UCRL-17351 errata

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EXPERIMENTAL STUDY OF $K^+ \to \pi^0 + e^+ + \nu$ DECAY

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June 1, 1967

ERRATUM

TO: All recipients of UCRL-17351

FROM: Technical Information Division

SUBJECT: UCRL-17351, "Experimental Study of $K^+ \rightarrow \pi^0 + e^+ + \nu$ Decay," George E. Kalmus and Anne Kernan, January 26, 1967

Please change the subject report as indicated.

Submitted to Physical Review

UCRL-17351
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

EXPERIMENTAL STUDY OF $K^+ \rightarrow \pi^0 + e^+ + \nu$ DECAY

George E. Kalmus and Anne Kernan

January 26, 1967

EXPERIMENTAL STUDY OF $K^+ \to \pi^0 + e^+ + \nu$ DECAY*

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January 26, 1967

ABSTRACT

An analysis of 515 decays $K^+ \to \pi^0 e^+ \nu$ in a freon-filled bubble chamber is described. The events were kinematically overdetermined so that a complete reconstruction of each decay was possible. All decay configurations were detectable except for those with positron momentum less than 5 MeV/c. The data is satisfactorily described by a pure vector interaction. The energy dependence of the vector form factor f_+ is given by $|f_+(q^2)| = |f_+(0)| \frac{M^2}{M^2 - q^2}$, with $M = 810^{+320}_{-140}$ MeV, or equivalently, $|f_+(q^2)| = |f_+(0)| [1 + \lambda (q^2/m_\pi^2)]$, with $\lambda = 0.028^{+0.013}_{-0.014}$. In the Callan-Treiman relationship for $K_{\ell 2}$ and $K_{\ell 3}$ decays this value of λ leads to the prediction that $\xi [\equiv \frac{f_-(0)}{f_+(0)}] = +0.43 \pm 0.14$ or -2.43 ± 0.14. Upper limits for scalar and tensor currents are given. Systematic biases inherent in the heavy-liquid-chamber technique are discussed.

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Professors W. J. Willis and R. J. Oakes have called our attention to an error in our interpretation of Callan and Treiman's paper. This error does not affect the results reported in the paper.

Section VI in the paper should be replaced as follows:

VI. CURRENT ALGEBRA PREDICTION FOR ξ

It is interesting to consider the q^2 dependence of f_+ in relation to the current algebra formula of Callan and Treiman,[9] which connects the $K_{\ell 2}$ and $K_{\ell 3}$ decay form factors, evaluated at zero four-momentum of the pion:

$$|f_K| = |2g_A\, m/g_r\, M_K|\ |f_+(M_K^2, 0) + f_-(M_K^2, 0)| \qquad (12)$$

In this expression g_A is the ratio of axial vector to vector β-decay coupling constants, g_r is the strong pion-nucleon coupling constant, and m is the nucleon mass. The experimental value[5] for f_K, which is proportional to the amplitude for $K_{\mu 2}$ decay, is 0.070 ± 0.001. The quantities $f_+(q^2, P_\pi^2)$ and $f_-(q^2, P_\pi^2)$ are the $K_{\ell 3}$ form factors defined in Sec. II; they depend on the variables $q^2 = (P_K - P_\pi)^2$ and P_π^2. Since P_π^2 equals m_π^2 and is not experimentally variable, the dependence of f_+ and f_- on this quantity is usually not shown explicitly. We rewrite Eq. (12)

$$|f_K| = 0.32\ |f_+(M_K^2, m_\pi^2)|\ |1 + \xi(M_K^2, m_\pi^2)|, \qquad (13)$$

where $\xi = f_-(M_K^2, m_\pi^2)/f_+(M_K^2, m_\pi^2)$. By evaluating f_+ and f_- at $P_\pi^2 = m_\pi^2$, we introduce an uncertainty of 10% in the relationship.[10]

Using the parameterization $f^+(q^2) = f^+(0)(1 + \lambda q^2/m_\pi^2)$, valid for $\lambda \lesssim 0.1$, one obtains

$$f_+(M_K^2, m_\pi^2) = 8.4\times 10^{-5}[\Gamma_{Ke3}]^{1/2}(1 + 11.7\lambda), \qquad (14)$$

where Γ_{Ke3} is the K_{e3} decay rate[5] in sec^{-1}; inserting (14) into (13) gives the relationship between $|\xi + 1|$ and λ displayed in Fig. 12(a). For $\lambda = 0.028$, as measured in this experiment, a value of $|\xi + 1| = 1.0 \pm 0.1$ is predicted by the Callan-Treiman relation; the error is the 10% uncertainty alluded to above. The circle in Fig. 12(b) shows the allowed range of values for complex ξ. A recent measurement of the transverse polarization in the $K^0_{\mu 3}$ decay gave Im $\xi = -0.0125 \pm 0.059$.[11] If we assume that ξ is purely real, then the current algebra relation predicts $\xi(M_K^2, m_\pi^2) = 0.0 \pm 0.1$ or 2.0 ± 0.1.

At present there are two incompatible measurements of ξ; $\xi = +0.27 \pm 0.23$ and $\xi = -1.25 \pm 0.3$. The former value was obtained from the momentum and angular distributions in $K_{\mu 3}$ decay combined with the $K_{\mu 3}/K_{e3}$ decay-rate ratio (method 1); the latter value was deduced from the measured polarization of the muon in $K_{\mu 3}$ decay (method 2). (These values are weighted means of the data in Ref. 6.)

Both values of ξ were obtained by analyzing the experimental data under the assumption that $f_-(q^2)/f_+(q^2)$ is constant. It is known that method 1 gives an incorrect value of ξ if this assumption is incorrect. For example, the value of $\xi = 0.72$ in Callahan et al.[12] is changed to 0.25 for $f_-(q^2)/f_+(q^2) = f_-(0)/[f_+(0)(1 + 0.028\, q^2/m_\pi^2)]$, i.e., assuming our measured value for λ and constant f_-.

The predicted values of $\xi = 0.0 \pm 0.1$ or -2.0 ± 0.1 from the Callan-Treiman relationship should also be substituted for the values quoted in the paper in the Abstract (p.1) and in the Conclusion p. 17 Section VII.4.

Finally new figure 12, (a) and (b), should replace old Fig. 12, and the figure legend should read:

Fig. 12 (a). The relationship between $|\xi + 1|$ and λ from Eqs. (13) and (14) as given by Callan and Treiman. The solid line is calculated for the $K_{\mu 2}$ and K_{e3} decay rates given in Ref. 5. The broken lines correspond to a $\pm 10\%$ uncertainty in the relationship. The shaded area indicates the range of values of $|\xi + 1|$ allowed by our measurement of $\lambda = 0.028 \pm {}^{+0.013}_{-0.014}$.

(b) The circles show the ξ values corresponding to the two values of $|\xi + 1|$ indicated by X_1 and X_2 in Fig. 12a.

structure of the form factors. Inspection of Eq. (3) shows that, for a pure vector interaction and setting $m_e = 0$, the q^2 dependence of f_+ can be inferred from the pion kinetic-energy distribution $\frac{dN}{dT_\pi} \propto f_+ p_\pi^3 dT_\pi$. Most theories predict that the form factors are slowly varying functions of q^2, so that a parameterization of the form

$$f_+(q^2) = f_+(0)(1 + \lambda q^2/m_\pi^2) \tag{4}$$

is reasonable. Alternatively, a parameterization of the form

$$f_+(q^2) = f_+(0) \frac{X}{X - q^2/m_\pi^2} \tag{5}$$

may be used, where $M(\equiv X^{1/2} m_\pi)$ is the mass of a $J = 1$, $I = 1/2$ intermediate K^* state. If X is found to be negative experimentally, the single-pole dominance assumption is not valid. For $|\lambda| \lesssim 0.1$, the expressions (4) and (5) are equivalent with $\lambda = 1/X$.

All the present evidence on muon capture and decay, and on beta decay of the free neutron and nuclei, suggests that the strangeness-conserving currents have a purely V, A character. For strangeness-changing currents as in $K_{\ell 2}$ and $K_{\ell 3}$ decay, the experimental data is less extensive, but it is also in satisfactory agreement with a purely V and A weak interaction.

The experiments on K_{e3} decay which have been published in final form are given in the list of References.[3] All these experiments are in satisfactory agreement with a purely vector interaction for K_{e3} decay. In addition, preliminary accounts of several K_{e3} studies have been reported at the Argonne Weak Interaction Conference, and the

XIII High-Energy Physics Conference at Berkeley.[4] The experimental results in references 3 and 4 are summarized in the proceedings of the aforementioned conferences.[5, 6]

III. EXPERIMENTAL DETAILS

The K^+ mesons were stopped in a chamber filled with freon C_3F_8 with a density of 1.22 g/cm^{-3}, and radiation length of 28 cm. A total of 240 000 pictures containing 3×10^6 stopped K^+'s were taken.

A. Scanning

A total of 70 000 pictures were scanned for the decay sequence

$$K^+ \rightarrow \pi^0 + e^+ + \nu$$
$$\hookrightarrow 2\gamma \rightarrow 2(e^+ + e^-) \; . \qquad (6)$$

The topology of the event is shown in Fig. 1.

The following scanning criteria were applied:

1. The track of the charged secondary from K^+ decay ended within the chamber, or it curled up sufficiently so that the vector R (Fig. 1) passed through a maximum value. In either case the track had minimum ionization.

2. Two electron-positron pairs pointed back to the K^+ decay vertex in all three stereo views, and the angle Ψ (in space) between them was greater than 45 deg. [The minimum value for Ψ in Reaction (6) is 61.6 deg.]

3. Neither of the pairs was tangential to the decay positron.

4. The K^+ ionization was consistent with a decay at rest, and

there were no kinks or changes of ionization along the K^+ track within 1 cm of the decay point.

Condition 1 ensured that the charged secondary was unambiguously identified as a positron. Condition 3 eliminated events in which one of the pairs was radiated by the decay positron. This criterion eliminated only 0.1% of genuine K_{e3} decays. Condition 4 was designed to reduce contamination by τ' and $K_{\mu 3}$ decays for which the $\pi^+\mu^+$ chain or the μ^+, respectively, were less that 1 cm in length. Background from this source is estimated at less than 1%.

The data was also contaminated by K_{e3} decays in flight in the K^+ momentum interval 0 to 200 MeV/c. Above 200 MeV/c, K^+ decays in freon can be identified by ionization of the primary track. The Monte Carlo program, described in Sec. IV, showed that for a pure vector interaction only 1-1/2% of these decays would fit the K^+_{e3} decay at rest hypothesis, this amounts to 8 events in our experiment. Subtraction of these events from the experimental data left our conclusions unchanged.

B. Measurement and Reconstruction

For each event the tracks of the decay positron and the two electron pairs were measured on a Franckenstein. Geometrical reconstruction and kinematical fitting were done with FOG-CLOUDY programs.

The events were kinematically overdetermined. The fit to the decay sequence (6) can be done in two ways: From the measured momenta of the two electron pairs the π^0 momentum can be obtained by a one-constraint fit to the reaction $\pi^0 \rightarrow \gamma + \gamma$. Then a one-constraint

fit to the process $K^+ \rightarrow \pi^0 e^+ \nu$ can be made using the calculated pion momentum and the measured decay positron momentum. Alternatively, a simultaneous two-vertex fit to the complete decay sequence (6) can be made. Both procedures were tried and gave the same answer.

Of 630 events processed through the geometry and kinematics programs, 529 fitted the K_{e3} decay hypothesis. Events in which the dip angle of the decay positron with respect to the film plane exceeded 60 deg, or the positron momentum was less than 5 MeV, were removed because of large measurement errors and possible scanning bias associated with these configurations. There remained 515 events which satisfied the selection criteria and fitted the K_{e3} hypothesis.

The estimation of electron momentum from curvature is complicated by the bremsstrahlung process. We used the prescription of Behr and Mittner:[7]

$$p = \frac{0.3\, B\, L^2}{8(\cos\delta)S} \left[1 + \frac{L y_0}{2 x_0 \log 2 (\cos\delta)} \right] \tag{7}$$

$$\frac{\Delta(1/p)}{1/p} = \left[\frac{L y_0^2}{6(\log 2)\, x_0 (\cos\delta)} + \left(\frac{42}{0.3B}\right)^2 \frac{\cos\delta}{6 L x_0} \right]^{1/2} \tag{8}$$

where p = probable momentum in MeV/c, B = magnetic field in kilogauss, L = projected chord length in cm, δ = dip of the track from the plane of the window, S = sagitta at the midpoint of L in cm, x_0 = radiation length in cm, and $y_0 = \log(E/E - h\nu_{max})$, where E is the energy of the electron at any point along its track. This formula applies under two conditions:

1. The length L does not exceed an optimum length L_0, which minimizes $\Delta(1/p)$.

2. The track is not measured beyond the point at which a fractional

bremsstrahlung loss $h\nu_{max}/E$ or a noticeable kink has occurred. The length L depends on the radiation length of the chamber liquid, and is $\approx$13 cm in this experiment. All track points measured beyond L_0 were discarded in the geometry program. We scrutinized a photograph of each event for radiation losses $h\nu_{max}/E$ greater than one half along the electron and positron tracks, and marked off the length of track to be measured.

Formulas (7) and (8) were tested by reconstructing the π^0-momentum in the $K^+_{\pi 2}$ decay sequence:

$$K^+ \rightarrow \pi^+ + \pi^0$$
$$\hookrightarrow 2\gamma \rightarrow 2(e^+ + e^-).$$

We measured the pair momenta $p_{\pi^0} = p_{\gamma 1} + p_{\gamma 2}$ for 162 $K^+_{\pi 2}$ decays; we found that the average π^0 momentum was 10% higher than the expected value of 205 MeV/c, and that $\Delta(1/p)$ was also overestimated. This was rectified by changing $h\nu_{max}/E$ from 1/2 to 1/3 in the expression $y_0 = \log(E/E - h\nu_{max})$. We concluded that we had been overzealous in eliminating large bremsstrahlung losses.

IV. EXPERIMENTAL BIASES

We now consider biases due to (a) variation of the detection efficiency $D(E_e, E_\pi)$, and (b) large measurement errors.

A. Detection Efficiency $D(E_e, E_\pi)$

The detection efficiency $D(E_e, E_\pi)$ is the probability that a K^+ decay with configuration (E_e, E_π) satisfies the scanning criteria in

Sec. III.A. The decay-positron detection efficiency for events satisfying the selection criteria varied from 100% at a few MeV to 40% at the maximum energy. The pion-detection efficiency ranged from 6.8 to 8.4%. The Monte-Carlo program used to evaluate $D(E_e, E_\pi)$ at 54 values of (E_e, E_π) is described below.

A total of 2200 fake K^+ decays was generated in the bubble chamber for each (E_e, E_π) configuration. The decay points were assigned according to the actual distribution of stopping K^+ mesons in the chamber. The spatial orientation of the event was randomized. For each event the electron was caused to undergo a series of random energy losses and changes in direction, through scattering and bremsstrahlung, until (a) it left the bubble chamber, (b) it ended in the chamber, or (c) the radius vector R from the K^+ decay vertex passed through a maximum. In case (a) $f = 0$, where f is the probability that the K^+ decay satisfies the selection criteria. In cases (b) and (c), the decay $\pi^0 \rightarrow \gamma_1 + \gamma_2$ was randomly generated, and the probability f_γ of each photon converting within the fiducial volume was evaluated analytically, giving $f = f_{\gamma 1} f_{\gamma 2}$. Then $D(E_e, E_\pi)$ equalled $\Sigma f/2200$.

The function $D(E_e, E_\pi)$ was obtained by fitting a fifth-order 2-dimensional polynomial to the 54 values of $D(E_e, E_\pi)$ determined in the Monte Carlo program.

B. Measurement Errors

If measurement errors were negligible, the true distributions could be obtained by giving each experimental event a weight $1/D(E_e, E_\pi)$. In general, measurement errors cause a broadening of the experimental distributions. We find in this experiment that they also caused systematic shifts in the distribution of the constrained variables.

These effects were studied empirically for π^0 reconstruction by measuring π^0 mesons in $K^+ \to \pi^+ + \pi^0$ decay. We found that the π^0 momentum was reconstructed correctly.

A total of 162 events fitting the decay sequence

$$K^+ \to \pi^+ \pi^0 \tag{9}$$

$$\hookrightarrow \pi^0 \to 2\gamma \to 2(e^+e^-) \tag{10}$$

was measured. The π^0 momentum was uniquely 205 MeV/c and $\theta_{\pi^+\pi^0}$ was 180 deg. Figure 2 shows (a) the π^0 momentum and (b) $\theta_{\pi^+\pi^0}$ both calculated from a one-constraint fit to reaction (10) of the measured electron and positron momenta and photon angles. The median value of π^0 momentum was 209 MeV/c, with an average error of ≈10%; we found that $\theta_{\pi^+\pi^0}$ peaked at 180 deg with an error of 6 deg. The relatively small errors are due to the negligible errors in the photon angles.

A Monte-Carlo program was used to examine $K^+ \to e^+ + \pi^0 + \nu$ reconstruction. The constrained lepton momenta in K_{e3} decay were found to be seriously distorted from the true values. This is not surprising in a one-constraint event with large measurement errors.

In this experiment, the K^+ decay was reconstructed by a one-constraint fit to the hypothesis $K^+ \to e^+\pi^0\nu$, using the measured positron momentum and the fitted π^0 momentum. The effect of measurement errors on the K^+ reconstruction was studied by generating 20 000 K_{e3} decays, with a pure vector interaction and a constant form factor. The positron and π^0 variables were assigned a typical measurement error and were shifted according to a Gaussian error

distribution. The modified variables were constrained to the K_{e3} hypothesis. We found that the contrained variables were systematically shifted from their original values.

The constraining procedure for the Monte-Carlo events was considerably simpler than the CLOUDY program used for the experimental data. The input quantities in CLOUDY for the $K^+ \rightarrow e^+ + \pi^0 + \nu$ fit are the vector momenta of the positron and π^0, expressed as p, α, and β, the scalar momentum, and polar and azimuthal angles. CLOUDY makes a least-squares adjustment of these parameters so as to satisfy the four-constraint equations of energy and momentum conservation. We discarded the inessential information on the spatial orientation of the decay, and used only the quantities p_e^m, p_π^m, and $\theta_{e\pi}^m$ and their errors, where the superscript m denotes a measured quantity. The single-constraint equation can be written as

$$\cos\theta_{e\pi} = \left[M_K^2 + m_\pi^2 - 2M_K(E_e + E_\pi) + 2E_eE_\pi\right]/2p_ep_\pi. \tag{11}$$

The least-squares solution is the (p_e, p_π) values which minimized the function

$$F(p_ep_\pi) = \left(\frac{p_e - p_e^m}{\Delta p_e^m}\right)^2 + \left(\frac{p_\pi - p_\pi^m}{\Delta p_\pi^m}\right)^2 + \left(\frac{\theta_{e\pi}(p_e, p_\pi) - \theta_{e\pi}^m}{\Delta\theta_{e\pi}^m}\right)^2 ,$$

where $\theta_{e\pi}(p_e, p_\pi)$ is given by Eq. (11).

The FORTRAN IV program SIPMIN search for the minimum of $F(p_e, p_\pi)$. It required approximately 7 minutes to generate and constrain 20 000 events on the CDC 6600 computer.

As a check, the experimental events were also constrained with this program. Because the constrained variables were identical with those calculated in CLOUDY, the equivalence of the two minimizing routines was established.

In Fig. 3 the lepton-momentum distributions for 20000 K^+_{e3} decays with vector interaction are compared with the corresponding distributions after error folding and constraint. The distributions of the constrained variables are clearly shifted from the original distributions, and the constrained electron-momentum plot is different from the constrained neutrino-momentum plot. In a vector interaction the lepton distributions are identical; a real difference between the electron and neutrino momentum spectra would indicate a scalar-tensor interference term in the matrix element. Measurement errors can thus simulate a scalar-tensor interaction term in the decay.

V. ANALYSIS OF THE EXPERIMENTAL DATA

A total of 515 events satisfied the selection criteria and fitted the K^+_{e3} hypothesis. Figure 4 shows the experimental distribution in $\cos\alpha$, T_{π^0}. The experimental distributions in $\cos\alpha$, T_{π^0}, p_e, and p_ν are shown in Figs. 5 through 8, respectively.

To test a given hypothesis, we compared the experimental (N_{iE}) and predicted (N_{ip}) Dalitz-plot densities in ($\cos\alpha$, T_π) and computed

$$\chi^2 = \sum_{i=1}^{20} \frac{(N_{ip} - N_{iE})^2}{N_{ip}}$$

where the experimental events were combined into 20 boxes as shown in Fig. 4. (N_{ip} was obtained, as described below, from the 20,000 Monte Carlo generated events.) No information on the K_{e3} decay rate was used in the analysis.

As described in Sec. IV, the experimental parameters are shifted from the true values due to various systematic effects. In order to make a comparison between theory and experiment, it was first necessary to distort the theoretical distribution in a fashion similar to the experimental ones. This was accomplished by the method described in Sec. IV and summarized below.

A total of 20 000 K^+_{e3} decays was generated by a Monte-Carlo method for a distribution function $\propto dE_e\ dE_\pi$. Events with $P_e < 5$ MeV/c were rejected so as to conform to the selection criteria. The kinematical variables were shifted according to the known measurement errors, and each event was then constrained to the K^+_{e3} decay hypothesis.

The Dalitz-plot density for a given hypothesis was then obtained by assigning to each constrained event a weight $N(E_e, E_\pi)\ D(E_e, E_\pi)$, where $N(E_e E_\pi)\ dE_e\ dE_\pi$ is the distribution function for the hypothesis under consideration, $D(E_e, E_\pi)$ is the detection efficiency (Sec. IV), and E_e, E_π are the variables of the generated event. By this means computer time was minimized, since any number of hypotheses could be tested with a single set of 20 000 constrained Monte Carlo events by changing the function $N(E_e E_\pi)$.

The calculated Dalitz-plot density in $(\cos\alpha,\ T_{\pi^0})$ for each hypothesis was normalized to a total of 515 events, and the theoretical distribution was then in a form suitable for comparison with experiment. Hereafter we shall refer to these adjusted normalized theoretical distributions as "fake" distributions.

In Fits 1 through 4, the vector form factor had an energy dependence $|f_+| = |f_+(0)|X/[X-q^2/m_\pi^2]$ as in (5), and the scalar and tensor form

factors f_S and f_T were considered constant. The comparison of the data with the different hypotheses is described below.

Fit 1 - Pure V interaction

A fake distribution was generated according to Eqs. (3) and (5), with $|f_S| = |f_T| = 0$, and $1/X$ in the range -0.12 to + 0.08. The fake distribution for each value of X was compared with the experimental distribution in Fig. 4, and χ^2 was computed. Figure 9 shows χ^2 as a function of $M (\equiv X^{1/2} m_\pi)$. The best agreement between theory and experiment occurs when $M = 810^{+320}_{-140}$ MeV. The upper and lower limits correspond to one standard deviation ($\chi^2_{min} + 1$). The 95% confidence band for M is 580 to ∞ MeV.

The probability for this fit is 0.10, corresponding to a χ^2 minimum of 26.0 for 18 degrees of freedom. The data is thus satisfactorily desecribed by a pure vector interaction.

Below, we describe the fits to the data with various combinations of V, S, and T. However, we emphasize that a good fit to the data is obtained with a pure vector interaction.

Fit 2 - V and S interaction

A total of 20 000 fake events was generated according to Eqs. (3) and (5) with $|f_T| = 0$, $|f_S|/|f_+|$ varying between 0 - 0.35 and $1/X$ in the same range as Fit 1. Figure 10 shows χ^2 as a function of $|f_S|/|f_+|$ and $1/X$.

The best fit to the data occurred for $|f_S/|f_+| = 0.2$, $1/X = -0.028$, with a probability of 13%.

Agreement between theory and experiment is thus slightly improved by inclusion of some scalar interaction.

If the interaction is V and S, the maximum value of $|f_S|/|f_+|$ compatible with the data is 0.30 at the 95% confidence level.

Fit 3 - V and T interaction

The Dalitz plot in (cos α, T_π) was generated according to Eqs. (3) and (5) with $|f_S| = 0$, $|f_T|/|f_+|$ varying between 0 and 1.1, and 1/X in the same range as before. These fake distributions were compared with the experimental distribution in the manner previously described; the resulting two-dimensional plot in χ^2 is displayed in Fig. 11. The χ^2 minimum at $|f_T|/|f_+| = 0$ and $1/X = +0.028$ shows that the fit to the data is not improved by adding a tensor component. For a V and T interaction, the upper limit for $|f_T|/|f_+|$ is 1.1 at the 95% confidence level.

Fit 4 - V, S, and T interaction

A fit to the data was made by varying X, $|f_S|/|f_+|$, and $|f_T|/|f_+|$ simultaneously. For fixed X and $|f_S|/|f_+|$, χ^2 was always minimized by $|f_T| = 0$.

Fit 5 - Pure V interaction and two-pole form factor

Lastly we fitted the data with a pure vector interaction in which the form factor was parameterized as

$$|f_+(q^2)| = |f_+(0)| \left[\frac{C_* M_*^2}{M_*^2 - q^2} + \frac{C_{**} M_{**}^2}{M_{**}^2 - q^2} \right]$$

as suggested by Dennery and Primakoff,[8] here M_* and M_{**} are the masses of two $J^P = 1^- K^*$ states. Values of C_*/C_{**} ranging from ±0.5 to ±10 were used, and M_* and M_{**} were varied from 3 m_π to 10m_π. We calculated χ^2 at 72 points within these limits; in no case was the fit improved over Fit. 1.

In comparing theory and experiment, we did not find it necessary to make any assumptions about the role of time-reversal invariance in weak interactions. If TRI is violated, the VS, VT, and ST interference terms in the matrix element are multiplied by a phase factor. However, for the values of $|f_S|/|f_+|$ and $|f_T|/|f_+|$ found in the experiment the interference terms are negligible, whatever the phase factor. This was verified by a Monte Carlo calculation. Since the VS, VT, and ST terms are very small, K_{e3} decay is sensitive only to the absolute values of f_V, f_S, and f_T [as is clear from Eq. (3)].

VI. CURRENT ALGEBRA PREDICTION FOR $f_-(0)/f_+(0)$

It is interesting to consider the q^2 dependence of f_+ in relation to the current algebra formula of Callan and Treiman[9] connecting the $K_{\ell 2}$ and $K_{\ell 3}$ decay form factors:

$$|f_K| = |2\, g_A m/g_r\, M_K|\,|f_+(0,0) + f_-(0,0)|. \tag{12}$$

In this expression g_A is the ratio of axial vector to vector β-decay coupling constants, g_r is the strong pion-nucleon coupling constant, and m is the nucleon mass. The experimental value for f_k, which is proportional to the amplitude for $K_{\mu 2}$ decay, is 0.70 ± 0.01 (Ref. 6). The quantities f_+ and f_- are the $K_{\ell 3}$ form factors defined in Sec. II; they depend on the variables $q^2 = (P_K - P_\pi)^2$ and m_π^2. Since m_π^2 is not experimentally variable, the dependence of f_+ and f_- on this quantity is usually not shown explicitly. We rewrite Eq. (12)

$$|f_K| = 0.32\, |f_+(0, m_\pi^2)|\,|1 + \xi|, \tag{13}$$

where $\xi = f_-(0, m_\pi^2)/f_+(0, m_\pi^2)$. By evaluating f_+ and f_- at $m_\pi^2 = m_\pi^2$, we introduce an uncertainty of 10% in the relationship.[10]

Using the parameterization $f^+(q^2) = f^+(0)\,(1 + \lambda q^2/m_\pi^2)$, valid for $|\lambda| \lesssim 0.1$, one obtains

$$f_+(0, m_\pi^2) = 8.4\times 10^{-5}\left[\frac{\Gamma_{K_{e3}}}{1+\lambda\, 3.3}\right]^{1/2}, \tag{14}$$

where $\Gamma_{K_{e3}}$ is the K_{e3} decay rate[5] in $\sec^{-1}$; inserting (14) into (13) gives the relationship between $|\xi + 1|$ and λ displayed in Fig. 12(a). For $\lambda = 0.028$, as measured in this experiment, a value of $|\xi + 1| = 1.43 \pm 0.14$ is predicted by the Callan Treiman relation; the error is the 10% uncertainty alluded to above. The circle in Fig. 12(b) shows the allowed range of values for complex ξ. A recent measurement of the transverse polarization in the $K^0_{\mu 3}$ decay gave $\mathrm{Im}\,\xi = -0.0125 \pm 0.059$.[11] If we assume that ξ is purely real, then the current algebra relation predicts $\xi = 0.43 \pm 0.14$ or -2.43 ± 0.14.

At present there are two incompatible measurements of ξ; $\xi = +0.27 \pm 0.23$ and $\xi = -1.25 \pm 0.3$. The former value was obtained from the momentum and angular distributions in $K_{\mu 3}$ decay combined with the $K_{\mu 3}/K_{e3}$ decay rate ratio (method 1); the latter value was deduced from the measured polarization of the muon in $K_{\mu 3}$ decay (method 2). (These values are weighted means of the data in Ref. 8.)

Both values of ξ were obtained by analyzing the experimental data under the assumption that $f_-(q^2)/f_+(q^2) = f_-(0)/f_+(0)$. It is known that method 1 gives an incorrect value of ξ if this assumption is incorrect. For example, the value of $\xi = 0.72$ in Callahan et al.[12] is changed to 0.25 for $f_-(q^2)/f_+(q^2) = f_-(0)/[f_+(0)\,(1 + 0.028\, q^2/m_\pi^2)]$, i.e., assuming our measured value for λ and constant f_-.

The value of ξ predicted by the Callan-Treiman relation with $\lambda = 0.028$ is in clear disagreement with the value of ξ obtained from polarization measurements.

VII. CONCLUSIONS

An analysis of 515 decays $K^+ \to \pi^0 e^+ \nu$ gave the following results:

1. The experimental data is satisfactorily described by a pure vector interaction with a form factor $|f_+(q^2)| = |f_+(0)| M^2/M^2 - q^2$ and $M = 810^{+320}_{-140}$ MeV. Equivalently, $|f_+(q^2)| = |f_+(0)|(1 + \lambda q^2/m^2_\pi)$, and $\lambda = 0.028^{+0.013}_{-0.014}$. The data does not require a two-pole fit. The K^+_{e3} decay is then in good agreement with the V-A theory of weak interactions.

2. The energy dependence of f_+ is compatible with previous reported estimates in K^+_{e3} and K^0_{e3} decay. The $|\Delta \underset{\sim}{I}| = 1/2$ leptonic rule requires that f_+ be the same for K^+_{e3} and K^0_{e3} decay.

3. Tests for scalar and tensor currents gave the following upper limits at the 95 % confidence level:

$\frac{|f_S|}{|f_+|} < 0.30$ and $\frac{|f_T|}{|f_+|} < 1.1$, where f_+, f_S, and f_T are defined in (2).

The kinematics of K_{e3} decay are insensitive to TRI violation because of the dominance of the vector current with a single form factor f_+.

4. We find that our value of λ when substituted in the Callan-Treiman equation relating $K_{\ell 2}$ and $K_{\ell 3}$ leads to the prediction that ξ is either $+0.43 \pm 0.14$ or -2.43 ± 0.14.

A comparison of these predictions with the experimental values of ξ is discussed in Sec. VI.

We note that in the reconstruction of K_{e3} decays in a heavy-liquid chamber, the constrained variables are systematically shifted from their true values.

ACKNOWLEDGMENTS

We thank Professor U. Camerini of the University of Wisconsin and Dr. C. Henderson of University College, London, for their contributions to this work. We are indebted to Professor S. Berman of Stanford Linear Accelerator Center for enlightening discussions on current algebra.

FOOTNOTES AND REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission.

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FIGURE CAPTIONS

Fig. 1. Schematic diagram of the decay sequence $K^+ \rightarrow \pi^0 e^+ \nu$ at vertex (1), followed by $\gamma \rightarrow e^+ + e^-$ at vertices (2) and (3).

Fig. 2. The π^0 momentum (a) and $\theta_{\pi^+\pi^0}$ (b), calculated by a one-constraint fit to the reaction $\pi^0 \rightarrow 2\gamma \rightarrow 2(e^+ + e^-)$ for 162 π^0's produced in the decay $K^+ \rightarrow \pi^+ + \pi^0$ at rest.

Fig. 3. Histogram of the constrained electron and neutrino momentum for 20 000 Monte-Carlo decays $K^+ \rightarrow \pi^0 + e^+ + \nu$. The events were generated for a pure vector interaction and distorted by the known experimental errors before constraint. The smooth curve is the theoretical distribution for pure vector. The distribution is normalizes to a total of 515 events.

Fig. 4. Experimental distribution in cos α and T_{π^0}.

Fig. 5. Histogram of cos α. The curves were obtained from the 20 000 "fake" events for a pure vector (V) (Fit 1), pure scalar (S), and pure tensor (T) interaction.

Fig. 6. Experimental distributions in pion kinetic energy T_{π^0}. The smooth curve is calculated from the solution to Fit 1.

Fig. 7. Experimental distribution in the electron momentum p_e. The smooth curve is the solution to Fit 1.

Fig. 8. Experimental distribution in the neutrino momentum p_ν. The smooth curve is the solution to Fit 1.

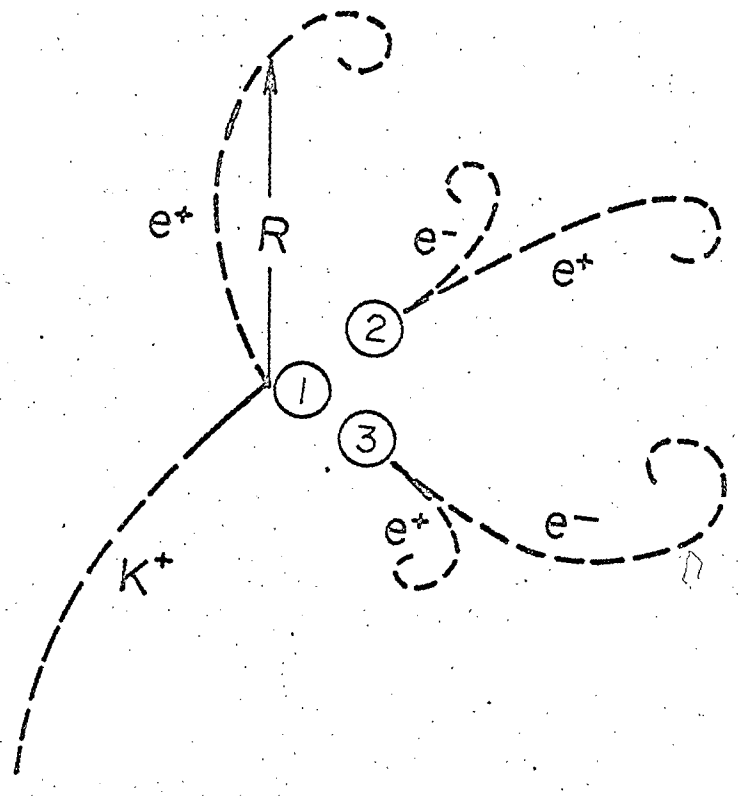
Fig. 9. Chi square (for 18 degrees of freedom) as a function of M in Fit 1. The experimental data is fitted with a pure vector interaction and form factor $f_+(q^2) = f_+(0) \dfrac{M^2}{M^2 - q^2}$. The value of M which minimizes χ^2 is 810^{+320}_{-140} MeV.

Fig. 10. Contours for $(\chi^2_{min} + 1)$ and $(\chi^2_{min} + 4)$ as a function of $\frac{|f_S|}{|f_+|}$ and $\frac{1}{X}$ in Fit 2. The χ^2 minimum at $\frac{|f_S|}{|f_+|} = 0.2$ and $\frac{1}{X} = -0.028$ is indicated by ⊗. The $(\chi^2_{min} + 1)$ contour defines the 68% probability interval; the $(\chi^2_{min} + 4)$ contour defines the 96% probability interval.

Fig. 11. Contours for $(\chi^2_{min} + 1)$ and $(\chi^2_{min} + 4)$ as a function of $\frac{|f_T|}{|f_+|}$ and $\frac{1}{X}$ in Fit 3. The χ^2 minimum at $\frac{|f_T|}{|f_+|} = 0$ and $\frac{1}{X} = +0.028$ is indicated by ⊗.

Fig. 12(a). The relationship between $|\xi + 1|$ and λ from Eqs. (V-2) and (V-3) as given by Callan and Treiman. The solid line is calculated for the $K_{\mu 2}$ and K_{e3} decay rates given in Ref. 6. The broken lines correspond to a ±10% uncertainty in the relationship. The shaded area indicates the range of values of $\zeta + 1$ allowed by our measurement of $\lambda = 0.028^{+0.013}_{-0.014}$.

Fig. 12(b). The circles show the ζ values corresponding to the two values of $|\xi + 1|$ indicated by X_1 and X_2 in Fig. 12(a).



XBL671-162

Fig. 1

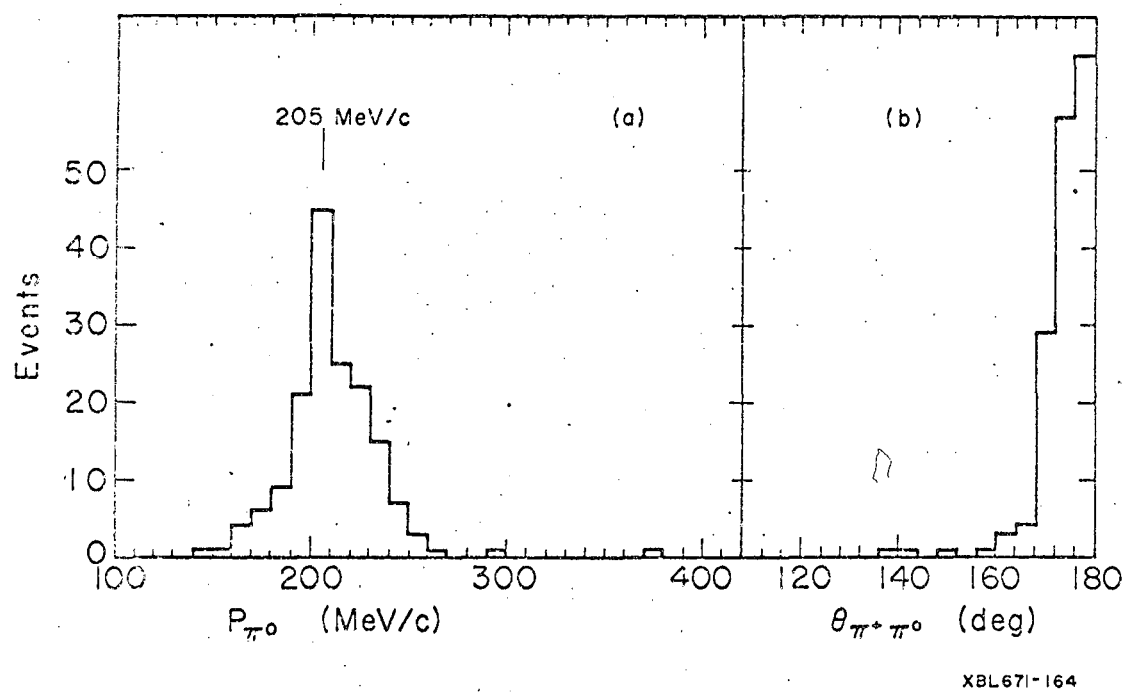

205 MeV/c
(a)
(b)
Events
50
40
30
20
10
0
100
200
300
400
120
140
160
180
P_{π^0} (MeV/c)
$\theta_{\pi^+\pi^0}$ (deg)
XBL671-164

Fig. 2

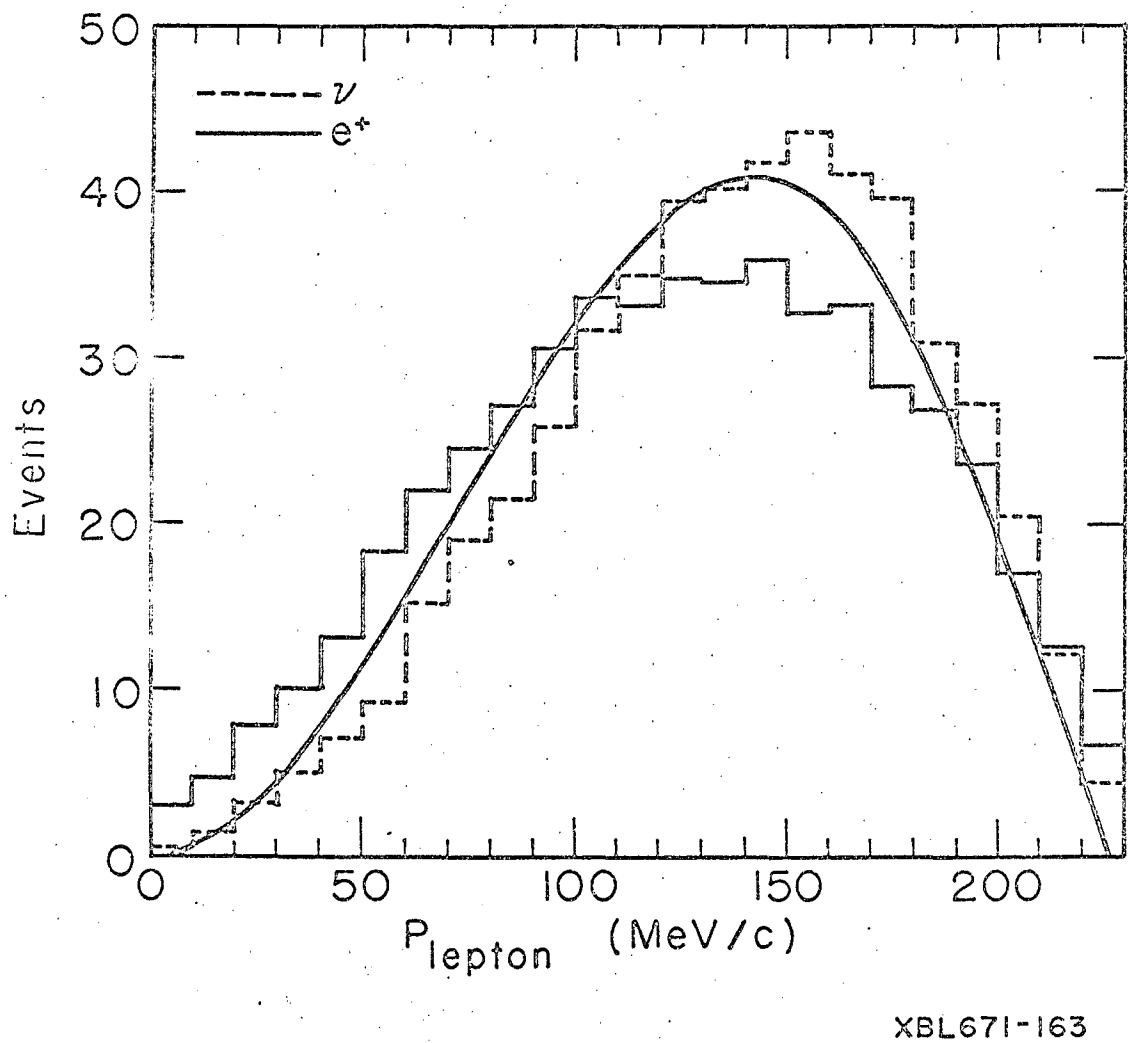
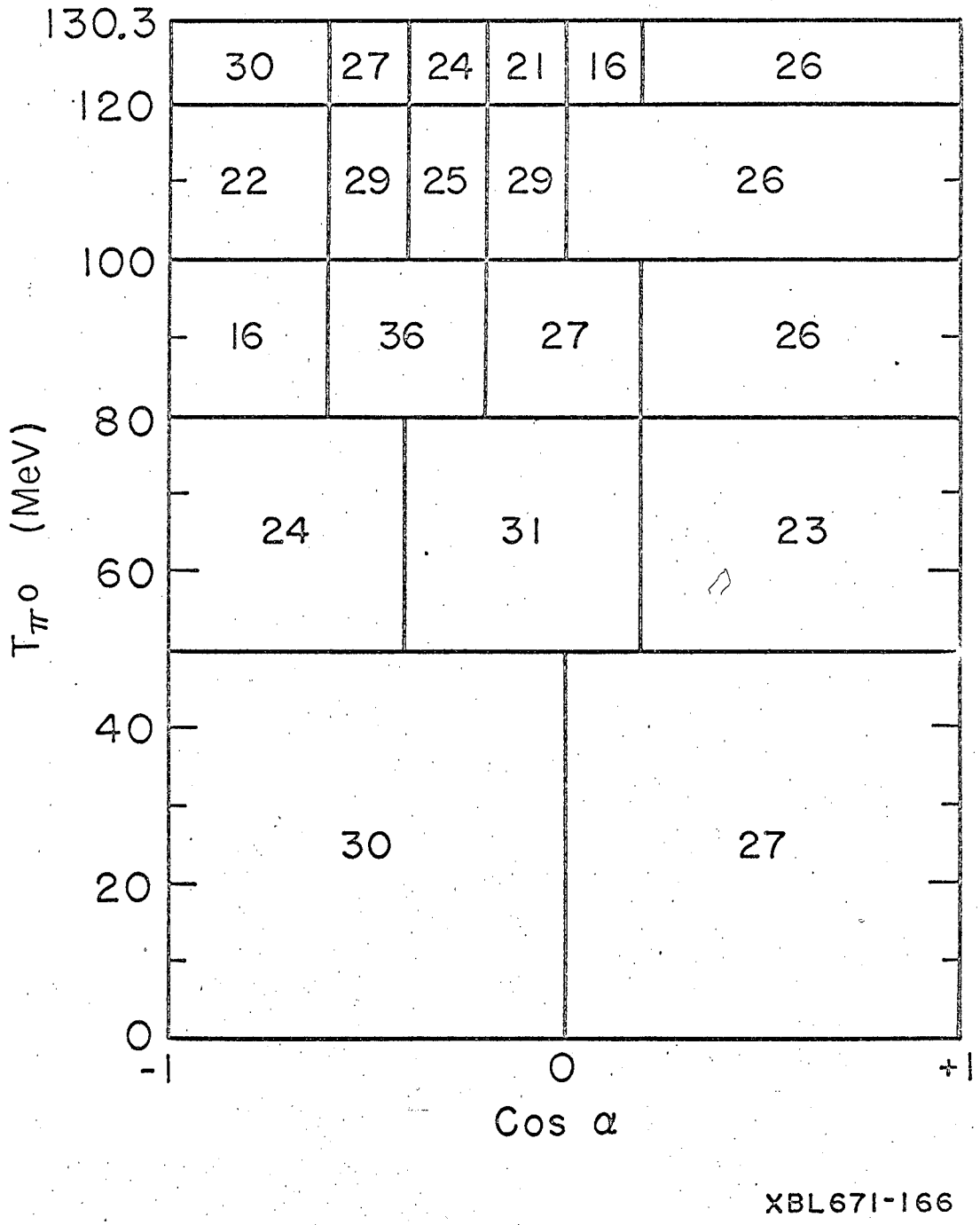

50
40
30
20
10
0
Events
ν
e+
0
50
100
150
200
P lepton (MeV/c)
XBL671-163

Fig. 3



XBL671-166

Fig. 4

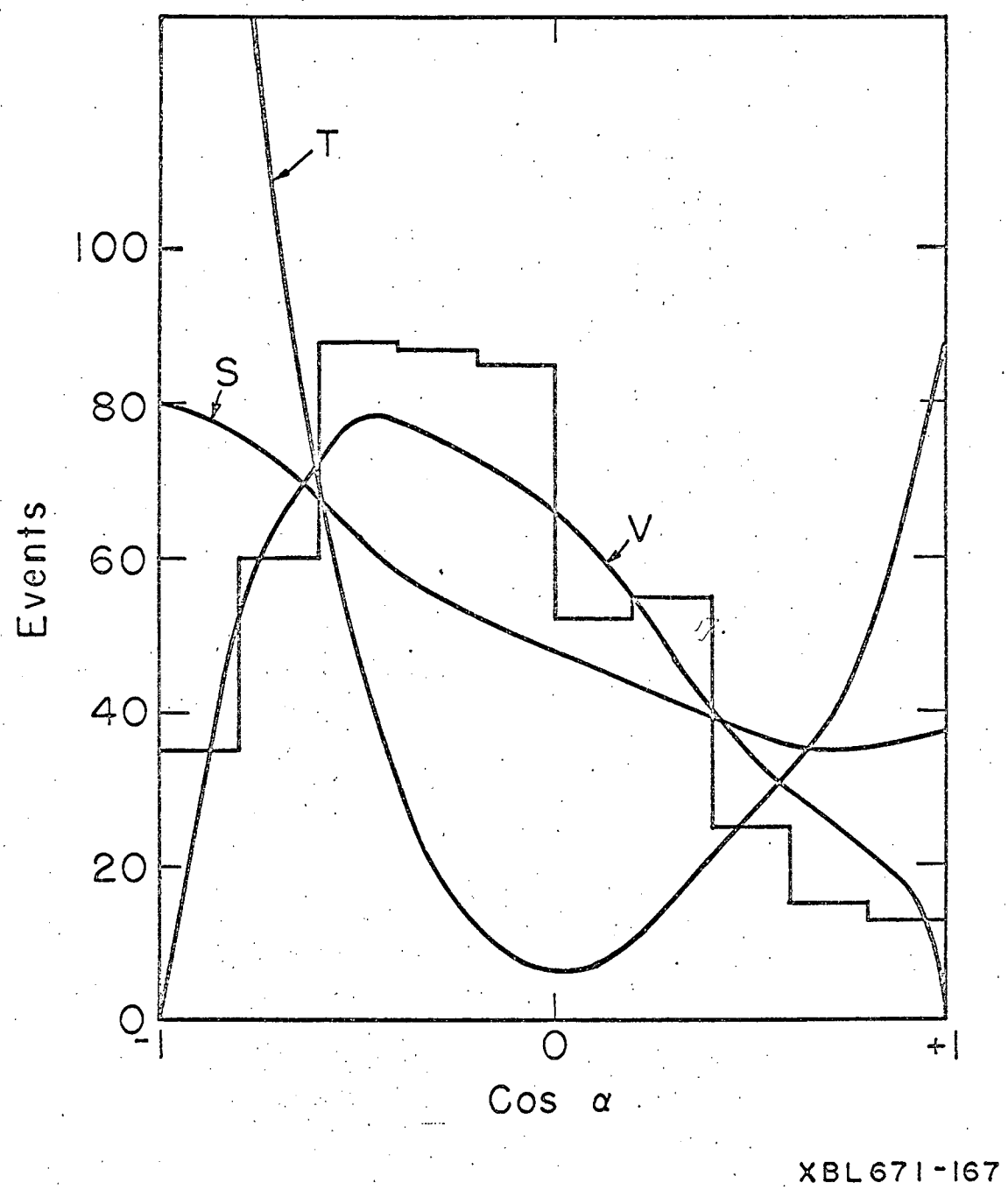


Fig. 5

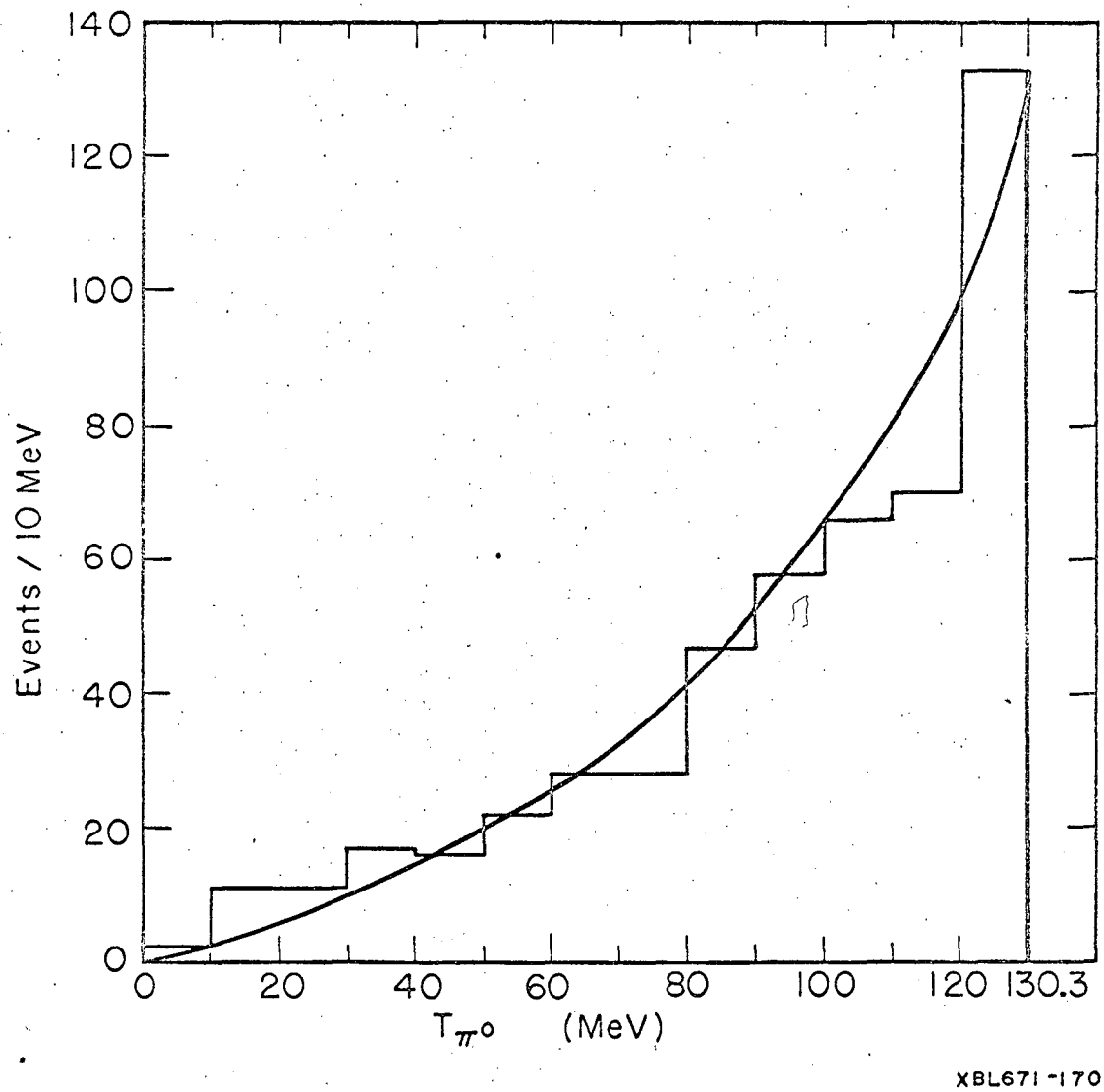

140
120
100
80
60
40
20
0
Events / 10 MeV
0
20
40
60
80
100
120
130.3
T_{π^0} (MeV)
XBL671-170

Fig. 6

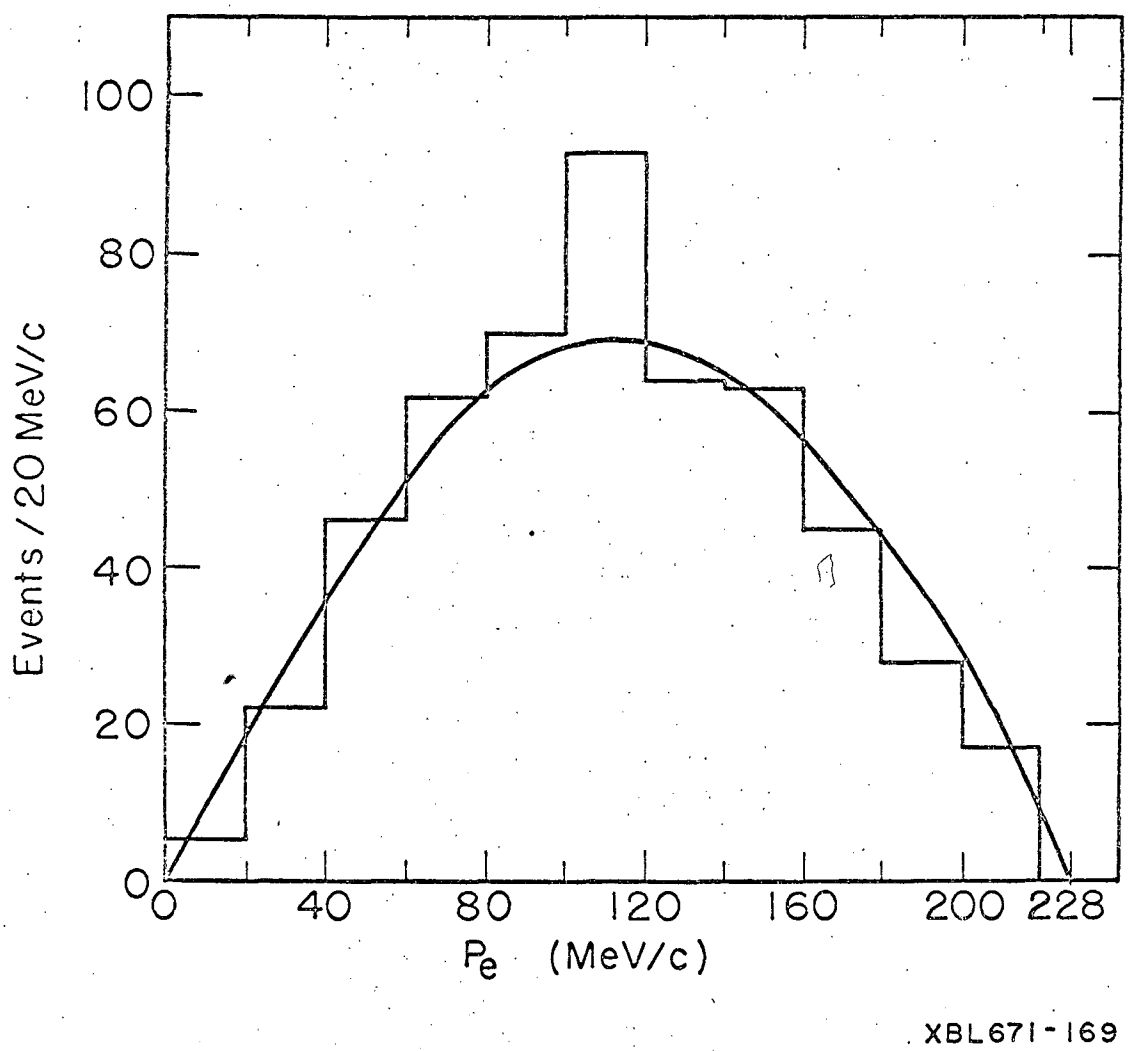

100
80
60
40
20
0
Events / 20 MeV/c
0
40
80
120
160
200
228
P_e (MeV/c)
XBL671-169

Fig. 7

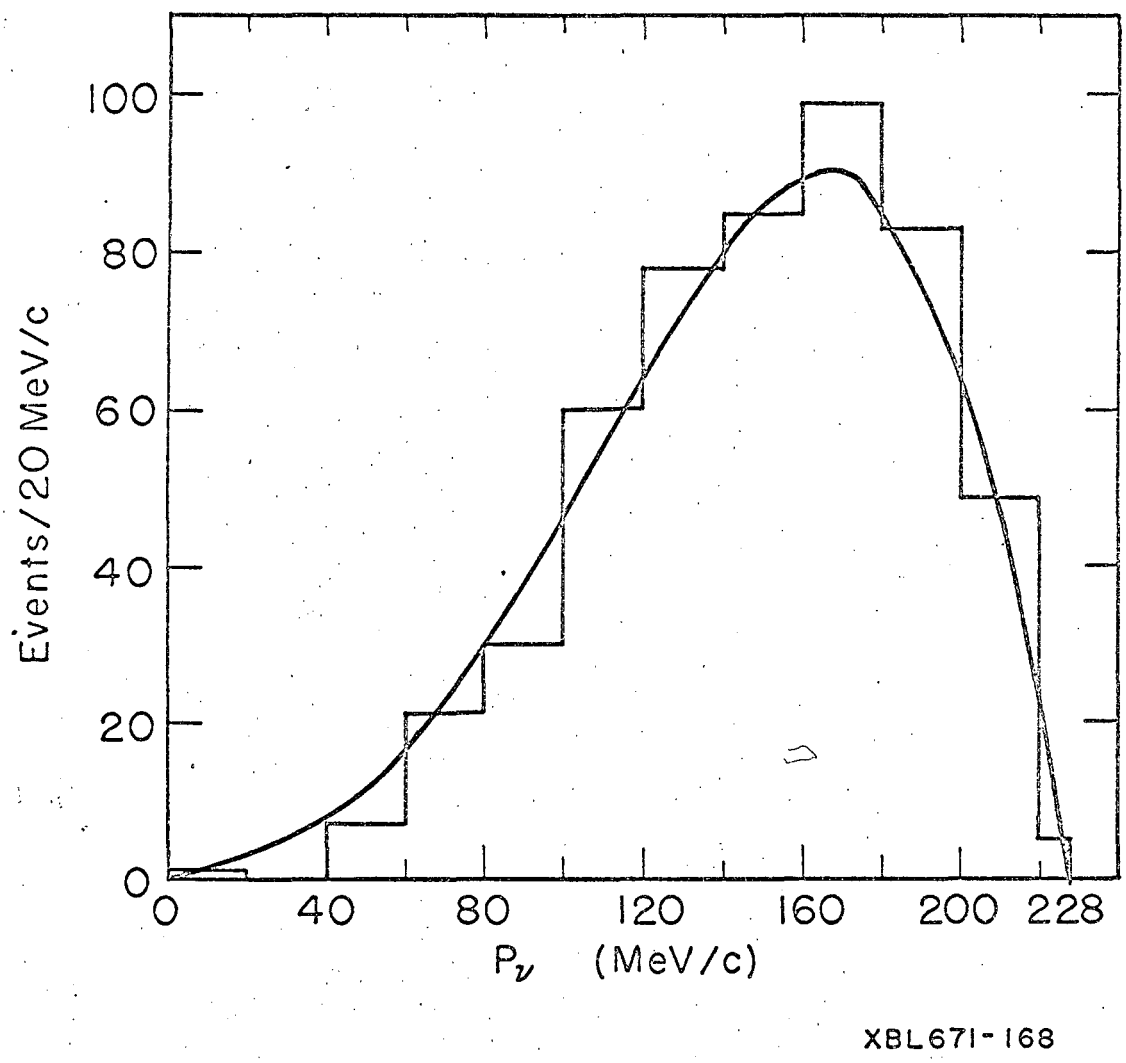
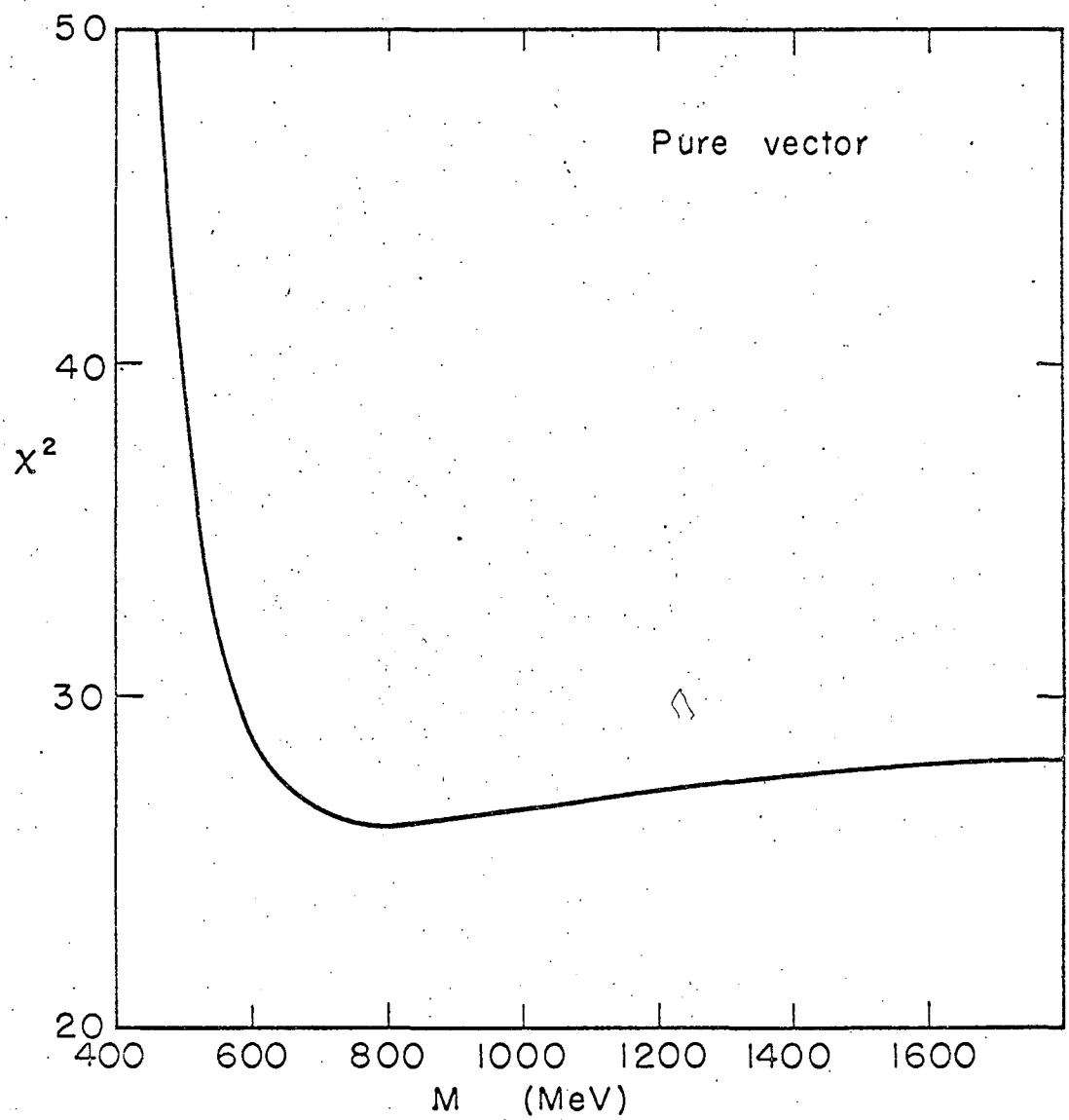
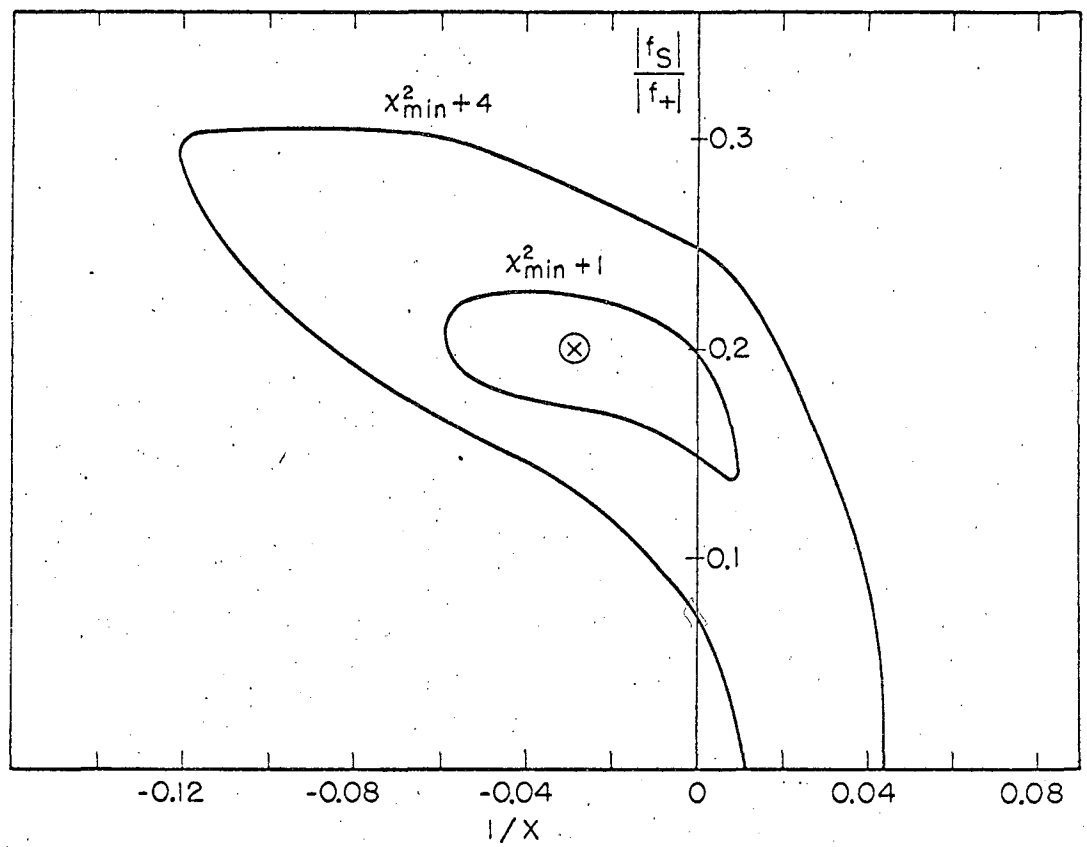

100
80
60
40
20
0
Events/20 MeV/c
0
40
80
120
160
200
228
P_ν (MeV/c)
XBL671-168

Fig. 8



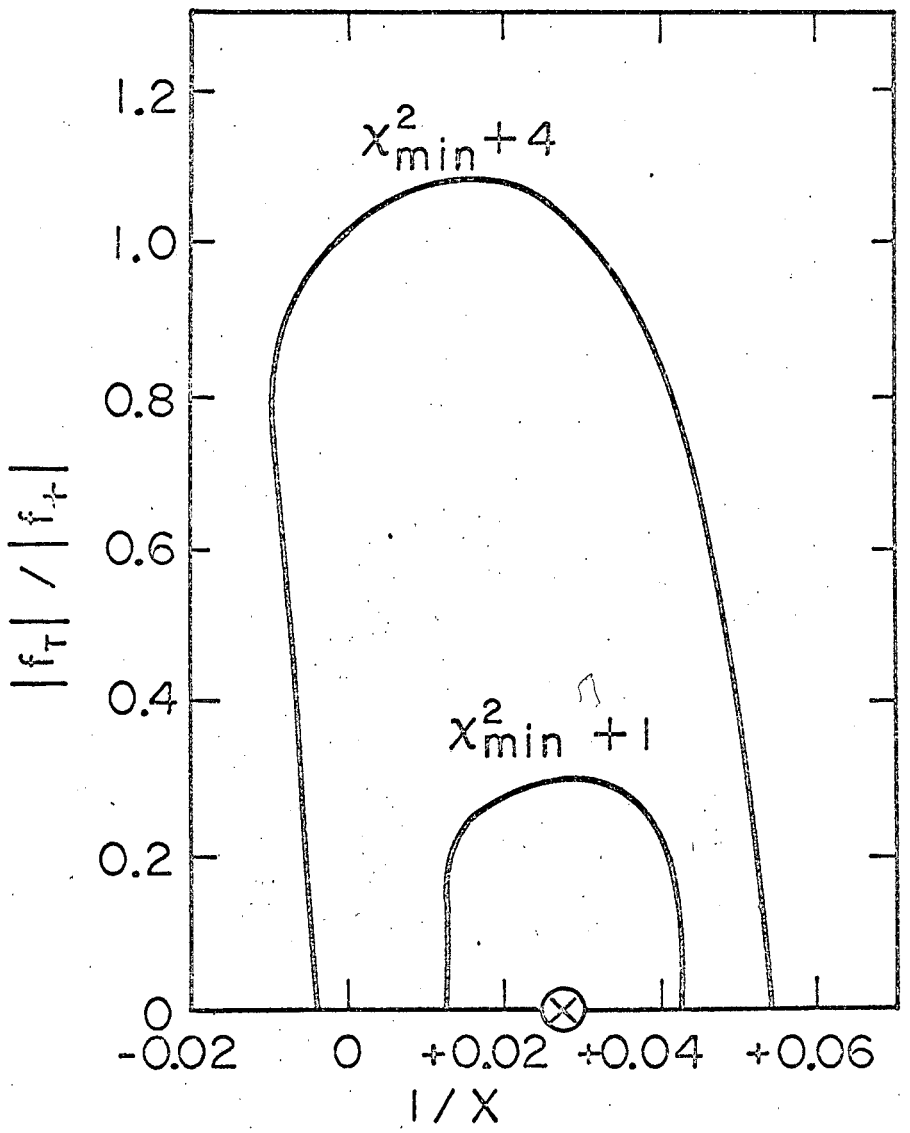
XBL671-165

Fig. 9



XBL671-172

Fig. 10



XBL671-171

Fig. 11

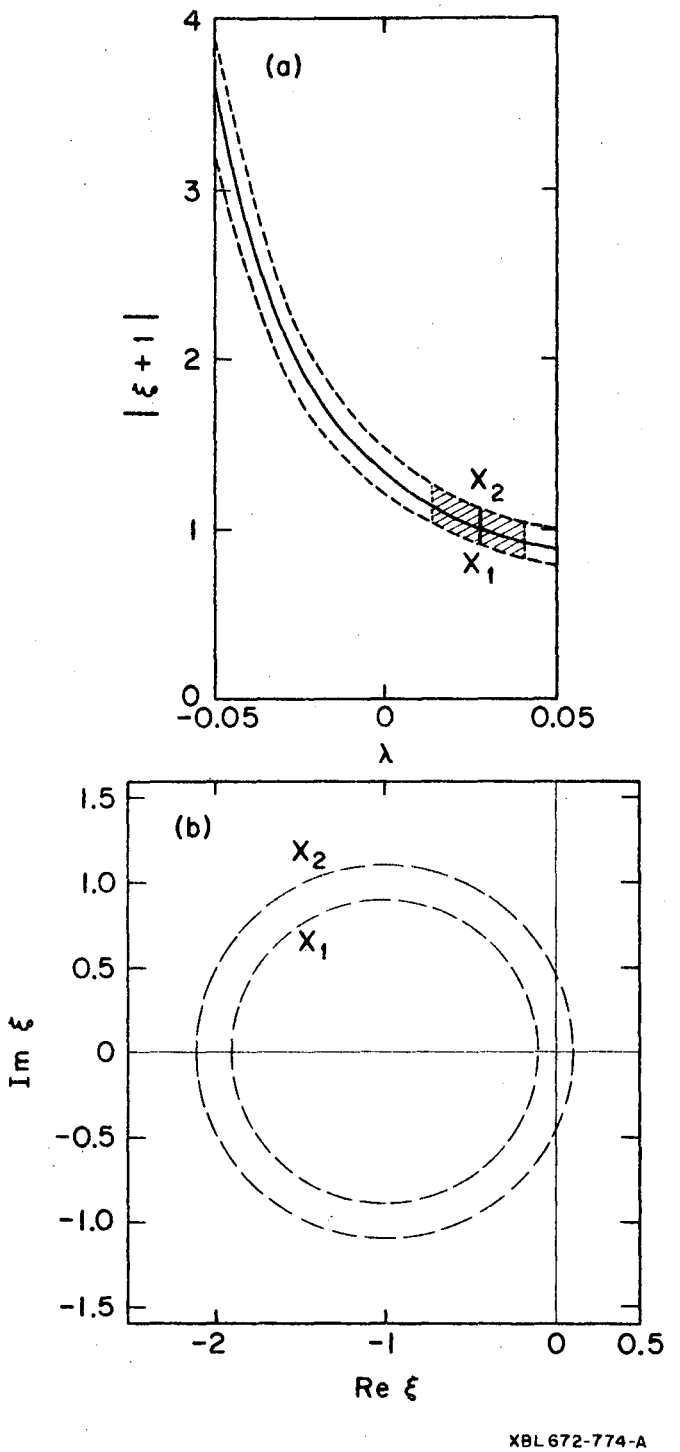

(a)
4
3
2
1
0
$|\xi + 1|$
X_2
X_1
-0.05
0
0.05
λ
(b)
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
Im ξ
X_2
X_1
-2
-1
0
0.5
Re ξ
XBL 672-774-A

Fig. 12

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