL BEING
C       CONSIDERED, INTO THEN RETURNS THE VALUE OF THE NUMERATOR INTEGRAND
C       DER, ALSO TS A DOUBLE PRECISION VALUE OF A
C
        CALL INTO(T,A,DER,TS,HPRINT)
        SWPR=.FALSE.
210     IF(SWPR) GO TO 220
        CALL INTS(T,A,DER,TS,SWPR)
C
C       INTS INTEGRATES DER OVER THE HPRINT INTERVAL
C
        GO TO 210
220     CONTINUE
C
```

```
C       A IS THE VALUE OF THE NUMERATOR INTEGRAL OVER THE HPRINT INTERVAL
C       AA IS THE VALUE OF THE INTEGRAL FROM E(N) TO E(N-I+1)
C
230     AA=AA+A
        A=AA
C
C       A IS NOW THE VALUE OF THE NUMERATOR INTEGRAL FROM E(N) TO E(1)
C
C       COMPARISION OF CALCULATED VALUE WITH MEASURED VALUE NOW BEGINS
C
300     F=AN-A/B
        F=F*F
        IF(IFLAG.EQ.1) GO TO 600
        GO TO 500
2       IFLAG=1
        GO TO 1
600     ANS=A/B
C
C       ANS = COMPUTED NUMBER OF GRAY PRONGS PER STAR
C
        IF (ITEST .EQ. 2) PRINT 1015
        PRINT 601
601     FORMAT(14H FINAL RESULTS)
        PRINT 603,NCALL
603     FORMAT(22HONO OF FUNCTION CALLS=,I5)
        PRINT 602,ANS,AN,X
602     FORMAT(27HOCOMPUTED GRAY PRONGS/STAR=,F15.9,
       1//27H MEASURED GRAY PRONGS/STAR=,F15.9,
       2//23H FINAL PARAMETER VALUE=,F15.9)
        IF (EJCT .EQ. EMAX) GO TO 2595
        PRINT 4575, ALPHA, BETA, GAMMA
4575    FORMAT(9HOALPHA = ,E20.8, 5X, 7HBETA = ,E20.8, 5X, 8HGAMMA = ,E20.
       $8)
2595    CONTINUE
        DELTA = (EJCT - EMIN) / 5.0
        PRINT 9550
9550    FORMAT(1H0, 20H        ENERGY        ,20H          FLUX        )
        DO 9575 I = 1,6
        W = EMIN + DELTA * FLOAT(I-1)
        F = 1.0 / W**X
9575    PRINT 8750, W, F
8750    FORMAT( 2E20.5)
        IF (EJCT .EQ. EMAX) GO TO 3890
        DELTA = (EMAX - EJCT) / 5.0
        PRINT 5002
        DO 9590 I = 1,6
        W = EJCT + DELTA * FLOAT(I-1)
        F = ALPHA * (1.0 + BETA * W / EMAX + GAMMA * (W/EMAX)**2)
9590    PRINT 8750, W, F
3890    CONTINUE
        IF (ENDATA .EQ. 0) GO TO 5000
        CALL EXIT
3       PRINT 700
700     FORMAT(41H1ROUNDING ERRORS CONVERGENCE NOT ACHIEVED)
        GO TO 2
4       PRINT 701,MAXFUN
701     FORMAT( 8H1MAXFUN=,I6,27H  CALLS WITHOUT CONVERGENCE)
        GO TO 2
        END
```

```
      SUBROUTINE DERI (T,D,F)
C
C     T IS THE LETHARGY VALUE BEING CONSIDERED AND F IS THE INTEGRAND
C     OF THE FUNCTION TO BE INTEGRATED. DERI CALCULATES THIS INTEGRAND
C     AND RETURNS IT
C
      COMMON /MD/ EMAX, EMIN, E(100), EN(100), N, Y, EJCT
      COMMON /PQ/ ALPHA, BETA, GAMMA
C
C     CHECK TO SEE IF T EXCEEDS THE MINIMUM VALUE IN THE TABLE AND IF
C     NOT SET F EQUAL TO THE INTEGRAND IN THE NUMERATOR
C
5     F = 0.0
      DO 10 I=1,N
      IF (E(I) .EQ. T) GO TO 25
10    IF(E(I).LT.T) GO TO 20
20    F=(T-E(I))*(EN(I-1)-EN(I))/(E(I-1)-E(I))
25    F = F + EN(I)
      F = EXP(F)
C
C     F IS NOW AN INTERPOLATED GRAY PRONGS PER STAR VALUE
C
      AQ = EMAX * EXP(-T)
      IF (AQ .GT. EJCT) GO TO 50
      F = F * AQ**Y
      RETURN
50    FQ = EXP(-T)
      F = (1.0 + BETA*FQ + GAMMA*FQ**2) * F
      F = F * ALPHA * AQ
C
C     F IS NOW THE INTEGRAND IN THE NUMERATOR
C
      RETURN
      END
```

```
      SUBROUTINE LINMIN(ITEST,X,F,MAXFUN,ABSACC,RELACC,XSTEP)
      GO TO (1,2,2),ITEST
    2 IS=6-ITEST
      ITEST=1
      IINC=1
      XINC=XSTEP+XSTEP
      MC=IS-3
      IF (MC) 4,4,15
    3 MC=MC+1
      IF (MAXFUN .GE. MC) GO TO 15
      ITEST=4
   43 X=DB
      F=FB
      IF (FB .LE. FC) GO TO 15
      X=DC
      F=FC
   15 RETURN
    1 GO TO (5,6,7,8),IS
    8 IS=3
    4 DC=X
      FC=F
      X=X+XSTEP
      GO TO 3
    7 IF (FC-F) 9,10,11
   10 X=X+XINC
      XINC=XINC+XINC
      GO TO 3
    9 DB=X
      FB=F
      XINC=-XINC
      GO TO 13
   11 DB=DC
      FB=FC
      DC=X
      FC=F
   13 X=DC+DC-DB
      IS=2
      GO TO 3
    6 DA=DB
      DB=DC
      FA=FB
      FB=FC
   32 DC=X
      FC=F
      GO TO 14
    5 IF (FB .LT. FC) GO TO 16
      IF (F .GE. FB) GO TO 32
      FA=FB
      DA=DB
   19 FB=F
      DB=X
      GO TO 14
   16 IF (FA .LE. FC) GO TO 21
      XINC=FA
      FA=FC
      FC=XINC
      XINC=DA
      DA=DC
```

```
      DC=XINC
   21 XINC=DC
      IF ((D-DB)*(D-DC) .LT. 0.0) GO TO 32
      IF (F .GE. FA) GO TO 24
      FC=FB
      DC=DB
      GO TO 19
   24 FA=F
      DA=X
   14 IF (FB .GT. FC) GO TO 29
      IINC=2
      XINC=DC
      IF (FB .EQ. FC) GO TO 45
   29 D=(FA-FB)/(DA-DB)-(FA-FC)/(DA-DC)
      IF (D*(DB-DC) .LT. 0.0) GO TO 33
      D=0.5*(DB+DC-(FB-FC)/D)
      IF ((ABS(D-X) .GT. ABS(ABSACC)) .AND. (ABS(D-X) .GT. ABS(D*RELACC)
     1)) GO TO 36
      ITEST=2
      GO TO 43
   36 IS=1
      X=D
      IF ((DA-DC)*(DC-D)) 3,26,38
   38 IS=2
      GO TO (39,40),IINC
   33 IS=2
      GO TO (41,42),IINC
   39 IF (ABS(XINC) .GE. ABS(DC-D)) GO TO 3
   41 X=DC
      GO TO 10
   40 IF (ABS(XINC-X) .GT. ABS(X-DC)) GO TO 3
   42 X=0.5*(XINC+DC)
      IF ((XINC-X)*(X-DC) .GT. 0.0) GO TO 3
      GO TO 26
   45 X=0.5*(DB+DC)
      IF ((DB-X)*(X-DC) .GT. 0.0) GO TO 3
   26 ITEST=3
      GO TO 43
      END
```

```
      SUBROUTINE INTO(X,Y,F,T,HPRO)
C
C     X IS THE LETHARGY VALUE BEING CONSIDERED AND HPRO IS THE LETHARGY
C     DIFFERENCE BETWEEN IT AND THE NEXT HIGHEST VALUE, Y IS THE
C     NUMERATOR INTEGRAL OVER THE HPRO INTERVAL AND T IS A DOUBLE
C     PRECISION VALUE OF Y, F IS THE VALUE OF THE NUMERATOR INTEGRAND
C
      COMMON /INTC/ IPMX,AREF,EMAX,SSSR,HFAC,SWAM,SWEX
      COMMON /INTP/ HPR,XX,N,EUB,ELB,IP,IT,NRKS,SWIN
      DIMENSION Y(1),F(1),T(8,1)
      LOGICAL SWAM,SWEX,SWIN
      INTEGER HFAC
      DOUBLE PRECISION T,HPRO,HPR,XX
      DATA IPMX,AREF,EMAX,SSSR,HFAC,SWAM,SWEX
     $    /1024,1.0,1.0E-6,100.0,2,.TRUE.,.TRUE./
C
      HPR=HPRO
C
C     DOUBLE PRECISION THE LETHARGY VALUE
C
      XX=DBLE(X)
      N=1
      EUB=EMAX
      ELB=EMAX/SSSR
      IP=1
      IT=0
      NRKS=0
      SWIN=SWEX
C
C     CALL DERI GIVING IT THE LETHARGY VALUE AND HAVING IT RETURN THE
C     INTEGRAND F IN THE NUMERATOR
C
      CALL DERI (X,Y,F)
C
C     DOUBLE PRECISION THE Y VALUE
C
      DO 9 I=1,N
      T(5,I)=DBLE(Y(I))
    9 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INTS(X,Y,F,T,SWPR)
      COMMON /INTC/ IPMX,AREF,EMAX,SSSR,HFAC,SWAM,SWEX
      COMMON /INTP/ HPR,XX,N,EUB,ELB,IP,IT,NRKS,SWIN
C
C     X IS THE LETHARGY VALUE BEING CONSIDERED AND Y IS THE VALUE
C     OF THE INTEGRAL OVER THE HPR INTERVAL, F IS THE VALUE OF THE
C     INTEGRAND, T IS A DOUBLE PRECISION VALUE OF Y
C
      DIMENSION Y(1),F(1),T(8,1)
      LOGICAL SWAM,SWEX,SWIN
      LOGICAL SWPR
      INTEGER HFAC
      DOUBLE PRECISION T,HPR,XX
      DOUBLE PRECISION D,H
 6000 FORMAT (36H0 CANNOT DECREASE H BECAUSE OF HMIN.    ,1PE16.8,I20)
C
 1    CONTINUE
      SWPR=.FALSE.
      TEST=0.0
      H=HPR/DBLE(FLOAT(IP*24))
      IF ((NRKS .LT. 3) .OR. (.NOT. SWAM)) GO TO 200
C
C     ADAMS-MOULTON STEP.
 100  CONTINUE
      DO 109 I=1,N
      D=DBLE(F(I))
      T(4,I)=D
      Y(I)=SNGL(T(5,I)+H*(
     X 55.0D0*D-59.0D0*T(3,I)+37.0D0*T(2,I)- 9.0D0*T(1,I) ))
 109  CONTINUE
      X=SNGL(XX+24.0D0*H)
      CALL DERI (X,Y,F)
      DO 119 I=1,N
      D=DBLE(F(I))
      D=(     T(5,I)+H*(
     X  9.0D0*D+19.0D0*T(4,I)- 5.0D0*T(3,I)+        T(2,I) ))
      T(6,I)=D
      E=ABS(SNGL(D)-Y(I))/14.0
      TEST=AMAX1(E/AMAX1(AREF,ABS(SNGL(D))),TEST)
 119  CONTINUE
C
      GO TO 300
C
C     ZONNEVELD STEP.
 200  CONTINUE
      DO 209 I=1,N
      D=DBLE(F(I))
      T(4,I)=D
C     1
      Y(I)=SNGL(T(5,I)+H*(
     X 12.0D0*D
 209  CONTINUE
      X=SNGL(XX+12.0D0*H)
      CALL DERI (X,Y,F)
      DO 219 I=1,N
      D=DBLE(F(I))
      T(6,I)=D
C     2
```

```
      Y(I)=SNGL(T(5,I)+H*(
     X 12.0D0*D                                                    ))
219   CONTINUE
     CALL DERI (X,Y,F)
      DO 229 I=1,N
      D=DBLE(F(I))
      T(7,I)=D
C     3
      Y(I)=SNGL(T(5,I)+H*(
     X 24.0D0*D                                                    ))
229   CONTINUE
      X=SNGL(XX+24.0D0*H)
     CALL DERI (X,Y,F)
      DO 239 I=1,N
      D=DBLE(F(I))
      T(8,I)=D
C     4
      Y(I)=SNGL(T(5,I)+H*(
     X 3.75D0*T(4,I)+5.25D0*T(6,I)+9.75D0*T(7,I)-0.75D0*D ))
239   CONTINUE
      X=SNGL(XX+18.0D0*H)
     CALL DERI (X,Y,F)
      DO 249 I=1,N
      D=DBLE(F(I))
      E=ABS(SNGL(H*(
     X -16.0D0*T(4,I)+48.0D0*T(6,I)+48.0D0*T(7,I)+48.0D0*T(8,I)
     X -128.0D0*D    )))
C     5
      D=(     T(5,I)+H*(
     X 4.0D0*T(4,I)+ 8.0D0*T(6,I)+ 8.0D0*T(7,I)+ 4.0D0*T(8,I)
     X ))
      T(6,I)=D
      TEST=AMAX1(E/AMAX1(AREF,ABS(SNGL(D))),TEST)
249   CONTINUE
C
C     BOTH ADAMS-MOULTON AND ZONNEVELD METHODS CONTINUE FROM HERE.
300   CONTINUE
     X=SNGL(XX+24.0D0*H)
      IF (TEST .LE. EUB) GO TO 310
      IF (IP*HFAC .GT. IPMX) GO TO 309
C
C     REPEAT STEP WITH SMALLER H.
      NRKS=0
      IP=IP*HFAC
      IT=IT*HFAC
      DO 305 I=1,N
      Y(I)=SNGL(T(5,I))
      F(I)=SNGL(T(4,I))
305   CONTINUE
      GO TO 1
C
C     CANNOT DECREASE H BECAUSE OF HMIN.
309   CONTINUE
     IF (.NOT. SWIN) GO TO 310
      PRINT 6000, X,IPMX
     SWIN=.FALSE.
C
310   CONTINUE
C
```

```
C
C       ACCEPT CURRENT STEP.
C
C       XX STILL HAS NOT BEEN CHANGED SINCE ENTRY.
C       YY(XX) IS STILL IN T(5, ).
C       F(YY) IS IN T(4, ).
C
        IT=IT+1
        XX=XX+HPR/DBLE(FLOAT(IP))
        NRKS=MIN0(NRKS+1,4)
        DO 319 I=1,N
        D=T(6,I)
        T(5,I)=D
        Y(I)=SNGL(D)
 319    CONTINUE
        X=SNGL(XX)
       CALL DERI (X,Y,F)
        IF (IT .LT. IP) GO TO 320
C
C       X IS A MULTIPLE OF HPRINT.
        SWPR=.TRUE.
        IT=IT-IP
C
 320    CONTINUE
        IF (TEST .GE. ELB) GO TO 330
        IF (MOD(IP,HFAC)+MOD(IT,HFAC) .NE. 0) GO TO 330
C
C       PROCEED TO NEXT STEP WITH LARGER H, USING ZONNEVELD METHOD.
        NRKS=0
        IP=IP/HFAC
        IT=IT/HFAC
        RETURN
C
C
C       PROCEED TO NEXT STEP WITH SAME H.
 330    CONTINUE
        DO 339 I=1,N
        T(1,I)=T(2,I)
        T(2,I)=T(3,I)
        T(3,I)=T(4,I)
 339    CONTINUE
        RETURN
        END
```

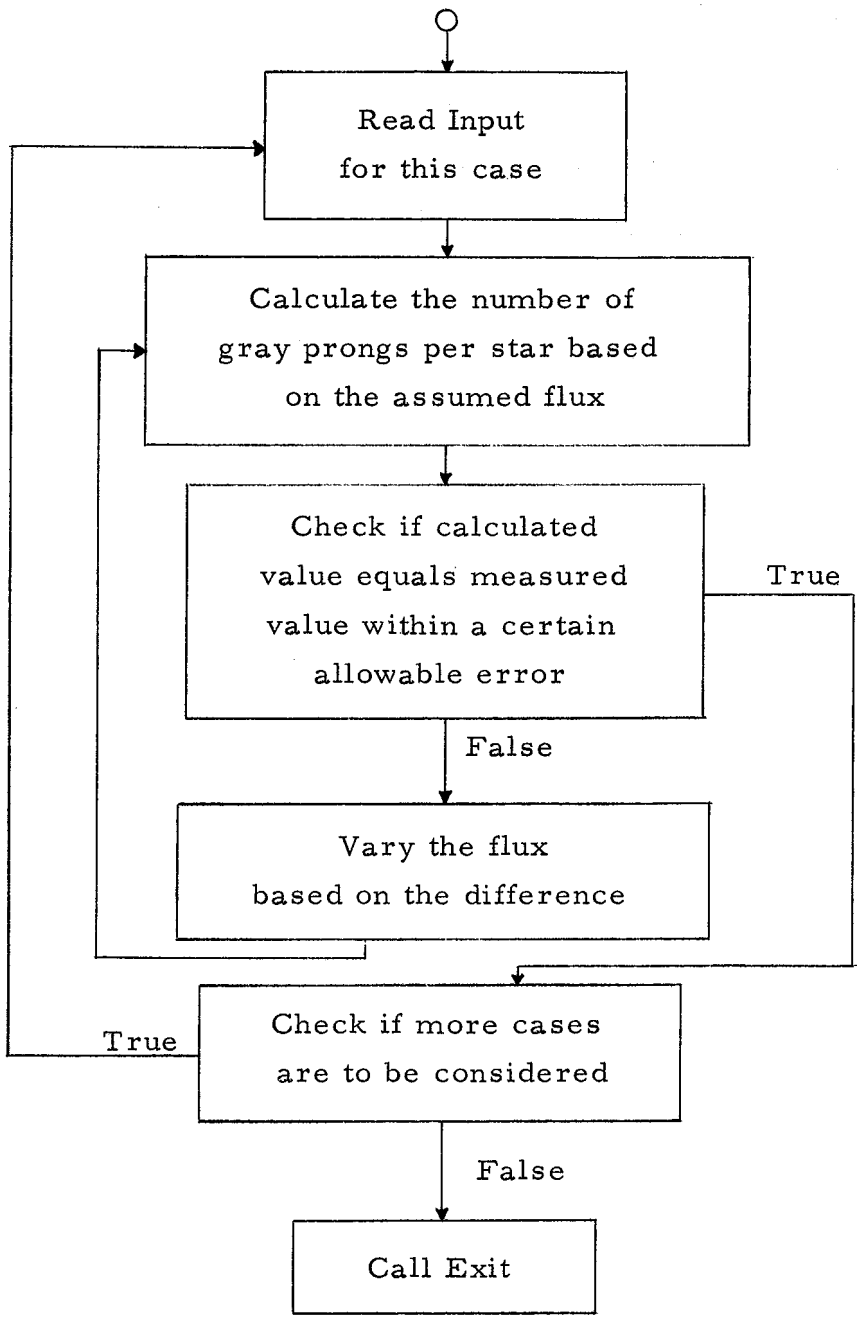
```
      SUBROUTINE SORT(N1, N2, N3, AZ)
      COMMON /MD/ EMAX, EMIN, E(100), EN(100), N, Y, EJCT
      DIMENSION AZ(100)
      NUT = N - 1
      DO 1000 I = 1,NUT
      A = AZ(I)
      B = AZ(I + 1)
      IF (A .LE. EMIN .AND. B .GT. EMIN) GO TO 2000
 1000 CONTINUE
 2000 N1 = I
      DO 3000 I = N1,NUT
      A = AZ(I)
      B = AZ(I+1)
      IF (A .LT. EJCT .AND. B .GE. EJCT) GO TO 4000
 3000 CONTINUE
 4000 N2 = I
      DO 5000 I = N2,NUT
      A = AZ(I)
      B = AZ(I+1)
      IF (A .LT. EMAX .AND. B .GE. EMAX) GO TO 6000
 5000 CONTINUE
 6000 N3 = I
      RETURN
      END
```

```
      SUBROUTINE COMPT(X, R, EJCT)
      COMMON /PQ/ ALPHA, BETA, GAMMA
      TOP = (2.0 + X) * R**2 - X
      BTM = (1.0 + X) * R - (2.0 + X) * R**2
      BETA = TOP / BTM
      GAMMA = - (1.0 + BETA)
      BTM = (EJCT**X) * (1.0 + BETA * R + GAMMA * R**2)
      ALPHA = 1.0 / BTM
      RETURN
      END
```

```
      SUBROUTINE POINT(N1, N2, N3, AZ, BZ)
      COMMON /MD/ EMAX, EMIN, E(100), EN(100), N, Y, EJCT
      DIMENSION AZ(100), BZ(100)
      E(1) = EMIN
      EN(1) = PDQ(N1, AZ, BZ, EMIN)
      J = 1
      DO 1000 I = 1,N
      A = AZ(I)
      IF (EMIN .LT. A .AND. EJCT .GT. A) GO TO 1500
      GO TO 1000
 1500 J = J + 1
      E(J) = AZ(I)
      EN(J) = BZ(I)
 1000 CONTINUE
      J = J + 1
      E(J) = EJCT
      EN(J) = PDQ(N2, AZ, BZ, EJCT)
      IF (EJCT .EQ. EMAX) GO TO 5000
      DO 2000 I = 1,N
      A = AZ(I)
      IF (EJCT .LT. A .AND. EMAX .GT. A) GO TO 2500
      GO TO 2000
 2500 J = J + 1
      E(J) = AZ(I)
      EN(J) = BZ(I)
 2000 CONTINUE
      J = J + 1
      E(J) = EMAX
      EN(J) = PDQ(N3, AZ, BZ, EMAX)
 5000 N = J
      RETURN
      END
```

```
      FUNCTION PDQ(NZ, AZ, BZ, EZ)
      DIMENSION AZ(100), BZ(100)
      A1 = ALOG(AZ(NZ))
      A2 = ALOG(AZ(NZ+1))
      B1 = ALOG(BZ(NZ))
      B2 = ALOG(BZ(NZ+1))
      BQ = ALOG(EZ)
      CQ = (BQ - A2) * (B1 - B2) / (A1 - A2)
      CQ = CQ + B2
      PDQ = EXP(CQ)
      RETURN
      END
```

FLOW DIAGRAM



FOOTNOTES AND REFERENCES

1. René Remy, Neutron Spectroscopy by the Use of Nuclear Stars from 20 to 300 MeV, UCRL-16325, Aug. 1965.

2. Alan R. Smith, Threshold Detector Applications to Neutron Spectroscopy at the Berkeley Particle Accelerators, UCRL-16312, Nov. 1965.

3. N. Metropolis, R. Bevins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Monte Carlo Calculations on Intranuclear Cascades. II. High-Energy Studies and Pion Processes, Phys. Rev. 110, 204 (1958).

4. Robert J. Ridell, (Lawrence Radiation Laboratory), private communication.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.