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# Ernest O. Lawrence Radiation Laboratory

LECTURES

INELASTIC SCATTERING AND NUCLEAR STRUCTURE

Norman K. Glendenning

INTERNATIONAL SCHOOL OF PHYSICS "ENRICO FERMI"

Course XL

26 June - 15 July 1967

Varenna (Como) Italy

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UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

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# INELASTIC SCATTERING AND NUCLEAR STRUCTURE

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I

# INELASTIC SCATTERING AND NUCLEAR STRUCTURE

Norman K. Glendenning

## I. INTRODUCTION

The scattering process is one of the most powerful means of investigating nuclear structure that we have at our disposal. Here I shall attempt to review the theory of inelastic scattering and its applications to the study of the properties of individual nuclear states.

The nature of the experimental observation can be described as follows. A beam of (ideally) monoenergetic particles, let us for the moment say protons, is fired at a target of some pure isotope. A counting system has been set up to record both the number of particles that are scattered at various angles, and their energies. A counter at a particular angle will record having seen protons in discrete energy groups, corresponding to excited states of the target, and then a continuous energy spectrum of protons corresponding to the closely spaced levels at high excitation in the nucleus. It is actually the description of events leading to the groups of protons corresponding to individual nuclear states of the target that we are interested in.

The intrinsic structure of the lower lying states of a nucleus generally can be thought of as not differring very much from that of the ground state. Those that do, lie at a much higher energy. That is to say, only a very few degrees of freedom are involved in the excitation. The relevant degrees of freedom may be those of a single nucleon, such as in $Pb^{209}$ which has a doubly magic number of neutrons and protons plus one extra neutron. The lowest half dozen levels or so differ from each other, essentially, only in the state of the odd neutron. On the other hand a nucleus like $Sm^{154}$ is strongly deformed and its low lying spectrum

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suggests that these levels all have the same intrinsic nucleonic structure and correspond merely to different rates of rotation of the system. This again, viewed in those terms, involves only the few degrees of freedom associated with the rotation.

In considering what physical process is involved in causing such a minimum sort of rearrangement, one is immediately led to consider a direct interaction between the projectile and the degrees of freedom in the nucleus that have been changed. In the $Pb^{209}$ example the interaction would be with the odd neutron. In the deformed nucleus case, with the part of the nuclear field arising from the deformation.

Are there competing mechanisms? Perhaps a compound state was formed which subsequently decayed to the observed levels. This is always possible. However if the bombarding energy in the experiment is sufficiently high, there will be so many states to which a compound state could decay, that the probability of decay to any one of them is very small. To make the contribution of this process small, we therefore shall consider only bombarding energies greater than, say 15 MeV, which, in a medium to heavy nucleus places millions of levels in competition with the ones of interest, as far as decay of a compound state is concerned.

The measurements at various angles, of the number of particles in each energy group, yields the differential cross-section for exciting each level. In a gross way, the angular distribution of scattered particles from a given energy level reflects the spin and parity of the level, or more accurately the spin and parity that was exchanged between the scattered particle and the nucleus in making the transition. This is because the particle must scatter in such a way as to obey the various conservation laws including angular momentum, and this imposes restrictions on the directions into which it can scatter. Just how directly

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the exchange of angular momentum and parity is reflected in the angular distribution depends on several circumstances. Let us for a moment consider some typical single-particle states whose wave functions we know have various forms inside the nucleus, but which all have similar exponential forms outside. These will be occupied by various numbers of nucleons and in such combinations that the nuclear wave functions are orthogonal to each other. Excitation of the nucleus from one state to another implies making the necessary rearrangement in the way in which the single-particle states are occupied. Inside the nucleus, because of the various forms of the single particle wave functions, one transition will be characterized, in general, by a different radial density than another. However in the exterior region all the radial densities will look similar to each other, aside from their normalization. Now it is known that when a composite particle, like an alpha, is scattered from a nueleus, it is mainly sensitive to the conditions in the exterior region. Hence its angular distribution will be quite insensitive to the details of the structure of the states but will depend essentially only on the angular momentum and parity transferred. The relationship is quite simple when the state is excited directly from the ground state, but is less direct when cascade transitions through intermediate states compete with the direct transition, as we shall see later. Even though the angular distributions of strongly absorbed particles are insensitive to interior details, the magnitude of the cross sections, which depend on the normalization of the transition density in the tail, are by continuity connected to the interior conditions.

In contrast to this situation, scattered nucleons can penetrate the interior of the nucleus and consequently their angular distributions may be quite strongly modulated by the interior structure. Experiments using nucleons clearly are less

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suitable for determining the spins and parities of levels. But they would be more appropriate than composite particles for studying the detailed nucleonic structure of nuclear states.

Since these lectures are concerned with the use of inelastic scattering in investigations of nuclear structure, it is appropriate to mention what particular aspects of this process supplement other means of investigation. First it is clear that only the simplest properties of nuclei can be inferred essentially directly from experiment. The position of energy levels is an example. For more complicated properties, such as the nucleonic structure of the energy levels, this is not so. The structure is not an observable. But it influences and reflects itself in observables. The approach therefore is to construct a model of the nucleus, and from the model calculate various observables. It should yield an energy spectrum in good agreement with observation. But since the Hamiltonian is stationary at its eigenstates it is possible that the energy levels of the model nucleus will come out in good agreement with observation even though the wave functions may be poor. They can be subjected to further tests by calculating additional properties such as electromagnetic transition rates. However the wave length of radiation emitted in the decay of a nuclear state is very large,

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi\hbar c}{\hbar\omega} \sim \frac{1000}{\hbar\omega} \qquad \text{(Fermi)}$$

so that the radiation is rather insensitive to radial details of the nuclear structure. Of course the selection rules are very important in a spectroscopic sense in determining gross properties such as spins and parities. Moreover the intensity does depend on the coherence involved in the nucleonic rearrangement in undergoing

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the transition from one nuclear state to another, which if constructive leads to the enhanced transitions commonly referred to as collective, or if destructive leads to hindered rates. These are important additional properties that a satisfactory model must reproduce. But even these properties are not highly sensitive to the particular microscopic details of the states involved. In any case it is clearly desirable to have a means of probing finer details than is allowed by the E.M. transitions. Inelastic scattering where the momentum transfer, $\underset{\sim}{q} = \underset{\sim}{k} - \underset{\sim}{k}'$, can be quite large, in general provides a finer probe, being sensitive to distances $\delta r \sim 1/q$. This sensitivity will register itself in the angular distributions which moreover will generally involve several multipoles in contrast with electromagnetic transitions, which are usually dominated by one.

Quite aside from the question of detail in radial distributions, scattering of various types of particles, offers a means of exploring various aspects or components of the nuclear field that exist by virture of the spin, isospin and tensor components of the nucleon-nucleon force.

During the last few years powerful theoretical techniques for handling the nuclear many-body problem have been developed, which have permitted a fairly detailed description of nuclei in certain regions of the periodic table.[1] In this way the collective and "single-particle" states are described on the same footing in terms of their nucleonic structure. While these detailed descriptions have succeeded fairly well in explaining the energy levels, and to some extent electromagnetic transition rates and their systematics in certain regions of the periodic table, it is important to have the additional checks that a detailed comparison with scattering data can provide.

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## II. COUPLED CHANNEL AND DISTORTED WAVE METHODS

The typical situation in attempting to extract nuclear information from a scattering experiment is the following. We have available a certain model of of the nucleus which provides wave functions for the nuclear states. Let $H_A$ denote the model Hamiltonian for the nucleus of mass A, whose internal coordinates we shall often denote also by $\underset{\sim}{A}$. We have available its eigenfunctions:

$$(H_A - E_{\alpha J})\, \Phi_{\alpha J}(\underset{\sim}{A}) = 0 \tag{1}$$

We want to determine whether the description of the nucleus provided by this model is consistent with a scattering experiment.

For the system comprising the nucleus and the scattered particle, let us say a proton, we therefore take the Hamiltonian

$$H = H_A + T + V(\underset{\sim}{r},\underset{\sim}{A}) \tag{2}$$

where $T$ is the kinetic energy of the proton, and $V$ is the interaction of the proton with the nucleus. If the nuclear Hamiltonian $H_A$ were written in terms of collective coordinates, then these would appear in $V$, whereas if we had a more fundamental description of the nucleus in terms of the nucleon coordinates $\underset{\sim}{r}_i\ \underset{\sim}{\sigma}_i\ \underset{\sim}{\tau}_i$, then $V$ would be a sum of two-nucleon potentials.

In any case, it is natural to seek a solution of our scattering problem, defined, with appropriate boundary conditions, by

$$(H - E)\, \Psi(\underset{\sim}{r},\underset{\sim}{A}) = 0 \tag{3}$$

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in the form of an expansion in terms of the nuclear wave functions $\Phi_{\alpha J}(\underset{\sim}{A})$, since we wish to determine their appropriateness for describing the nucleus.

To this end introduce the spin-orbit functions for the proton

$$\mathcal{Y}_{\ell s j}^{\ m} = [Y_\ell(\hat{\underset{\sim}{r}})\ \eta_s(\underset{\sim}{\sigma})]_j^m \tag{4}$$

where $\hat{\underset{\sim}{r}}$ denotes the polar coordinates of $\underset{\sim}{r}$ and $\eta$ is the spin function for the proton. Since we now have many quantum numbers to keep track of, let us denote by $c$ the whole collection of quantum numbers which define the intrinsic state of nucleus and proton and their relative angular momenta before the collision,

$$c \equiv \alpha J\ \ell s j$$

and by $c'$ some other state of intrinsic or relative motion that may be achieved as a result of the collision. Using the spin-orbit functions for the proton, and the nuclear wave functions $\Phi_{\alpha J}$ we form the functions of total angular momentum $I$ and parity $\pi$

$$\begin{aligned} \underset{\sim}{I} &= \underset{\sim}{j} + \underset{\sim}{J} \\ \pi &= (-)^\ell \pi_\alpha \end{aligned} \tag{5}$$

namely

$$\phi_{c\pi I}^{\ M}(\hat{\underset{\sim}{r}},\underset{\sim}{\sigma},\underset{\sim}{A}) = [\mathcal{Y}_{\ell s j}\ \Phi_{\alpha J}]_I^M \tag{6}$$

In terms of these we now write an expansion for a solution of (3) having given parity and total angular momentum $\pi$ and $I$.

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$$\Psi^{M}_{c\pi I}(\underset{\sim}{r},\underset{\sim}{A}) = \frac{1}{r}\sum_{c'} u^{c\pi I}_{c'}(r)\phi_{c'}{}^{M}_{\pi I}(\hat{\underset{\sim}{r}},\underset{\sim}{\sigma},\underset{\sim}{A}) \tag{7}$$

I have put a $c \equiv \alpha J \ell s j$ on the state functions $\Psi$ and on the radial function $u(r)$ to remind us that the initial state of the system corresponded to the nucleus in a definite state $\alpha J$ (the ground) with an incident proton having angular momenta $\ell s j$.

There are two improtant approximations implied when we write the expansion (7). Since it always contains the proton as a free particle, we are neglecting any specific effect on the inelastic channels that we are interested in, of reactions such as pickup. The expansion is in terms only of the elastic and inelastic channels containing the proton. Moreover we neglect exchange of the proton with one in the target, which may take place, either by virtue of the indistinguishability of identical particles or by the exchange nature of the forces.

As to the first approximation we may note that the elastic and inelastic processes typically represent the largest part of the direct cross-section. We may be prepared then to treat the other channels implicity through use of an absorptive part in the diagonal matrix elements of the interaction.

The neglect of exchange is less well founded, but is presumed, on the strength of overlap arguments, to introduce little error particularly for sufficiently high bombarding energies.

II

Coupled equations.[2] Inserting now the expansion (7) for $\Psi$, into the Schroedinger equation, (3), and making use of the orthonomality of the functions $\phi$,

$$\langle \phi_{c'\pi' I'}^{M'} | \phi_{c\pi I}^{M} \rangle = \delta_{cc'} \, \delta_{\pi\pi'} \, \delta_{II'} \, \delta_{MM'} \tag{8}$$

we obtain, for each total angular momentum and parity $I,\pi$ of the system, a set of coupled equations for the radial functions $u(r)$ of the scattered proton. For each channel $c'$ they have the form

$$(T_{c'} + V_{c'c'}^{I}(r) - E_{c'}) \, u_{c'}(r) = - \sum_{c'' \neq c'} V_{c'c''}^{I}(r) \, u_{c''}(r) \tag{9}$$

where

$$T_c = \frac{\hbar^2}{2m} \left( - \frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} \right)$$

$$k_c^{\,2} = \frac{2m}{\hbar^2} E_c \tag{10}$$

$$E_c = E - E_{\alpha J}$$

with $E$ denoting the bombarding energy. Finally

$$V_{c'c''}^{I}(r) = \langle \phi_{c'\pi\, I}(\hat{\underset{\sim}{r}},\underset{\sim}{A}) \, | V(\hat{\underset{\sim}{r}},\underset{\sim}{A}) \, | \phi_{c''\pi\, I}(\hat{\underset{\sim}{r}},\underset{\sim}{A}) \rangle \tag{11}$$

denotes an integration over all internal coordinates and the polar angles of $\underset{\sim}{r}$, leaving a function of radius $r = |\underset{\sim}{r}|$. It is of course diagonal in $\pi$ and $I$, and independent of $M$, since $V$ is scalar. All the functions $u$ should be understood to have the additional common labels shown in (7).

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As it stands, the system of equations is still infinite in number so that we shall make a further approximation: of all the inelastic channels leading to bound or continuum states of the nucleus, we shall keep only those terms in the expansion (7) which correspond to states having large cross-sections, plus any others which may interest us. Now the system of equations is finite in number, and solvable numerically with a computer. We shall have to discuss the approximation we have made in considering only a few of the many channels. In particular we shall see in what way the finite system can be modified so as to include some of the effects of the channels that were explicitly eliminated. This will be taken up in the next section.

Now let us discuss a little what is involved in numerically solving the coupled equations. Each function $u_c$ is subject to two boundary conditions; one is at the origin where the functions must vanish. Actually the behaviour in the vicinity of the origin is that of a spherical Bessel function. This fact can be used in numerically integrating the equations on a computer, so as to avoid a very bad starting condition that could cause the solutions to grow in magnitude beyond the range of the computer, as the integration progress to larger radii. So starting the integration very close to the origin with an infinitesimal value for the function in the incident channel, one would <u>guess</u> its slope to be that of the appropriate Bessel functions. Of course the point is that we will not know the actual slope until we reach the exterior region where the second boundary condition applies.

In the exterior region, where the nuclear potentials have fallen to zero, the equations become uncoupled and are just the equations whose solutions are related to the Bessel functions in the case of neutrons, and Coulomb functions in the case of protons. Hence

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$$u_c \rightarrow \alpha F_c + \beta G_c \tag{12}$$

where F and G denote the regular and irregular Coulomb functions. In the absence of charge they become

$$F_\ell \rightarrow \left(\frac{\pi\rho}{2}\right)^{1/2} J_{\ell + 1/2}(\rho)$$

$$G_\ell \rightarrow (-)^\ell \left(\frac{\pi\rho}{2}\right)^{1/2} J_{-\ell-1/2}(\rho) \tag{13}$$

$$\rho = kr$$

It is actually more convenient to use instead of F and G the combinations of these functions which asymptotically behave like outgoing and incoming spherical waves, $O_\ell$ and $I_\ell$,

$$I_\ell^* = O_\ell = G_\ell + i F_\ell \rightarrow e^{i(kr-\eta \ln(2kr)-\ell\pi/2+\sigma_\ell)} \tag{14}$$

where $\eta = mZZ'e^2/\hbar k$, and $\sigma_\ell$ is the Coulomb phase shift. The boundary conditions now can be stated precisely. In the channel c there are both incoming and outgoing spherical waves at infinity, corresponding to the fact that there is an incident wave in this channel. However in all other channels, there are only outgoing waves. Hence we require:

$$u_{c'} \rightarrow \delta_{c'c} I_c(k_c r) - \left(\frac{k_c}{k_{c'}}\right)^{1/2} S^I_{c'c} O_{c'}(k_{c'} r) \tag{15}$$

In general the integrated solutions will have both incoming and outgoing waves in all channels at infinity because the integration had to be started at the origin. Therefore a linearly independent set of solutions must be generated.

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Let us number the various channels c' by 1,2, ... N with 1 denoting the target channel c. (There will be only one target channel for given $I\pi$ when the target has J=0. This case we now specialize to for simplicity of notation.) Let us place two subscripts on each solution. The one will denote the channel number just mentioned and the other which of N different initial conditions could be chosen for the integration. Thus $u_{kp}$ will denote the k'th channel of the system solved according to the boundary condition numbered p. These conditions we can choose to be

$$\lim_{r\to o} u_{kp}(r) = \begin{cases} \dfrac{(Kr)^{\ell_p+1}}{(2\ell_p+1)!!} & , p = k \\ 0 & p \neq k \end{cases}$$

and the derivative of this function. Thus the behaviour of the solutions is such that all but one channel function has zero value and <u>slope</u> at the origin. By solving the system N times with the boundary conditions p = 1,...N we generate N distinct sets of solutions, some linear combination of which satisfies the required boundary conditions at an arbitrary exterior point R. This is expressed by the linear algebraic equations

$$\begin{array}{l} a_1 u_{11} + a_2 u_{12} + \ldots a_N u_{1N} + S_1 O_{\ell_1} + 0 \ldots 0 = I_{\ell_1} \\ a_1 u_{21} + a_2 u_{22} + \ldots a_N u_{2N} + 0 + S_2 O_{\ell_2} \ldots 0 = 0 \\ \vdots \\ a_1 u_{N1} + a_2 u_{N2} + \ldots a_N u_{NN} + 0 + 0 \ldots S_N O_{\ell_N} = 0 \end{array} \qquad (18)$$

where all functions are evaluated at R. Writing down also the derivatives of these equations would yield us 2N equations which can be solved for the N

coefficients a and the N scattering matrix elements S. From the latter the cross-sections for exciting each of the nuclear states can be calculated.

It may be noted that the number of coupled equations far exceeds the number of nuclear states considered. This is because c labels not only the nuclear state $\alpha J$ but the angular momentum of the scattered particle. Thus if the total angular momentum and parity of the system are $I,\pi$ then corresponding to one nuclear state of spin and parity $J\,\pi_\alpha$ there will be scattered particles of all angular momenta $\ell,j$ satisfying the conditions of Eq. (5), of which there are in general $2J+1$ (when $J < I$, as is so except for the lowest partial waves). Thus for <u>each</u> angular momentum and parity $I\pi$ there are

$$N = \sum_{\text{states}} (2J+1)$$

channels, where the sum is over the states of the nucleus. The experimental conditions correspond to a beam of particles incident on the target, which is a superposition of many angular momentum states. They range from $\ell=o$ up to some maximum value, given approximately by $2kR$, where R, is the radius beyond which the interactions are effectively zero. For each $\ell$ there are two j states, therefore the total number of differential equation to be solved is about $4kRN^2$.

<u>Distorted wave Born approximation</u>. A modest scattering problem for 20 MeV protons scattered on the nickel isotopes, and taking into account the lowest 6 or 7 states involves solving about 20,000 second order coupled differential equations in the method described above. It is not surprising therefore that until recently the problem as stated above was solved only to first order in the interaction V. This is called the distorted wave Born approximation and was first considered by Mott and Massey in their classic book on scattering, and was first applied to nuclear problems by Horowitz and Messiah and by Tobocman and Kalos.[3]

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Referring to Fig. 1, transitions which lead from the ground to an excited state occur in first order in $V$, whereas other transitions leading back to the ground or connecting excited states, occur only in second or higher order. Therefore, to first order, the differential equation, of the system (9) which refer to scattering on the ground state have no source term on the right, while those referring to excited states have a single source term corresponding to the direct transition from the ground. Thus to first order the coupled system (9) becomes,

$$(T_c + V^I_{cc} - E_c)u^o_c = 0 \qquad \text{(target channels)} \qquad (19a)$$

$$(T_{c'} + V^I_{c'c'} - E_{c'})u_{c'} = - V^I_{c'c}\, u^o_c \qquad \text{(excited state channels)} \qquad (19b)$$

The superscript "zero" on the solution to the first equation will serve to remind us, whenever it appears, that it is a solution to an uncoupled equation. The subscript $c$ refers to those channels which have the nucleus in the ground state, and the projectile in any one of various angular momentum states $\ell j$, while $c'$ refers only to excited states of the nucleus. The system is much simpler than before, since each excited state is coupled only to ground state channels. Hence each excited state can be considered separately from the others in this approximation. Therefore let $c'$ for the remainder of the discussion denote the nucleus in a particular excited state $\alpha' J'$. Consequently the various channels $c'$ refer to various angular momentum states of the projectile $\ell' j'$, while the nucleus is in this definite excited state.

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The equations could be solved numerically as before. This would involve solving for each $\ell j$, $\ell' j'$, four second order equations. However the solution can be obtained more simply by solving two uncoupled equations and then evaluating an integral. This is what is referred to as the distorted wave method.

To obtain this solution to the problem we introduce the Green's function corresponding to (19b)

$$(T_{c'} + V^{I}_{c'c'} - E_{c'})\, G_{c'}(r,r') = \frac{\hbar^2}{2m}\, \delta(r-r') \tag{20}$$

This Green's function can be constructed from particular solutions, $O_{c'}$ and $u^{o}_{c'}$, of the homogeneous equation

$$(T_{c'} + V^{I}_{c'c'} - E_{c'})\, y = 0 \tag{21}$$

which are respectively irregular and regular at the origin and have the assymptotic forms, for chargeless particles

$$O_{c'} \rightarrow \exp i\,(k_{c'}r - \ell'\pi/2) \tag{22a}$$

$$u^{o}_{c'} \rightarrow I_{c'} - e^{2i\delta_{c'}}\, O_{c'}$$

$$= -2i\, e^{i\delta_{c'}} \sin(k_{c'}r - \ell'\pi/2 + \delta_{c'}) \tag{22b}$$

(For charged particles, see Eq. (14)). Here $\delta_{c'}$ is the phase shift produced by $V_{c'c'}$. The normalizations have been chosen for later convenience. The desired Green's function is

$$G_{c'}(r,r') = \frac{-1}{2ik_{c'}}\, u^{o}_{c'}(r_<)\, O_{c'}(r_>) \tag{22c}$$

where $r_<$ denotes the smaller of $r,r'$ .

II

In view of (21) it obviously satisfies (20) except at $r = r'$. For this singular point we can make the proof by integrating the equation over $r$ from $r' - \delta$ to $r' + \delta$, and finally use the value of the Wronskian of $u_c^o$ and $O_c$ that can be obtained from (21) and (22).

Now we can show by direct substitution into (19b) and using (20) that

$$u_{c'}(r) = -\frac{2m}{\hbar^2} \int_o^\infty G_{c'}(r,r')\, V^I_{c'c}(r')\, u_c^o(r')\, dr' \tag{23}$$

is a particular solution of (19b). It is in fact the desired solution because as $r$ goes to infinity it has outgoing waves only, which corresponds to the physical conditions:

$$u_{c'}(r) \rightarrow \frac{2m}{\hbar^2} \frac{O_{c'}(r)}{2ik_{c'}} \int_o^\infty u^o_{c'}\, V^I_{c'c}\, u^o_c\, dr'$$

$$= -\left(\frac{k_c}{k_{c'}}\right)^{1/2} S_{c'c}\, O_{c'}(r) \tag{24a}$$

where we have written

$$S^I_{c'c} = -\frac{2m}{\hbar^2} \frac{1}{\sqrt{k_c k_{c'}}} \int_o^\infty u^o_{c'}\, V^I_{c'c}\, u^o_c\, dr \tag{24b}$$

This is the distorted wave Born approximation to the S-matrix. In this approximation, it can be obtained by finding the two functions which describe the elastic scattering from the potentials $V_{c'c'}$ and $V_{cc}$, respectively, and then performing the integrations indicated in (24b) involving the coupling potential $V_{c'c}$.

As the problem has been stated so far, $V_{cc}(r)$ is a real interaction since it is a diagonal matrix element of the two-body interaction. We know that, to re-

produce the elastic scattering of a nucleon from a nucleus, a complex (optical) potential is needed. In the next section we shall see that the effects of the truncation of the infinite system are properly taken into account by using in place of $V_{cc}$ a new effective interaction which is complex valued. The procedure for using the distorted wave approximation consists then of searching for a parameterization of the effective interaction represented as an optical potential,

$$V_{cc}(r) \rightarrow V(r) + i\, W(r) + V_{SO}(r)\langle \ell \cdot s \rangle_c \tag{25}$$

so as to reproduce the observed elastic cross sections. The elastic scattering functions $u_c^o$ thus determined, are then used in (24b) to calculate transitions out of the ground state.

The distorted wave approximation includes only such transitions as 1 and 2 indicated in Fig. 1 and ignores all second order transitions 3 to 6 and higher. If however any of these direct transitions from the ground is <u>too</u> strong, let us say 1, then the population of level "a" becomes sufficiently high that the transition, 3, back to the ground state (which is of course of the same strength as 1) cannot be ignored. This is a qualitative statement, but it seems, as a rule, that the collective transitions in vibrational nuclei are not "too" strong, whereas the transitions in strongly deformed nuclei are. The B(E2) for electromagnetic transitions from the collective $2^+$ state in the former case runs about 10 single-particle units while it is as large as 200 in the permanently deformed region. This provides a rough guide as to when the DWBA could be valid.

Clearly the method cannot be used to calculate the cross section to level "b" <u>if</u> transition 2 is weak compared to 1 and 5. This situation could arise through the action of some selection rules arising from

II

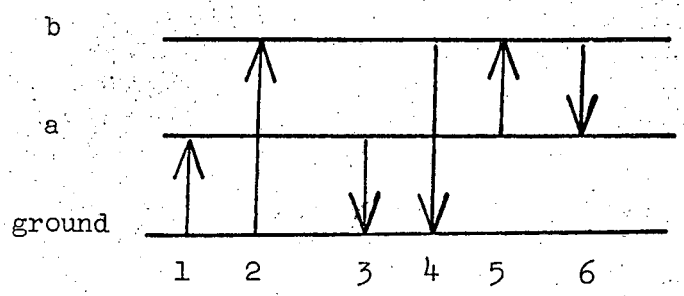


Fig. 1. The DWBA takes account of only transitions 1 and 2. The others can occur only after the excited states have been populated, and so correspond to higher order corrections to the scattering. The coupled channel calculations includes all transitions to all orders.

the particular structure of the levels, as in the case of harmonic vibrations. Thus even if none of the transitions is <u>too</u> strong in the sense described above, the method may still not work for <u>all</u> levels in a nucleus.

The absence of the complex conjugate sign on $u_{c'}^{o}$ in (24b) should be remarked on. According to (22b) it has outgoing scattered waves at infinity so that it is the complex conjugate of a function having incoming scattered waves.

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## III. TRUNCATION AND THE EFFECTIVE INTERACTION

Our main interest in inelastic scattering is in the information it can yield about the structure of the lower states of nuclei. We were therefore quite willing to reduce the infinite number of coupled equations describing the whole system of nucleus and scattered particle to a finite set of equations which refers explicitly only to the nuclear levels that are strongly coupled to the ground state, and any others of special interest. We did this simply by dropping the other terms in the expansion of the wave function that refer to the higher levels of the nuclear spectrum. This is of course completely analogous to the shell model treatment of a residual interaction, which is diagonalized within a finite subspace drawn from the lowest single-particle levels. In the shell model, this truncation procedure is justified on the grounds that the single-particle level spacings, and gaps between major shells are relatively large, say of the order of the spacing between the lower levels of the system itself so that most configurations lie distant in energy. In the scattering case however, the kinetic energy available for exciting the nucleus is of the order of 20 MeV or more for the experiments we are interested in. This is more than enough to excite the nucleus into the region of high level density. States of the system consisting of the nucleus in an excited state of interest, coexist at the same total energy with states in which the nucleus is in the continuum region of its spectrum. Thus there is no natural cut off in our expansion, arising from the energetics.

We therefore cannot hope to justify our procedure as discussed so far and must therefore investigate whether it can be modified. We shall see in fact that the effect of the excluded channels can be formally incorporated into the finite system which refers explicitly only to the small subspace of interesting channels. This is accomplished by a suitable modification of the interaction V.

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For the discussion of this matter we shall use a less explicit notation than that employed in the previous section, and shall use operator methods.[4] First we derive an expression for the transition matrix which corresponds to the approximations made in the previous section.

Relation of truncated to exact solution. Let us denote the part of the Hamiltonian(II-2) exclusive of the interaction V, by

$$H_o = H_A + T \tag{1}$$

so that the complete Hamiltonian is

$$H = H_o + V \tag{2}$$

Solutions to the unperturbed problem

$$(E - H_o)\,\phi = o \tag{3a}$$

have therefore the structure

$$\phi_{\alpha Jk} = \Phi_{\alpha J}(\underset{\sim}{A})e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}} \tag{3b}$$

$$k^2 = \frac{2m}{\hbar^2}(E - E_{\alpha J}) \tag{3c}$$

We can formally write a solution to the exact problem defined by

$$(E - H)\,\Psi = o \tag{4}$$

as

$$\Psi_\alpha^{(+)} = \phi_\alpha + \frac{1}{E-H+i\epsilon}V\phi_\alpha \tag{5a}$$

$$= \omega[V]\,\phi_\alpha \tag{5b}$$

This embodies the two boundary conditions that there is a plane wave in channel $\alpha$ and that there are outgoing spherical waves at infinity as is assured by the $i\epsilon$

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We have defined the operator for later convenience.

$$\omega[X] = 1 + \frac{1}{E-H+i\epsilon} X \tag{5c}$$

From the work of Gell-Mann and Goldberger[4] we know that the transition matrix for the problem defined by Eq. (2) is

$$\mathcal{T}_{\alpha'\alpha} = \langle \phi_{\alpha'} \, |V| \, \Psi_{\alpha}^{(+)} \rangle \tag{6}$$

However in the previous section we did not obtain a solution to the complete problem but to a truncated one. Let P denote the group of nuclear states retained in the expansion of $\Psi$ and let it also denote a projection operator onto this part of the vector space. The solution that we can obtain by solving the truncated set of coupled equations, which contain the actual interaction V, we shall denote by $\Psi(T)$. It is the solution to the approximation problem

$$(E - H_T) \, \Psi(T) = o \tag{7}$$

where

$$H_T = H_o + V_{PP} \tag{8a}$$

$$V_{PP} \equiv PVP \tag{8b}$$

Clearly $H_T$ does not cause any excitations out of P into Q and defines therefore the truncated problem of the previous section.

As above we can write a formal <u>exact</u> solution to this <u>approximate</u> problem as

$$\Psi_{\alpha}^{(+)}(T) = \Omega[V_{PP}]\phi_{\alpha} \tag{9a}$$

$$\Omega[X] = 1 + \frac{1}{E-H_o-X+i\epsilon} X \tag{9b}$$

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Similarly the transition matrix for the problem defined by (8) is

$$\mathcal{T}_{\alpha'\alpha}(T) = \langle \phi_{\alpha'} | V_{IP} | \Psi_{\alpha}^{(+)}(T) \rangle \tag{10}$$

$$= \langle \phi_{\alpha'} | V | \Psi_{\alpha}^{(+)}(T) \rangle$$

As we discussed earlier, we do not expect this to give reliable results, because it does not take account of the effect of the eliminated channels, even in an approximate way. On the other hand the exact result embodied in Eq. (6) is unattainable. Before deriving a compromise result let us derive the connection between the exact and approximate expressions for $\mathcal{T}$. This is easily done. First introduce the potential $U$ that links the two parts of the space

$$V \equiv (P+Q)\ V(P+Q) = V_{PP} + U$$

$$U \equiv V_{PQ} + V_{QP} + V_{QQ} \tag{11}$$

and take note of the identity

$$\omega[V] = \omega[U + V_{PP}] \equiv \omega[U]\ \Omega[V_{PP}] \tag{12}$$

This is readily proved by inserting the definitions of the operators on the right, carrying out the multiplication, and using the operator identity

$$\frac{1}{X} - \frac{1}{Y} = \frac{1}{X}(Y-X)\frac{1}{Y} = \frac{1}{Y}(Y-X)\frac{1}{X} \tag{13}$$

With this result we can now express the exact solution $\Psi$ in terms of the solution of the truncated problem

$$\Psi = \omega[V]\phi = \omega[U]\ \Omega[V_{PP}]\phi$$

$$= \omega[U]\ \Psi(T) \tag{14}$$

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Using this in Eq. (6) we have for transitions between any two states $\alpha$ and $\alpha'$ of P

$$\mathcal{T}_{\alpha'\alpha} = \langle \phi_{\alpha'} | V\omega_U | \Psi_\alpha^{(+)}(T)\rangle$$

$$= \langle \phi_{\alpha'} | V + V \frac{1}{E-H+i\epsilon} U | \Psi_\alpha^{(+)}(T)\rangle$$

$$= \mathcal{T}_{\alpha'\alpha}(T) + \langle \phi_{\alpha'} | V \frac{1}{E-H+i\epsilon} U | \Psi_\alpha^{(+)}(T)\rangle \tag{15}$$

Inserting the decomposition of V and U and remembering that $\alpha$ and $\alpha'$ both belong to P gives us

$$\mathcal{T}_{\alpha'\alpha} = \mathcal{T}_{\alpha'\alpha}(T) + \langle \phi_{\alpha'} | (V_{PQ} + V_{PP}) \frac{1}{E-H+i\epsilon} V_{QP} | \Psi_\alpha^{(+)}(T)\rangle \tag{16}$$

The second term on the right describes the effects which are omitted in the simple truncation employed. These effects obviously consist of excitation out of P into Q and back again.

Modified truncation. We now seek a formulation of the problem that is more amenable to incorporation of some of the effects due to the explicit elimination of most of the vector space. Following Feshbach[5] we shall use the same projection operators P and Q mentioned above and insert into the Schroedinger equation (4) the unit operator

$$(E-H)(P+Q)\Psi = 0 \tag{17}$$

Multiplying by Q on the left yields

$$(E-H_{QQ})(Q\Psi) = H_{QP}(P\Psi) \tag{18}$$

which can be solved formally as

$$Q\Psi = \frac{1}{E-H_{QQ}+i\epsilon} H_{QP}(P\Psi) \tag{19}$$

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We have not added a solution to the homogeneous equation $(E-H_{QQ})\,X = 0$ because any solution to this homogeneous equation has zero net flux through any closed surface about the origin and hence violates the conditions of the experiment, which correspond to outgoing waves only in all but the target channel.

Now multiply Eq. (17) on the left by $P$ to get

$$(E-H_{PP})(P\Psi) = H_{PQ}(Q\Psi) \tag{20}$$

Into this insert the solution (19) for $Q\Psi$ to get

$$[E-H_{PP} - H_{PQ}\,\frac{1}{E-H_{QQ}+i\epsilon}\,H_{QP}]\,(P\Psi) = 0 \tag{21}$$

which is the equation for the part of $\Psi$ that contains only parts belonging to the truncated space. It contains all the information necessary for calculating transitions among states belonging to $P$. Denote it more conveniently by

$$\Psi(P) \equiv P\Psi \tag{22}$$

Since $PQ \equiv 0$ we can rewrite Eq. (21) as

$$(E - H_o - \mathcal{V})\,\Psi(P) = 0 \tag{23}$$

where

$$\mathcal{V} = V_{PP} + V_{PQ}\,\frac{1}{E-H_{QQ}+i\epsilon}\,V_{QP} \tag{24a}$$

The problem defined by (23) differs from the one that defined our original truncated problem, (7), by precisely the second term of (24). Since we are interested in solving (23) in the space P only, we do not need the outer projection operators on (24).

$$\mathcal{V} = V + V\,\frac{Q}{E-H+i\epsilon}\,V \tag{24b}$$

[Note: The action of Q in this equation is to be effected as in (24a).]

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From the identity $\Psi = P\Psi + Q\Psi$ and Eq. (19) we have the entire solution

$$\Psi = \left( 1 + \frac{1}{E-H_{QQ}+i\epsilon} V_{QP} \right) \Psi(P) \tag{25}$$

whence according to Eq. (6)

$$\mathcal{T}_{\alpha'\alpha} = \langle \phi_{\alpha'} | V + V \frac{Q}{E-H+i\epsilon} V | \Psi_{\alpha}^{(+)}(P) \rangle$$

$$= \langle \phi_{\alpha'} | \mathcal{V} | \Psi_{\alpha}^{(+)}(P) \rangle \tag{26}$$

We can of course also write down the formal solution to Eq. (23)

$$\Psi^{(+)}(P) = \phi + \frac{1}{E-\mathcal{H}+i\epsilon} \mathcal{V} \phi \tag{27a}$$

with

$$\mathcal{H} = H_o + \mathcal{V} \tag{27b}$$

The statement was made earlier that from $\Psi(P)$ any transitions within P could be computed. If that statement was not obvious it is now proven since the transition matrix for the problem (27) is given by the exact result, (26).

To summarize the results so far, we have obtained a formal equation (23) which refers explicitly only to the truncated space P, and from which the exact transition matrix can be calculated according to (26). The equation for $\Psi(P)$ contains, not the original interaction V, but an effective interaction, effective within P, which includes the effect of the eliminated channels, Q. While the exact problem is actually no more tractable now than before, it is at least in a more instructive form, a form which is more suitable for making the approximations which will lead to a tractable problem, while retaining some of the important contributions from the eliminated part of the space.

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The effective interaction. Whereas the original scattering problem governed by $H = H_o + V$ led to an infinite set of coupled equations corresponding to all channels, a modified problem governed by $\mathcal{H} = H_o + \mathcal{V}$ which is to be solved only within a finite subspace of channels, can be defined which leads to the same T-matrix elements between channels of the subspace. Of course this is a statement which is only formally correct, since to calculate the effective interaction $\mathcal{V}$, we would need the complete spectrum of $H_{QQ}$. At the energies of interest to us, there are millions of open channels in medium to heavy nuclei so that a calculation of $\mathcal{V}$ is not feasible. Nevertheless it is a very important conceptual result, since we can make certain statements concerning the nature and behaviour of $\mathcal{V}$. For example the usual optical potential for elastic scattering corresponds just to the case that P projects on the ground state alone. Only the elastic channel is treated explicitly, all others entering implicitly through $\mathcal{V}$. While we cannot calculate the optical potential from first principles, it can be parameterized very successfully. From its definition (24), $\mathcal{V}$ is explicitly dependent on the energy, is complex valued, and non-local. The second term makes a large contribution to diagonal matrix elements but is probably small for off-diagonal ones. This can be seen by introducing the eigenstates of $H_{QQ}$ in a typical diagonal matrix element,

$$\begin{aligned} \mathcal{V}_{\alpha\alpha} &\equiv \langle \Phi_{\alpha J}(\underset{\sim}{A}) | \mathcal{V} | \Phi_{\alpha J}(\underset{\sim}{A}) \rangle \\ &\equiv V_{\alpha\alpha}(\underset{\sim}{r}) + \sum_C \frac{\langle \Phi_{\alpha J} | V | \Psi_C \rangle \langle \Psi_C | V | \Phi_{\alpha J} \rangle}{E - E_C + i\epsilon} \end{aligned} \tag{28}$$

Here $\Psi_C$ is intended to denote the wave function for the $A + 1$ particles while $\Phi_\alpha$ denotes the wave function of the nucleus $A$ only. The brackets denote integrations over the $A$ particles and remain functions of the remaining coordinate.

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The sum on C runs over the space of Q and includes continuum and discrete states. The numerator is positive and the denominator changes sign only once. There is little chance therefore that the sum is small. However the individual terms in the second part of $\mathcal{V}$ generally will be very small since they involve matrix elements connecting the two parts of the space P and Q whose states will usually be of quite a different structure, since the latter lie at higher excitation in the nucleus A.

This means that the sum, which is over an enormous number of terms (a continuum built on each state Q of the A-particle system) is not dominated by any particular one of them, so long as no states of Q are strongly coupled to any of P. With the proviso that the effect of such strong coupling be removed from $\mathcal{V}$ by treating them explicitly in P, then the sum C in (28) will be dominated by the high excitation region because of the high level density. From this fact we see that $\mathcal{V}$ will be slowly varying from nucleus to nucleus, and that the diagonal matrix elements in the space P will be similar. We do know that the optical model treatment of elastic scattering is generally satisfactory. This suggests that we treat the <u>diagonal</u> matrix elements of $\mathcal{V}$ by a similar phenomenological parameterization.

For the off-diagonal matrix elements of $\mathcal{V}$, on the other hand, the numerator of the second term is not positive but will fluctuate in sign from term to term so that in general we may expect it to give a small contribution in comparison with the first.

<u>Non-locality</u>. To demonstrate the non-local nature of the second term of $\mathcal{V}$, which for convenience we call $\mathcal{W}$, we examine the coordinate representation of $\mathcal{W}$,

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$$\langle \underset{\sim}{r},\underset{\sim}{A}|\mathcal{W}|\underset{\sim}{r}',A'\rangle = \sum_C \frac{\langle \underset{\sim}{r},\underset{\sim}{A}|V|C\rangle\langle C|V|\underset{\sim}{r}',\underset{\sim}{A}'\rangle}{E - E_C + i\epsilon}$$

$$= \sum_C \frac{V(\underset{\sim}{r},\underset{\sim}{A})\ \Psi_C(\underset{\sim}{r},\underset{\sim}{A})\ \Psi_C^*(\underset{\sim}{r}',\underset{\sim}{A}')\ V(\underset{\sim}{r}',\underset{\sim}{A}')}{E - E_C + i\epsilon} \tag{29}$$

where we have used

$$\Psi_C(\underset{\sim}{r},\underset{\sim}{A}) \equiv \langle \underset{\sim}{r},\underset{\sim}{A}|C\rangle \tag{30}$$

to denote a wave function of $H_{QQ}$ for the A + 1 particle composite, and the fact that $V$ is local (i.e. diagonal in the coordinate representation)

$$V(\underset{\sim}{r},\underset{\sim}{A}) \equiv \langle \underset{\sim}{r},\underset{\sim}{A}|V|\underset{\sim}{r},\underset{\sim}{A}\rangle \tag{31}$$

Now we can write the effect of $\mathcal{W}$ acting on the wave functions we have been using. In a coordinate representation, we have

$$\langle \underset{\sim}{r},\underset{\sim}{A}|\mathcal{W}|u,\phi\rangle = \sum_C V(\underset{\sim}{r},\underset{\sim}{A})\ \Psi_C(\underset{\sim}{r},\underset{\sim}{A})$$

$$\times \frac{1}{E - E_C + i\epsilon}\int d\underset{\sim}{r}'d\underset{\sim}{A}'\ \Psi_C^*(\underset{\sim}{r}'\underset{\sim}{A}')\ V(\underset{\sim}{r}',\underset{\sim}{A}')\ u(r')\ \phi(r',\underset{\sim}{A}') \tag{32}$$

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## IV. MICROSCOPIC THEORY OF INELASTIC SCATTERING

We emphasized in the introduction that the remarkable development of theoretical techniques for handling the nuclear many-body problem, have led to fairly detailed descriptions of nuclei in terms of their nucleonic structure. The enhanced E2 transition rates of the first excited $2^+$ states is understood in terms of a coherent superposition of single-particle transitions. The energy level systematics through certain regions of the periodic table can be reproduced, at least qualitatively. It is important to subject these descriptions to the further stringent tests that inelastic scattering sets, to see to what degree they succeed here. In this section we develop the necessary apparatus for confronting nuclear structure theory with inelastic scattering data.[6]

The interaction. According to the development of the last section we understand that the diagonal matrix elements of the interaction between the incident particle and nucleus would be best parameterized as a non-local complex-valued optical-model potential. However we know from the work of Perey and Buck on elastic scattering, that generally an equivalent local potential can be found and this we take advantage of. Hence for the diagonal matrix elements we have

$$\mathcal{V}_{cc}(r) = V\, f_V(r) + i[W\, f_W(r) - 4\, W_D\, a_D\, f_D'(r)] + \langle \underset{\sim}{\ell}\cdot\underset{\sim}{\sigma}\rangle_c \left(\frac{\hbar}{m_\pi c}\right)^2 V_{SO}\, \frac{1}{r}\frac{d}{dr}\, f_{SO}(r) + V_{Coulomb}(r) \tag{1}$$

where

$$f_x(r) = \left[1 + \exp\left(\frac{r - r_x A^{1/3}}{a_x}\right)\right]^{-1}, \quad f' = df/dr \tag{2}$$

is the Woods-Saxon form factor. This is the standard parametrization of the

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optical potential that is used at Oak Ridge, where so much of the elastic scattering analysis is performed.

In principle the diagonal matrix elements $\mathcal{V}_{cc}$ are different in each intrinsic state. That is, the optical potentials in the excited states may be different from each other and from that of the ground state. However referring to the structure of $\mathcal{V}$ (III-24) it seems very plausible that, unless there are states belonging to Q which are very strongly coupled to one or several, but not all states of P, then the optical potentials in each state of P wil be similar. The reason for believing this is that the sum over intermediate states Q, involved in the evaluation of $\mathcal{V}$, extends over such an enormous number of levels that it will not matter much if the initial state is the ground state or one nearby.

We need also the off-diagonal matrix elements $\mathcal{V}_{c'c''}$ $(c' \neq c'')$ which we shall often refer to as the direct interaction, since it is responsible for the transitions. We learned that it may not differ too much from the nucleon-nucleon matrix elements $V_{c'c''}$. We shall represent V in the form

$$V(\underset{\sim}{r},\underset{\sim}{A}) \equiv \sum_{i=1}^{A} V(\underset{\sim}{r},\underset{\sim}{r}_i) \tag{3a}$$

$$V(\underset{\sim}{r},\underset{\sim}{r}_i) = (V_0 + V_1\, \underset{\sim}{\sigma}\cdot\underset{\sim}{\sigma}_i)\, g(|\underset{\sim}{r}-\underset{\sim}{r}_i|) \tag{3b}$$

where the well depths $V_0$ and $V_1$ may also depend upon $\underset{\sim}{\tau}\cdot\underset{\sim}{\tau}_i$. Let us make a Legendre expansion of g

$$g(|\underset{\sim}{r}-\underset{\sim}{r}_i|) = \sum_L \frac{4\pi}{2L+1}\, v_L(r,r_i)\; Y_L(\hat{r})\cdot Y_L(\hat{r}_i) \tag{4}$$

Moreover, in order to treat both spin-dependent and independent terms on the same footing let us define the tensors

$$\mathcal{Y}_{LSJ}(\hat{r},\underset{\sim}{\sigma}) = [Y_L(\hat{r})\Sigma_S]_J \tag{5}$$

where $Y_L$ is a spherical harmonic, $\Sigma_0 = 1$, $\Sigma_1 = \underset{\sim}{\sigma}$ and the square bracket denotes vector coupling. Then we can write the multipole expansion of $V(\underset{\sim}{r},\underset{\sim}{A})$

$$V(\underset{\sim}{r},\underset{\sim}{A}) = \sum_{LSJ} (-)^{L+S+J} V_S \, \mathfrak{T}_{LSJ}(r,\underset{\sim}{A}) \cdot \mathfrak{Y}_{LSJ}(\hat{r},\underset{\sim}{\sigma}) \tag{6a}$$

where

$$\mathfrak{T}_{LSJ}(r,\underset{\sim}{A}) = \frac{4\pi}{2L+1} \sum_{i=1}^{A} v_L(r,r_i) \mathfrak{Y}_{LSJ}(\hat{r}_i \underset{\sim}{\sigma}_i) \tag{6b}$$

is a tensor operator in the nuclear space and depends only parametrically on $r = |r|$.

The multipole expansion is actually much more general than the particular form chosen for the interaction, so that some of the following formulation, though not the numerical results, are more general.

Now we want matrix elements of $V$

$$V^I_{c_1 c_2}(r) \equiv \langle \phi_{c_1 I}^{M} | V | \phi_{c_2 I}^{M} \rangle$$

$$= \langle [\mathcal{Y}_{\ell_1 s\, j_1}(\hat{r}) \, \Phi_{\alpha_1 J_1}(\underset{\sim}{A})]^M_I \, |V| \, [\mathcal{Y}_{\ell_2 s\, j_2}(\hat{r}) \, \Phi_{\alpha_2 J_2}(\underset{\sim}{A})]^M_I \rangle \tag{7}$$

Inserting the multipole expansion for $V$, we note that the tensor operator $\mathfrak{Y}_{LSJ}$ acts on the space of the scattered nucleon and the other operator, $\mathfrak{T}$, on the nuclear space. Such matrix elements can be evaluated by Racah techniques[7] and the result written as,

$$V^I_{c_1 c_2}(r) = \sum_{LSJ} V_S \, C^I_{LSJ}(c_1 c_2) \langle \alpha_1 J_1 \| \mathfrak{T}_{LSJ} \| \alpha_2 J_2 \rangle \tag{8}$$

where $C$ is purely geometrical in character

$$C^I_{LSJ}(c_1,c_2) = (-)^{L+S+J+j_2+J_1+I} \begin{Bmatrix} I & J_1 & j_1 \\ J & j_2 & J_2 \end{Bmatrix} \langle j_1 \| \mathfrak{Y}_{LSJ} \| j_2 \rangle \tag{9}$$

The curly bracket is a 6-j symbol. Rach's definitions of reduced matrix elements is used, which differs by a factor from another definition also in common

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use. For the reduced matrix element of the spin-orbit tensor we find

$$\langle(\ell_1\ 1/2)j_1\|\mathcal{Y}_{LSJ}\|(\ell_2\ 1/2)j_2\rangle = (-)^{\ell_a}\left[\frac{\hat{j}_1\hat{j}_2\hat{\ell}_1\hat{\ell}_2\ \hat{L}\hat{S}\hat{J}}{2\pi}\right]^{1/2}$$

$$\times\begin{pmatrix}\ell_1 & L & \ell_2\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\ell_1 & 1/2 & j_1\\ \ell_2 & 1/2 & j_2\\ L & S & J\end{Bmatrix} \tag{10}$$

where the last two symbols are a 3-j and 9-j coefficients, and $\hat{L} \equiv 2L+1$.

Form factors. The matrix element of the interaction, (8) is now expressed in terms of the purely geometrical quantities C, the strengths of the interaction $V_0$ and $V_1$ and nuclear reduced matrix elements which remain a function of the radial coordinate r of the scattered particle. They are referred to as form factors for the transition between nuclear states $\alpha_1 J_1$ and $\alpha_2 J_2$.

$$\mathcal{F}^{\alpha_1\alpha_2}_{LSJ}(r) \equiv \langle\alpha_1 J_1\|\mathcal{F}_{LSJ}(r,\underset{\sim}{A})\|\alpha_2 J_2\rangle \tag{11}$$

These are the quantities that carry the nuclear information that is relevant to the inelastic scattering process. We now discuss them in some detail.

The structure of the nuclear form factors (11) is actually rather simple as follows from the fact that the scattered particle acts as a one-body operator on the nuclear coordinates because the interaction between nucleons is two-body. So $\mathcal{F}$ is a sum of one-body operators as seen in (6b), and the nuclear form factor must be expressible as a sum of form factors for single-nucleon excitations. Thus

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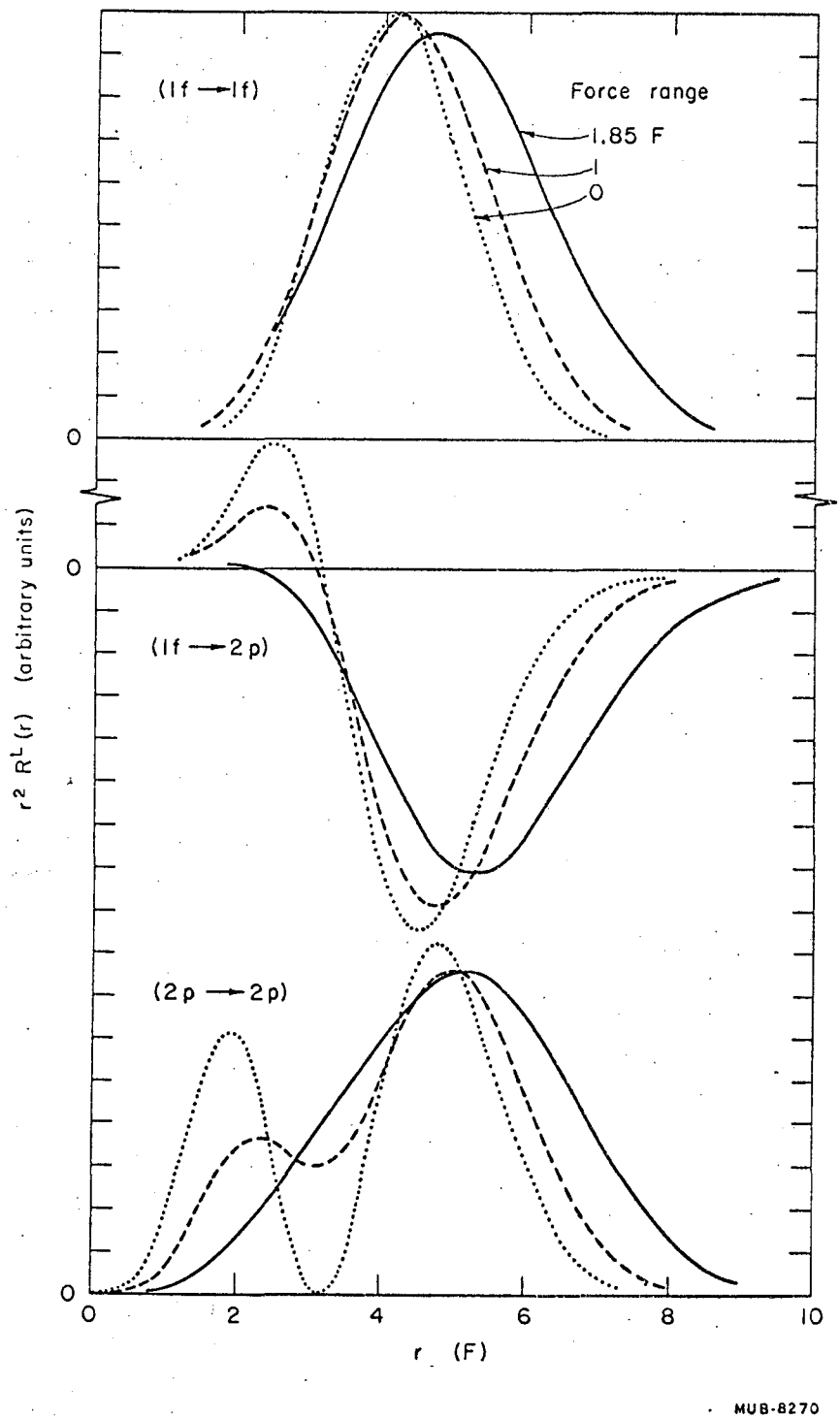


Fig. 2. Shapes of several single-particle form factors that contribute to excitation of $2^+$ states in nickel isotopes are shown for several force ranges of the direct interaction. The oscillator parameter $\nu = m\omega/\hbar \doteq 0.25\ F^{-2}$. (Ref. 6b).

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$$\mathcal{F}^{\alpha_1\alpha_2}_{LSJ}(r) = \sum_{a,b} S^{a,b}_{LSJ}(\alpha_1\alpha_2)\, F^{ab}_{LSJ}(r) \tag{12}$$

where

$$F^{ab}_{LSJ} = \langle \psi_a(\underset{\sim}{r}') \| \mathcal{T}_{LSJ}(r,\underset{\sim}{r}') \| \psi_b(\underset{\sim}{r}') \rangle \tag{13a}$$

is the form factor corresponding to the promotion of a single nucleon from the state a to b. We use a to denote the collection of quantum numbers for a single-particle state in the nucleus, $a \equiv n_a \ell_a j_a$, and $\psi_a(\underset{\sim}{r})$ denotes a single-particle wave function with radial part $u_a(r)$. From the definition of $\mathcal{T}$, Eq. (6b)

$$F^{ab}_{LSJ}(r) = 4\pi\, R^{ab}_L(r)\, \langle j_a \| \mathcal{Y}_{LSJ} \| j_b \rangle \tag{13b}$$

with the radial integral

$$R^{ab}_L(r) = \frac{1}{2L+1} \int u_a(r')\, v_L(r,r')\, u_b(r')\, r'^2 dr' \tag{13c}$$

This defines the radial shape of the form factor. We can gain an immediate impression of the general shape by considering a zero-range interaction. In that case the Legendre transform in Eq. (4) is

$$v_L(r,r') = \frac{2L+1}{4\pi}\, \frac{\delta(r-r')}{r\, r'} \qquad \text{(zero range)} \tag{14a}$$

and

$$R^{ab}_L(r) \to u_a(r)\, u_b(r)/4\pi \tag{14b}$$

Several single-particle form factors for transitions relevant to the nickel region are shown in the Fig. 2 for various force ranges. The zero-range results would be easily surmised from (14b) but a force range of realistic size destroys most of the fine detail.

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For the Gaussian shaped potential

$$g = \exp-\left(\frac{|\mathbf{r}-\mathbf{r}'|}{\rho}\right)^2 \tag{15}$$

the Legendre transform is

$$v_L(r,r') = (2L+1)\, i^L e^{-(r^2+r'^2)/\rho^2}\, j_L\left(\frac{-2irr'}{\rho^2}\right) \quad , \quad \text{(Gaussian)} \tag{16}$$

Using harmonic oscillator radial functions $u_a$, a closed form for the form factors can also be found. It has the form

$$R_L^{ab}(r) = e^{-\gamma r^2} \sum_{m=0}^{\bar{m}} G_m^L(a,b)\left(\frac{r}{\rho^2\sqrt{\nu}}\right)^{2m+L} \tag{17}$$

where $\nu = m\omega/\hbar$ is the oscillator parameter, $\gamma = \nu/(\nu\rho^2+1)$ and $\bar{m} = 1/2(N_a+N_b-L)$ with $N$ denoting the oscillator quantum number, $N = 2(n-1)+\ell$. The coefficients $G$ are rather complex but are expressible in closed form as a finite double sum.[6b] It is very convenient for calculational purposed to have available such a closed form as (17), because it means that all the nuclear information that is relevant, can be stored in a few expansion coefficients for the various form factors (12).

Nuclear structure and amplitudes for elementary transitions. The form factors for a transition between any pair of nuclear states can be expressed as a linear combination of elementary transitions just discussed, as expressed in (12). The amplitudes with which the various elementary transitions contribute can be calculated from the wave functions for the nuclear states. The same amplitudes arise in other contexts also, since they merely express how the matrix element of a one-body operator taken between a many-body state, is written in terms of the one-particle matrix elements of the operator. We illustrate their calculation by several examples.

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Two-quasiparticle states. One of the methods used for handling many-particle nuclear systems can be briefly described as follows.[1] The pairing effects of the residual interaction are isolated by solving the BCS equations. This corresponds to a transformation from the real nucleons of the system to quasi-particles, which, as far as the pairing effects of the interaction are concerned are almost non-interacting. The remaining part of the interaction is treated approximately by diagonalizing it in the quasiparticle space. Often the diagonalization is restricted to two-quasiparticle configurations. The wave function in such a calculation have the form

$$|\alpha JM\rangle = 1/2 \sum_{a,b} \eta_{ab}^{\alpha J} A_{JM}^{+}(ab) \, |0\rangle \tag{18}$$

where the $\eta$'s are configuration amplitudes, $|0\rangle$ denotes the ground state which is here the vacuum for quasiparticles, of which a pair creation operator is denoted by

$$A_{JM}^{+}(a,b) = (-)^{j_a - j_b + J} A_{JM}^{+}(b,a) = -\left[\alpha_a^{+} \alpha_b^{+}\right]_J^M \tag{19}$$

The quasiparticle creation operator is denoted by $\alpha^{+}$. We need also the quasi-particle scattering operator

$$N_{JM}^{+}(a,b) = (-)^{j_a - j_b + M} N_{J-M}(b,a) = -\left[\alpha_a^{+} \tilde{\alpha}_b\right]_J^M , \tag{20}$$

where

$$\tilde{\alpha}_{am_a} = (-)^{j_a - m_a} \alpha_{a-m_a} \tag{21}$$

The transformation connecting the particles $\beta^{+}$ to the quasiparticles $\alpha^{+}$ is

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$$\beta^{+}_{am} = U_a \alpha^{+}_{am} + V_a \tilde{\alpha}_{am} \tag{22a}$$

where

$$U^2 + V^2 = 1 \quad , \quad UV(-)^{\ell} \geq 0 \tag{22b}$$

Since $\mathfrak{T}_{LSJ}$, Eq. (6b) is a one-body operator in the <u>particle</u> space, to evaluate its matrix elements between states expressed in terms of <u>quasiparticles</u> we would want to re-express it in these latter terms. The results for any such operator is found after some Racah algebra to be

$$\begin{aligned}\mathfrak{T}^{M}_{LSJ} = \sum_{ab} (2J+1)^{-1/2} \langle a \| \mathfrak{T}_{LSJ} \| b \rangle \Big\{ & V_a^{\,2} (2j_a+1)^{1/2} \delta_{ab} \delta_{J0} \\ & + \frac{1}{2}\Big[U_a U_b - (-)^{J+\sigma} V_a V_b\Big]\Big[N^{+}_{JM}(a,b) + (-)^{M+\sigma} N_{J-M}(a,b)\Big] \\ & - \frac{1}{2}\Big[U_a V_b + (-)^{J+\sigma} V_a U_b\Big]\Big[A^{+}_{JM}(a,b) + (-)^{M+\sigma} A_{J-M}(a,b)\Big]\Big\}\end{aligned} \tag{23}$$

where the phase $\sigma$ is defined by

$$\langle a \| \mathfrak{T}_{LSJ} \| b \rangle = (-)^{j_a - j_b + \sigma} \langle b \| \mathfrak{T}_{LSJ} \| a \rangle \tag{24}$$

and in particular for the operators (5) and (6b), $\sigma = L + S + J$. The reduced matrix element in (23), for our situation, is the previously defined single-particle form factor (13).

To evaluate the form factor corresponding to transitions from the ground state $|0\rangle$ to an excited state such as (18), one sees that only the $A^{+}_{JM}$ term in (23) contributes. Using the readily proven fact that

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$$\langle 0|A_{\lambda\mu}(a,b)\ A^{+}_{\lambda'\mu'}(a',b')|0\rangle$$

$$= \delta_{\lambda\lambda'}\ \delta_{\mu\mu'}(\delta_{aa'}\ \delta_{bb'} + (-)^{j_a - j_b + \lambda}\ \delta_{ab'}\ \delta_{a'b}) \tag{25}$$

we obtain

$$\mathcal{F}_{LSJ}(r) \equiv \langle \alpha J \| \mathfrak{F}_{LSJ}(r,\underset{\sim}{A}) \| 0\rangle$$

$$= -\frac{1}{2} \sum_{ab} \eta^{\alpha J}_{ab} \left[ U_a V_b + (-)^{L+S}\ V_a U_b \right] F^{ab}_{LSJ}(r) \tag{26}$$

It is interesting to look at this in some detail for a collective state, say the $2_1^+$ state of $Ni^{60}$. The wave functions of Arvieu, Salusti and Veneroni have been used.[8] Considering first the case of S=0 in (26) which we refer to as the scalar form factor, one finds that for this state their wave function is fully coherent. Each term, corresponding to the various single-particle excitations that lead from the ground to the $2_1^+$ state has the same sign and so gives rise to the large form factor shown in the Fig. 3 which is very similar to the one used in macroscopic treatments of vibrational states. The individual terms are also shown. This is in contrast to the form factors for other $2^+$ states for which the various elementary transitions interfere destructively, giving rise to smaller form factors which sometimes oscillate as illustrated in Fig. 4. (Note that the absolute values are plotted on a logarithmic scale which accounts for the difference in appearance between Fig. 3 and the top curve of Fig. 4).

There are three vector form factors (S=1) corresponding to L=J, J±1. However, since L is connected to the parity change between the nuclear states,

$$(-)^L = \pi_1\ \pi_2$$

then only the L=J vector form factor, together with the scalar, just discussed, can contribute to the so-called natural parity states whose parity is given by

IV

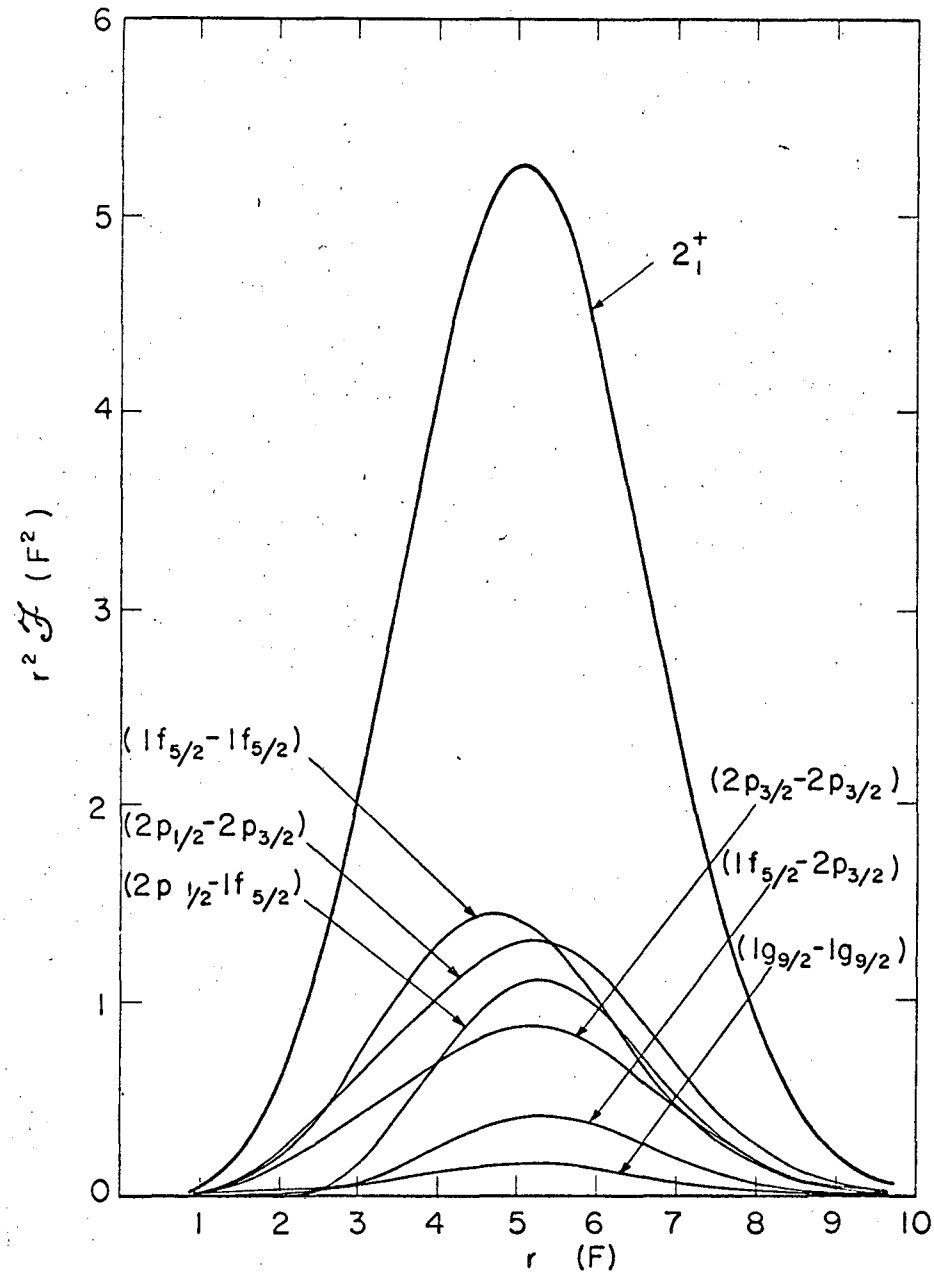


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Fig. 3. Scalar form factor for the lowest $2^+$ state in $Ni^{60}$ is shown together with the single-particle form factors that contribute to it. They all have such phases that they add constructively for the lowest $2^+$ but must therefore be destructive for all other $2^+$ states. (Ref. 6b).

their spin as $\pi = (-)^J$. The vector form factor is small compared to the scalar for the collective $2_1^+$ state, since it is incoherent. The dashed lines of Fig. 4 illustrate the vector form factors for several $2^+$ states. It is actually possible that another type of collectivity may exist, based, not on the singlet-even part of the force as the $2_1^+$ state is, but on the triplet-even part of the n-p interaction. Evidently single-closed shell nuclei, such as Ni isotopes would not be a favourable place to find them.

Only form factors such as (26) which connect the ground state to an excited state, are needed in the distorted wave Born approximation. However the coupled channel method takes account also of transitions between excited states. Such transitions become exceedingly important if one of the states is very weakly coupled to the ground, particularly if the coupling to a collective state like the $2_1^+$ is strong by comparison.

For transitions between two excited states having the structure (18) the amplitudes appearing in the expression (12) for the form factor is

$$S^{ab}_{LSJ}(\alpha_1 J_1, \alpha_2 J_2) = (-)^{J_1+J_2+J+1} \left[(2J_1+1)(2J_2+1)\right]^{1/2}$$

$$\times \left(U_a U_b - (-)^{L+S} V_a V_b\right) \sum_d \eta^{\alpha_1 J_1}_{da} \eta^{\alpha_2 J_2}_{bd} \begin{Bmatrix} J_2 & J & J_1 \\ j_a & j_d & j_b \end{Bmatrix} \tag{27}$$

<u>Two-phonon states</u>. The available experimental evidence concerning so-called two-phonon states indicates considerable departures from the vibrational model. The evidence consists of the sometimes large splittings between the members of the two-phonon triplet, deviations from the predicted intensity rules for E2 radiation, and the finite static quadrupole moment of the two-phonon $2^+$ state in those nuclei for which the measurements have been made. Nevertheless some of the two-phonon character is expected to persist, though it may be shared with other neighboring states. The type of nuclear structure calcu-

IV

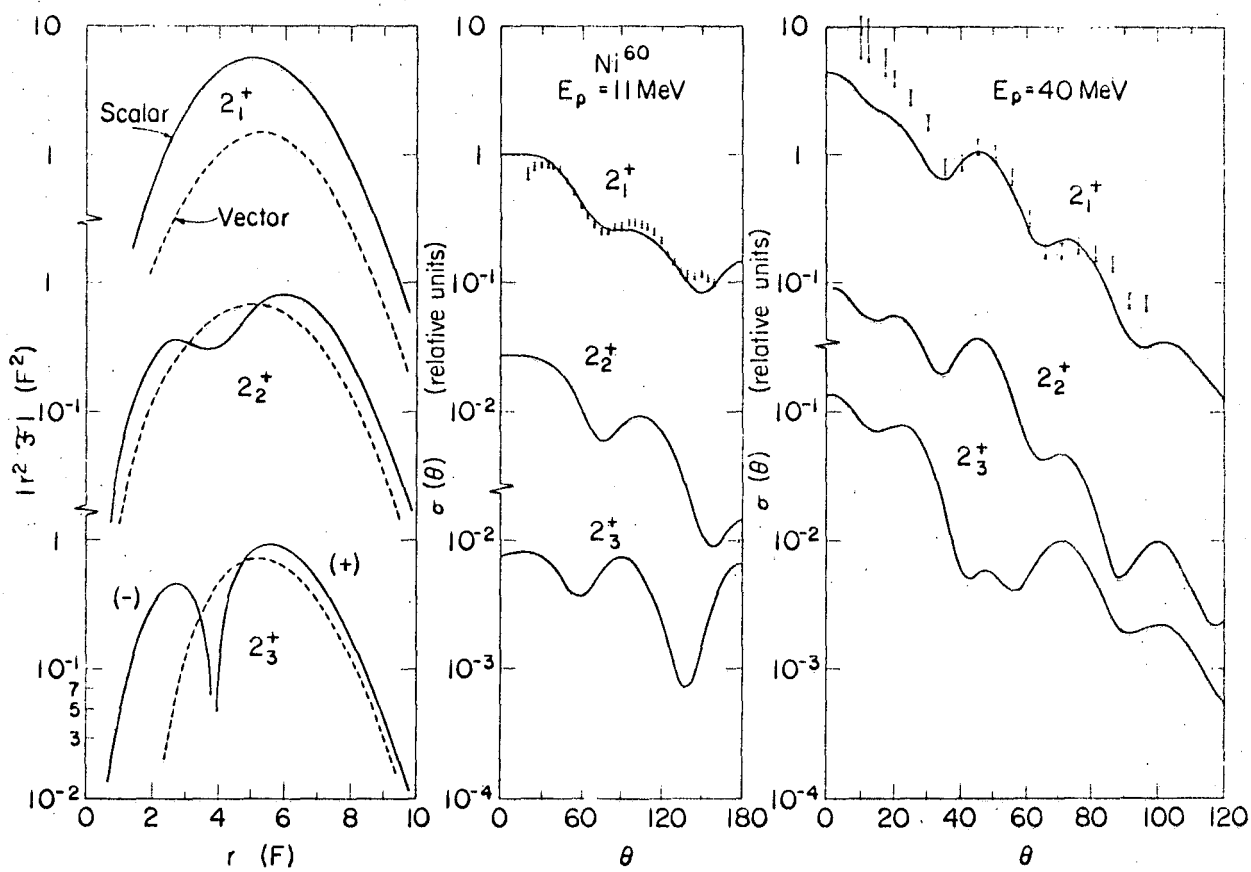


Fig. 4. Scalar and vector form factors for $2^+$ states in $Ni^{60}$ are shown at the left. The absolute values are plotted and oscillations are indicated by (+) and (-). Corresponding cross-sections at 11 and 40 MeV for proton scattering calculated in the distorted wave approximation are also shown. (Ref. 6b).

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lation which truncates the problem at two quasiparticle configurations does not produce two-phonon states because they involve four-quasiparticle configurations. However we can use the coherent operator that produces the $2_1^+$ state to generate a "two-phonon" triplet of wave functions. Presumably these are not eigenfunctions of the problem, and would mix with neighboring two-quasiparticle states, but we can consider them as an extreme limit. The form factors by which each member of the triplet is connected to the collective $2_1^+$ state can be obtained easily upon using the quasiboson commutation rules for the two-quasiparticle operators introduced in (19), which are given in fact by the right side of (25). The result is that the form factor connecting each member of the triplet to the collective $2^+$ state is proportional to the form factor for the ground state to $2_1^+$ state just previously discussed. The proportionality constant is $\left[2(2J_2+1)/5\right]^{1/2}$ where $J_2$ is the spin of the two phonon state.

Two-particle states. For conventional shell model wave functions describing two particles beyond closed shells, the structure amplitudes can be obtained from (27) by choosing $U_a = U_b = 1$ and $V_a = V_b = 0$. Whereas most shell model wave functions are described as

$$\sum_{a\leq b} C_{ab} \, |(ab)J\rangle$$

with the ket antisymmetrized, our sum over configurations (18) is unrestricted as to order of a,b. Hence

$$\eta_{ab} = (1 + \delta_{ab})^{1/2} \, C_{ab}$$

$$\eta_{ba} = (-)^{j_a - j_b - J} (1 + \delta_{ab})^{-1/2} \, C_{ab}$$

Other cases can also be obtained from (27) as special cases.

V

## V. APPLICATION OF MICROSCOPIC THEORY

Effective interaction. As we have already discussed, the diagonal matrix elements of the effective interaction are certain to be very complicated. To calculate them would involve approximating the wave functions for nuclear states ranging all the way up into the continuum. The results of such a calculation would certainly not be as reliable as an optical model parameterization. For the special case when all excited levels are eliminated, the effective interaction so parameterized is the usual optical potential for elastic scattering, and this can be found in any particular instance quite easily by use of one of the automatic search routines. We shall see in fact, that for spherical nuclei, where the coupling to excited states is not too strong, the same optical potential for pure elastic scattering can be used with little or no change when a few of the excited states are explicitly treated. This is illustrated in Fig. 5 for $Ni^{60}$. This is definitely not true for deformed nuclei, whose coupling to the $2_1$ state, as measured in single-particle units of the reduced B(E2) is about 100-1000 in contrast to a typical figure of 10 in spherical (vibrational) nuclei. However in this section we are concerned with spherical nuclei.

Concerning the off-diagonal matrix elements of the effective interaction, we have some reason to hope that they may not differ very much from those of the free two-body interaction, or some simple parameterization thereof. Nevertheless this is an open question. In principle it may be energy dependent, nonlocal, and complex valued, in addition to having all the usual complexities of the two-body interaction. By its nature, anything that can be determined about it for one energy, or one small region of the periodic table cannot be carried over with any assurance to another.

V

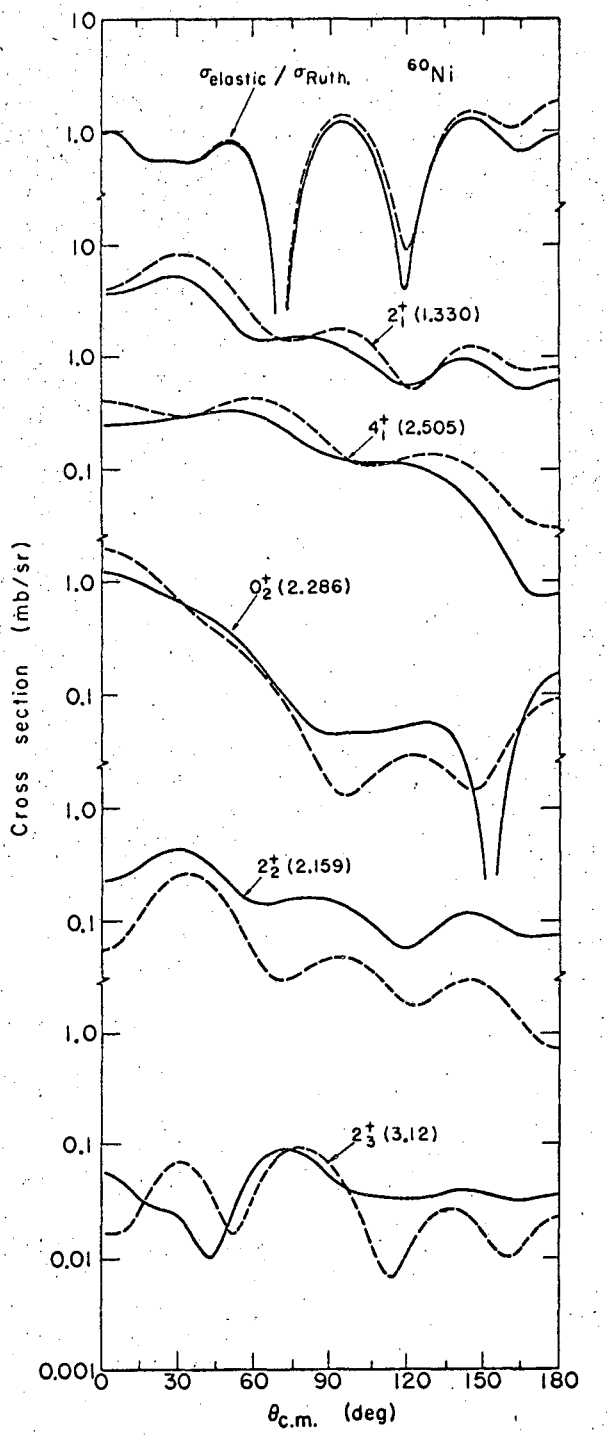


Fig. 5. Coupled channel (solid) is compared with optical model calculation of elastic and distorted wave calculation of inelastic proton scattering (dashed). Note that elastic is barely effected by the additional coupling. While several cross-sections are only slightly changed others are significantly modified. (Ref. 6i).

V

Keeping these reservations in mind we now discuss results based on a simple force parameterized as in the last section. We shall in fact keep its range fixed at 1.85 F such as is often used in shell model calculations which employ a Gaussian potential, and the two well depths $V_0$ and $V_1$ of (IV-3) will be adjusted.

Two quasiparticle description of nickel and tin. We gave a very brief idea of what is involved in this method of approximately solving the nuclear problem, and it has been discussed thoroughly elsewhere.[1] Here we shall see to what extent it succeeds in describing the scattering of protons. From Fig. 5 we see that while the distorted wave method would be adequate for some levels, in general we must solve the coupled equations, since the collective $2_1^+$ state is an important intermediate step in the excitation of some of the levels.

It is worth considering for a moment the exchange nature of the residual interaction for such nuclei, where all valence nucleons are of the same type. The only relevant parts are singlet-even (SE) and triplet-odd (TO). The former is attractive and the latter smaller and possibly repulsive. If we rewrite the central potential in terms of 1 and $\sigma_1 \cdot \sigma_2$ we have

$$V = \frac{SE + 3\ TO}{4} + \frac{TO - SE}{4}\ \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2$$

From this we see that the $\underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2$ part is typically repulsive and that the attraction comes from the spin-independent part. The lowest lying states will as a consequence possess such correlations as exploit this part best. The position of a state in which the nucleons are so correlated will not be very sensitive to the strength of $\underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2$ part.

V

From the discussion we can understand why the lowest $2^+$ state has a fully coherent scalar form factor while its vector form factor is small by comparison (Fig. 4). The higher lying $2^+$ states because they do lie higher evidently have not exploited the same part of the force so successfully, and their scalar and vector form factors are more comparable in magnitude.

Because the scalar form factor for the collective $2^+$ state is so much larger than the vector, this state can be used to fix $V_0$. When this is done and $V_1$ is set to zero the resulting cross-sections are shown in Fig. 6. Two nuclear structure calculations for $Ni^{60}$ have been used one corresponding to a Gaussian residual interaction[8] and one to a surface delta interaction.[9] They both give essentially the same result for the collective $2^+$ state and $4_1^+$, where the agreement is not too bad. For the other states the magnitude of the cross-sections[10] are correct to within a factor of about three, but the details are not well reproduced. Trying different values for the spin dependent part $V_1$ does not significantly improve the detailed agreement.[6j]

The polarization for the collective state has also been measured[11] though at a slightly different energy than the cross-sections. This is shown in Fig. 7. The agreement is fair.

For $Sn^{120}$ the cross-sections[10] are shown in Fig. 8 and again the collective $2^+$ is well reproduced but the magnitudes of the others are in error by as much as a factor two or three. As it turns out for this nucleus, all the levels shown could be calculated in the distorted wave Born approximation with little error.

<u>Two-phonon states</u>. We can use the approximate microscopic description of two-phonon states mentioned at the end of section IV to see if this provides a better description than the two-quasiparticle description. The results are

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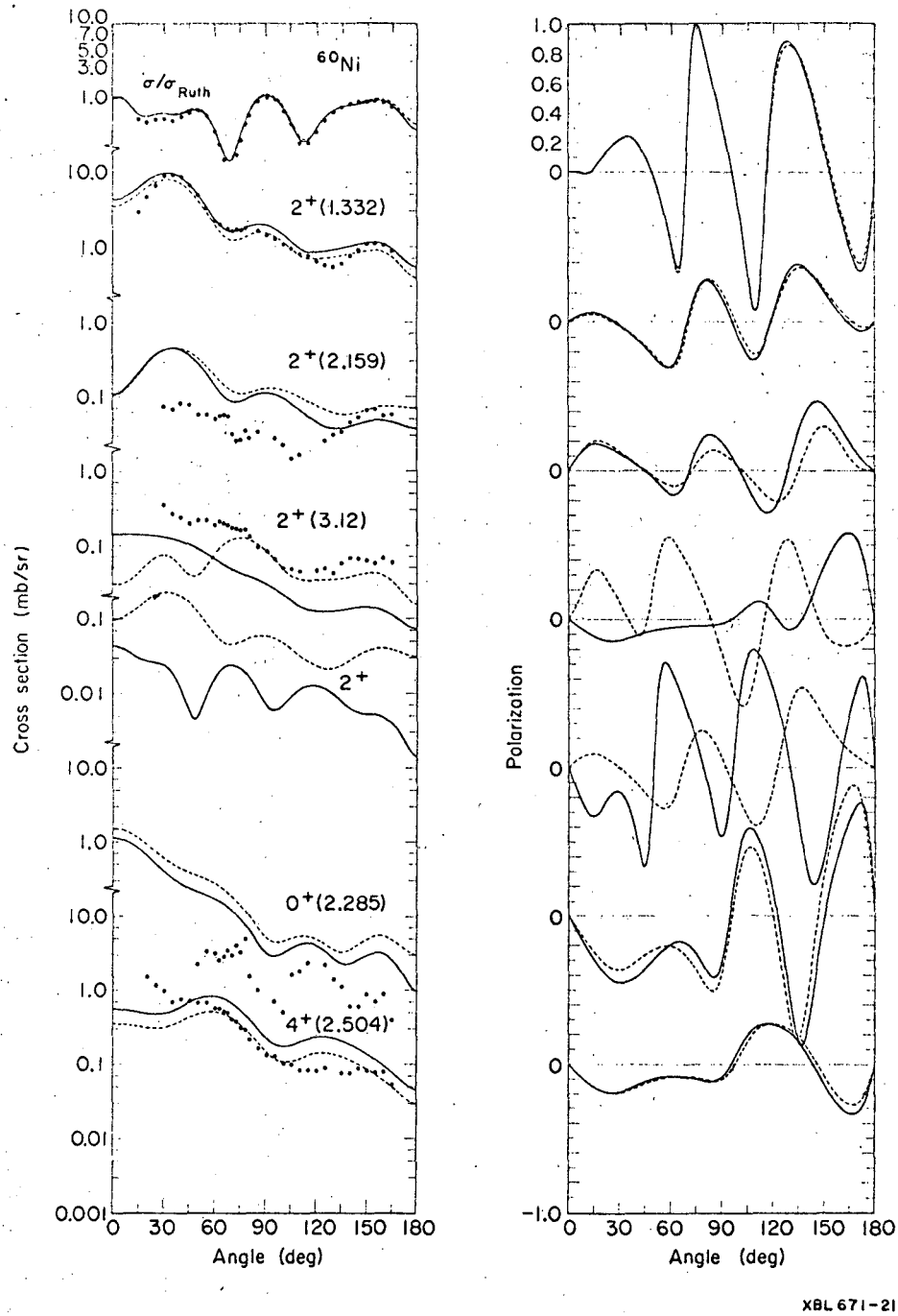


Fig. 6. Cross-sections and polarizations for 17.8 MeV protons. Curves are coupled equation calculations using microscopic description employing a surface delta interaction Ref. 9 (solid line) and a volume finite range interaction Ref. 8 (dashed line). Direct interaction parameters are $V_0$ = -55 MeV, $V_1$ = 0 and $\rho$ = 1.85 F of Eq. (3) and (15). (Ref. 6j).

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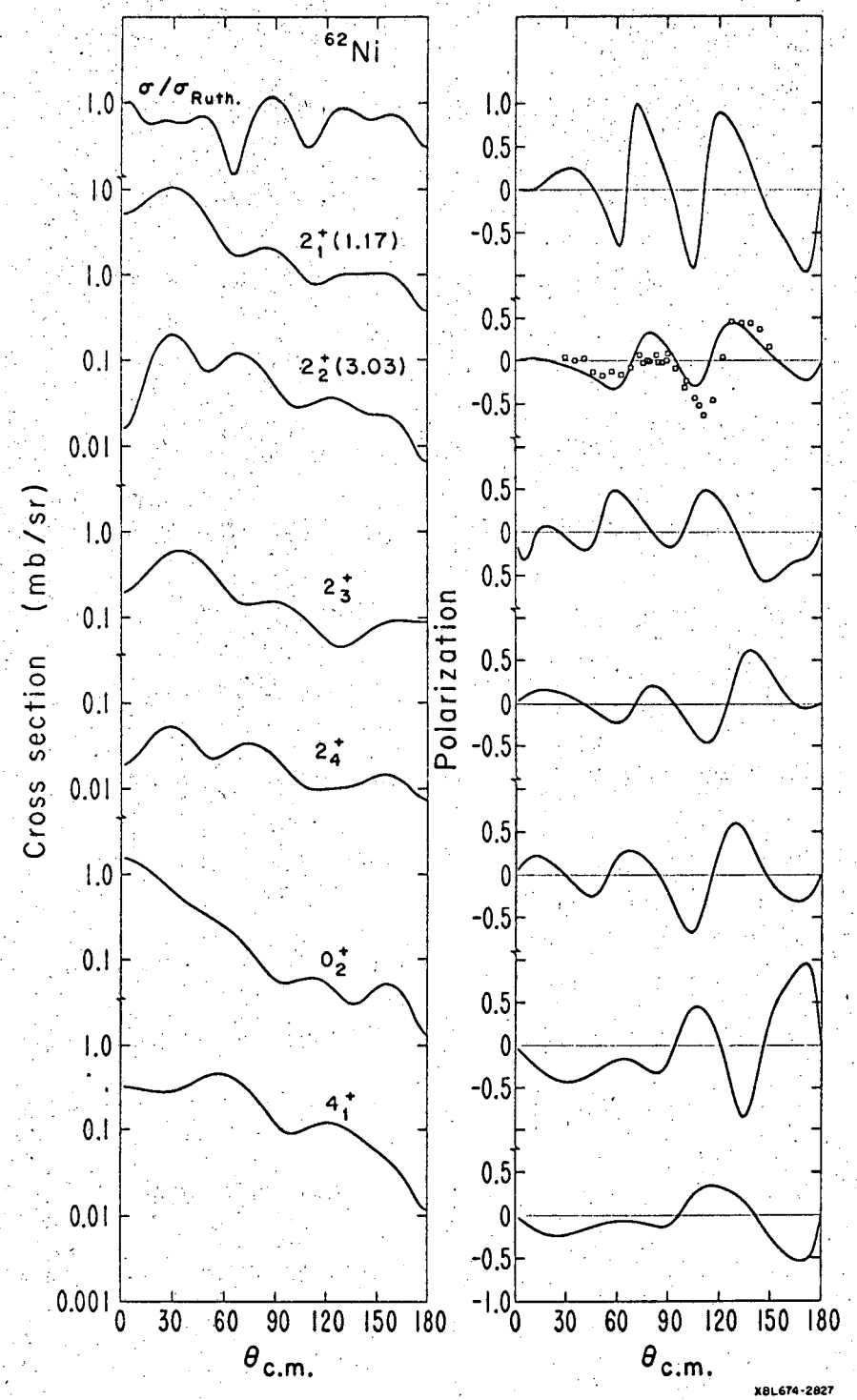


Fig. 7. Cross sections and polarizations for 18.6 MeV protons. The experimental polarization was measured at Saclay. The solid lines are coupled channel calculations based on the microscopic theory of Ref. 8.

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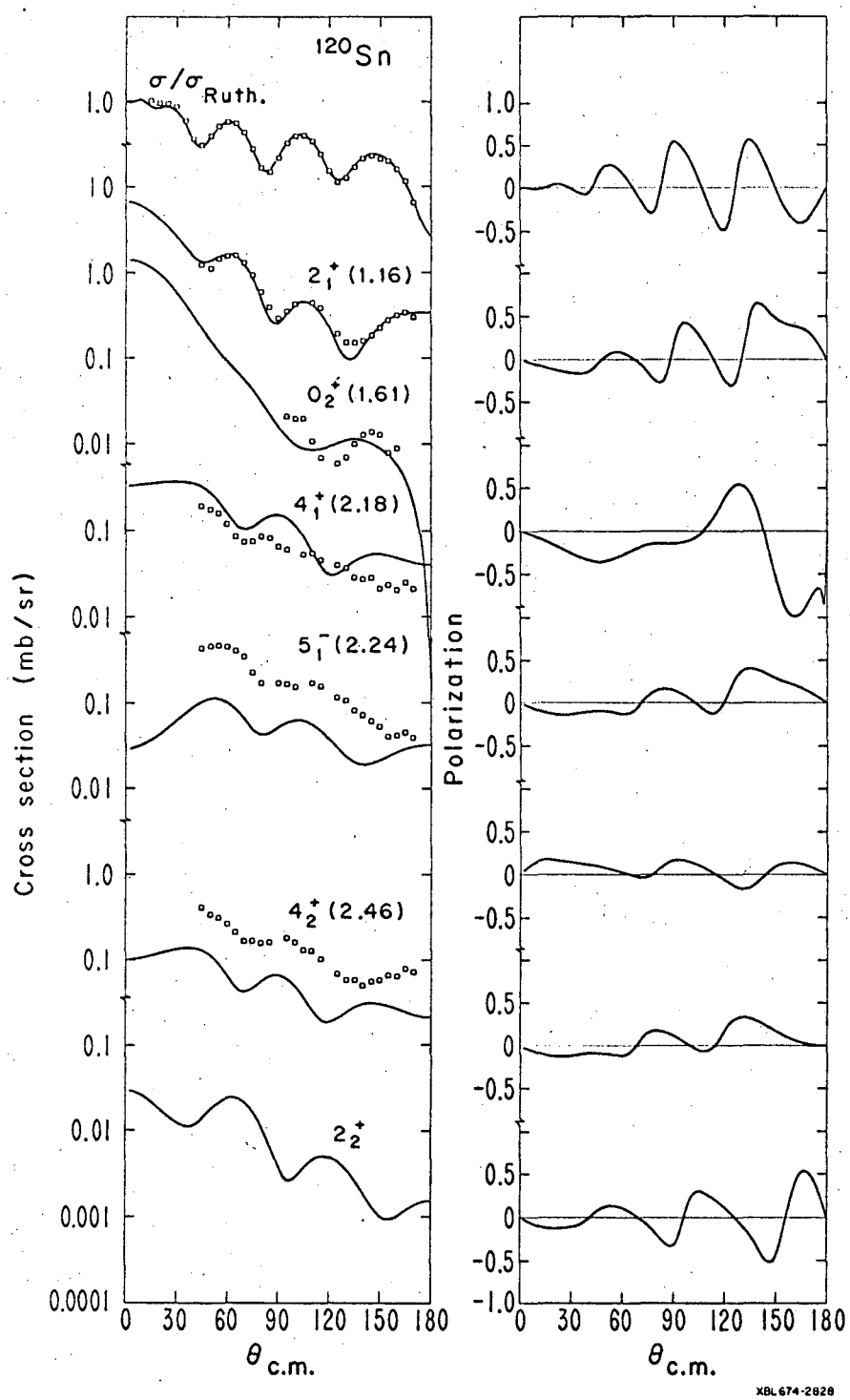


Fig. 8. Cross-sections and polarizations for 17.8 MeV protons, measured at Berkeley. Solid lines represent coupled channel calculation based on microscopic theory of Ref. 8.

V

shown in Ref. 6j and indicate that the extreme phonon picture is not in better agreement.

Summary. From these calculations we have seen that the microscopic description of the collective $2^+$ state seems to be fairly good. At least we calculate a cross section in good accord with experiment. This gives us some confidance in thinking that the direct interaction that was used in the scattering calculation is not grossly wrong for these nuclei and at this energy (~20 MeV).

In that case we conclude that the description of the other states is generally not very good. It is not an accident that if we are able to obtain any agreement at all, it should be for the collective state. Since it is coherent with respect to the various configurations it is not as sensitive to the particular amplitudes as an incoherent state where the small form factor results from cancellations. In a phrase, it is easier to describe the nuclear order as embodied in a collective state than the chaos of an incoherent state. Neither the two-phonon description, nor the two quasiparticle description of the higher lying states gives a very good account of the observed scattering. This is rather dissapointing, since more accurate structure calculations are much more difficult. Nevertheless if a realistic description of any but the collective states in even spherical nuclei is to be obtained, these difficulties must be overcome.

VI

# VI. SCATTERING FROM ROTATIONAL NUCLEI

The rotation of strongly deformed nuclei, especially in their lower states, is slow compared to the velocities of the individual nucleons. In this situation, as was discussed long ago by Bohr and Mottelson, the rotation and the nucleonic motion may be considered independently of each other, to good approximation, as is intuitively clear. Then for every intrinsic nucleonic state there will be a spectrum of rotational states.

In the last sections we considered the excitation of the nucleon degrees of freedom in spherical nuclei. For deformed nuclei the lowest lying states are rotational and all belong to the same intrinsic state of motion. It is the excitation of the rotational states that we wish to consider now. Such a study is useful in measuring the shape of the nucleus.

Adiabatic approximation.[12] If the orientation of the nucleus does not change appreciably during the passage of the scattered particle over it, the adiabatic approximation may apply also to the scattering process. In this case the Hamiltonian

$$H = T + T_\Omega + V(\underset{\sim}{r},\Omega) \tag{1a}$$

may be approximated by

$$H_a = T + V(\underset{\sim}{r},\Omega) \tag{1b}$$

where we are neglecting the rotational energy $T_\Omega$ of the nucleus. The $\Omega$ stands for the Euler angles defining the orientation of the nucleus. We shall not consider explicitly the intrinsic excitations, so that the interaction $V$ is an effective interaction in the sense described earlier, since it must carry the effects of the eliminated intrinsic excitations and their rotations. As before it will be parameterized as a complex optical potential, which now has a non-spherical shape.

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Equation (1b) describes the elastic scattering of the particle from the nucleus having fixed orientation $\Omega$. The desired solution has the form

$$\psi^{(+)} = \chi_{\underset{\sim}{k}}^{(+)}(\underset{\sim}{r})\ \Phi_{JKM}(\Omega) \quad , \tag{2a}$$

$$\chi_{\underset{\sim}{k}}^{(+)}(\underset{\sim}{r}) \rightarrow e^{ikz} + f(\theta,\phi,\Omega)\ \frac{e^{ikr}}{r} \tag{2b}$$

Where $\Phi$ denotes the target nuclear state. The problem obviously does not possess spherical symmetry but may be solved conveniently as described by Barrett.[13]

On the other hand, considering the original problem governed by (1a), the scattering amplitude for the transition from the state J to J' may be written according to Gell-Mann and Goldberger,[4] as

$$f^{K}_{JM,J'M'} = \frac{-m}{2\pi\hbar^2}\ \langle \phi_{\underset{\sim}{k}',J'KM'} | V | \Psi^{(+)}_{\underset{\sim}{k},JKM} \rangle \tag{3a}$$

where, as earlier,

$$\phi_{k',J'KM'} = e^{i\underset{\sim}{k}'\cdot\underset{\sim}{r}}\ \Phi_{J'KM'} \tag{3b}$$

is a solution in the absence of V, while $\Psi^{(+)}$ is a solution of H. If we use the adiabatic solution (2) as an approximation to the exact solution, $\Psi^{(+)}$, we obtain

$$f^{K}_{JM,J'M'} \simeq \frac{-m}{2\pi\hbar^2}\int d\Omega\ \Phi_{J'KM'}(\Omega)^{*}\langle e^{-i\underset{\sim}{k}'\cdot\underset{\sim}{r}} | V(\underset{\sim}{r},\Omega)\ | \chi_{\underset{\sim}{k}}^{(+)}(\underset{\sim}{r})\rangle \Phi_{JKM}(\Omega) \tag{4}$$

The bracket would be just the expression for the amplitude f of the adiabatic problem, (2) if k'=k. The adiabatic approximation neglects this difference, so we have

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$$f^{K}_{JM,J'M'}(\theta,\phi) \simeq \int d\Omega \Phi_{J'KM'}(\Omega)^{*} f(\theta,\phi,\Omega)\Phi_{JKM}(\Omega) \tag{5}$$

This result may seem strange at first glance, since it indicates that by solving a certain elastic scattering problem and taking matrix elements of the resulting scattering amplitude, the amplitudes for transitions to excited states can be calculated. To understand that this is a sensible result we return to Eq. (2), which, remember, is the elastic scattering solution to the fictitious problem in which the nucleus has a fixed orientation. As such it represents a superposition of the various rotational states, and we may write its expansion,

$$f(\theta\phi\Omega)\Phi_{JKM}(\Omega) = \sum_{J'M'} f^{K}_{JM,J'M'}(\theta,\phi)\Phi_{J'KM'}(\Omega) \quad . \tag{6}$$

Because of this close connection between the elastic and inelastic scattering to states possessing the same internal nucleon structure, which is manifested in the adiabatic expression (5), this approximation provides a convenient framework within which to discuss the striking regularities observed in the scattering of strongly absorbed particles like alphas. Austern and Blair[14] have discussed this very thoroughly in their paper.

<u>The interaction</u>. The nuclear shape will be parameterized by the usual expression

$$\begin{aligned} R &= R_0[1 + \sum_{\lambda,\kappa} \alpha_{\lambda\kappa} Y_{\lambda\kappa}(\theta',\phi')] \\ &= R_0 + \delta R \end{aligned} \tag{7}$$

with the sums restricted to even values. Here $\theta',\phi'$ refer to the body fixed

axis taken to be the symmetry axis of the nucleus. So that R will be a real number, we require $\alpha_{\lambda\kappa} = \alpha_{\lambda-\kappa}$.

We paramerize V as an optical potential and expand it about the spherical shape. The nuclear part is

$$V_{Nuc}(r-R) = V_{Nuc}(r-R_0) + \sum_{n=1}^{\infty} \frac{(-\delta R)^n}{n!} \frac{\partial^n V}{\partial r^n} \tag{8a}$$

where

$$\mathrm{Re}\ V(r-R_0) = V_0\left[1 + \exp\left(\frac{r-R_0}{a}\right)\right]^{-1} \tag{8b}$$

and similarly for the other parts of V. Since

$$\frac{\delta R}{R_0} = \sum_{\lambda\kappa} \alpha_{\lambda\kappa}\ Y'_{\lambda\kappa} \tag{9}$$

we obtain by use of the addition theorem for spherical harmonics an expression in terms of the generalized deformation parameters $\delta$

$$\left(\frac{\delta R}{R_0}\right)^n = \sum_{LK} \delta_{LK}^{(n)}\ Y'_{LK}$$

$$\delta_{LK}^{(n)} = \sum_{\lambda k \Lambda\kappa} \left\{\frac{\hat{\lambda}\hat{\Lambda}\hat{L}}{4\pi}\right\}^{1/2} \begin{pmatrix} \Lambda & \lambda & L \\ \kappa & k & -K \end{pmatrix} \begin{pmatrix} \Lambda & \lambda & L \\ 0 & 0 & 0 \end{pmatrix} \delta_{\Lambda\kappa}^{(n-1)} \alpha_{\lambda k} \quad , \quad (\text{for } n > 1) \tag{10}$$

where $\delta_{\Lambda\kappa}^{(1)} \equiv \alpha_{\Lambda\kappa}$ and $\hat{L} = 2L + 1$. Now we have

$$V_{Nuc}(r-R) = V_{Nuc}(r-R_0) + \sum_{LK} N_{LK}(r)\ Y_{LK}(\theta',\phi') \tag{11a}$$

where

$$N_{LK}(r) = \sum_{n=1}^{\infty} \frac{(-R_0)^n}{n!} \delta_{LK}^{(n)} \frac{\partial^n V(r)}{\partial r^n} \tag{11b}$$

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If the deformation parameters $\alpha$ are not too large, the series converges rapidly since $\delta^{(n)}$ is of order $\alpha^n$.

The coulomb potential due to the charge distribution $\rho(\underset{\sim}{r})$ is

$$V_{Coul}(r') = ZZ'e^2 \int \frac{\rho(\underset{\sim}{r}'')}{|\underset{\sim}{r}' - \underset{\sim}{r}''|} \, d\underset{\sim}{r}'' \tag{12}$$

where the coordinates are expressed in the body-fixed frame. Using

$$\frac{1}{|\underset{\sim}{r}' - \underset{\sim}{r}''|} = \sum_{\lambda} \frac{4\pi}{2\lambda+1} \frac{r_<^{\lambda}}{r_>^{\lambda+1}} \; Y_\lambda(\theta',\phi') \cdot Y_\lambda(\theta'',\phi'')$$

we get a similar expression for $V_{Coul}$ in terms of $\delta^{(n)}$. In fact

$$V_{Coul}(\underset{\sim}{r}) = V_{Coul}(r) + \sum_{LK} C_{LK}(r) \; Y_{LK}(\theta',\phi') \tag{13a}$$

where

$$C_{LK}(r) = \frac{3ZZ'e^2}{2L+1} \; C_\delta \begin{cases} \dfrac{r^L}{R_C^{L+1}} \, I_{LK} & \text{if } r < R_1 \\[2ex] \dfrac{R_C^L}{r^{L+1}} \, O_{LK} & \text{if } r > R_2 \end{cases} \tag{13b}$$

Here $R_1$ and $R_2$ are the smallest and largest value of $R$ obtained in (7). Between these points no analytic form exists, but $C(r)$ is continuous.[15] The constant $C_\delta$ is the ratio of densities for the deformed shape (7) and the sphere of radius $R_C$

$$C_\delta = \rho_\delta / \rho_0 = \left[ 1 + \frac{3\delta_0^{(2)} + \delta_0^{(3)}}{\sqrt{4\pi}} \right]^{-1} \tag{13c}$$

and the monopole part of the Coulomb field is given by the usual expression

$$V_{Coul}(r) = \begin{cases} C_\delta \dfrac{3ZZ'e^2}{2\,R_C} \left[ 1 + \dfrac{\delta_0^{(2)}}{\sqrt{4\pi}} - \dfrac{1}{3} \left( \dfrac{r}{R_C} \right)^2 \right], & r < R_1 \\[2ex] \dfrac{ZZ'e^2}{r}, & r > R_2 \end{cases} \tag{13d}$$

where we employ a uniform charge density with radius parameter $R_C$ having a deformed shape of the form (7). In this case the constants I and O of (13b) are,

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$$I_{LK} = -\sum_{n=1}^{\infty} \frac{(-)^n}{n!} \frac{(L+n-3)!}{(L-2)!} \delta_{LK}^{(n)} \quad , \quad I_{00} = 0$$

$$O_{LK} = \sum_{n=1}^{L+3} \frac{(L+2)!}{n!(L+3-n)!} \delta_{LK}^{(n)} \quad , \quad O_{00} = 0 \tag{13e}$$

The potentials are still expressed in terms of the body-fixed frame. Using the D-coefficients to express our results in terms of the laboratory frame and adding to-gether the nuclear and Coulomb parts gives us finally

$$V[r-R(\theta,\phi)] = V(r-R_0) + \sum_{LM} Y_{LM}(\theta,\phi) \sum_{K\geq 0} V_{LK}(r) \frac{D^L_{MK} + D^L_{M-K}}{1+\delta_{K0}} \tag{14a}$$

where we have used the fact that $V_{LK} = V_{L-K}$ as follows from the property of the $\delta_{LK}^{(n)}$, and

$$V_{LK} = N_{LK} + C_{LK} \tag{14b}$$

The first term of (14a), being merely a spherical optical potential, has only diagonal matrix elements whereas the second, non-spherical part, gives rise to excitations of the nucleus from one rotational state to another.

<u>Form factors</u>. Following our former general definition of form factors we have from (14) that

$$\mathcal{F}^{\alpha_1\alpha_2}_{LK}(r) = V_{LK}(r) \langle J_1K_1 \| \frac{D^L_{,K} + D^L_{,-K}}{1+\delta_{K0}} \| J_2K_2 \rangle \tag{15}$$

where, as earlier, we let $\alpha$ denote the quantum numbers describing the nuclear

state. Since the wave functions for the rotations are given in terms of D-functions, the reduced matrix element can be readily evaluated as integrals over three D-functions with the result (for $K \geq 0$, and even)

$$\langle J_1 K_1 \| \frac{D^L_{,K} + D^L_{,-K}}{1 + \delta_{K0}} \| J_2 K_2 \rangle$$

$$= \begin{cases} (\hat{J}_1 \hat{J}_2)^{1/2} \begin{pmatrix} J_1 & L & J_2 \\ 0 & 0 & 0 \end{pmatrix} \delta_{K0} , & \text{if } K_1 = K_2 = 0 \\ (2 \hat{J}_1 \hat{J}_2)^{1/2} \begin{pmatrix} J_1 & L & J_2 \\ 0 & -K_2 & K_2 \end{pmatrix} \delta_{KK_2} , & \text{if } K_1 = 0, K_2 \neq 0 \\ \frac{(-)^{J_1} (\hat{J}_1 \hat{J}_2)^{1/2}}{1 + \delta_{K,0}} \left\{ \begin{pmatrix} J_1 & L & J_2 \\ -K_1 & K & K_2 \end{pmatrix} + \begin{pmatrix} J_1 & L & J_2 \\ -K_1 & -K & K_2 \end{pmatrix} + (-)^{J_2} \begin{pmatrix} J_1 & L & J_2 \\ -K_1 & K & -K_2 \end{pmatrix} \right\} , & \text{if } K_1 \neq 0, K_2 \neq 0 \end{cases} \tag{16}$$

It will be instructive to discuss the form factors in terms of the possible excitations of an even-even nucleus; for which $J_1 = 0$, and whose excited states have spin 2, 4,... . Therefore the L'th term in (14a) leads to <u>direct</u> excitation of the state with spin L. In addition it can be reached by many different cascade routes through intermediate states. Now the radial shape of the form factor (15) is given by $V_{LK}$ and its general character can be surmised readily from (11b) (since the Coulomb part is small although it has a long tail and is especially important at forward angles when ZZ' is large). If, as an example the intrinsic nuclear shape is characterized as pure L=2, ($\alpha_2$ finite, all others zero) then the leading term of $V_2$ is of order $\alpha_2$ and has the familiar first derivative shape of the (Woods-Saxon) optical potential (Fig. 9). The

V -

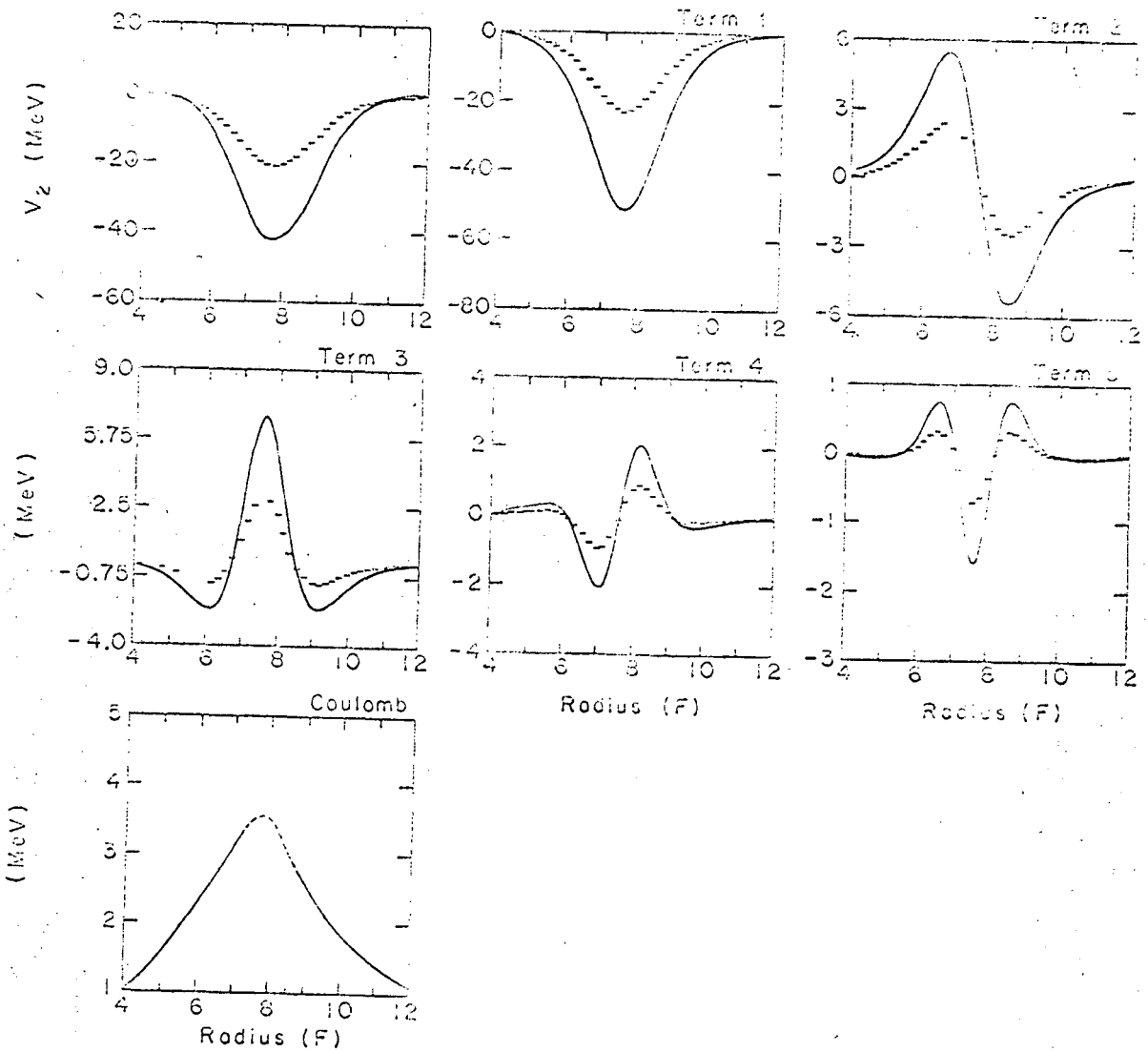


Fig. 9. The $V_2$ form factor together with contributions to it from various orders 1 through 5 in $\beta_2$ including the Coulomb part. The leading term is the well known first derivative but the higher terms yield altogether a correction of about 20% in the form factor for $\beta_2$ = .27. Note the changing scale for higher order terms. The Coulomb form factor is also shown. Dashed lines are the imaginary part.

leading term of $V_4$ is of order $\alpha_2^2$ and has the second derivative shape, and so on (see Fig. 10). If there is some intrinsic $\alpha_4$ shape present, then $V_4$ has also a first derivative component of order $\alpha_4$ so that we may expect that a careful analysis of the $4^+$ state may lead to a measurement of $\alpha_4$. No single state can be considered in isolation from the others however, since the coupling between excited states for strongly deformed nuclei cannot be neglected. Hence the distorted wave approximation is not valid.

Coupled equations for alpha scattering. Alpha particles should be very useful for studying the nuclear surface because, being strongly absorbed, we are assured that most alphas that have excited a low lying rotational state, were involved in a surface reaction. Here we will discuss scattering from strongly deformed even nuclei.

Much interesting work based on various approximate calculations of the scattering amplitude within the framework of the adiabatic approximation have been published by Drozdov, Inopin and Blair.[16] However with the constantly improving experimental techniques, the information contained in the data is more precise than can be extracted by use of the earlier approximate methods. More recently the scattering amplitude has been obtained by exact numerical integration of the differential equations both in the adiabatic approximation, which is certainly valid at the energies we have in mind ($E_\alpha \gtrsim 20$ MeV), and by solving the coupled equations after truncating the space of rotational states. The two methods agree very well for all levels one or two removed from the point of truncation. We shall use the latter method since it corresponds to our earlier discussion.

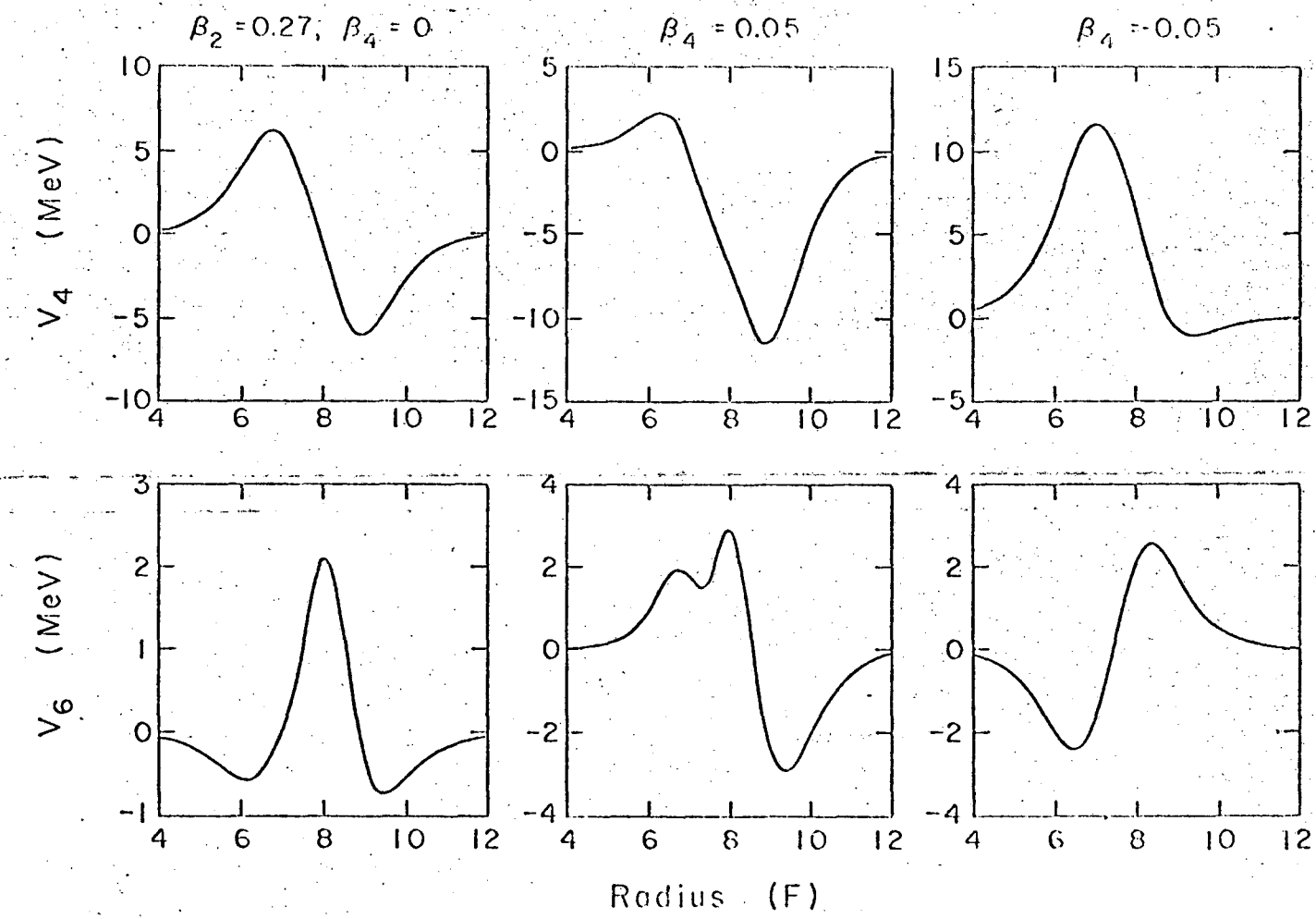


Fig. 10. The form factors $V_4$ and $V_6$ are shown for three values of $\beta_4$.

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The coupled equations describing scattering from a deformed nucleus would look quite similar to those already considered except for the interpretation of the matrix elements. Here we are interested only in rotations of the ground state. Consequently the effective interaction carries the effect of intrinsic excitations and their rotations (as well as the higher rotations of the ground state that are dropped in the truncation). All the matrix elements that appear, are therefore diagonal with respect to the intrinsic structure and so, for practical reasons, according to our earlier discussion, must be parameterized as a complex optical potential. The potential is of course deformed. While we shall loosely refer to its shape as being the nuclear shape, it should be kept in mind that we are really talking about the particular component of the field generated by the nucleus, to which the projectile is sensitive. Various projectiles such as protons, neutrons and alpha particles all should feel different fields. How different their shapes and surface structure are is not à priori known, nor are the experimental data definitive.

In contrast to the earlier discussion of intrinsic excitations in spherical nuclei where the diagonal matrix elements, or optical potential, could in principle be different in each nuclear channel, they are identical in all channels of the present problem. The channel quantum numbers are

$$c' \equiv \alpha' K' J' \ell' \tag{16}$$

where $\alpha'$ denotes nuclear quantum numbers additional to $K'$ and $J'$ like the parity of the level $\pi'$, while $\ell'$ is the angular momentum of the alpha particle. Its values are restricted by conservation of angular momentum and

parity. Since the target spin and parity is $J=0^+$, the channel spin $I$ of our earlier discussion is just the angular momentum $\ell$ of the incident alpha. Then

$$\underset{\sim}{\ell}' = \underset{\sim}{\ell} + \underset{\sim}{J}'$$

$$(-)^{\ell+\ell'} = \pi' \tag{17}$$

The off-diagonal matrix elements of the interaction, whose multipole decomposition is given by (14) can be found from the results of section IV by setting $s = 1/2 \to 0$, for our present case of spinless alpha particles, or otherwise one can obtain them directly by the same Racah methods.

$$V^{\ell}_{c_1 c_2}(r) = \sum_{LK} C^{\ell}_{L}(c_1,c_2)\, \mathcal{F}^{\alpha_1\alpha_2}_{LK}(r) \tag{18}$$

where, as before $C$ is purely geometrical

$$C^{\ell}_{L}(c_1,c_2) = (-)^{\ell+J_1+\ell_1} \left(\frac{\hat{\ell}_1\ \hat{L}}{4\pi\ \hat{\ell}_2}\right)^{1/2} \begin{pmatrix} \ell_1 & L & \ell_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell_1 & J_1 & \ell \\ J_2 & \ell_2 & L \end{Bmatrix} \tag{19}$$

and $\mathcal{F}$ is the form factor (15) that was discussed already. Actually we shall be talking only about the ground state band of rotational states all of which have therefore $K = 0$ so that only $K = 0$ contributes to (18). In general two values of $K$ could contribute, namely $K = |K_1 \pm K_2|$.

Now the coupled equations (II-9) are completely defined. For each incident partial wave $\ell$ there are $J + 1$ outgoing waves for each nuclear state $J$ when $\ell > J$ so altogether there are

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$$N = \sum_{J} (J + 1)$$

where the sum is over the spins of the states considered. The method of solving them was qualitatively discussed, but the detailed numerical techniques are not interesting to us here.

## VII. ALPHA SCATTERING IN THE DEFORMED RARE EARTH REGION[17]

The excitation of rotations of the ground state in strongly deformed even nuclei is exceptional in that the intrinsic structure of the states is not changed in the reaction. The scattering is thus defined by one object, $\langle 0|\mathcal{V}|0\rangle$, the diagonal matrix element of the effective interaction in the intrinsic ground state. This is in contrast with the excitation of vibrations and single-particle states in which a number of matrix elements which are non-diagonal in the internal structure are also needed. As we mentioned in the previous section, we shall work within the subspace of ground state rotations so that $\mathcal{V}$ carries implicitly the effects of all the intrinsic excitations, and we parameterized $V \equiv \langle 0|\mathcal{V}|0\rangle$ as an optical potential having a deformed shape. Its expansion in a multipole series was written in (VI-14). The L'th multipole is responsible for directly exciting the J=L rotational state, so that a careful study of the various excited states will yield us the multipole expansion of $V$ whose various terms are related to the optical parameters $V_0$, $W_0$, $r_0$, $a$ of (VI-8) and the shape parameters of the nucleus (VI-7) through equations (VI-10,11,13). A careful measurement of the cross-sections of the various rotational states provides a great deal of data with which to determine $V$. Each angular distribution constitutes not one piece of data, but many, being characterized by such features as the frequency of oscillation, phase relative to elastic pattern, slope of the envelope of the maxima, amplitude of the oscillations and magnitude of the cross-section. One can expect therefore to measure the optical and shape parameters very accurately. Perhaps even the well known ambiguity in the depths of the optical parameters can be resolved, although we can not state this, since our analysis has not been completed.

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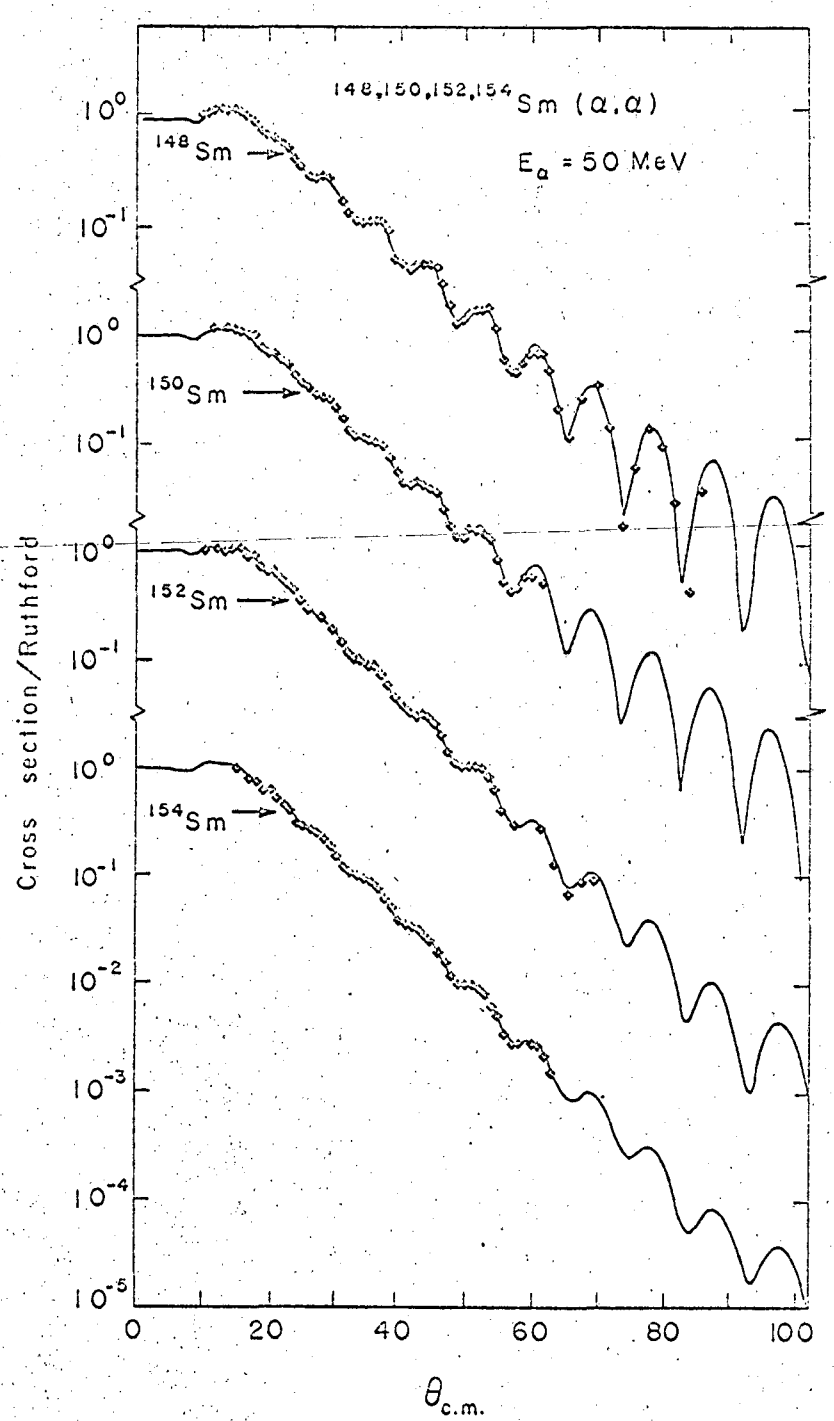


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Fig. 11. Elastic cross-sections for four samarium isotopes. The strongly dampted oscillations correspond to the rotational nuclei. Optical model cross-sections were obtained by N. Jarvis.

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Here we shall apply the picture of scattering by an ideal rotor to experiments performed at Berkeley[17] on various nuclei in the rare earth region of mass $A \simeq 160$. In these experiments differential cross sections for 50 MeV alpha particles were measured with very high precision. The elastic cross sections for a series of four even samarium isotopes are shown in Fig. 11. They exhibit a very important qualitative difference. The oscillations which are very distinct in the lighter two nuclei are almost absent in the heavier. In fact it is known that the lighter two are vibrational while the latter two have rotational spectra. The B(E2) for the $2^+ \rightarrow 0^+$ transition is much larger for the latter nuclei[18] and it is the stronger coupling to the surface modes that is reflected so clearly in the elastic cross-sections. The scattering back to the elastic state from the $2^+$ cannot be neglected when the coupling becomes too strong.

The curves shown in Fig. 11 are cross-sections computed from an optical model potential in which the elastic scattering is treated in isolation from the other states.[19] From our discussion of the effective interaction in Section III it is evident that the parameters (shown in Table 1) will be very different for the rotational nuclei than for the vibrational ones. This is because it must take into account the effects on the elastic channel not only of the intrinsic excitations which are also present in the spherical nucleus, but of the rotations as well, which are absent in the latter. When excitation of the rotations is explicitly calculated by solving the coupled equations, their contribution to $\mathcal{V}$ is removed and we find that essentially the same optical parameters (see Table 1) apply to both the spherical and deformed nuclei, since now they both take account of the intrinsic excitations only. It is true that near the ground state the spectrum and structure of intrinsic excitations are generally different, but of course the higher energy non-collective excitations of which there

are millions at the energy of the experiment, dominate the effective interaction. It is possible that the small disagreement that does still persist could be eliminated if the effect of the collective vibrational level in $Sm^{148}$ were removed from $\mathcal{V}$ in the same way. Thus a rather consistent picture of the effective interaction emerges.

So far we have a complete calculation only for one rotational nucleus, $Sm^{154}$. The experimental results are shown in Fig. 12 together with a coupled channel calculation using the formulation of the preceeding section. The multipole expansion of the potential (VI-11) in both nuclear and Coulumb parts was carried out to L=8, and in the Taylor expansion of each multipole we kept all terms up to and including n=8, which we confirmed led to completely convergent results in the cross-section. We also checked the effect of truncation within the space of rotational states. We confirmed that eliminating the $6^+$ state led to no detectable change in computed cross-sections to the lower states. Eliminating both $4^+$ and $6^+$ did produce small but detectable differences. So we can be assured that truncation beyond the $6^+$ level introduces no errors in our deduced parameters. In fact the calculated cross-sections must be regarded as numerically exact within an error too small to be detected even by the very precise experiments which are more accurate than indicated by the experimental points marked on the graphs.

The excellent agreement shown in Fig. 12 corresponds to the optical parameters listed in Table 1 (set b) and values of the deformation parameters:

$$\beta_2 = \alpha_{20} = 0.225$$

$$\beta_4 = \alpha_{40} = 0.05$$

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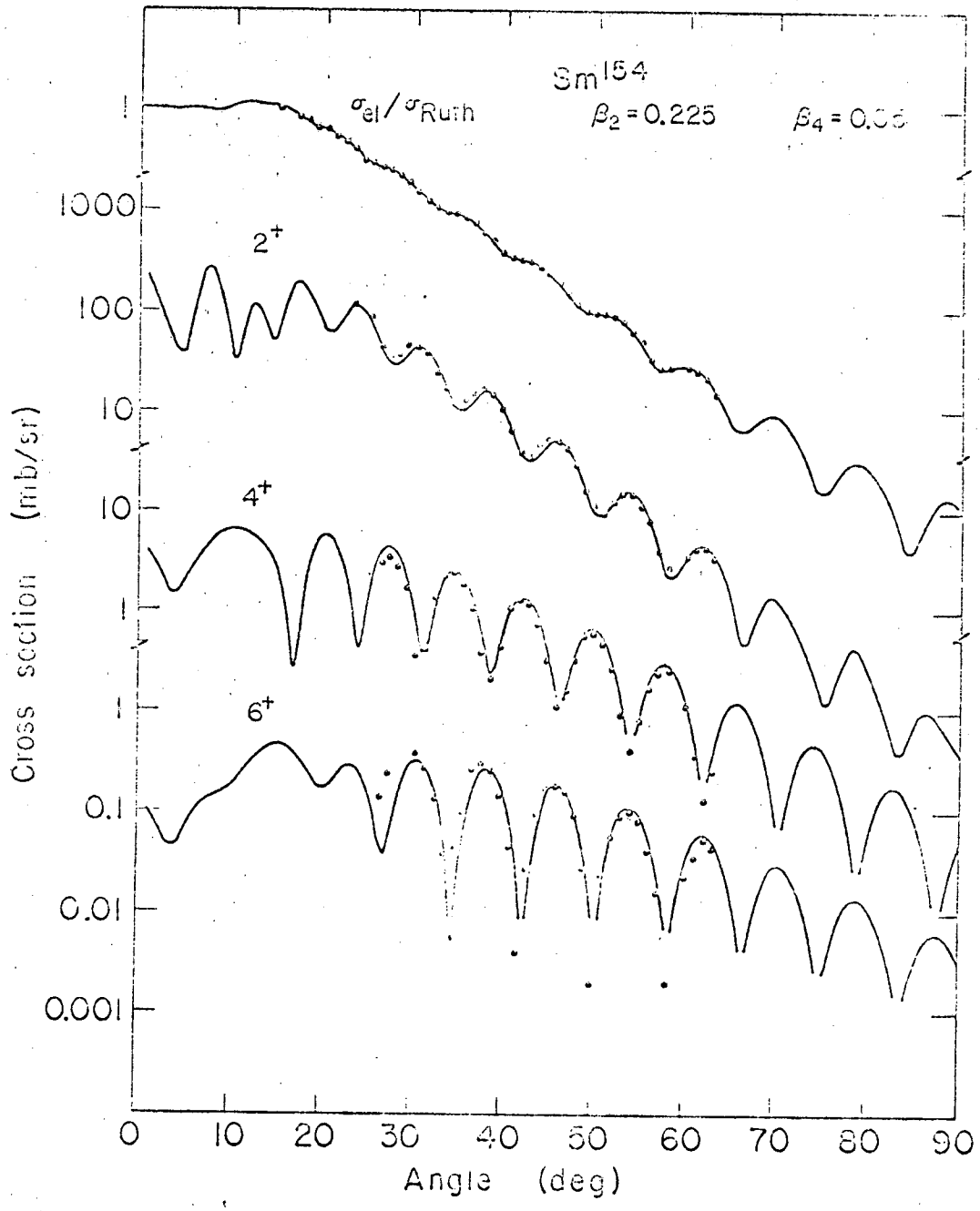


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Fig. 12. Cross-sections for 50 MeV alpha exciting ground state rotational band of $Sm^{154}$. Curves are coupled channel calculation as described in text. The data were taken at the Berkeley 88 inch cyclotron (Ref. 17).

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The sensitivity to the value and sign of $\beta_4$ is indicated in Fig. 13. There we see that both the phase and magnitude of cross-sections to the $4^+$ and $6^+$ are dramatically effected by $\beta_4$ as was previewed in the form factors $V_4$ and $V_6$ of Fig. 10. There is some suggestion, because of the slight disagreement of the $6^+$ cross-section at forward angles, of a small $\beta_6$ component to the shape. The $8^+$ state will have to be measured to pin this down.

Although in this strong coupling situation the distorted wave Born approximation should not be applied, it is instructive to see what results if it is used. The computed cross-sections are shown in Fig. 14. The main qualitative difference is that the large amplitude oscillations in the higher spin states are no longer reproduced. In fact the deep minima are produced by an interference between the direct transitions which the DWBA takes account of, and the cascade transitions, which it does not. (The DWBA does not take account of the backward going transitions either, being of first order in V, and these have a very strong effect on the elastic cross-section). But nevertheless if one accepts the magnitude of the cross-section as a means of determining $\beta_2$ the value of .300 is obtained which is in closer accord with Coulomb excitation work.

However our parameters reproduce the experiment so well, as seen in Fig. 12, that we have considerable confidence in them. It does remain to be seen whether they are unique. These calculations are in progress but incomplete.

Though the main field responsible for the excitations is the nuclear one, the Coulomb excitation cannot be ignored, as is indicated by the results shown in Fig. 15. The erroneous value $\beta_2 = .27$ is deduced, compared to .225 when the Coulomb excitation is accounted for.

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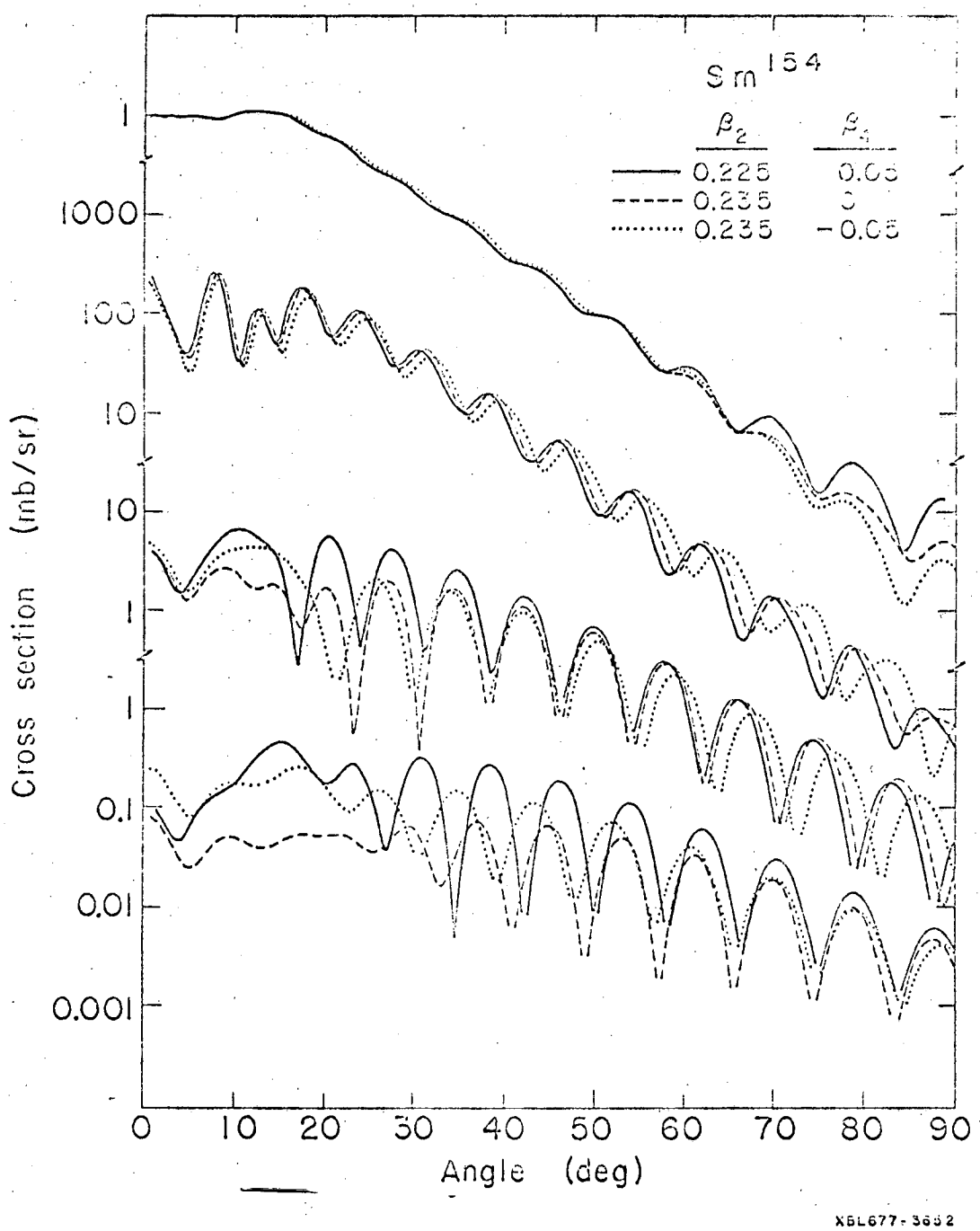


Fig. 13. Extreme sensitivity of $4^+$ and $6^+$ to sign and value of $\beta_4$ is indicated in 50 MeV alpha scattering from $Sm^{154}$.

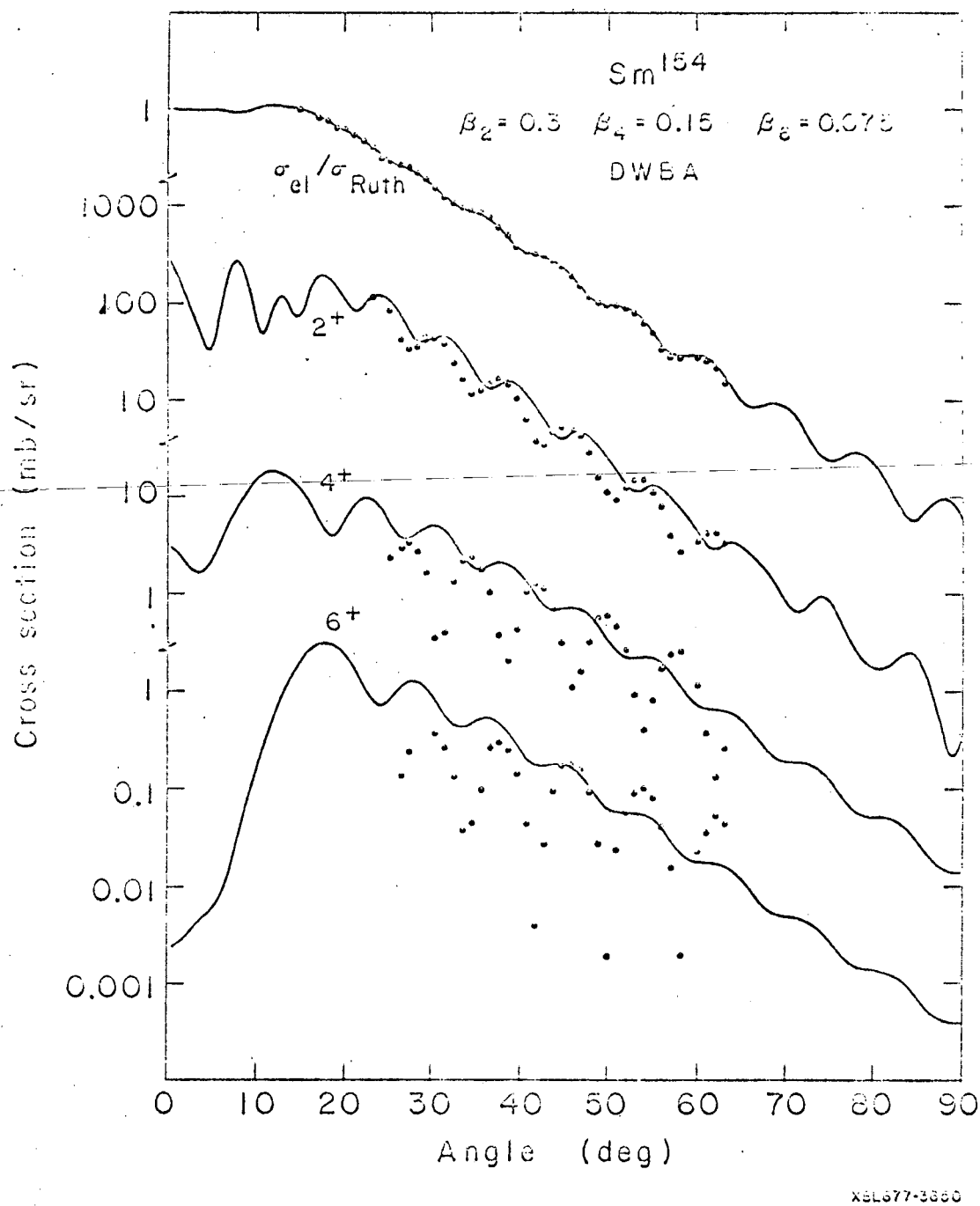


Fig. 14. Distorted wave calculation with optic parameters set (a) of Table 1.

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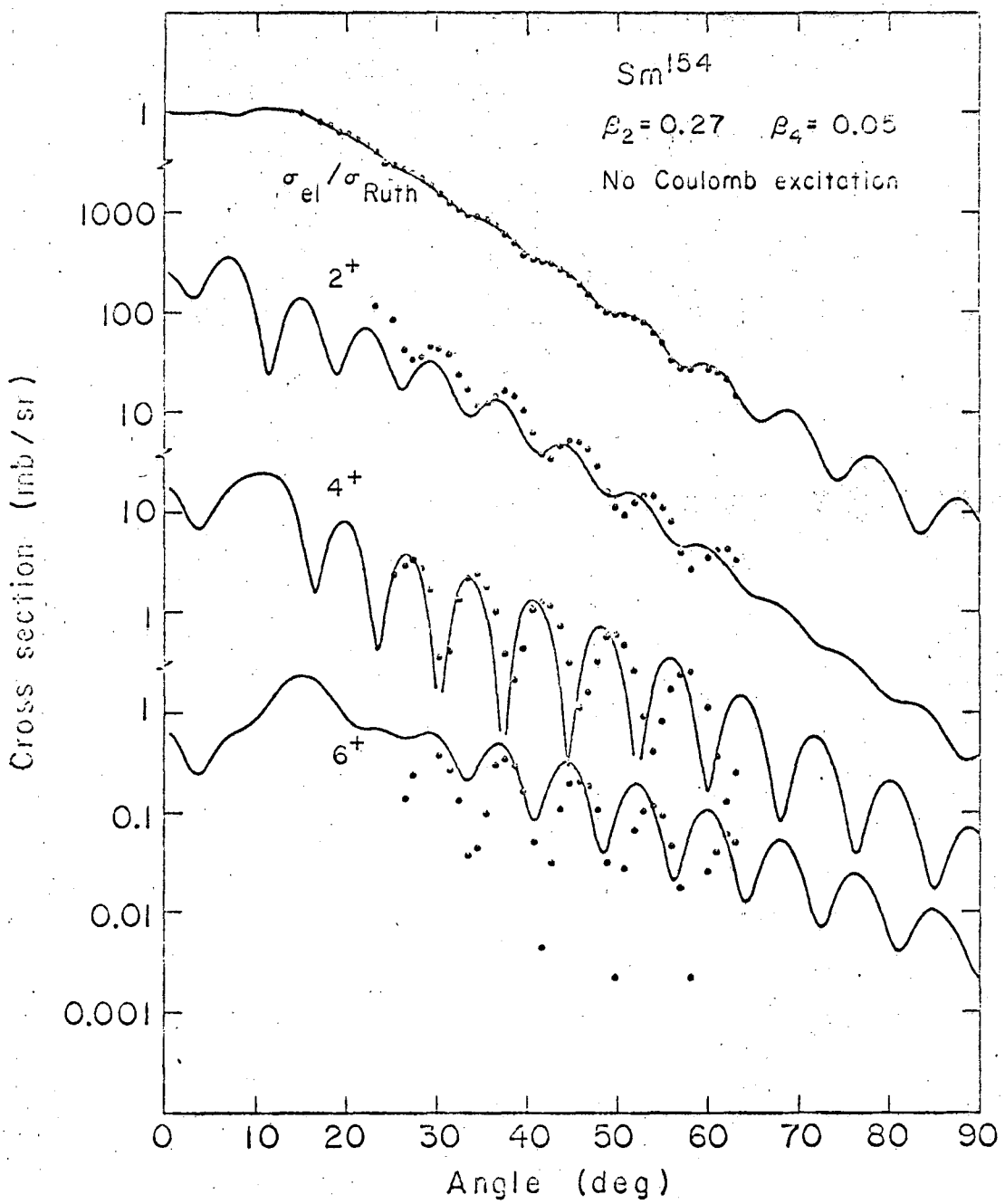


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Fig. 15. Coulomb excitation is neglected here. Comparing with Fig. 12 where it is included illustrates its importance, even though the main mechanism is the nuclear excitation. Note also the interference between nuclear and Coulomb excitation in Fig. 12 at small angles in the $2^+$. To compensate for the neglect of Coulomb excitation a larger value of $\beta_2$ is required, namely $\beta_2$ = .27 compared to .225. Even so the data cannot be well reproduced.

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Table 1. Optical model parameters for 50 MeV alpha particles. The 148 parameters and set (a) for 154 are pure elastic whereas set b) include explicit coupling to the ground state rotational band through solution of the coupled equations. Most important difference is diffuseness which in set (a) is very large corresponding to the implicit treatment of the rotations.

| | | V | W | r | a | $r_c$ |
|---|---|---|---|---|---|---|
| $Sm^{148}$ | | 65.5 | 29.8 | 1.427 | .671 | 1.4 |
| $Sm^{154}$ | a | 34.6 | 29.4 | 1.404 | .819 | 1.4 |
| | b | 65.9 | 27.3 | 1.440 | .637 | 1.440 |

Trends in $\beta_4$. The excellent agreement obtained with the $Sm^{154}$ data and the high sensitivity of the $4^+$ and $6^+$ states to $\beta_4$ lead us to believe that we can make a very precise measurement of the nuclear shape. Referring to Fig. 12 for $Sm^{154}$ we see that the states are alternating in phase. The $4^+$ is out of phase and the $6^+$ in phase with the $2^+$. However in $Yb^{176}$ as seen in Fig. 16 the maxima of the $4^+$ fall between the maxima and minima of the $2^+$ and the $6^+$ is out of phase with the $2^+$. This, as seen in Fig. 13, approximately describes the situation for $\beta_4 = -.05$. According to that figure, we also see that the amplitude for $\beta_4 = +.05$ than for $\beta_4 = 0$ or $-.05$. Comparing Fig. 12 and 16 we see that this feature is present in the experiments also. We must do an explicit analysis of this nucleus, but it seems safe to conclude that $Yb^{176}$ has a negative $\beta_4$ deformation near $-.05$. Hendrie,[20] following the example of Harada,[21] computed the $\beta_4$ deformation on the basis of the Nilsson model. This calculation took account only of the nucleons outside the last closed shells and consequently the origin of the $\beta_4$ deformation is not known, since the core presumeably also makes a contribution. However using our measurement on $Sm^{154}$ the origin was fixed and the resulting computed $\beta_4$ shown in Fig. 18. We note that the predicted $\beta_4$ for $Yb^{176}$ is negative in agreement with our conclusion above.

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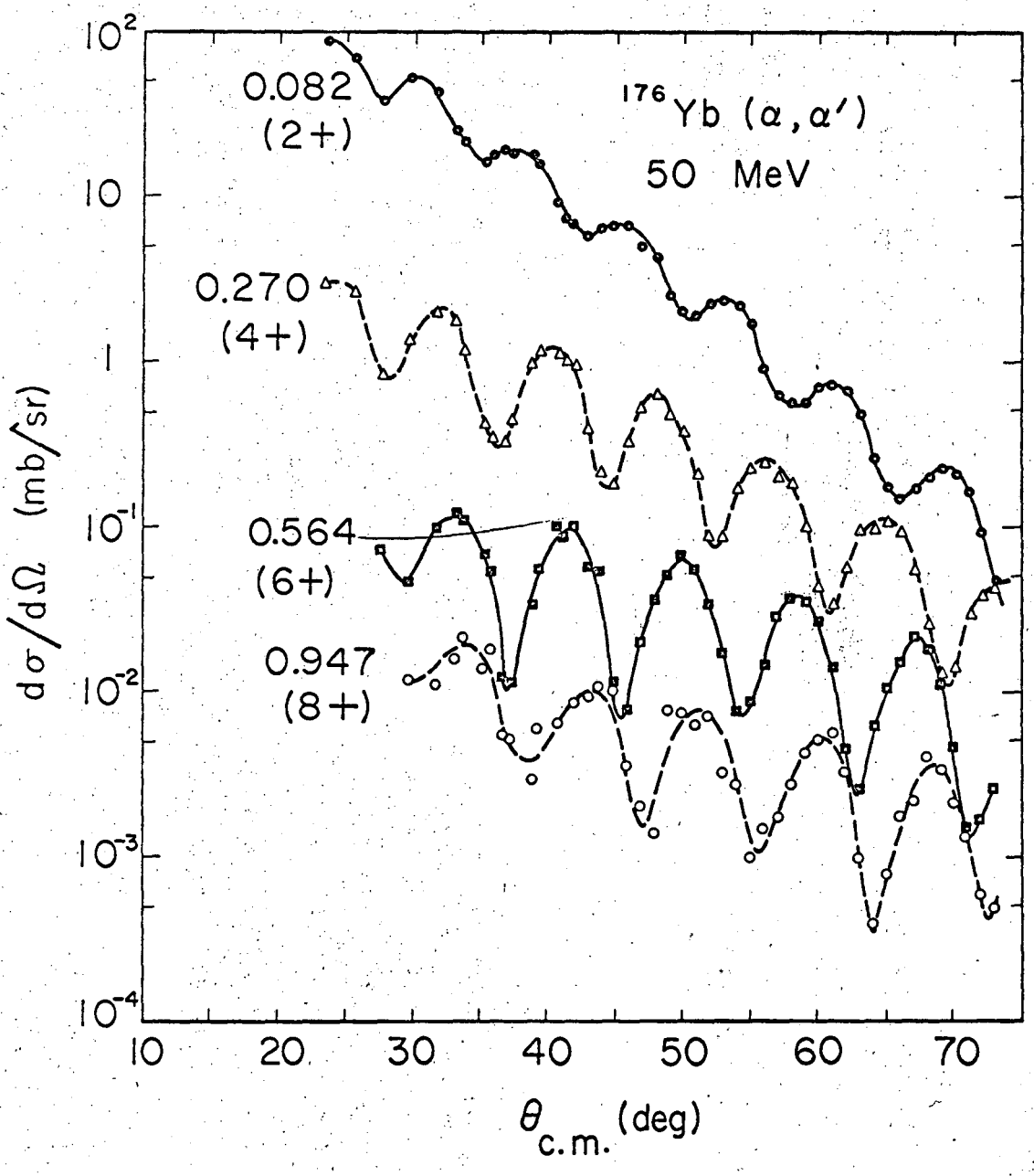


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Fig. 16. Cross-sections for 50 MeV alpha particles on $Yb^{174}$ as measured at Berkeley (Ref. 17). The curve is merely a line through the points.

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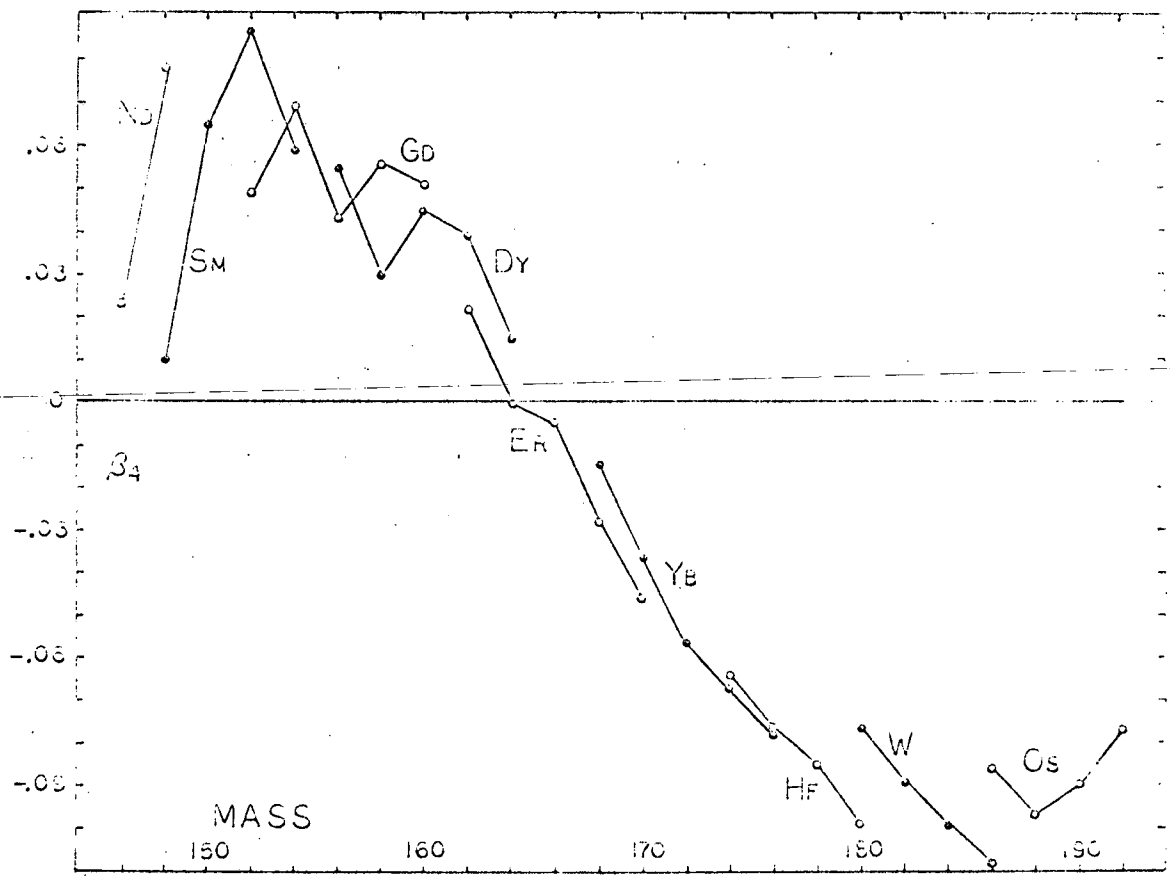


Fig. 17. The $\beta_4$ deformation computed by Hendrie on the basis of the Nilsson scheme, by the method suggested by Harada.

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Summary. We have computed the scattering of alpha particles by a perfect rotor. In view of the sensitivity to the nuclear shape parameters, we believe that by demanding essentially perfect agreement with experiment one can measure the shape very precisely. This was achieved for $Sm^{154}$. Analysis of other nuclei is underway but the trend of the $\beta_4$ deformation in the rare earth region appears already evident, and is portrayed in Fig. 17.

Finally there is a question as to the uniqueness of our parameterization of the nuclear field. This has still to be settled, but if there is any hope at all in finding a unique potential it is surely in this situation where there is so much data available which is sensitive to the various multipole moments.

## APPENDIX TO SECTION II

Cross-sections. For the sake of completeness let us recall how the cross-sections can be obtained from the asymptotic behaviour of the wave function. From (7) and (15) we have for one channel

$$\Psi^{M}_{c\pi I} \to \frac{1}{r} \sum_{c'} \phi^{M}_{c'\pi I} \left\{ \delta_{c'c}(I_c - O_c) + \left(\frac{k_c}{k_{c'}}\right)^{1/2} (\delta_{c'c} - S^{I}_{c'c}) O_{c'} \right\}$$

The total wave function is

$$\Psi = \Sigma A^{M}_{c\pi I} \Psi^{M}_{c\pi I}$$

and we must choose the coefficients $A$ to correspond to the fact that there is a plane (or Coulomb distorted) wave in the target channel $\alpha J$. That is in the absence of scattering ($S \equiv 0$) we should have

$$\Psi^{M,m}_{0} = \Phi^{M}_{\alpha J}(A)\ \chi^{m}_{1/2}(\sigma) e^{ikz}$$

or in the presence of the Coulomb field alone

$$\Psi^{M,m}_{C} = \Phi^{M}_{\alpha J}(A) \chi^{m}_{1/2} \sum_{\ell} \sqrt{4\pi(2\ell+1)}\, i^{\ell} e^{i\sigma_\ell} \frac{O_\ell - I_\ell}{2ikr} Y^{0}_{\ell}$$

$$= \sum_{\ell j I} C^{\ell 1/2 j}_{0mm}\ C^{jJI}_{mMM+m} \sum_{\ell} \sqrt{4\pi(2\ell+1)}\, i^{\ell} e^{i\sigma_\ell} \frac{O_\ell - I_\ell}{2ikr} \phi^{M+m}_{c\pi I}$$

whence

$$A^{M+m}_{c\pi I} = -\frac{1}{2ik} C^{\ell 1/2 j}_{0mm}\ C^{jJI}_{mMM+m} \sqrt{4\pi(2\ell+1)}\ i^{\ell} e^{i\sigma_\ell}$$

Inserting this into $\Psi$ and employing the asymptotic expressions for $O_c$ and $I_c$ we obtain, the general form,

$$\Psi^{M,m} \to \Psi_C^{M,m}$$

$$+ \sum_{\alpha' J' M' m'} \frac{e^{i\left[k'r-\eta' \ln(2k'r)\right]}}{r} \mathcal{f}^N_{\alpha JMm,\alpha' J' M' m'}(\hat{r})\ \Phi_{\alpha' J'}^{M'}(A)\ \chi_{1/2}^{m'}(\sigma)$$

The first term has a Coulomb scattered wave, or in the absence of charge is just a plane wave and the second term represents the scattered waves arising from the nuclear interactions. The explicit form of $\mathcal{f}^N$ can be obtained by performing the algebra implied above.

The current associated with $\mathcal{f} e^{2k'r}/r$ is $|\mathcal{f}|^2 v'/r^2$ so that the flux through the surface $r^2 d\Omega$ in the direction $\Omega$, is $|\mathcal{f}|^2 v' d\Omega$. The incident flux is $v$ so

$$\left(\frac{d\sigma}{d\Omega}\right)_{\alpha J \to \alpha' J'} = \frac{1}{2(2J+1)} \left(\frac{v'}{v}\right) \sum_{MmM'm'} |\mathcal{f}|^2$$

where

$$\mathcal{f} = \mathcal{f}^C\ \delta_{\alpha\alpha'}\ \delta_{MM'}\ \delta_{mm'} + \mathcal{f}^N_{\alpha JMm,\alpha' J' M' m'}$$

with $\mathcal{f}^C$ denoting the Coulomb amplitude. Asymptotically we can replace $\hat{r}$ by $\hat{k}'$.

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FIGURE CAPTIONS

Fig. 1. The DWBA takes account of only transitions 1 and 2, whereas the coupled channel calculations includes such additional ones shown and all of them iterated.

Fig. 2. Shapes of several single-particle form factors that contribute to excitation of $2^+$ states in nickel isotopes are shown for several force ranges of the direct interaction. The oscillator parameter $\nu = m\omega/\hbar = 0.25\ F^{-2}$. (Ref. 6b).

Fig. 3. Scalar form factor for the lowest $2^+$ state in $Ni^{60}$ is shown together with the single-particle form factors that contribute to it. They all have such phases that they add constructively for the lowest $2^+$ but must therefore be destructive for all other $2^+$ states. (Ref. 6b).

Fig. 4. Scalar and vector form factors for $2^+$ states in $Ni^{60}$ are shown at the left. The absolute values are plotted and oscillations are indicated by (+) and (-). Corresponding cross-sections at 11 and 40 MeV for proton scattering calculated in the distorted wave approximation are also shown (Ref. 6b).

Fig. 5. Coupled channel (solid) is compared with optical model calculation of elastic and distorted wave calculation of inelastic proton scattering (dashed). Note that elastic is barely effected by the additional coupling. While several cross-sections are only slightly changed others are significantly modified (Ref. 6i).

Fig. 6. Cross-sections and polarizations for 17.8 MeV protons. Curves are coupled equation calculations using microscopic description employing a surface delta interaction Ref. 9 (solid line) and a volume finite range interaction Ref. 8 (dashed line). Direct interaction parameters are $V_0 = -55$ MeV, $V_1 = 0$ and $\rho = 1.85$ F of Eq. (3) and (15) (Ref. 6j).

Fig. 7. Cross-sections and polarization for 18.6 MeV protons. The experimental polarization was measured at Saclay. The solid lines are coupled channel calculations based on the microscopic theory of Ref. 8.

Fig. 8. Cross-section and polarization for 17.8 MeV protons, measured at Berkeley. Solid lines represent coupled channel calculation based on microscopic theory of Ref. 8.

Fig. 9. The $V_2$ form factor together with contributions to it from various orders 1 through 5 in $\beta_2$ including the Coulomb part. The leading term is the well known first derivative but the higher terms yield altogether a correction of about 20% in the form factor for $\beta_2 = .27$. Note the changing scale for higher order terms. The Coulomb form factor is also shown. Dashed lines are the imaginary part.

Fig. 10. The form factors $V_4$ and $V_6$ are shown for three values of $\beta_4$.

Fig. 11. Elastic cross-sections for four samarium isotopes. The strongly dampted oscillations correspond to the rotational nuclei. Optical model cross-sections were obtained by N. Jarvis.

Fig. 12. Cross-sections for 50 MeV alpha exciting ground state rotational band of $Sm^{154}$. Curves are coupled channel calculation as described in text. The data were taken at the Berkeley 88 inch cyclotron (Ref. 17).

Fig. 13. Extreme sensitivity of $4^+$ and $6^+$ to sign and value of $\beta_4$ is indicated in 50 MeV alpha scattering from $Sm^{154}$.

Fig. 14. Distorted wave calculation with optic parameters set (a) of Table 1.

Fig. 15. Coulomb excitation is neglected here. Comparing with Fig. 12 where it is included illustrates its importance, even though the main mechansim is the nuclear excitation. Note also the interference between nuclear and Coulomb excitation in Fig. 12 at small angles in the $2^+$. To compensate for the neglect of Coulomb excitation a larger value of $\beta_2$ is required namely $\beta_2 = .27$ compared to .225. Even so the data cannot be well reproduced.

Fig. 16. Cross-sections for 50 MeV alpha particles on $Yb^{174}$ as measured at Berkeley (Ref. 17). The curve is merely a line through the points.

Fig. 17. The $\beta_4$ deformation computed by Hendrie on the basis of the Nilsson scheme, by the method suggested by Harada.

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