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January 11, 1967

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UCRL-17325

AMBIGUITIES IN THE PHENOMENOLOGICAL DETERMINATION OF REGGE POLE PARAMETERS*

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Berkeley, California

January 11, 1967

ABSTRACT

We attempt to eliminate some of the ambiguities of the phenomenological determination of Regge pole parameters. To this end, we study charge-exchange reactions involving the ρ and R trajectories. We find that we are not able to resolve these ambiguities with the available data.

I. INTRODUCTION

In the last few years there have been many successful phenomenological Regge pole fits to high-energy data. In particular, it has been possible to understand the energy dependence of total cross sections,[1] interference effects between different trajectories or between trajectories and direct channel resonances,[2] and some features of the differential cross sections.[1,3] Some of these attempts have shed light on the properties of the residue and trajectory functions of the poles involved, but only after certain assumptions are made about their structure in momentum transfer. It is, of course, of great interest to see to what extent these assumptions can be checked by confrontation with the experimental data.

The purpose of this paper is to investigate the possibility of phenomenologically eliminating some of the theoretical ambiguities in these fits by detailed study of some simple scattering processes to which only one or two trajectories contribute. To this end, we have chosen to fit some meson-nucleon data which are quite abundant and accurate, and at the same time presumed to be controlled by the ρ and R (or A_2) Regge trajectories. We find, unfortunately, that we can get good fits to experiment while making qualitatively different assumptions about the behavior of the trajectories and residues. In particular there are important theoretical questions (such as ghost-killing mechanisms) which cannot be settled by the presently available data. Our efforts, however, illuminate the experimental requirements for settling such questions.

In Section II we discuss the processes considered and the formulas which relate the Regge parameters to the physically measurable quantities. We also discuss in detail the type of ambiguities present in our fits and in particular the alternative ghost-killing mechanisms. In Section III we exhibit some of the best fits we have obtained to the data.

-II-

The reactions considered are: $\pi^- p \to \pi^0 n$,[4] which is dominated by the exchange of the ρ trajectory, $\pi^- p \to \eta n$,[5] which involves only the R trajectory, and $K^- p \to \bar{K}^0 n$[6] which depends both on ρ and on R. Other possible candidates such as $K^+ n \to K^0 p$ and $\pi N \to \pi\Delta$ were not considered because experimental information is scarce or at too low an energy to be reliably expressed in terms of a simple Regge-pole expansion. The differences between the total cross sections of $\pi^- p$ and $\pi^+ p$, $K^+ n$ and $K^+ p$, and $K^- p$ and $K^- n$ (Ref. 7) were also included. In all, 185 experimental points were considered with the laboratory-system momentum of the incident meson ranging from 5 to 20 GeV/c.

To obtain the Regge pole formulas, we have used helicity amplitudes and followed the method developed by L. L. Wang for the determination of kinematic singularities.[8] This method has the advantage of making all kinematic singularities of the amplitude quite explicit, so that the remaining residue functions can be parameterized by smooth functions. We have ignored problems

arising from the second-order terms in the Regge expansion or from secondary trajectories, in accordance with the current philosophy in phenomenological Regge pole work.

For the s-channel processes in question, namely reactions of the form $a + b \to c + d$, where a and c are spin zero particles and b and d have spin 1/2, we can write the t-channel $(a + \bar{c} \to \bar{b} + d)$ helicity amplitudes as

$$f_{++} = \sum_i f_{++}^{\ i} , \quad f_{+-} = \sum_i f_{+-}^{\ i} ,$$

with

$$f_{++}^{\ i} = g_1(s,t)\, A_1^i(t) \frac{1 \pm e^{-i\pi\alpha_i(t)}}{\sin \pi\alpha_i(t)} \left(\frac{E}{E_0}\right)^{\alpha_i(t)} b_{++}^{\ i}(t) ,$$

$$f_{+-}^{\ i} = g_2(s,t)\, A_2^i(t) \frac{1 \pm e^{-i\pi\alpha_i(t)}}{\sin \pi\alpha_i(t)} \left(\frac{E}{E_0}\right)^{\alpha_i(t)-1} b_{+-}^{\ i}(t) .$$

Here g_1 and g_2 are kinematical factors and for the special case $m_b = m_d = M$ are given by

$$g_1(t) = \frac{1}{2(4M^2 - t)^{1/2}} ,$$

$$g_2(s,t) = [t-(m_a - m_c)^2]^{1/2} \; [t-(m_a + m_c)^2]^{1/2} \; \sin \theta_t .$$

E is the laboratory-system energy of the incident particle and E_0

is a scaling factor which is usually taken to be 1 GeV.[9] The factor $\alpha_i(t)$ is the ith Regge pole trajectory, while $b^i(t)$ is proportional to the corresponding residue function. The parameterization of α_i and b_i will be discussed later. The factor $(1 \pm e^{-i\pi\alpha})$ is the signature factor, + or - referring to even or odd trajectories. The functions A^i are of the form $\alpha^n(1 + \alpha)$, where n is some positive integer. In the Regge pole expansion, one finds that all amplitudes contain a factor $X(\alpha) = \frac{(2\alpha + 1)\,\Gamma(\alpha + 1/2)\,(1 \pm e^{-i\pi\alpha})}{\Gamma(\alpha + 1)\,\sin \pi\alpha}$. The poles of $(2\alpha + 1)\,\Gamma(\alpha + 1/2)$ will be cancelled via Mandelstam symmetry. The zeros of $1/\Gamma(\alpha + 1)$ will serve to cancel the spurious poles of $1/\sin \pi\alpha$ for negative integers. Since in our analysis $\alpha(t)$ is never smaller than -1, we have written out a factor $(\alpha + 1)$ from the expansion of $1/\Gamma(\alpha + 1)$ explicitly and included the rest of the factros, which are smoother in the region of interest, in the residue functions. The factor α^n is part of the reduced residue that we isolate for convenience. It differs for different ghost-killing mechanisms and is obtained from the following arguments.[10]

Let us label channels by sense or nonsense according to their behavior at $\alpha(t) = 0$ (sense refers to channels with total helicity zero and nonsense to channels with absolute value of the helicity greater than or equal to one). Then using factorization, for each pole residue we can write

$$\beta_{++} = \gamma_S \, \xi_S \qquad \beta_{+-} = \gamma_S \, \xi_N ,$$

where γ_S corresponds to the meson channel and ξ_S and ξ_N to the sense or nonsense nucleon-antinucleon channels respectively. The functions β are the residues of the pole before any factors were taken out. For example, neglecting factors of α arising from ghost-killing,

$$b_{++} = -(4M^2 - t)^{1/2} \left(\frac{p_t \, q_t}{2ME_0} \right)^{-\alpha} \pi^{\frac{1}{2}} \frac{(2\alpha + 1)\,\Gamma(\alpha + 1/2)}{(\alpha + 1)\,\Gamma(\alpha + 1)} \beta_{++} .$$

We can also write the corresponding residues for nucleon-nucleon scattering in conventional notation,[11]

$$\beta_{11} = \xi_S^{\,2} \quad \beta_{12} = \beta_{21} = \xi_S \, \xi_N \qquad \beta_{22} = \xi_N^{\,2} .$$

In the Regge expansions, one finds that the helicity amplitudes are proportional to the following factors of α and $(\alpha + 1)$ [in addition to the usual $X(\alpha)$] originating from the d-functions of the partial wave expansion:

sense-sense amplitudes: $f_{++} \propto \beta_{++}$, $f_{11} \propto \beta_{11}$;

sense-nonsense amplitudes: $f_{+-} \propto [\alpha/(\alpha + 1)]^{\frac{1}{2}} \beta_{+-}$, $f_{12} \propto [\alpha/\alpha+1)]^{\frac{1}{2}} \beta_{12}$;

nonsense-nonsense amplitudes: $f_{22} \propto 1/(1 + \alpha)\, \beta_{22}$.

Since the amplitudes are analytic functions of t in this region, the residues should be proportional to certain factors which would cancel the branch point of f_{SN} or the pole of f_{NN}. From the analytic properties of partial waves it can be shown that β_{12} and β_{+-} are proportional to $[\alpha(\alpha + 1)]^{\frac{1}{2}}$. Since f_{22} is proportional to $1/(\alpha + 1)$, the simplest alternative satisfying factorization is to assume that $\xi_N \propto (\alpha + 1)^{\frac{1}{2}}$, so that $\beta_{22} = \xi_N^{\ 2} \propto (\alpha + 1)$, cancelling the pole in f_{22}. We then consider the following alternatives for handling the factor $\alpha^{\frac{1}{2}}$.

(a) Suppose ξ_N is proportional to $\alpha^{\frac{1}{2}}$, that is, the coupling to the nonsense channel vanishes at $\alpha(t) = 0$, or the trajectory "chooses sense." Then we will have:

$$\beta_{11}, \beta_{++} \propto b_{11}, b_{++} ,$$

$$\beta_{12}, \beta_{+-} \propto [\alpha(\alpha + 1)]^{\frac{1}{2}} (b_{12}, b_{+-}) ,$$

$$\beta_{22} \propto \alpha(\alpha + 1)\, b_{22} ,$$

where the functions b approach a nonzero constant as $\alpha \to 0$ or $\alpha \to 1$. While this arrangement is sufficient for the odd trajectories, for even signature we also have to kill a pole due to the factor

$$(1 + e^{-i\pi\alpha}) \Big/ \sin \pi\alpha .$$

We can do this by assuming that for even signature ξ_S, ξ_N, and γ_S are all proportional to an extra factor of $\alpha^{\frac{1}{2}}$, thus still

satisfying analyticity and factorization. For even trajectories we will then have

$$\beta_{11}, \beta_{++} \propto \alpha(b_{11}, b_{++}),$$

$$\beta_{12}, \beta_{+-} \propto [\alpha(\alpha + 1)]^{\frac{1}{2}} (b_{12}, b_{+-}),$$

$$\beta_{22} \propto \alpha^2 (\alpha + 1) b_{22}.$$

This ghost-killing mechanism has become known as the Chew mechanism.[12]

(b) We could associate the factor of $(\alpha)^{\frac{1}{2}}$ with ξ_S and γ_S, thus assuming that the trajectory chooses nonsense. Then

$$\beta_{11}, \beta_{++} \propto \alpha(b_{11}, b_{++}),$$

$$\beta_{12}, \beta_{+-} \propto [\alpha(\alpha + 1)]^{\frac{1}{2}} (b_{12}, b_{+-}),$$

$$\beta_{22} \propto (\alpha + 1) b_{22}.$$

We would still have the same kind of difficulty with even trajectories for lower-order terms in the nonsense-to-nonsense amplitude (the first-order term of f_{22} is already proportional to α). However, Gell-Mann has proved that if β_{22} does not vanish at $\alpha(t) = 0$, then there exist other trajectories to cancel the higher order terms exactly. We will refer to this alternative as the Gell-Mann mechanism.[13]

Considering the above two ghost-killing mechanisms for the ρ and R trajectories we studied the following cases:[14]

Case 1. Both trajectories obey the Chew mechanism:

$$A_1^{\rho} = (1 + \alpha_\rho), \qquad A_2^{\rho} = \alpha_\rho(1 + \alpha_\rho) ,$$

$$A_1^{R} = \alpha_R(1 + \alpha_R), \qquad A_2^{R} = \alpha_R^{2}(1 + \alpha_R) .$$

Case 2. ρ obeys the Chew mechanism but R follows the Gell-Mann mechanism:

$$A_1^{\rho} = (1 + \alpha_\rho), \qquad A_2^{\rho} = \alpha_\rho(1 + \alpha_\rho) ,$$

$$A_1^{R} = A_2^{R} = \alpha_R(1 + \alpha_R) .$$

Case 3. Both trajectories obey the Gell-Mann mechanism:

$$A_1^{\rho} = A_2^{\rho} = \alpha_\rho(1 + \alpha_\rho) ,$$

$$A_1^{R} = A_2^{R} = \alpha_R(1 + \alpha_R) .$$

The fourth combination was not studied, since it would not lead to any new information not deducible from the above three cases.

For our normalization the cross sections are given by the following relations:

(a) Since the crossing matrix is orthogonal:

$$\frac{d\sigma}{dt} = \frac{1}{\pi} \frac{1}{4s\ p_s\ q_s} \{|f_{++}|^2 + |f_{+-}|^2\} ,$$

where

$$(4s)^{\frac{1}{2}}\, q_s = \{[s - (M + m_a)^2]\ [s - (M - m_a)^2]\}^{1/2} ,$$

$$(4s)^{\frac{1}{2}}\, p_s = \{[s - (M + m_c)^2]\ [s - (M - m_c)^2]\}^{1/2} .$$

(b) The total cross section:

$$\sigma^{total} = \frac{1}{q_s\ s^{\frac{1}{2}}}\ \mathrm{Im}\ f_{++}{}^{t}\ (t = 0).$$

(c) polarization of the recoiling nucleon:

$$P\ \frac{d\sigma}{dt} = -\ \mathrm{Im}\ (f_{++}{}^{t}\ f_{+-}{}^{t*}) \Big/ 4\pi\ q_s{}^{2}\ s.$$

If we use the subscripts π, η, and K to denote the three processes $\pi^- p \to \pi^0 n$, $\pi^- p \to \eta\ n$, and $K^- p \to \bar{K}^0 n$ (or $K^+ n \to K^0 p$), then

$$f\ (\pi^- p \to \pi^0 n) = {}_{\pi}f^{\rho} ,$$

$$f\ (\pi^- p \to \eta n) = {}_{\eta}f^{R} ,$$

$$f(K^- p \to \bar{K}^0 n) = {}_{K}f^{\rho} + {}_{K}f^{R} ,$$

$$f(K^+ n \to K^0 p) = -{}_{k}f^{\rho} + {}_{K}f^{R} ,$$

with the following constraints due to factorization:

$$\pi^{b^{\rho}}_{+-} \Big/ \pi^{b^{\rho}}_{++} = K^{b^{\rho}}_{+-} \Big/ K^{b^{\rho}}_{++} ,$$

$$\eta^{b^{R}}_{+-} \Big/ \eta^{b^{R}}_{++} = K^{b^{R}}_{+-} \Big/ K^{b^{R}}_{++} .$$

We constrained our parameters so that the above two relations were always satisfied.

From isotopic spin considerations we also have:

$$\sigma^{tot}_{\pi^+p} - \sigma^{tot}_{\pi^-p} = \frac{2^{\frac{1}{2}}}{q_s \; s^{\frac{1}{2}}} \, \mathrm{Im} \, {}_{\pi}f^{\rho}_{++} \; (t = 0) ,$$

$$\sigma^{tot}_{K^-n} - \sigma^{tot}_{K^-p} = \frac{1}{q_s \; s^{\frac{1}{2}}} \, \mathrm{Im} \, [{}_{K}f^{\rho}_{++} \; (t = 0) + {}_{K}f^{R}_{++} \; (t = 0)] ,$$

$$\sigma^{tot}_{K^+p} - \sigma^{tot}_{K^+n} = \frac{1}{q_s \; s^{\frac{1}{2}}} \, \mathrm{Im} \, [- {}_{K}f^{\rho}_{++} \; (t = 0) + {}_{K}f^{R}_{++} \; (t = 0)] .$$

The residue and trajectory functions $b(t)$ and $\alpha(t)$ were chosen in the simplest way consistent with experimental information. They were assumed to be real analytic functions, with only a branch cut on the physical region of the crossed (t) channel, and thus real in the region of interest $(t < 0)$. This explicitly assumes that there are no trajectory crossings. We have chosen $\alpha(t)$ to be a linear function of t, which seems to be suggested by experience. The inclusion of curvature would only increase the uncertainties of our fits, and it does not seem to be demanded by

the data in the small region of $t(-1.5\ \mathrm{GeV}^2/c^2 < t < 0)$. The t dependence of the residue functions has been taken as an exponential multiplied by a linear polynomial in t. Again we believe that this choice is at the same time simple and general enough to represent $b(t)$ in the limited t region of our fit. Our study of ambiguities arising in Regge pole fitting is therefore in the spirit of the parameterization usually made in such analyses.

III. RESULTS

Within the framework of linear trajectories, exponential residues, and the three ghost-killing mechanisms discussed above, we have produced 12 good fits to the data. In two of them (case 1b) we have assumed a zero in the residues b^{ρ}_{++} and b^{R}_{++} for reasons discussed below. In all cases the χ^2 values were good, and the plotted fits were quantitatively similar.

The existence of 6 of the 12 fits is due to the fact that the ρ and R trajectories turned out to be close to each other. The solutions are defined by taking ${}_K b^{\rho}_{+-}$ alternately positive and negative. The K charge-exchange spin-flip amplitudes for these two cases are then roughly

$$\left| \left| {}_K b^{\rho}_{+-} \right| (1 - e^{-i\pi\alpha}) + {}_K b^{R}_{+-} (1 + e^{-i\pi\alpha}) \right|^2$$

and

$$\left| - \left| {}_K b^{\rho}_{+-} \right| (1 - e^{-i\pi\alpha}) + {}_K b^{R}_{+-} (1 + e^{-i\pi\alpha}) \right|^2$$

the magnitudes being equal, regardless of the values of $|{}_K b^{\rho}_{+-}|$ and ${}_K b^{R}_{+-}$. (Other possibilities using different signs of the non-spin-flip residues are ruled out by the constraints imposed by total cross sections). Measurements of $K^+n \to K^0p$ differential cross sections and polarization measurements at high energies would resolve this ambiguity, the solutions for ${}_K b^{\rho}_{+-} > 0$ and ${}_K b^{\rho}_{+-} < 0$ predicting different results. Figures 1 through 5 illustrate a typical fit to the data, including the predicted K^+n cross sections. The broad shoulders in the η production and K charge-exchange and the bump in the π^0 charge-exchange cross sections near the forward direction always required magnitudes of the spin-flip residues comparable to those of the non-spin-flip residues. Certain general characteristics of the parameters are shared by all fits. The ρ trajectory is essentially unique, as is the $t = 0$ intercept of the R trajectory.

Case 1a. The two fits using the Chew mechanism without zeroes in the residue functions gave χ^2 values of 196 and 200. The major feature of these fits was that the slope of the R trajectory was quite small, $(\alpha'_R(0) \leqslant 0.5)$. This is presumed due to the absence of a dip in the $\pi^- p \to \eta\, n$ cross sections; the Chew mechanism predicts such a dip when $\alpha_R = 0$. Such a small slope, together with the more or less unique intercept $\alpha_R(0) \propto 0.5$, is inconsistent with a straight trajectory passing through the A_2 meson.

Case 1b. To explore the uniqueness of the R trajectory slope, zeros were placed in the residues ${}_K b^{\rho}_{++}$, ${}_\pi b^{\rho}_{++}$, ${}_K b^{R}_{++}$, and ${}_\eta b^{R}_{++}$, there being

some evidence from elastic scattering that ${}_{\pi}b^{\rho}_{++}$ $(t \approx -0.2) = 0$. Factorization then implies a zero in ${}_{K}b^{\rho}_{++}$ at the same place. The actual position of this zero was allowed to vary in the fits, but tended to remain around $t = -0.25$. With a zero also placed in ${}_{\eta}b^{R}_{++}$ (and thus in ${}_{K}b^{R}_{++}$ by factorization), slopes of up to 0.85 could be obtained for the R trajectory. The position of this zero tended to fall around $t = -0.1$. With the zeros, solutions having $\chi^2 = 198$ and 198 were obtained for $\alpha'_R = 0.85$; the value of α'_R held fixed. Without the zeros, and with α'_R held fixed at 0.7, solutions with high χ^2 of 212 and 218 were obtained. The mechanism through which the zeros allow the high R slope is a rather complicated interference effect, the zero in ${}_{\eta}b^{R}_{++}$ allowing a larger value of ${}_{\eta}f^{R}_{++}$ to compensate for the zero in ${}_{\eta}f^{R}_{+-}$ at $\alpha_R = 0$.

Thus it appears that such a basic parameter as the slope of the R trajectory cannot uniquely be determined, and is dependent on the choice of forms for the residue functions.

Case 2: Since the Gell-Mann mechanism predicts no dips for even trajectories, solutions with high values of $\alpha'_R(0)$ $(\approx 1.)$ were obtained both with and without zeros in the residue functions. The χ^2 were 203 and 204 for the no-zero case, and 197 and 197 for the zero cases.

Case 3: Since a zero at $\alpha_\rho = 0$ was placed in the ρ amplitude, large χ^2 values were obtained for the few points around $\alpha_\rho = 0$ in πN charge exchange. The χ^2 values for the solutions obtained were

of the order of 300, with $\alpha'_R = 1$. Since some sort of background effect would have to fill in the zero at $\alpha_\rho = 0$, the following functional form was added to the cross sections for $\pi^- p \to \pi^0 n$:

$$\sigma_{background} = (a + b\ t)\ s^\gamma\ ;$$

the zero at $\alpha_\rho = 0$ was thus removed. With this function, the fits gave χ^2 of 195 and 195 for residue functions with no zeros, and 197 and 196 for residue functions with zeros. The value of γ was usually about -2.0, with a and b varying according to the fit. There is obviously no justification for the form of the added function except that it crudely attempts to model effects of direct channel resonances or secondary trajectories which become important when the ρ contribution vanishes. We have neglected any phase relation there may be between this function and the amplitudes. Fit 1b is exhibited in Figs. 1 through 5. It is a typical fit to the data; all other plotted fits differ from it in very minor details. We also include the parameters involved in the fit 1b in Table I. In all fits the $\eta \to 2\gamma$ branching ratio has been taken as 0.35. In general most of the parameters contributing to the $t = 0$ amplitudes did not vary appreciably between fits. The main difference was usually in the t dependence of the residue functions.

IV. CONCLUSIONS

We have obtained statistically good fits to the reactions considered from a variety of significantly different assumptions about the residues and trajectories. In particular, although the slope of the ρ trajectory is well determined, that of the R remains uncertain within a factor of two, and the behavior of residues near the point $\alpha = 0$ is still a wide-open question. Experiments on $K^+n \rightarrow \bar{K}^0p$ and polarization measurements may eventually help clean up many of these uncertainties. However, at present exact deuteron corrections cannot be made for the K^+n data, and Regge pole polarization calculations are notoriously unreliable, since they can be greatly affected by small contributions from direct channel resonances or background terms. If, on the other hand, one is willing to make some hypothesis about the behavior of the residue and trajectory functions based on theoretical arguments, these ambiguities can be greatly diminished. For instance, if one is willing to take the "exchange degeneracy hypothesis", then the ρ and R trajectories should be essentially overlapping, and thus set 1a is ruled out as it predicts a fairly flat R trajectory. If one further assumes that, as indicated in potential theory, the residue functions cannot vanish, then alternative 1 is completely ruled out. If should be pointed out, however, that such vanishing of the nonflip residue functions seems to be demanded, in the case of the ρ trajectory, by the crossover of π^+p and π^-p elastic differential cross sections. Thus it is not unlikely that a similar vanishing could occur in the case of the R trajectory,

making alternative 1b possible.

An approximate exchange degeneracy would also imply that the ghost-killing mechanism is probably the same for both trajectories, so that alternative 2 could be excluded on these grounds, thus making case 3 the most likely one. However, this cannot be concluded from phenomenological examination of the data.

To summarize, a number of quite different types of behavior for the residue functions and trajectories remain open from this strictly phenomenological standpoint. More experiments are required before unique deductions become feasible.

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FOOTNOTES AND REFERENCE

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1. R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

2. V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

3. F. Arbab and C. Chiu, Phys. Rev. 147, 1095 (1966).

4. Saclay-Orsay Collaboration, (a) Phys. Rev. Letters 14, 763 (1965); (b) Phys. Letters 20, 75 (1966).

5. Saclay-Orsay Collaboration, Phys. Letters 18, 200 (1965); Michael Wahlig (private communication).

6. P. Astbury, et al., CERN Preprint 66/1057/5.

7. W. Galbraith et al., Phys. Rev. 138, B913 (1965).

8. Ling Lie Wang, Phys. Rev. 142, 1187 (1966).

9. Actually we should write

$$4\, p_t\, q_t \cos\theta_t = (4\, M\, E^{lab} + t + m_a^{\,2} - m_c^{\,2}) .$$

However we neglect the terms of order

t/E_{lab}, since $t \leqslant 1.5\ \mathrm{GeV}^2/c^2$.

10. We would like to thank Dr. L. L. Wang for explaining the different ghost-killing mechanism to us. The reader is also referred to her paper (UCRL-17053, 1966).

11. M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

12. G. F. Chew, Phys. Rev. Letters 16, 30 (1966).

13. M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

14. Actually, as the reader can easily see, these two choices are not the only possible ones. However other alternatives will only contribute further ambiguities to the calculation.

Table I. Parameters for Solution 1 of Case 1b.

$\chi^2 = 198$

${}_{\pi}b^{\rho}_{++}$	$= -6.24\ e^{0.25t}$ [a]	$\alpha_R(0)$	$= 0.50$
${}_{\pi}b^{\rho}_{+-}$	$= 8.17\ e^{0.21t}$	$\alpha'_R(0)$	$= 0.85$ [c]
${}_{\eta}b^{R}_{++}$	$= 10.21\ e^{1.44t}$ [b]	$\alpha_\rho(0)$	$= 0.57$
${}_{\eta}b^{R}_{+-}$	$= 14.61\ e^{0.77t}$ [b]	$\alpha'_\rho(0)$	$= 0.97$
${}_{K}b^{\rho}_{++}$	$= -3.42\ e^{0.03t}$	R zero	$= -0.09$
${}_{K}b^{R}_{++}$	$= 13.01\ e^{1.88t}$	ρ zero	$= -0.26$

a. Notice that some exponential t dependence is in the $E^{\alpha(t)}$ factor.

b. Assumes $\eta \to 2\gamma$ branching ratio of 0.35.

c. Held fixed in this fit.

FIGURE CAPTIONS

1. Fit 1b to the $\pi^- p \to \pi^0 n$ data at several energies.

2. Fit 1b to the $\pi^- p \to \eta\, n$ data at several energies.

3. Fit 1b to the $K^- p \to \bar{K}^0 n$ data at several energies.

4. Differential cross sections for $K^+ n \to K^0 p$.

5. Fit 1b to the difference of total cross sections of several processes.

   Top: $\sigma_{total}\,(\pi^- p) - \sigma_{total}\,(\pi^+ p)$

   Middle: $\sigma_{total}\,(K^- p) - \sigma_{total}\,(K^- n)$

   Bottom: $\sigma_{total}\,(K^+ n) - \sigma_{total}\,(K^+ p)$

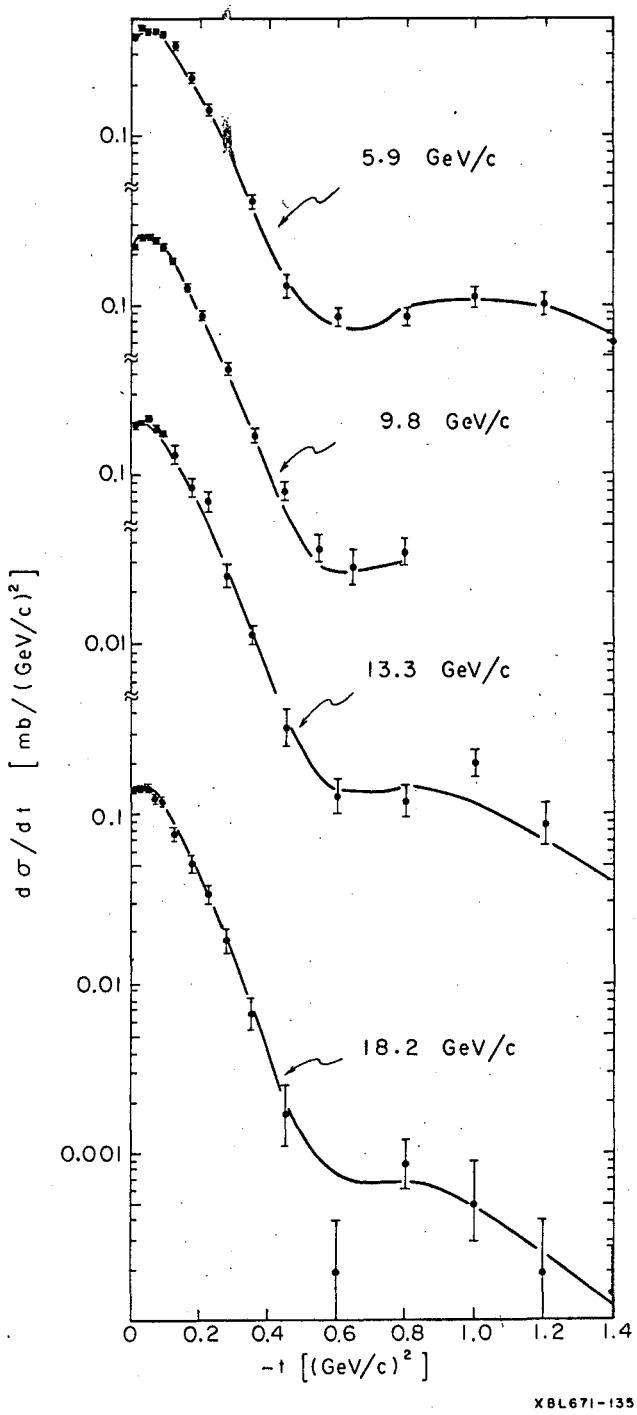
5.9 GeV/c
9.8 GeV/c
13.3 GeV/c
18.2 GeV/c
0.1
0.1
0.1
0.01
0.1
0.01
0.001
dσ/dt [mb/(GeV/c)²]
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
-t [(GeV/c)²]
XBL671-135

Fig. 1

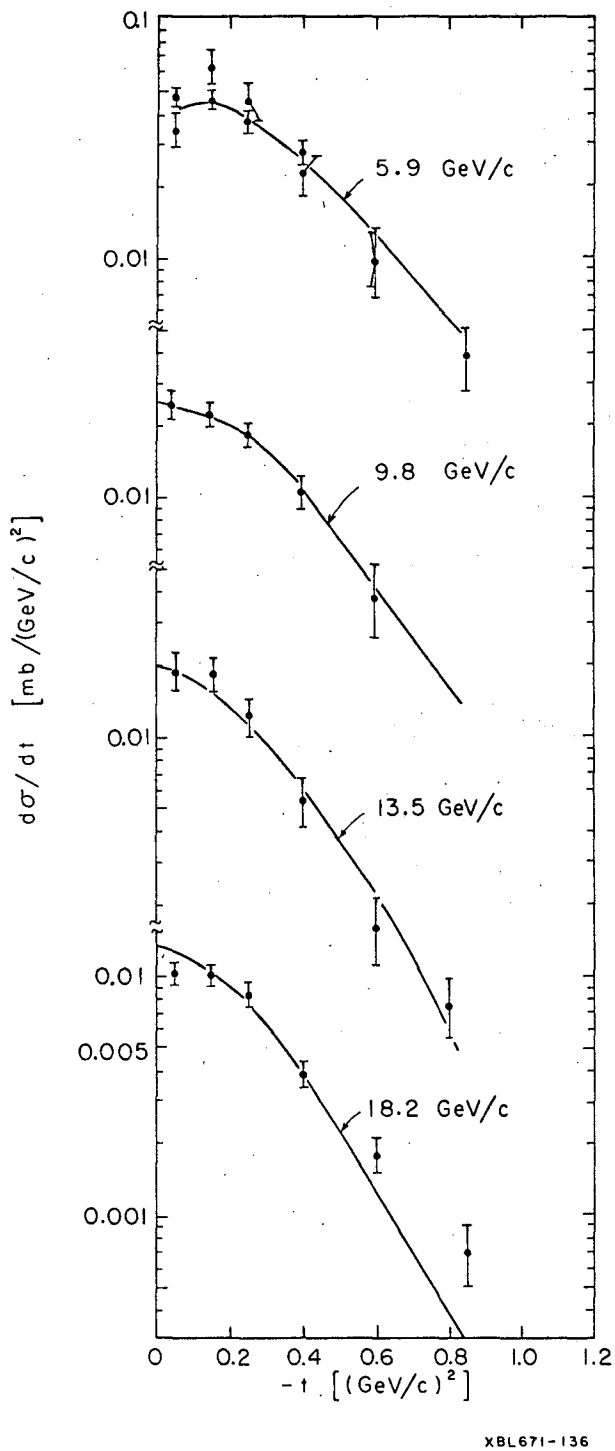
0.1
0.01
0.01
0.01
0.01
0.005
0.001
5.9 GeV/c
9.8 GeV/c
13.5 GeV/c
18.2 GeV/c
dσ/dt [mb/(GeV/c)²]
0
0.2
0.4
0.6
0.8
1.0
1.2
-t [(GeV/c)²]
XBL671-136

Fig. 2

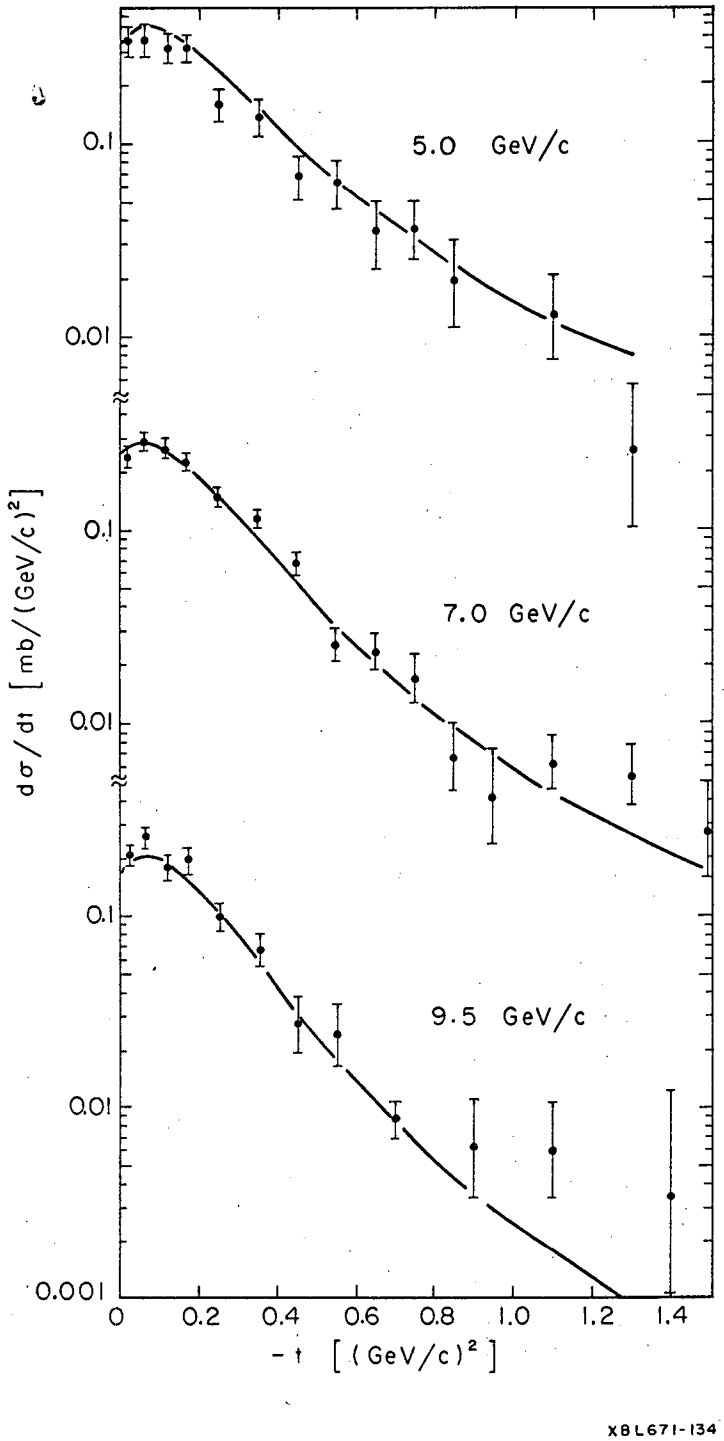
5.0 GeV/c
7.0 GeV/c
9.5 GeV/c
0.1
0.01
0.001
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
dσ/dt [mb/(GeV/c)²]
-t [(GeV/c)²]
XBL671-134

Fig. 3

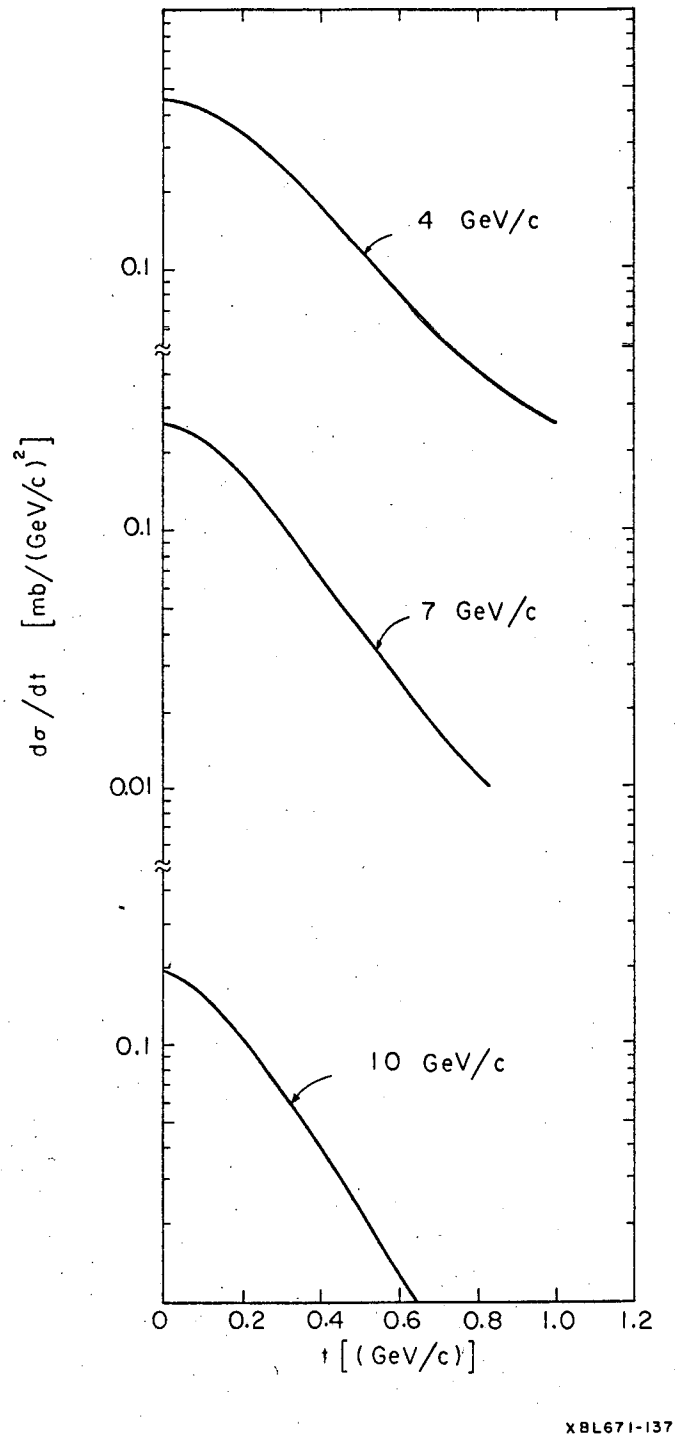
4 GeV/c
7 GeV/c
10 GeV/c
0.1
0.1
0.01
0.1
dσ/dt [mb/(GeV/c)²]
0
0.2
0.4
0.6
0.8
1.0
1.2
t [(GeV/c)]
XBL671-137

Fig. 4

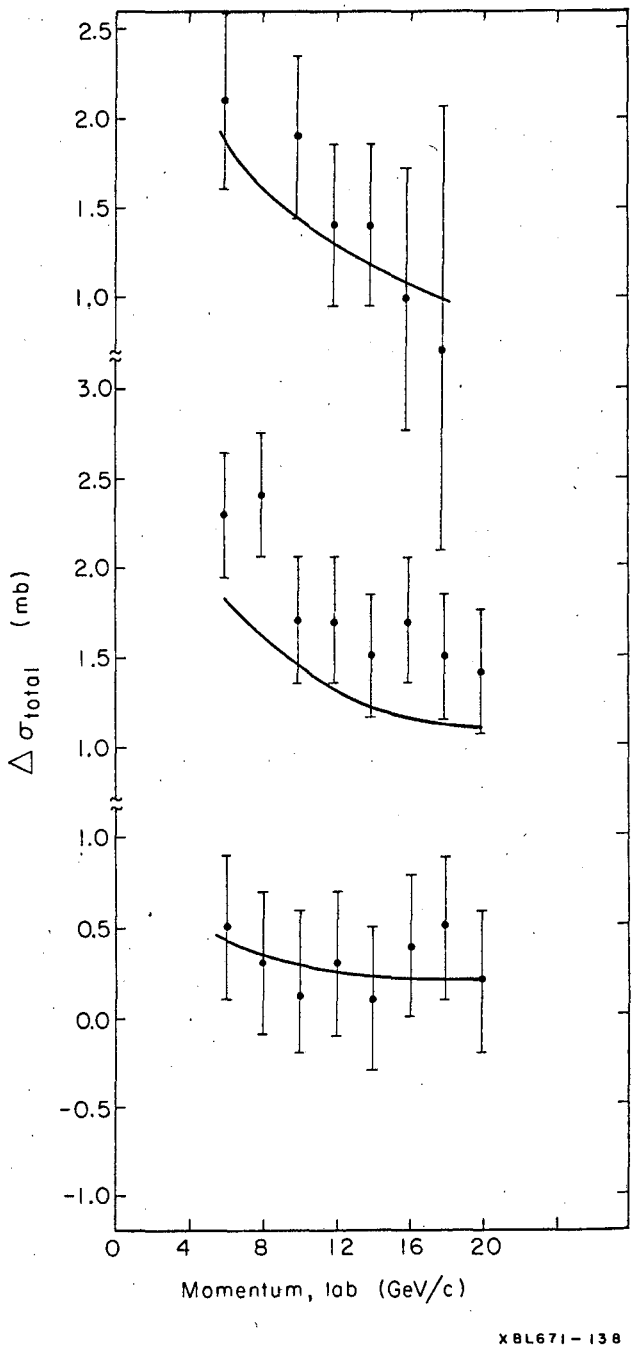
2.5
2.0
1.5
1.0
3.0
2.5
2.0
1.5
1.0
1.0
0.5
0.0
-0.5
-1.0
$\Delta\sigma_{total}$ (mb)
0
4
8
12
16
20
Momentum, lab (GeV/c)
XBL671-138

Fig. 5

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