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A PROOF OF THE LORENTZ POLE HYPOTHESIS

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Jerome Finkelstein and Jiunn-Ming Wang

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Jerome Finkelstein and Jiunn-Ming Wang

Lawrence Radiation Laboratory
University of California
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Recently there have been several suggestions that at vanishing momentum transfer $(t = 0)$, Regge poles occur in infinite families, which have been called "daughter sequences." This behavior has been established in two specialized models. In one of these,[1,2] one finds that Mandelstam analyticity, the Regge pole hypothesis, and certain other technical assumptions lead to the existence of daughters in spinless amplitudes for which either initial or final particles in the t reaction have unequal masses. The other model in which the existence of Regge families has been verified is the spinless Bethe-Salpeter equation in the ladder approximation.[3,4,5,1]

There have also been suggestions that the existence of these Regge families follows from Lorentz invariance.[6,7] This argument may be outlined as follows: for positive t, the t-channel center-of-mass amplitude is invariant under the rotation group $O(3)$, and so is diagonal in the basis states of that group, which are the familiar angular momentum states. For pairwise equal external masses, at $t = 0$ the center-of-mass amplitude is invariant under

the full Lorentz group $O(3,1)$, or, by analytic continuation, under the four-dimensional rotation group $O(4)$, and so is diagonal in "four-dimensional angular momentum." The same assumptions which lead to Regge behavior at nonzero values of t also lead to the existence of simple poles (called "Lorentz poles") in the complex plane of this four-dimensional angular momentum, which control the asymptotic behavior in s for $t = 0$. Furthermore, a single Lorentz pole can be shown to correspond to an infinite family of Regge poles; thus Lorentz invariance at $t = 0$ seems to imply the existence of Regge families.

Unfortunately, it is also true that a single Regge pole corresponds to an infinite family of Lorentz poles. One might therefore worry that the Lorentz poles are arranged so that all but the single leading member of the Regge family cancels out; in this case we might say that a family of Lorentz poles has "counter conspired"[8] to give a single Regge pole. In previous work,[6,7] the rejection of this possibility was an added assumption, based on the feeling that it would be accidental for the Lorentz poles to appear in families. Because of this extra assumption, and the technical assumptions made in the alternative proof in the unequal-mass case,[1,2] there has been a certain amount of uneasiness about the existence of daughter trajectories.

In this paper we point out that this additional assumption is not necessary, and therefore establish that the existence of daughters is a consequence of Lorentz invariance and Regge pole

assumptions themselves; no counter conspiracy, not even an accidental one, can remove the necessity for infinite families of Regge poles. To show this, we observe that if members of different $O(4)$ representations interfere destructively in one amplitude, they must interfere constructively in some other amplitude. That is, let us suppose that the Regge daughters are in fact absent in some amplitude; this will give us conditions on the counter-conspiring Lorentz poles. We can then find another amplitude, related to the first by $O(4)$ but not by $O(3)$, in which the daughters <u>must</u> appear. Thus the daughter trajectories must exist. We need only assume that the parent trajectory does not itself decouple at $t = 0$, but this is an assumption within Regge, not Lorentz pole, theory.

We will present the explicit verification of the existence of Regge families, following the outline of the preceding paragraph, for two examples. As the first example, we consider any trajectory (such as the ρ) whose exchange contributes to the diffference of total cross sections of different helicity states in πp scattering. (If there were no such trajectories, then this difference would disappear faster than any power of s.) We will work within the framework of $O(4)$ symmetry as formulated in Ref. 7 (hereinafter called I). Below we shall discuss daughters in the $N\bar{N}$ amplitude, but before we proceed with this first example, we must discuss the $O(4)$ kinematics of πp scattering.

Denote by $F_{\lambda_1\lambda_2}(s)$ the conventionally defined t-channel helicity amplitude at $t = 0$, and by $f^j_{\lambda_1\lambda_2}$ its partial wave; λ_1 and λ_2 are the helicities of the ρ meson. The corresponding amplitudes in the s channel at $t = 0$ are denoted by $G_{\lambda_1\lambda_2}(s)$ and $g^j_{\lambda_1\lambda_2}$. $F_{\lambda_1\lambda_2}(s)$, and $G_{\lambda_1\lambda_2}(s)$ are related by[9]

$$
\begin{aligned}
F_{11}(s) &= 1/2\left[G_{11}(s) + G_{00}(s)\right], \\
F_{10}(s) &= (\sqrt{2}/4)\left[G_{-1-1}(s) - G_{11}(s)\right] = 0, \\
F_{00}(s) &= G_{11}(s), \\
F_{1-1}(s) &= 1/2\left[G_{11}(s) - G_{00}(s)\right], \qquad (1)
\end{aligned}
$$

since $G_{10}(s) = G_{1-1}(s) = 0$ from geometry. It has been shown in I that Lorentz invariance at $t = 0$ implies the following expansion for $f^j_{\lambda_1\lambda_2}$:

$$
\begin{aligned}
f^j_{\lambda_1\lambda_2} = (2\pi)^{-2}\,(-1)^{1+\lambda_2} \sum_{\substack{\kappa=0\\ \kappa\ \text{even}}}^{\infty} \sum_{s=0,2} (2s+1)^{-\frac{1}{2}}\, C(1,1,s;\lambda_1,-\lambda_2) \\
\times (j+\kappa+1)^2\, T_s^{\,j+\kappa}\, d^{(j+\kappa,0)*}_{js(\lambda_1-\lambda_2)}(\pi/2) \\
\times\ d^{(j+\kappa,0)}_{j00}(\pi/2), \qquad (2)
\end{aligned}
$$

where $T_s^{\,n}$ is the $O(4)$ partial wave amplitude. The quantum number M defined in I is always zero here because the pion has no spin. Because of Bose statistics, j and n are always even

(odd) when the total isospin in the t-channel is even (odd). Also note that because of parity conservation, $s = 1$ amplitudes do not exist, and that for $\lambda_1 - \lambda_2 = 1$, $s = 2$ does not contribute, since $d^{n0}_{j21}(\pi/2) = 0$ for $n - j$ even, from Eq. (A-11) of I.

By use of the techniques of I, Eq. (2) can be extended to complex j. We use

$$d^{n0}_{j20}(\delta) = -\tfrac{1}{2}\left[\frac{5}{(n-1)\ n(n+2)\ (n+3)}\right]^{\frac{1}{2}}\left[3\frac{d^2}{d\delta^2} + n(n+2)\right] d^{n0}_{j00}(\delta)$$

$$d^{n0}_{j22}(\delta) = -\frac{1}{4}\left[\frac{3}{(n-1)\ (n+3)\ (j-1)\ (j+2)}\right]^{\frac{1}{2}} \times$$

$$\left[-e^{-i\delta}\left(n-1-i\frac{d}{d\delta}\right)\left(n+3+i\frac{d}{d\delta}\right) + e^{i\delta}\left(n-1+i\frac{d}{d\delta}\right)\left(n+3-i\frac{d}{d\delta}\right)\right] d^{n0}_{j11}(\delta)\ ; \tag{3}$$

other relevant $d^{n0}_{js\lambda}(\delta)$ are given in I. The signature of $f^{j}_{\lambda_1\lambda_2}$ and of $T_s^{\ n}$ is given by $(-1)^I$, where I is the total isospin in the t channel. The continued $O(4)$ amplitudes are given by

$$T_0^{\ n} = \frac{2}{n+1}\int_{x_0}^{\infty} dx\ B_{00}(x)\ D_n^{\ 1}(x)\ ,$$

$$T_2^{\ n} = \frac{2}{n+1}\left[\frac{5}{(n+3)\ (n+2)\ n(n-1)}\right]^{\frac{1}{2}}\int_{x_0}^{\infty} dx\ B_{20}(x)$$

$$\times\left[n(n-1)\ D_n^{\ 1}(x) - \frac{3}{2(x^2-1)}\left\{(n-1)\ D_n^{\ 1}(x) - (n+1)\ D^1_{n-2}(x)\right\}\right]\ ,$$

$$x_0 \equiv \frac{3m_\pi^{\ 2} - m_\rho^{\ 2}}{2m_\pi\ m_\rho}\ , \tag{4}$$

where $B_{s\lambda}(x)$ is the u-channel absorptive part of $T_{0s\lambda}(\delta)$; $T_{s's\lambda}(\delta)$ is defined in Eq. (40) of I. The contributions coming from the s-channel absorptive part $A_{s\lambda}(x)$ have already been taken into account in Eq. (4) by using $A_{s\lambda}(x) = (-1)^I B_{s\lambda}(-x)$, which follows from Bose statistics. The integral in the second of Eq. (4) converges because $B_{20}(x)$ behaves like[10] $(x - 1)$ near $x = 1$. From Eqs. (3) and (4) we can show, just as was done in I in the $NN \to NN$ case, that the series in Eq. (2) converge uniformly in the j plane, and then, by using Carlson's theorem, that the RHS and LHS of Eq. (2) coincide for complex j. We wish to emphasize that to establish this we need to use only Lorentz invariance and the forward $(t = 0)$ dispersion relation, since for κ large enough $T_s^{j+\kappa}$ is always in the region of holomorphy in the n plane.

The proof of Eq. (2) for complex j is the analogue of the proof given in I for the NN case; we are now ready to discuss the existence of daughters in the $\pi\rho$ amplitude. First, we notice that for given j, f_{11}^j and f_{00}^j receive the same contribution from Lorentz poles with $s = 0$, and different contributions from poles with $s = 2$ (the ratio of these latter contributions being $- C(1,1,2,1,-1)/C(1,1,2,0,0) = - \frac{1}{2}$). Thus contributions of $s = 0$ cannot cancel those of $s = 2$ in both f_{11}^j and f_{00}^j; there will surely be a pole in either f_{11}^j or f_{00}^j, or both, unless the contribution of $s = 2$ Lorentz poles to f_{11}^j vanishes.

Now let us assume that $j = \alpha$ gives the position of the leading Regge pole which contributes to the difference of total cross sections corresponding to different initial helicities in the s channel. From the optical theorem and the last of Eqs. (1), we see that this must be the leading pole in f^j_{1-1}. From Eq. (2), this implies that $T_2{}^n$ has its leading pole at $n = \alpha$, because on the one hand, if $T_2{}^n$ had its leading pole at $n = \alpha + r$, for $r > 0$, then f^j_{1-1} would have its leading pole at $j = \alpha + r$; and on the other hand, if $T_2{}^n$ were analytic at $n = \alpha + r$ for all $r \geq 0$, then $f^j_{1,-1}$ would be analytic at $j = \alpha$. In fact, we need not have required that the pole in f^j_{1-1} be the leading one, as long as there is no other pole an even integer higher.

This Lorentz pole of $T_2{}^n$ at $n = \alpha$ produces daughter poles in $f_{00}{}^j$, $f_{11}{}^j$, and f^j_{1-1}. Let us first show that the second daughter, at $j = \alpha - 2$, must actually appear in at least one of these amplitudes. Since $s = 0$ does not contribute to f^j_{1-1}, and since $T_2{}^n$ has no poles for $n > \alpha$, the only way in which the pole in $f_{1-1}{}^j$ at $j = \alpha - 2$ could be cancelled would be if there were another pole of $T_2{}^n$, at $n = \alpha - 2$. The assumption that this cancellation occurs fixes the ratios of the residues of these two $s = 2$ poles, which enables us to see that together they must make a nonzero pole contribution to f_{11} and f_{00} at $j = \alpha - 2$. Remembering that $s = 0$ and $s = 2$ cannot cancel in both $f_{11}{}^j$ and $f_{00}{}^j$, we see that the assumption

that the second daughter is cancelled from f^j_{1-1} leads to the conclusion that it must be present either in f^j_{11} or in f^j_{00}; this establishes that the second daughter exists.

This result could have been obtained in another way. From Eqs. (1) we see

$$F_{11}(s) + F_{1-1}(s) - F_{00}(s) = 0 , \tag{5}$$

which implies

$$f_{11}^{j-1} - f_{11}^{j+1} - \left(\frac{j(j-1)}{(j+1)\ (j+2)}\right)^{\frac{1}{2}} f^{j-1}_{1-1} + \left(\frac{(j+1)\ (j+2)}{j(j-2)}\right)^{\frac{1}{2}} f^{j+1}_{1-1}$$

$$- f_{00}^{j-1} + f_{00}^{j+1} = 0 . \tag{6}$$

Equation (6) is the analogue for $\pi\rho$ of the original conspiracy equation derived by Volkov and Gribov for the NN case.[11] Like the Volkov-Gribov equation, it can be satisfied by a finite number of trajectories, and in fact is consistent with the absence of all daughters beyond the second. However, Eq. (2), which is a direct consequence of Lorentz invariance, is stronger than this; the Regge family must be infinite, as we now demonstrate.

The fourth and sixth daughters, at $j = \alpha - 4$ and $\alpha - 6$, could receive contributions from counter-conspiring Lorentz poles at $n = \alpha, \alpha - 2, \alpha - 4,$ and $\alpha - 6$. The vanishing of fourth and sixth daughters in both f^j_{1-1} and f^j_{11} would constitute four conditions on the four possible $s = 2$ Lorentz poles (remember that $s = 0$ contributions can at most shift a pole from

$f_{11}{}^{j}$ to $f_{00}{}^{j}$), which can be shown to have the unique solution that all of the Lorentz residues are zero. Thus under the assumption that the parent Regge trajectory at $j = \alpha$ is present, we see that either the fourth or the sixth daughter must exist. We can continue the argument to show that, for any N, at least $N/2$ of the first N even daughters must exist; thus the family is infinite.

We have established that Regge trajectories which couple to f^{j}_{1-1}, such as the ρ, are members of infinite families. Unfortunately, this method of proof does not apply to those Regge poles which do not couple to f^{j}_{1-1}. It has been argued[12] that the Pomeranchuk pole does not couple to f^{j}_{1-1} at $t = 0$, since $\alpha_P(0) = 1$ is a point of nonsense and wrong signature for this helicity amplitude.[13] However, it may be that the fixed poles in the j plane[14] would allow the Pomeranchon to couple to f^{j}_{1-1}, in which case our argument would establish the existence of even daughters of the Pomeranchon. On the other hand, these fixed poles are in the right position to themselves be the even daughters of the Pomeranchon!

As a second example, consider the A_1 trajectory in the $N\bar{N}$ amplitude. The A_1 trajectory, if it indeed exists, would appear in the amplitude $f_1{}^{j}$. Just as in the first example, as long as the A_1 is coupled to $N\bar{N}$ at $t = 0$, the Volkov-Gribov equation[11] tells us that some other conspiring trajectory must exist, but $O(4)$ tells us more; the conspiring family must be infinite.

The $O(4)$ kinematics of NN scattering have been presented in I. It was shown there that Lorentz poles with $s = 0$ contribute only to $f_{11}{}^{j}$, and hence they do not counter conspire with poles with $s = 1$. A pole in $T_{s=1}^{n,M=0}$ at $n = \alpha$ leads to poles in $f_1{}^{j}$ at $j = \alpha, \alpha - 2, \alpha - 4, \cdots$ and in $f_0{}^{j}$ at $j = \alpha - 1, \alpha - 3, \alpha - 5 \cdots$; a pole in $T_{s=1}^{n,M=1}$ at $n = \alpha + 1$ leads to poles in $f_1{}^{j}$ at $j = \alpha, \alpha - 2, \alpha - 4, \cdots$, in $f_0{}^{j}$ at $j = \alpha + 1, \alpha - 1, \alpha - 3 \cdots$, and in $f_{22}{}^{j}$ at $j = \alpha + 1, \alpha - 1, \alpha - 3, \cdots$. Under the assumption that the A1 is coupled to $N\bar{N}$, there must be a pole in $T_1^{n,0}$ or in $T_1^{n,1}$ with nonzero residue; let $n = \alpha$ be the position of the leading pole. Then it can be shown that for any integer N, in the $2N$ units below $n = \alpha$ at least $N + 1$ Regge conspirators must exist, and once again the Regge family is infinite.

In these two example, although the Regge families have been shown to be infinite, the possibility has still been left open that some of the daughters might be cancelled out. This is because, for each j, the number of helicity amplitudes in which the daughters might exist, and in which the counter conspirators must therefore try to cancel, is fairly small. This situation should improve in amplitudes with more exotic external spins. Alternatively, if one were to consider amplitudes involving more than four external particles, the finite degree of freedom we have been exploiting, is going from one helicity amplitude to another, would become a continuous degree of freedom, and then the existence of all daughters could presumably be established.

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FOOTNOTES AND REFERENCES

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10. Using the results of second paper of Ref. 9 one can show that for pairwise equal mass and equal spin scattering in the t channel, those c.m. amplitudes which do not vanish at $t = 0$ do not have kinematical singularities in s at $t = 0$, and then one can use Eq. (9) of I to prove the same thing for $T_{s's\lambda}(\delta)$. In a similar fashion, one can show that $B_{20}(x)$ behaves like $(x - 1)$ near $x = 1$.

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