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EXTENSION OF THE PLASMA KINETIC EQUATION
TO SMALL WAVE-NUMBERS

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ABSTRACT

The equation for the evolution of the electron velocity distribution, in a stable uniform electron-ion plasma, is extended to wave-numbers so small that collisional damping exceeds Landau damping for electron plasma waves.

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It is generally realized[1] that the standard plasma kinetic equation for a stable uniform Coulomb plasma, that of Lenard, Balescu, and Guernsey,[2] has a serious defect in principle. Its derivation is based on the Bogolyubov synchronization assumption, that the electron correlation function $g(\vec{k})$ relaxes to an asymptotic state at a rate much faster than the rate of variation of the electron velocity distribution $f(\vec{v})$. However, in the various derivations of the L-B-G equation, one finds that $g(\vec{k})$ relaxes at the rate $\gamma_L(\vec{k})$ (the Landau damping rate) while $f(\vec{v})$ characteristically relaxes at the rate $\nu(v)$, the electron-ion collisional frequency. Comparing the two rates, one observes that for

$$k < k_D \, (\ell n \Lambda)^{-\frac{1}{2}} \tag{1}$$

(where k_D is the Debye wave-number and Λ is the Debye parameter), $\gamma_L(\vec{k})$ is <u>exceeded</u> by $\bar{\nu}$ [the mean collisional frequency, defined in Eq. (18)], so that the Bogolyubov assumption is violated.

It is also realized[1] that $g(\vec{k})$ actually relaxes not at the rate $\gamma_L(\vec{k})$ predicted by the standard derivation, but rather at the rate $\gamma(\vec{k})$, the true damping coefficient for an electron plasma wave $\vec{k}$. The latter is known to be[3]

$$\gamma(\vec{k}) = \gamma_L(\vec{k}) + \tfrac{1}{2}\bar{\nu} \, , \tag{2}$$

for $k \ll k_D$. Thus, as $k \to 0$, the rate $\gamma(\vec{k}) = \frac{1}{2}\bar{\nu}$ is appropriate for the relaxation of $g(\vec{k})$.

The present paper utilizes this idea, heretofore based on intuition, to derive a kinetic equation valid for arbitrarily small k . The Bogolyubov assumption is now valid at small k for <u>fast</u> electrons [such that $\nu(v) \ll \bar{\nu}$]; this limitation is acceptable, since only fast electrons can resonate with <u>fast</u> plasma waves $(k < k_D (\ell n\Lambda)^{-\frac{1}{2}})$, and only for the latter is the present modification in theory needed.

The validity of the Bogolyubov assumption implies that the momentum $\vec{p}$ of an electron is a Markov process. Neglecting large-angle scattering, we may thus utilize the standard Fokker-Planck equation,[4] for the momentum probability density $f(\vec{p})$:

$$\frac{\partial}{\partial t} f(\vec{p};t) = -\frac{\partial}{\partial \vec{p}} \cdot \vec{J}(\vec{p}) , \tag{3}$$

$\vec{J}(\vec{p}) = \vec{F}(\vec{p})\, f(\vec{p}) - \frac{\partial}{\partial \vec{p}} \cdot [\underset{\sim}{D}(\vec{p})\, f(\vec{p})]$. Here $\vec{J}$ is the flux density in momentum space, $\vec{F}$ is the dynamic friction , and $\underset{\sim}{D}$ is the momentum diffusion tensor. As is shown by Thompson and Hubbard,[5] these quantities may be expressed as

$$\vec{J}(\vec{p}) = \vec{F}^P(\vec{p})\, f(\vec{p}) - \underset{\sim}{D}(\vec{p}) \cdot \frac{\partial}{\partial \vec{p}} f(\vec{p}) , \tag{4}$$

$$\vec{F}^P(\vec{p}) = 4\pi e^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}}{k^2} \operatorname{Im} K^{-1}(\vec{k}, \vec{k}\cdot\vec{v}) , \tag{5}$$

$$\underset{\sim}{D}(\vec{p}) = \tfrac{1}{2}\, e^2 \int \frac{d^3k}{(2\pi)^3}\ \vec{k}\,\vec{k}\ \ C(\vec{k},\ \vec{k}\cdot\vec{v})\ . \tag{6}$$

Here $\vec{F}^P$ is the contribution of polarization to dynamic friction, $K(\vec{k},\omega)$ is the dielectric function, and $C(\vec{k},\omega)$ is the micro-potential spectral density:

$$C(\vec{k},\omega) = \int d^3s \int_{-\infty}^{+\infty} \langle \phi(\vec{r},t)\ \phi(\vec{r}+\vec{s},\ t+\tau)\rangle \exp(-i\vec{k}\cdot\vec{s} + i\omega\tau)\ . \tag{7}$$

The L-B-G equation is obtained if one replaces K by the Vlasov function K_V, and C by the expression[5] obtained from the test-particle theorem.[6]

For collisionally-damped electron[7] plasma waves, the damping is <u>weak</u> $(\gamma \ll \omega_e)$. Hence, in (5), one may use the resonance approximation for the range (1):

$$\mathrm{Im}\ K^{-1}(\vec{k},\omega) = -\pi\,\delta\,[\mathrm{Re}\ K(\vec{k},\omega)]\ . \tag{8}$$

Since the inclusion of collisions does not modify $\mathrm{Re}\ K$, we see that $\vec{F}^P$ is insensitive to the damping mechanism, as one would expect on intuitive grounds.

However, $\underset{\sim}{D}$ is modified by collisions. That the Rostoker expression[5,6] is no longer valid for C is clear, as one can check by testing it, in the case of thermal equilibrium, with the fluctuation-dissipation theorem:[5]

$$C(\vec{k},\omega) = -\frac{8\pi\Theta}{k^2\omega} \operatorname{Im} K^{-1}(\vec{k},\omega) , \tag{9}$$

where, for $k \to 0$,

$$K(\vec{k},\omega) = 1 - \frac{\omega_e^{\,2}}{(\omega+i\bar{\nu})^2} . \tag{10}$$

To obtain a valid expression for C in the range (1), we again make the resonance approximation, now in the form:

$$C(\vec{k},\omega) = \frac{4\pi^2}{k^2} [\mathcal{E}(\vec{k}) \, \delta(\omega - \omega_{\vec{k}}) + \mathcal{E}(-\vec{k}) \, \delta(\omega + \omega_{-\vec{k}})] , \tag{11}$$

where $\omega_{\vec{k}}$ is the positive frequency of the wave $\vec{k}$, and

$$\mathcal{E}(\vec{k}) \equiv 2 \frac{k^2}{4\pi} \int_0^\infty d\omega \, C(\vec{k},\omega) \tag{12}$$

is the energy of the wave[8] $(\vec{k},\omega_{\vec{k}})$. The contribution of the range (1) to the diffusion tensor (6) is thus

$$\underset{\sim}{D}_1(\vec{p}) = 4\pi^2 e^2 \int \frac{d^3k}{(2\pi)^3} \, \hat{k}\,\hat{k} \, \delta(\omega_{\vec{k}} - \vec{k}\cdot\vec{v}) \, \mathcal{E}(\vec{k}) , \tag{13}$$

where the integration is over the range (1). Not surprisingly, this is identical to the expression obtained in the quasi-linear theory,[9] although the conventional derivation and conditions of validity of the latter are quite different.

The wave energy $\mathcal{E}(\vec{k})$, in turn, may be obtained from

the steady-state condition:

$$\overset{\circ}{\mathcal{E}}(\vec{k}) = 2\,\gamma(\vec{k})\;\mathcal{E}(\vec{k})\;, \tag{14}$$

where $\overset{\circ}{\mathcal{E}}$ is the emission rate. The part of the latter due to longitudinal bremsstrahlung may be calculated, by the method of Mercier,[10] to be

$$\dot{\mathcal{E}}_B(\vec{k}) = \frac{e^2\omega_e^{\,2}}{3}\,\langle v^{-1}\rangle \ln\Lambda\;, \tag{15}$$

where $$\langle v^{-m}\rangle = \int d^3p\; v^{-m}\, f(\vec{p})\;. \tag{16}$$

In addition, there is the collisionless emission rate, $\dot{\mathcal{E}}_C(\vec{k})$, the longitudinal Cerenkov radiation,[11] which vanishes with k. From (14) and (2), we obtain

$$\mathcal{E}(\vec{k}) = \frac{\dot{\mathcal{E}}_B(\vec{k}) + \dot{\mathcal{E}}_C(\vec{k})}{\bar{\nu} + 2\gamma_L(\vec{k})} \xrightarrow[k \text{ in } (1)]{} \frac{\dot{\mathcal{E}}_B(\vec{k})}{\bar{\nu}}\;. \tag{17}$$

The mean collisional frequency $\bar{\nu}$ depends functionally[12] on f:

$$\bar{\nu} = \frac{e^2\omega_e^{\,2}}{3m}\,\langle v^{-3}\rangle. \tag{18}$$

As a check on (17), we may evaluate it for a Maxwellian $f(\vec{p})$, and obtain the Rayleigh-Jeans formula $\mathcal{E}(\vec{k}) = \Theta$. Noting that $\dot{\mathcal{E}}_B(\vec{k})$ is independent of k, we may define an <u>effective</u>

electron temperature for non-Maxwellian $f(\vec{p})$, by $\Theta_{eff} \equiv \mathcal{E}(\vec{k})$ in the range (1).

Substituting (17) into (13), we may easily evaluate $\underset{\sim}{D}_1(\vec{p})$. We find that, for $v \gg \langle v \rangle \ln\Lambda$,

$$\underset{\sim}{D}_1(\vec{p}) = \frac{ne^4}{\ln\Lambda} \frac{\underset{\sim}{I} - \hat{v}\hat{v}}{v}, \tag{19}$$

valid approximately for any $f(\vec{p})$. In contrast, the standard theory, using $\mathcal{E}(\vec{k}) = \dot{\mathcal{E}}_C(\vec{k}) / 2\gamma_L(\vec{k})$, would yield a result highly sensitive to $f(\vec{p})$. We note that, in magnitude, the contribution (19) is smaller, by a factor $(\ln\Lambda)^{-1}$, than the contribution of <u>all</u> waves $k < k_D$ to $\underset{\sim}{D}$, and, by a factor $(\ln\Lambda)^{-2}$, than the "dominant" contribution, from $k > k_D$, representing collisions within the Debye sphere.

Thus the present considerations represent a correction of the same order in Λ^{-1} as the terms in the L-B-G equation,[13] but of higher order in $(\ln\Lambda)^{-1}$.

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FOOTNOTES

1. E. A. Frieman, The Present State of Plasma Kinetic Theory (Princeton University PPL-AF-1, 1966), pp. 19-20.

2. D. C. Montgomery and D. A. Tidman, Plasma Kinetic Theory (McGraw-Hill Book Co., New York, 1964), Chap. 6; T. Y. Wu, Kinetic Equations of Gases and Plasmas (Addison-Wesley Publishing Co., Reading, Mass., 1966), Chap. 6.

3. J. Dawson and C. Oberman, Phys. Fluids 5, 517 (1962). In addition to damping caused by electron-ion collisions, there is a contribution to wave damping from the elastic scattering of the wave by ions. The latter is smaller than the former by a factor of the order $(k/k_D)\ (\ell n\ \Lambda)^{-1}$, which is less than $(\ell n\ \Lambda)^{-3/2}$ in the range (1). Hence, the latter contributions will be neglected here.

4. S. Gasiorowicz, M. Neuman, and R. Riddell, Phys. Rev. 101, 922 (1956).

5. W. B. Thompson and J. Hubbard, Rev. Mod. Phys. 32, 714 (1960); J. Hubbard, Proc. Roy. Soc. A260, 114 (1961).

6. N. Rostoker, Phys. Fluids 7, 491 (1964).

7. We ignore ion waves in the present discussion. Their properties are quite sensitive to collisions [see R. Kulsrud and C. Shen, Phys. Fluids 9, 177 (1966)], and must be treated with great care.

8. We are tempted to call this non-macroscopic wave a "micro-wave."

9. For a critical summary, see I. Bernstein and F. Engelmann, Phys. Fluids 9, 937 (1966).

10. R. P. Mercier, Proc. Phys. Soc. 83, 819 (1964).

11. N. Rostoker, Nuc. Fus. 1, 101 (1961).

12. In evaluating (18) with (16), one must impose a lower limit of $\Lambda^{-1}\langle v\rangle$ on the speed v.

13. We note that the divergence of the L-B-G equation at large k has been treated by a number of authors, leading to a correction of order $(\ell n\ \Lambda)^{-1}$.

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