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K^+n CHARGE EXCHANGE AND THE ρ' REGGE TRAJECTORY

Bertram M. Schwarzschild

June 8, 1967

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UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

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Bertram M. Schwarzschild
(Ph. D. Thesis)

June 8, 1967

DEDICATION

Would that

LUDWIG WOLFGANG SCHWARZSCHILD and SHULAMITH GOLDHABER were alive to receive my thanks. To their memory I humbly dedicate this work. May the remembrance of the righteous serve us as a blessing. ז"צ"ל

K^+n CHARGE EXCHANGE AND THE ρ' REGGE TRAJECTORY

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University of California
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Abstract

Finding that the Regge exchange of ρ and A_2 fails to describe our K^+n charge-exchange data at 2.3 GeV/c (the highest energy available to date) when a simultaneous fit is attempted with higher energy data on K^-p and π^-p charge exchange and $\pi^-p \to \eta^0 n$, we introduce a second, lower lying ρ-type trajectory (ρ'). This also provides a possible mechanism for the puzzling π^-p charge-exchange polarization. We find that we are then able simultaneously to fit all these data (including the polarization), together with related total cross-section differences up to 20 GeV/c, with a ρ' whose spin 1 mass is 1.0 GeV [perhaps the $\delta(965)$?] and whose $t = 0$ intercept, 1.1 units of angular momentum below the ρ, agrees roughly with the ρ' proposed by Högaasen and Fischer to describe forward $\bar{p}p$ and np charge exchange, where the (ρ, A_2) model also fails. Our ρ and A_2 trajectories turn out to be essentially traditional. In the fit we permit only small SU_3 breaking between the $K\bar{K}$ and $\pi\pi$ (or $\eta^0\pi$) couplings to the trajectories. We further constrain the fit to obey the sum rule of Igi and Matsuda. In fitting our K^+ data at 2.3 GeV/c, we include a deuteron correction, and employ exact Legendre functions rather than the high-energy asymptotic Regge forms. We offer predictions for higher energy K^+ charge exchange.

INTRODUCTION

Considerable success has previously been achieved in fitting the high-energy differential cross sections for the reactions:

$$\pi^- p \to \pi^0 n \tag{1}$$

$$K^- p \to \overline{K}^0 n \tag{2}$$

$$\pi^- p \to \eta^0 n \tag{3}$$

in the peripheral region $[-t \lesssim 1\ (\mathrm{GeV}/c)^2]$, by the t-channel Regge exchange of the $\rho(C^-)$ and $A_2(C^+)$ trajectories.[1,2] Together with

$$K^+ n \to K^0 p, \tag{4}$$

these constitute all the isotopically independent reactions

$$\mathrm{PS8} + \mathrm{N} \to \mathrm{PS8} + \mathrm{N} \qquad (\mathrm{PS8} = \text{Pseudoscalar Octet},\ \mathrm{N} = \text{Nucleon}) \tag{5}$$

which require $I = 1$ exchange in the t channel. For t-channel exchange, reaction (4) differs from (2) only by a relative sign change in the C^+ and C^- exchange amplitudes. Thus, if the (ρ, A_2) model is adequate, the previous fits ought to determine high-energy K^+n peripheral charge exchange.

We therefore compared our 2.3 GeV/c K^+n charge-exchange data[3] [See Appendix A. These remain the highest energy data on reaction (4) available to date] with the previous fits of Rarita and Phillips,[1] and attempted a simultaneous fit of data on all four reactions to the assumption of only ρ and A_2 Regge exchange in the peripheral region. The attempt was in part motivated by the correct prediction of this model that, in contrast to K^-p, the K^+n charge exchange ought to show $|\mathrm{Re}\, f(t=0)/\mathrm{Im}\, f(t=0)| \gg 1$. Despite correction for the deuteron effect in (4), and use of exact Legendre function forms because of the relatively

low energy of our K^+ data, we found that the model always gave a cross section only about half as large as that observed for K^+n charge exchange. A kinder fate would have been expected considering the relative absence of resonant activity in the K^+ nucleon system above about 1.3 GeV/c.

The model has other difficulties. It predicts, contrary to fact, zero polarization for π^-p charge exchange. Further, Högaasen and Fischer find that it fails to describe the energy dependence of forward pn and $\bar{p}p$ charge exchange.[4] There seems to be a simultaneous plausible remedy for all three difficulties, namely, the exchange of a second, lower lying ρ' trajectory having the same quantum numbers as the ρ. Lying well below the ρ trajectory, its relative contribution to the differential cross sections would decrease rapidly with increasing energy, thus explaining the earlier fits which ignored it.

Adding this ρ' trajectory to the C^- exchange amplitude, which we then subject to the constraint of a recent superconvergent sum rule of Igi and Matsuda,[5] we have achieved a good simultaneous fit to extensive data on processes (1) through (4), together with related total cross-section differences and π^-p charge-exchange polarization. This fit involves only small SU_3 symmetry breaking, the ρ' being assumed to belong to an octet. The resulting ρ' trajectory is consistent with the intercept found by Högaasen and with the δ(965 MeV) meson at $\alpha = 1$.

FORMALISM

All the processes (5) (see Fig. 1) require that the t-channel exchanged object satisfy $C = P = (-1)^J = G(-1)^I$. Anticipating Reggeization and SU_3, we separate the $I = 1$ exchange into two parts:

$$C^+(P^+,\ G^-\ J \text{ even}) \text{ and } C^-(P^-,\ G^+,\ J \text{ odd}).$$

Clearly, for reaction (1) only the C^- exchange is possible, while for (3) we have only C^+. The $K^{\pm}N$ charge exchanges admit both. Therefore we define the t-channel helicity nonflip amplitudes

$$A(K^-p \to \overline{K}^0 n) = {}_{\kappa}A^+ + {}_{\kappa}A^- \tag{6}$$

$$A(\pi^-p \to \pi^0 n) = {}_{\pi}A^- \tag{7}$$

$$A(\pi^-p \to \eta^0 n) = {}_{\pi}A^+, \tag{8}$$

giving

$$A(K^+n \to K^0 p) = {}_{\kappa}A^+ - {}_{\kappa}A^- \tag{9}$$

with similar formulae for the helicity flip amplitudes B. The superscript sign refers to both charge conjugation and J parity (or signature) $(-1)^J = C$.

Now let each amplitude be a sum over contributing Regge trajectories, e.g.,

$${}_{\pi}A^- = \sum_i {}_{\pi}A_i^- \text{ etc.} \tag{10}$$

Then we have from the factorization theorem:

$${}_{\pi}A_i/{}_{\kappa}A_i = {}_{\pi}B_i/{}_{\kappa}B_i \equiv F_i(t) \tag{11}$$

If SU_3 is unbroken, and all contributing trajectories are octet members, we have

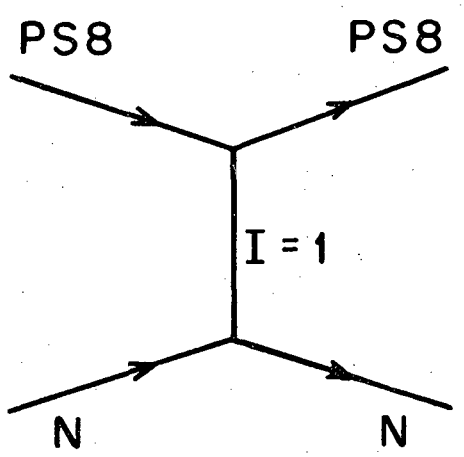

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Fig. 1. Reactions (1) through (4), for which we investigate peripheral scattering data, constitute a complete, isotopically independent, set of the reactions (pseudoscalar octet + nucleon → pseudoscalar octet + nucleon) which require isovector exchange in the t (peripheral) channel.

$$F_i(t) = \begin{cases} F^+ = (2/3)^{1/2} \text{ for } C_i = +1, \\ F^- = -\sqrt{2} \text{ for } C_i = -1 \end{cases} \tag{12}$$

independent of t.

The experimentally observed quantities are given in terms of the helicity amplitudes by:

$$\frac{d\sigma}{dt} = \frac{1}{\pi}\left(\frac{1}{4p}\right)^2 \left\{\left(1-\frac{t}{4M^2}\right)|A|^2 - \frac{t}{4M^2}\left[\frac{s+P^2}{1-(t/4M^2)} - s\right]|B|^2\right\} \tag{13}$$

$$\sigma_T(K^\pm p) - \sigma_T(K^\pm n) = \mathrm{Im}({}_\kappa A^+ \mp {}_\kappa A^-)_{t=0}/p \tag{14}$$

$$\sigma_T(\pi^+ p) - \sigma_T(\pi^- p) = \sqrt{2}\,\mathrm{Im}({}_\pi A^-)_{t=0}/p \tag{15}$$

$$P\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) =$$

$$\frac{1}{\pi}\left(\frac{1}{4p^2}\right)\left[\left(1-\frac{t}{4M^2}\right)\right]^{1/2}\left\{\frac{-t}{4M^2}\left[\frac{s+p^2}{1-(t/4M^2)} - s\right]\right\}^{1/2} {}_\pi\underset{\sim}{A}^- \times {}_\pi\underset{\sim}{B}^- \tag{16}$$

where M is the nucleon mass, s and t are the invariant squares of energy and momentum transfer, p is the incoming lab momentum, and P is the $\pi^- p$ charge-exchange polarization. $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are vectors in the complex plane.

There is only one reasonably well established trajectory for each of C^+ and C^-, namely the A_2 and ρ. As discussed above, however, they do not seem to suffice for all the processes under consideration. Including now the speculative ρ', we parametrize the various high-energy asymptotic Regge helicity amplitudes as follows:

$${}_\kappa A^- = {}_\kappa A_\rho + {}_\kappa A_{\rho'} \text{ and } {}_\kappa A^+ = {}_\kappa A_{A_2} \equiv {}_\kappa A_R \tag{10'}$$

with similar relations for the ${}_\kappa B$, ${}_\pi A$, and ${}_\pi B$ amplitudes.

$$
\begin{aligned}
{}_{\kappa}A_i^{\pm} &= -C_i(t)\,(\alpha_i+1)\;\frac{(e^{-i\pi\alpha_i}\pm 1)}{\sin\pi\alpha_i}\;\left(\frac{E}{E_0}\right)^{\alpha_i} \\
{}_{\kappa}B_i^{\pm} &= -D(t)\,\alpha_i(\alpha_i+1)\;\frac{(e^{-i\pi\alpha_i}\pm 1)}{\sin\pi\alpha_i}\;\left(\frac{E}{E_0}\right)^{\alpha_i-1},
\end{aligned}
\tag{17}
$$

where E/E_0 is the lab energy of the incoming meson in GeV.

We take

$$
\begin{aligned}
C_\rho(t) &= C_\rho^0(1+C_\rho^1 t) & D_\rho(t) &= D_\rho^0\exp(D_\rho^1 t) \\
C_{\rho'}(t) &= C_{\rho'}^0\exp(C_{\rho'}^1 t) & D_{\rho'}(t) &= D_{\rho'}^0\exp(D_{\rho'}^1 t) \\
C_R(t) &= \alpha_R C_R^0\exp(C_R^1 t) & D_R(t) &= \alpha_R D_R^0\exp(D_R^1 t)
\end{aligned}
\tag{17'}
$$

$$\alpha_i^0(t) = \alpha_i^0 + \alpha_i' t \tag{18}$$

$$F_i(t) = F_i^0\exp(F_i^1 t). \tag{11'}$$

The distribution of the factors of α in Eqs. (17) implies a specific mechanism for the required vanishing of flip residue functions and the "ghost killing" for even-signature amplitudes at $\alpha = 0$. We assume that the "nonsense" vertices (see Fig. 2) each provide a factor $\sqrt{\alpha}$ for all exchanged trajectories, and that in the case of even-trajectory exchange, every vertex provides an additional factor $\sqrt{\alpha}$. This is the so-called Chew ghost-killing mechanism, with all trajectories "choosing sense." Alternate mechanisms have been suggested, but there is some evidence in favor of our choice.[6]

For $C_\rho(t)$, we chose originally the form $C^0\{[G+1]\exp(C^1 t)-G\}$ to provide a possible mechanism for the crossover of the π^+p and π^-p elastic differential cross sections via a sign change in $C_\rho(t)$. The fitting program always chose the case $G \gg 1 \gg C^1$. Hence the form in (17').

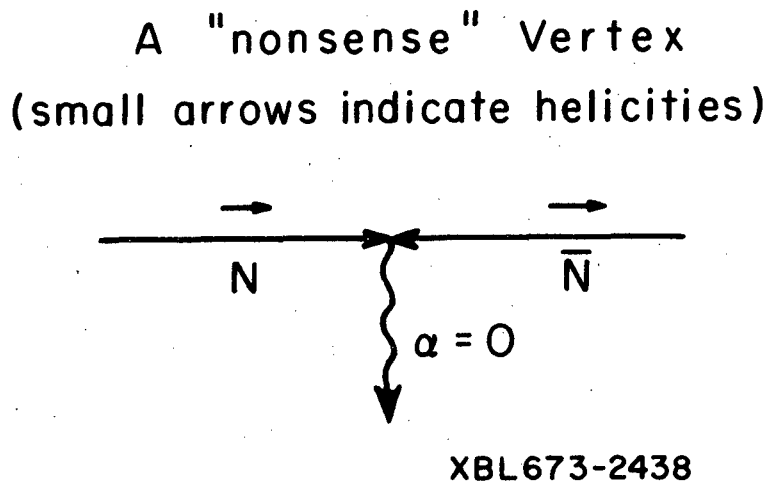

Fig. 2. If one thinks of $\alpha(t)$ as the spin of the exchanged "particle" for a given t, then the t-channel helicity flip vertex above violates angular momentum conservation when $\alpha = 0$. We assume that each such "nonsense" vertex contributes a factor $\sqrt{\alpha}$ to the t-channel helicity flip amplitude.

Table I shows the data to be fitted. Of the 24 parameters in Eqs. (17') to (11') varied to fit these data, one degree of fitting freedom is lost via the constraint of the Igi sum rule discussed below, which relates α^0_ρ, $\alpha^0_{\rho'}$, C^0_ρ, $C^0_{\rho'}$, F^0_ρ, and $F^0_{\rho'}$ in an equation of constraint. We have also constrained the six SU_3 breaking parameters F^0_i and F^1_i to give symmetry breaking of less than 25%. Thus we are left effectively with 17 free parameters and 6 restricted parameters.

SPECIAL TREATMENT OF LOW-ENERGY K^+N DATA

The K^+n charge-exchange data at 2.3 GeV/c require special treatment, (a) because of the relatively low energy for the applicability of the high-energy asymptotic forms (17), and (b) because the target neutron is bound in a deuteron.

A. Explicit Legendre Function Formulation

The high-energy Regge behavior ($\sim E^\alpha$) of the amplitudes comes from the asymptotic behavior of the Legendre functions:

$$\left.\begin{array}{l} \alpha > -1/2,\ P_\alpha(w) \\ \alpha < +1/2,\ M_\alpha(w) \equiv \dfrac{-\tan\pi\alpha}{\pi}\, Q_{-\alpha-1}(w) \end{array}\right\} \xrightarrow[w\to\infty]{} \frac{(2w)^\alpha}{\sqrt{\pi}}\ \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} \tag{19}$$

where P and Q are the Legendre functions of the first and second kind. Following a suggestion of Read et al.[7] based on Mandelstam's extension[8] of the Regge formalism to $\alpha < -1/2$, we will use P_α for $\alpha \geq 0$ and M_α for $\alpha < 0$, denoting this generically by L_α.

Table I. Data.

Reaction	Lab momenta (GeV/c) and references
Differential cross section	
$K^+n \to K^0p$	2.3[a]
$K^-p \to \bar{K}^0n$	(5, 7, 9.5)[b]
$\pi^-p \to \pi^0n$	(5.9, 9.8, 13.3, 18.2)[c]
$\pi^-p \to \eta^0n \times B(\eta \to 2\gamma)$	(5.9, 9.8, 13.3, 18.2)[d, e, f]
Total cross-section differences	
$\sigma_T(K^+p)-\sigma_T(K^+n)$	2.3,[g] (6, 8, 10, 12, 14, 16, 18, 20)[h]
$\sigma_T(K^-p)-\sigma_T(K^-n)$	(6, 10, 12, 14, 16, 18)[h]
$\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$	(5, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4)[i]
	(8, 10, 12, 14, 16, 18, 20)[h]
Polarization	
$\pi^-p \to \pi^0n$	(5.9, 11.2)[j]

a. I. Butterworth, J. Brown, G. Goldhaber, S. Goldhaber, A. Hirata, J. Kadyk, B. Schwarzschild, and G. Trilling, Phys. Rev. Letters 15, 734 (1965), Thesis Appendix A.

b. P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. West, P. Zanella, W. Beuch, W. Fischer, B. Gobbi, M. Peppin, E. Polgar, C. Verkerk, and M. Pouchon, CERN Preprint 66/1057/5, 1966.

c. Saclay-Orsay Collaboration: A. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters 14, 763 (1965); Phys. Letters 20, 75 (1966).

d. Saclay-Orsay Collaboration (see Ref. c), Phys. Letters 18, 200 (1965).

e. Branching ratio ($\eta \to 2\gamma/\eta \to$ all neutrals) = 0.416, as given in G. DiGiugno, R. Querzoli, G. Triose, F. Vanoli, M. Giorgi, and P. Schiavon, Phys. Rev. Letters 16, 767 (1966).

f. Branching ratio ($\eta \to$ all neutrals) = 0.729, as given in A. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

g. T. Kycia, Bull. Am. Phys. Soc., April 1967 (JB2).

h. W. Galbraith, E. Jenkins, T. Kycia, B. Leontic, R. H. Phillips, A. Read, and R. Rubinstein, Phys. Rev. 138, B913 (1965).

i. A. Citron, W. Galbraith, T. Kycia, B. Leontic, R. H. Phillips, A. Rouset, and P. Sharp, Phys. Rev. 144, 1101 (1966).

j. P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri, and L. Vincelli, Phys. Letters 23, 501 (1966).

Retreating from asymptotically high energies, we may write the nonflip amplitudes more generally as

$$A_i^{\pm} = -a(t)\left(\frac{-qq'}{ME_0}\right)^{\alpha}(\alpha+1/2)\, L_{\alpha}(w)\,\frac{(e^{-i\pi\alpha}\pm 1)}{\sin\pi\alpha} \tag{20}$$

where q, q', and $w = -\cos\theta_t$ are the (nonphysical) momenta and scattering angle cosine in the t channel. These are given by

$$\begin{aligned} t &= 4(q^2+M^2) = 4(q'^2+m^2) \\ -s &= q^2+q'^2-2qq'\cos\theta_t \end{aligned} \tag{21}$$

giving

$$w = \frac{2(s-M^2-m^2)+t}{(4M^2-t)^{1/2}(4m^2-t)^{1/2}} \quad . \tag{22}$$

$m = (E^2-p^2)^{1/2}$ is the meson mass. $(-qq'/ME_0)^{\alpha} = \{[(4M^2-t)(4m^2-t)]^{1/2}/4ME_0\}^{\alpha}$ is factored out of the residue function $a(t)$ to cancel the anomalous t-channel threshold singularities in w. From (19) we have then

$$A_i^{\pm} \xrightarrow[E\to\infty]{} -a(t)\left(\frac{2E}{E_0}\right)^{\alpha}\frac{(\alpha+1/2)}{\sqrt{\pi}}\;\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\;\frac{(e^{-i\pi\alpha}\pm 1)}{\sin\pi\alpha} \quad . \tag{23}$$

Comparing (23) with (17) we have finally in terms of the fitted parameters,

$$\kappa A_i^{\pm} = -C_i(t)\,(\alpha_i+1)\,\frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{1}{2})}\,\frac{\sqrt{\pi}}{2^{\alpha_i}}\left(\frac{-qq'}{ME_0}\right)^{\alpha_i} L_{\alpha_i}(w)\left(\frac{e^{-i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}\right) . \tag{20'}$$

Similarly we write the general form for the helicity flip amplitudes

$$B_i^{\pm} = -b(t)\left(\frac{-qq'}{ME_0}\right)^{\alpha-1}(\alpha+\tfrac{1}{2})\,\frac{d}{dw}\,L_{\alpha}(w)\,\frac{(e^{-i\pi\alpha}\pm 1)}{\sin\pi\alpha} \quad . \tag{24}$$

Invoking the general property $d/dw L_\alpha(w) = \alpha(wL_\alpha - L_{\alpha-1})/(w^2-1)$, we get

$$B_i^{\pm} \xrightarrow[w\to\infty]{} -b(t)\left(\frac{2E}{E_0}\right)^{\alpha-1} \frac{2\alpha(\alpha+\frac{1}{2})}{\sqrt{\pi}} \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} \frac{(e^{-i\pi\alpha_i}\pm 1)}{\sin\pi\alpha} , \qquad (25)$$

which by comparison with (17) gives

$$_\kappa B_i^{\pm} = -D_i(t)\,(\alpha_i+1)\frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{1}{2})} \frac{\sqrt{\pi}}{2^{\alpha_i}} \left(\frac{-qq'}{ME_0}\right)^{\alpha_i-1} \frac{d}{dw} L_{\alpha_i}(w)\left(\frac{e^{-i\pi\alpha_i}\pm 1}{\sin\pi\alpha_i}\right). \qquad (24')$$

The factor $[\Gamma(\alpha+1)]^{-1}$ in (19) serves to cancel the unwanted poles in the signature factors for negative α. In our formalism we have effectively replaced it by $(\alpha+1)$. So long as we have $\alpha^+ > -2$ and $\alpha^- > -3$ (which turns out to be the case), the empirical factors C(t) and D(t) can make up the difference.

We replace Eqs. (17) by (20') and (24') when fitting the K^+ charge-exchange and total cross-section difference data at 2.3 GeV/c. To all the rest of our data, which are above 5 GeV/c, we apply the asymptotic forms.

B. Deuteron Correction

We must now express the observed $K^+d \to K^0p(p)$ distribution $(d\sigma/dt)_d$ in terms of the free neutron K^+n charge-exchange cross section $(d\sigma/dt)_n$, and thus in terms of the Regge amplitudes. The data $(d\sigma/dt)_d$ had been determined by attributing to each K^0 the lab momentum p' it would have if its observed direction $\hat{p}'$ had resulted from a collision with a stationary free neutron.[3] Then the impulse and closure approximation gives[9]:

$$\left(\frac{d\sigma}{dt}\right)_d = \left(\frac{d\sigma}{dt}\right)_n \frac{[1 - H + R(1 - H/3)]}{(1+R)} , \tag{26}$$

where

$$R(t) = \frac{(d\sigma/dt)_n^{\text{spin flip}}}{(d\sigma/dt)_n^{\text{nonspin flip}}}$$

and

$$H(t) = \int d^3\underset{\sim}{r}\, |\Psi_d(\underset{\sim}{r})|^2 \exp[i(\hat{p}'p' - \underset{\sim}{p})\cdot \underset{\sim}{r}] , \tag{27}$$

$\Psi_d(\underset{\sim}{r})$ being the Hulthén deuteron wave function. (See Appendix B.)

For $t = 0$ we have $H = 1$ and $R = 0$, causing $(d\sigma/dt)_d$ to vanish in the forward direction whether or not the two-body cross section vanishes. With increasing $-t$, H approaches zero, falling to about 0.1 at $t = -0.13$ GeV/c, causing $(d\sigma/dt)_d$ to approach $(d\sigma/dt)_n$.

It remains to make the connection between R and the t-channel helicity amplitudes A, B. $R = f_2^2/f_1^2$ where f_1 and f_2 are, in conventional notation, given by $f(\theta) = f_1 + f_2\, \sigma\cdot\hat{n}$. Now the s-channel helicity amplitudes g_1 and g_2 are given by $f = g_1 + g_2 \sigma\cdot\hat{k}_f\, \sigma\cdot\hat{k}_i$. Comparing the two representations and using well known properties of the Pauli matrices σ, we get

$$R = \left| \frac{g_2 \sin\theta}{g_1 + g_2 \cos\theta} \right|^2 , \tag{28}$$

where θ is the c.m. scattering angle. Finally, having followed the formalism of Singh[10] for the t-channel helicity amplitudes, we have

$$g_{\substack{1\\2}} = \frac{(k^2+M^2)^{1/2} \pm M}{8\pi\sqrt{s}} \left[\pm A' + (\sqrt{s} \mp M)B\right] \tag{29}$$

$$A = A' + \frac{E + (t/4M)}{1-(t/4M^2)} B ,$$

k being the c.m. momentum.

In summary then, the K^+n charge-exchange data $(d\sigma/dt)_d$ are fitted to Eq. (26) in which the factor $(d\sigma/dt)_n$ is given by the Regge two-body formula (13). Note that the free neutron cross section, especially near the forward direction, cannot be extracted from the deuterium data without the aid of a model which gives R(t).

A SUPERCONVERGENT SUM RULE

To test the validity of additional Regge poles (and cuts) with the quantum numbers of the ρ, proposed to explain the π^-p charge-exchange polarization, Igi and Matsuda have obtained a superconvergent sum rule.[5] From a dispersion relation for that part of the $t = 0$ amplitude which vanishes at infinity faster than E^{-1} (i.e., $\alpha < -1$), they get the sum rule

$$4\pi f^2 = \frac{1}{2\pi} \int_{E=m}^{\infty} \{p[\sigma_T(\pi^+p) - \sigma_T(\pi^-p)] - \sqrt{2} \sum_i \beta_i L_{\alpha_i^0}(\frac{E}{m})\} dE, \tag{30}$$

where $\Sigma\beta L$ is the sum, over contributing singularities with $\alpha^0 > -1$ (in our case ρ and ρ'), of the imaginary parts of the forward Regge π^-p charge-exchange scattering amplitudes. $f^2 = 0.081$ is the πN coupling constant squared.

$$\beta_i L_{\alpha_i^0}\left(\frac{E}{M}\right) \xrightarrow[E\to\infty]{} \mathrm{Im}_\pi A_i(t=0)^{\text{asymptotic}}$$

giving

$$\beta_i = F_i^0 C_i^0(\alpha_i^0+1)\frac{\Gamma(\alpha_i^0+1)}{\Gamma(\alpha_i^0+\frac{1}{2})}\sqrt{\pi}\left(\frac{m}{2}\right)^{\alpha_i^0} . \qquad (30')$$

The integrand of (30) vanishes at energies sufficiently high that all contributions other than ρ and ρ' become negligible [cf. optical theorem, Eq. (15)]. We take the integral up to 39m ($\approx$5.5 GeV/c), using Igi's numerical determination of $\int p\Delta\sigma_T dE$. Then Eqs. (30) relate C_ρ^0, $C_{\rho'}^0$, α_ρ^0, $\alpha_{\rho'}^0$, F_ρ^0, and $F_{\rho'}^0$ in an equation of constraint, which we impose upon the fitting.

RESULTS

The fitted parameters resulting from a least-squares fit to the data are given in Table II. In Figs. 3 through 9 the resulting theoretical curves are shown, superimposed upon the data. For 194 data points we have a χ^2 of 191. The ρ and A_2 trajectories turn out essentially traditional, i.e., not unlike the results of the usual fits to the high-energy cross sections without a ρ'. The ρ' intercept, -0.48, is reasonably consistent with Högaasen's determination, -0.6, from the $\bar{p}p$ and pn data.[4] The ρ' slope, 1.44/(GeV)^2, gives a mass, at $\alpha = 1$, of 1.01 GeV, suggesting the $\delta(965)$, about which little is known except that it is an isovector. Relative to the ρ, the ρ' contribution near the forward direction is given roughly by

Table II. Fitted parameters.

	Trajectories		KN residue parameters				$\pi N/KN$ SU_3 breaking	
	α^0	α'	C^0 (mb×GeV)	D^0 (mb)	C^1 $(GeV)^{-2}$	D^1 $(GeV)^{-2}$	$F^0/F^{\pm}$	F^1 $(GeV)^{-2}$
$\rho(-)$	0.58	0.92	1.30	22.7	2.92	0.26	1.10	-0.006
$\rho'(-)$	-0.48	1.44	5.02	-264	4.4	2.95	0.80	+0.20
$A_2(+)$	0.37	0.41	5.50	-116	0.42	0.66	1.01	-0.07

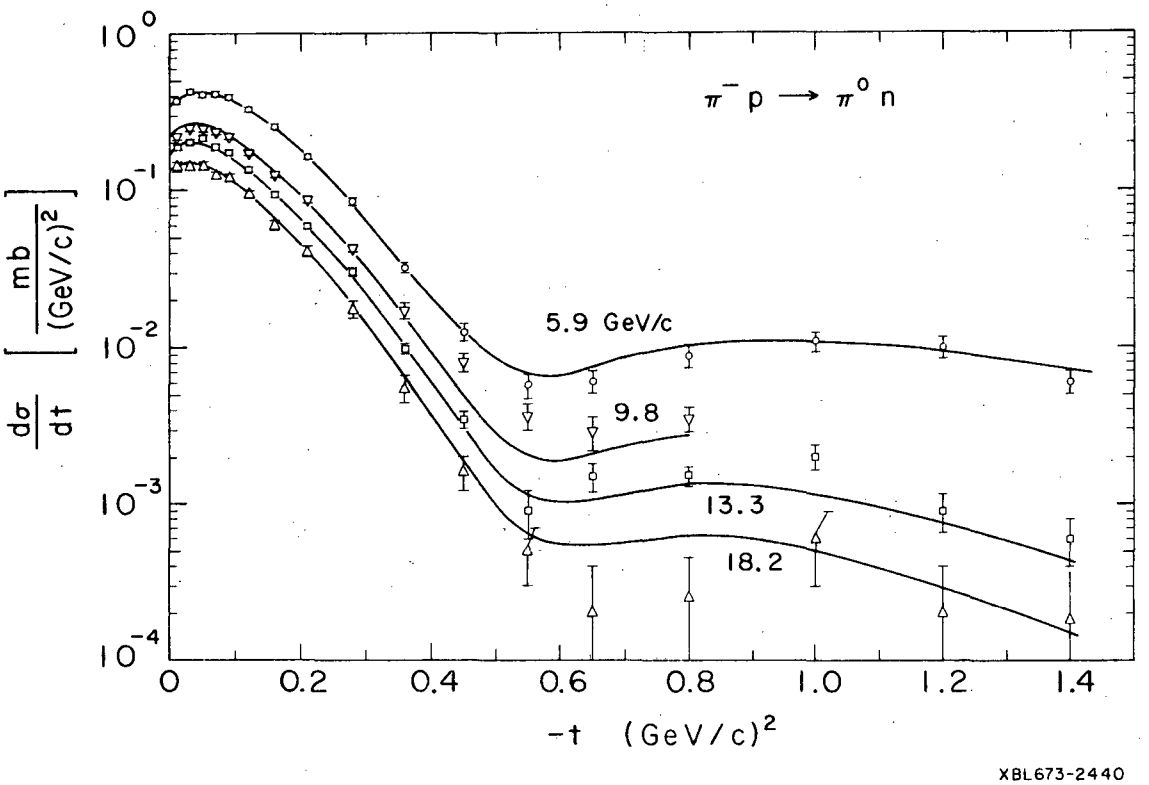

Fig. 3. $\pi^- p$ charge-exchange differential cross sections, incoming lab momenta from 5.9 to 18.2 GeV/c. Data from Saclay–Orsay Collaboration, Stirling et al. (Table I, ref. c). Solid curves are our Regge fits, Table II parameters.

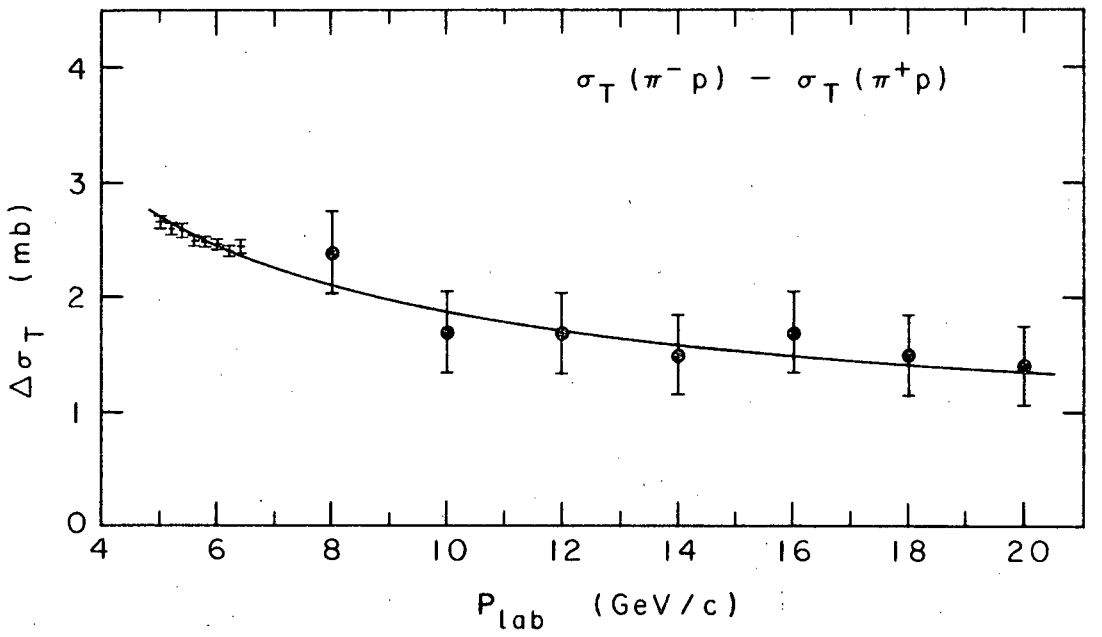

Fig. 4. πp total cross-section difference from 5 to 20 GeV/c. Data from Galbraith et al., Brookhaven (Table I, Refs. h and i). Solid curve is our Regge fit, Table II parameters.

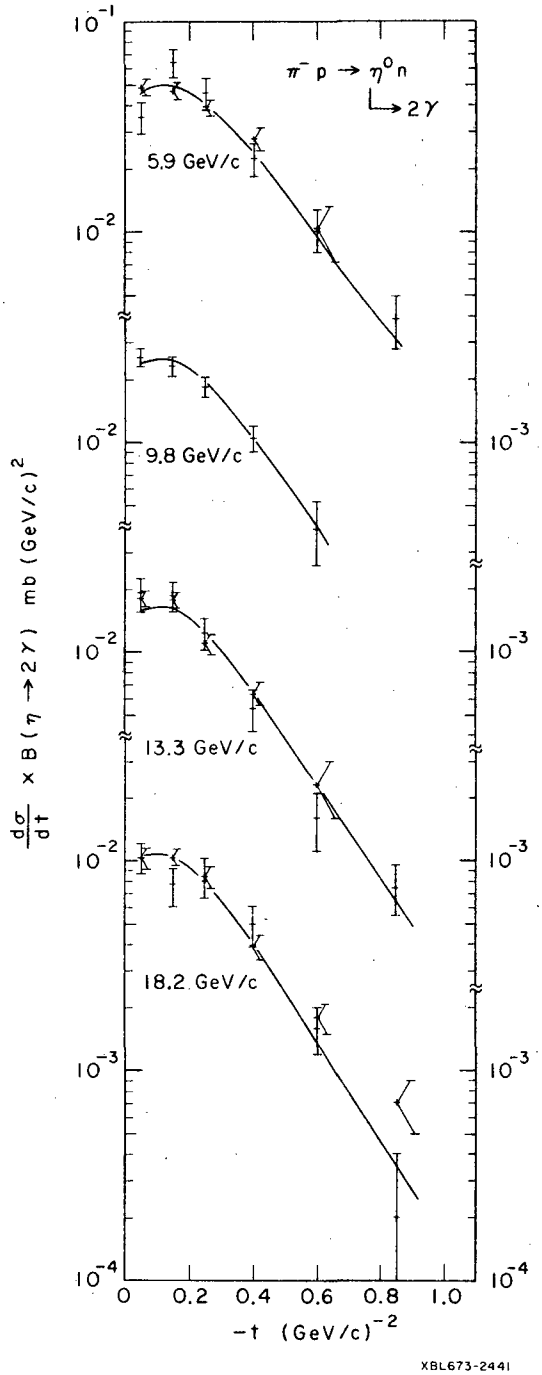

Fig. 5. $\pi^- p \to \eta^0 n$ differential cross sections reduced by branching ratio $B(\eta^0 \to 2\gamma)$, which is the only mode observed in these data, Stirling et al., Saclay–Orsay Collaboration (Table I, Ref. d). Solid curves are our Regge fits, Table II parameters. To make contact with the KN normalization parameters via SU_3, we use for the branching ratio 0.303 (Table I, Refs. e and f).

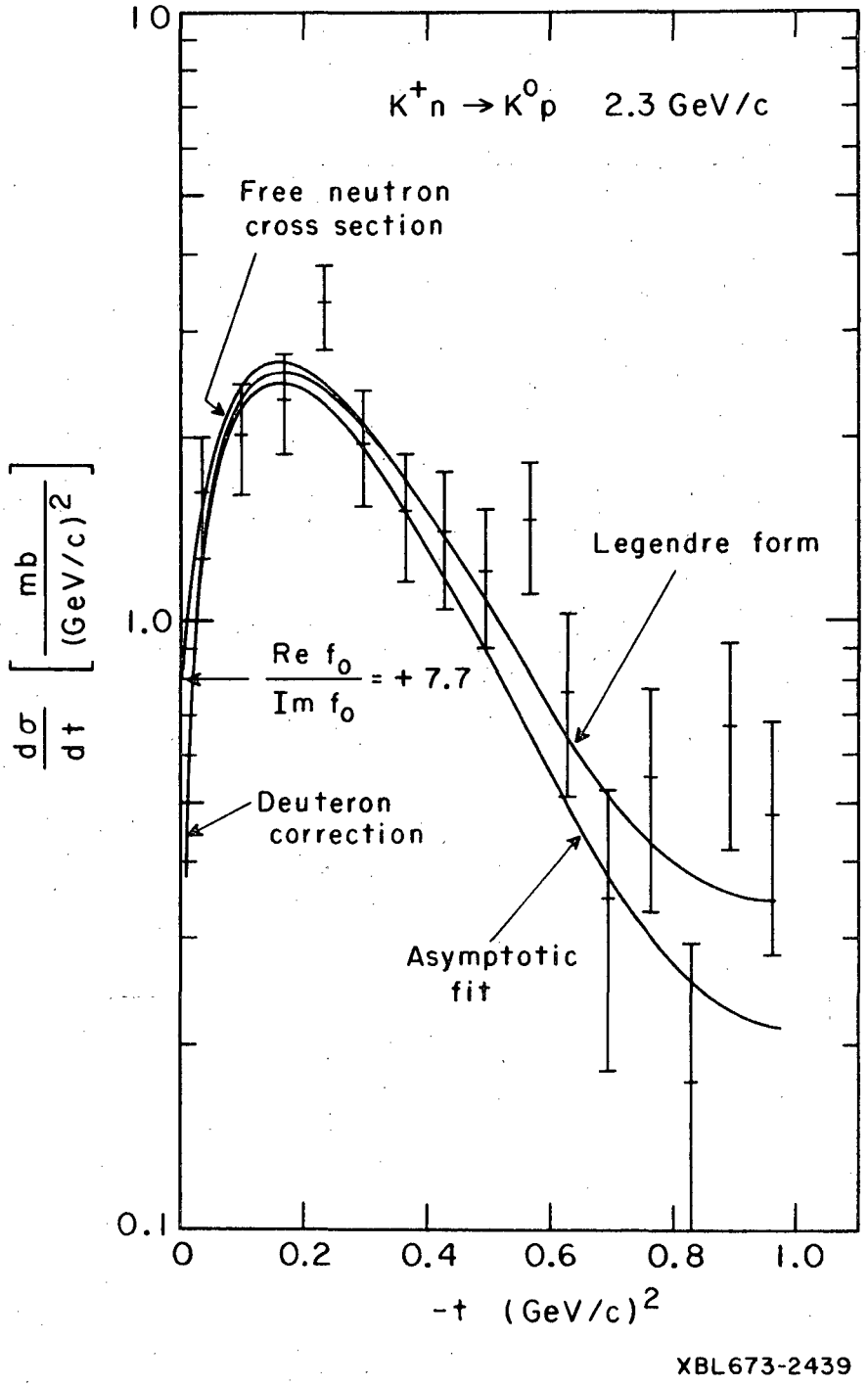

Fig. 6. K^+n charge exchange at 2.3 GeV/c, the highest energy data available to date (Butterworth, Goldhaber, Schwarzschild, et al., Berkeley, Table I, Ref. a). In the forward region we show the Regge fit, with deuteron correction, to the deuterium data, as well as the free neutron cross section deduced from the fitted parameters, Table II. At larger angles one sees the correction due to the use of exact Legendre functions, as well as the fit using the high-energy asymptotic forms.

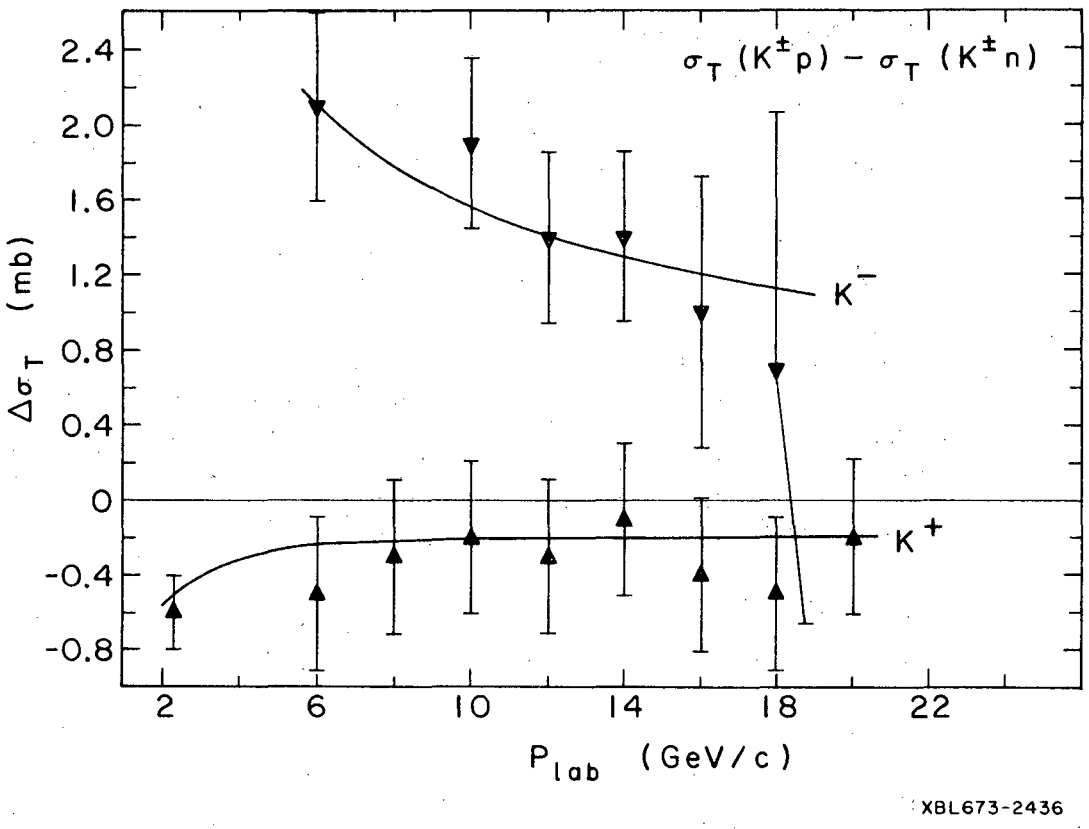

Fig. 7. K^+N and K^-N total cross-section differences, 6 to 20 GeV/c. $\sigma_T(K^-p)-\sigma_T(K^-n) > 0$, and $\sigma_T(K^+p)-\sigma_T(K^+n) < 0$. Data of Galbraith et al., Brookhaven (Table I, Ref. h). We include also the K^+ datum at 2.3, for which we indicate here (and use in fitting) an uncertainty given by the amplitude of the small wiggles of $\Delta\sigma_T(K^+)$ in this region, the quoted experimental error being considerably smaller (Kycia, Brookhaven, Table I, Ref. g). The solid curves are our Regge fits, Table II parameters.

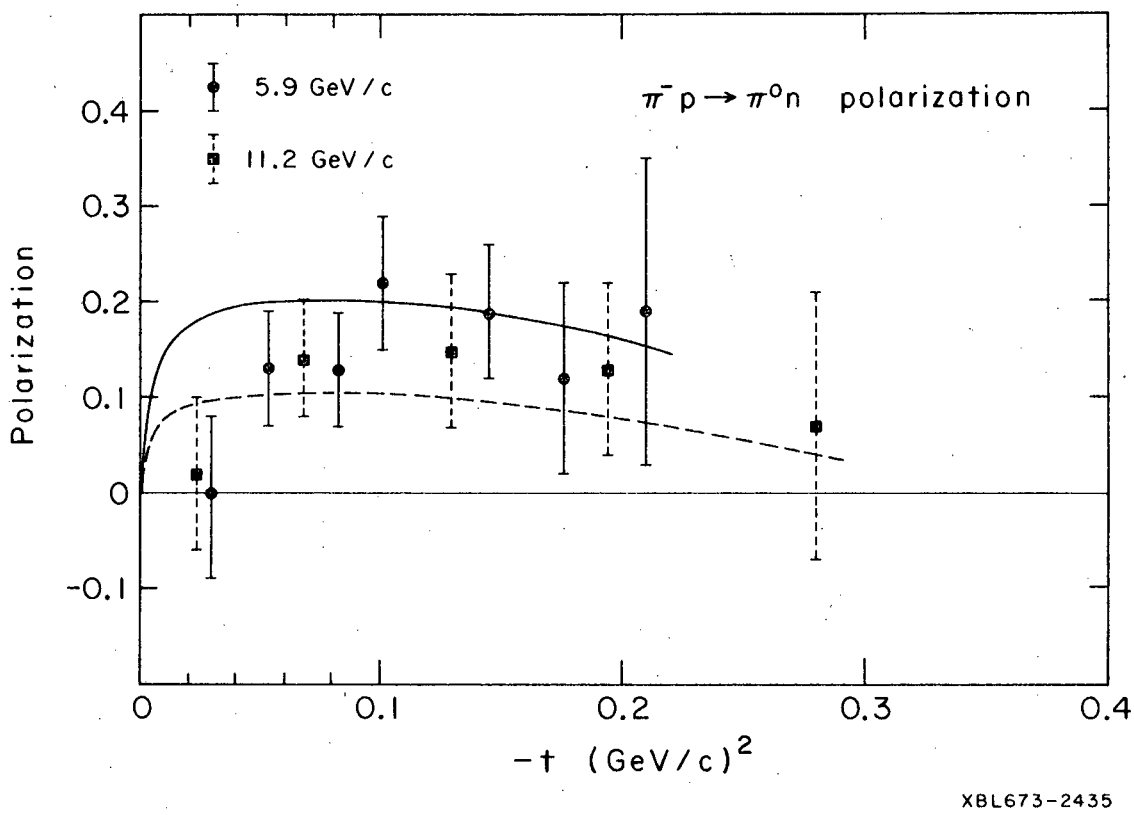

Fig. 8. $\pi^- p$ charge-exchange polarization, 5.9 GeV/c (solid error bars and fitted curve) and 11.2 GeV/c (dashed bars and curve). Data of Bonamy et al., Saclay-Orsay-Pisa Collaboration (Table I, Ref. j).

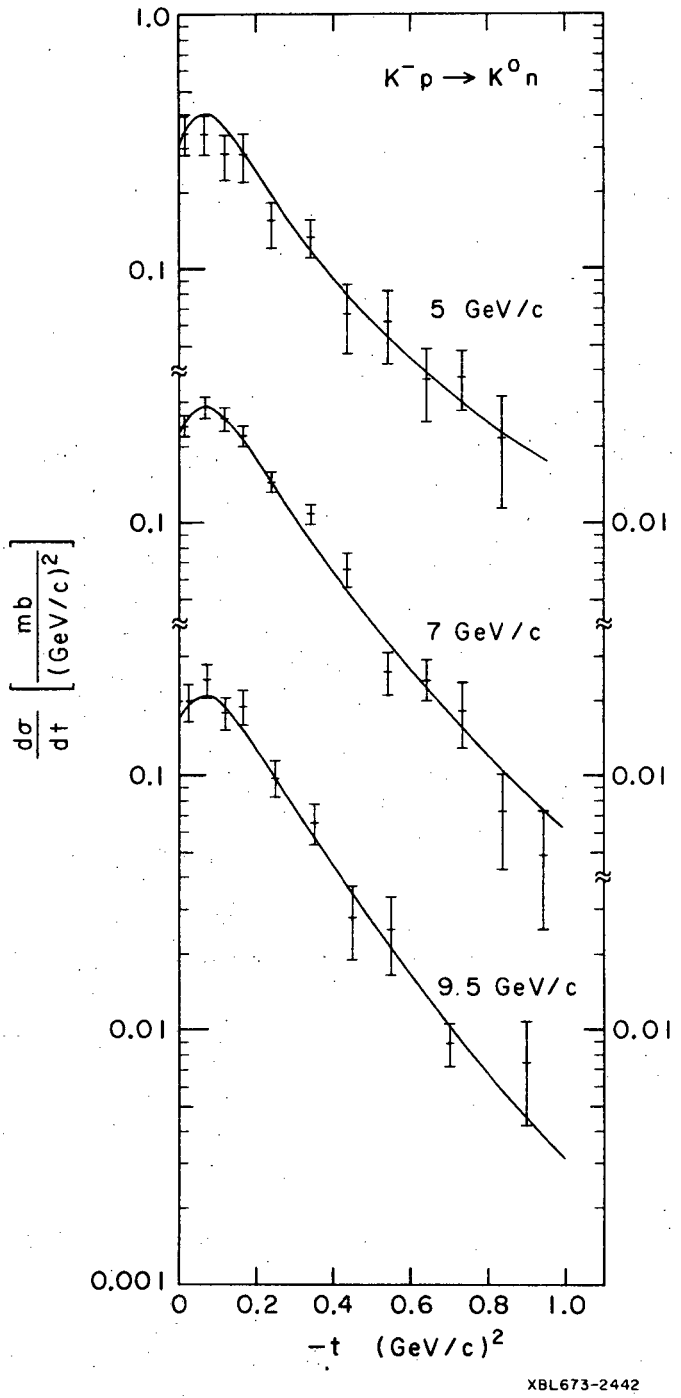

Fig. 9. K^-p charge-exchange differential cross sections, incoming lab momenta 5 to 9.5 GeV/c. Data of Astbury et al., CERN-ETH Zürich (Table I, Ref. b). Solid curves are our Regge fits, Table II parameters.

$$\mathrm{Im}A_{\rho'}/\mathrm{Im}A_{\rho} \approx e^{3.7t}/E^{\Delta\alpha}$$

$$\mathrm{Im}B_{\rho'}/\mathrm{Im}B_{\rho} \approx 3e^{0.3t}/E^{\Delta\alpha}$$

$$\mathrm{Re}A_{\rho} \approx \mathrm{Im}A_{\rho};\ \mathrm{Re}A_{\rho'} \approx -\mathrm{Im}A_{\rho'}$$

where E is in GeV, and $\Delta\alpha \equiv \alpha_{\rho} - \alpha_{\rho'}$.

For the high-energy cross-section data we see that the ρ' plays only a small role. In Fig. 3 we have the usual dip in the $\pi^- p$ charge-exchange cross section at $t \approx -0.6$, independent of energy, due to $\alpha_{\rho} = 0$. Our "ghost killing and sense choosing" mechanism predicts an analogous dip in $\pi^- p \to \eta^0 n$ for $\alpha_R(t \approx -0.9) = 0$. Figure 5 shows that our data do not extend to sufficiently large momentum transfer to serve as evidence in this matter. The "nonsense choosing" mechanism of Gell-Mann would not require such a dip for even-signature trajectories.[6] Note that our η production data measure only etas decaying to two photons. Therefore, to arrive at the SU_3 breaking parameter F_R^0, one needs to know the branching ratio for $\eta \to 2\gamma$. Using[11] $B(\eta \to 2\gamma) = 0.303$ we find F_R^0 differing from the unbroken F^+ for the A_2 by only 1%. For ρ and ρ' we have ten and twenty percent SU_3 breaking in F_i^0. In each case the symmetry breaking is slightly greater for $t \neq 0$ due to the nonvanishing F_i^1. The zero of $C_{\rho}(t)$ occurs at $t = -0.34$, roughly the first inflection point in the $\pi^- p$ charge-exchange cross sections.

In the K^+N system at 2.3 GeV/c (Figs. 6 and 7), we see the ρ' asserting itself. In the forward direction ${}_{\kappa}A_{\rho}$ and ${}_{\kappa}A_R$ are, roughly speaking, equal, and lie at $\pi/4$ and $3\pi/4$ in the complex plane. Thus their imaginary parts subtract and real parts add for K^+ charge exchange (vice versa for K^-). The ρ' does not significantly alter the traditional

result that the forward $K^+(K^-)$ amplitude is predominantly real (imaginary). We find $\mathrm{Re/Im}\ f_0(K^+) = +7.7$. But when we include the $\Delta\sigma_T(K^+N)$ datum at 2.3 GeV/c and apply the optical theorem (Eq. 14), we see the ρ' at work. At high energies the cancellation of $\mathrm{Im}({}_\kappa A_R - {}_\kappa A_\rho)_{t=0}$ results in $\Delta\sigma_T(K^+N) \approx 0$, as seen in Fig. 7. This trend would continue down to 2.3 but for the ρ' term, $\sim -1/E$, which emerges at lower energies and causes $\Delta\sigma_T$ to increase negatively. This is required by the datum at 2.3. The error bar here is given by the amplitude of the small wiggles (presumably not a t-channel Regge effect) in the $\Delta\sigma_T(K^+N)$ data in this region.[12] The quoted experimental error is considerably smaller.

To see the contribution of the flip amplitudes to $d\sigma/dt$ (Eq. 13), we write for small t.

$$\frac{\left\{\frac{-t}{4M^2}\left[\frac{s+p^2}{1-(t/4M^2)}\right]-s\right\}^{1/2} B_i}{(1-t/4M^2)^{1/2}\ A_i} \approx \sqrt{-t}\ \frac{D_i^0}{C_i^0}\ \frac{\alpha_i^0}{2}\ \exp[D_i^1 - C_i^1 + (\alpha'/\alpha^0)_i]t.$$

For each trajectory, the t-channel helicity flip contribution becomes comparable to the nonflip at $-t \lesssim 0.1\ (\mathrm{GeV})^2$. This rapid rise from zero in the forward direction is responsible for the initial rise in each of the different cross sections. In the case of K^+n charge exchange at 2.3 (GeV/c), ρ' adds significantly and positively to the flip amplitude, producing a considerable initial rise and increasing the cross section in the peak region by about a factor of two. Without the ρ', the theoretical curve in this region had stubbornly remained a factor of two below the data.

Figure 6 also shows the improvement in the fit at larger angles (smaller $-\cos\theta_t$) due to the use of the exact Legendre functions rather

than the high-energy asymptotic expressions at this energy. Near the forward direction, where the deuteron effect is significant, the free neutron cross section is also shown. Without knowing R from the fitted Regge parameters, one could not say to what extent the forward dip in the data reflects the two-body cross section rather than the forward vanishing required by the deuteron effect.

For the $\pi^- p$ charge-exchange polarization (Fig. 8), the ρ' is of course, in our model, the <u>sine qua non</u>. For $-t < 0.34$, where our $C_\rho(t)$ goes through zero, we have

$$_{\pi}A^- \times {}_{\pi}B^- = \sin\frac{\pi}{2}(\alpha_\rho - \alpha_{\rho'})(|{}_{\pi}A_\rho||{}_{\pi}B_{\rho'}| - |{}_{\pi}A_{\rho'}||{}_{\pi}B_\rho|).$$

The data require, and we find, $A_\rho B_{\rho'} > A_{\rho'} B_\rho$, giving positive polarization. The approximate orthogonality of the ρ and ρ' amplitudes for small t gives roughly maximal polarization, given the magnitudes and signs of the amplitudes. If the polarization is in fact due to a ρ' trajectory lying about one unit of angular momentum below the ρ, we have perforce a $\sim 1/E$ falloff in the polarization. The data, being quite uncertain, are consistent with this, but are also consistent with an energy-independent polarization. Better polarization data would constitute a severe test of our model. The unsightly high shoulder in the theoretical curves at the smallest t data emerged upon imposition of the sum rule constraint. Previously the two data around $-t = 0.03$ had been better fit. The good fit for $\Delta\sigma_T(\pi p)$, shown in Fig. 4, guarantees the convergence of the sum rule integral (Eq. 30) above 5 GeV/c. The ρ' term turns out to make 1/15th the contribution of the ρ term to this integral.

Finally, in Fig. 10 are shown some higher energy K^+n charge-exchange cross-section predictions from our fitted parameters. As the ρ' contribution wanes with increasing energy, the forward turnover diminishes, but, as in the case of K^-p charge exchange (Fig. 9), some turnover persists at high energies. For comparison with experiment, the deuteron effect, which causes the observed cross section to vanish at $t = 0$, is also shown. Except for the 9.5 GeV/c prediction, where the difference has become negligible, exact Legendre functions were used here in place of the asymptotic forms.

POSTSCRIPT

Our Regge fit gives for the real part of the forward K^+n charge-exchange scattering amplitude at 2.3 GeV/c

$$k \operatorname{Re} f(t = 0) = -0.69.$$

Note that, unlike the imaginary part, this is not a directly observable quantity in deuterium. In a Phys. Rev. Letter just published [18, 801 (1967)], A. A. Carter, using forward K^+N dispersion relations, calculates for this quantity the value -0.70 ± 0.01.

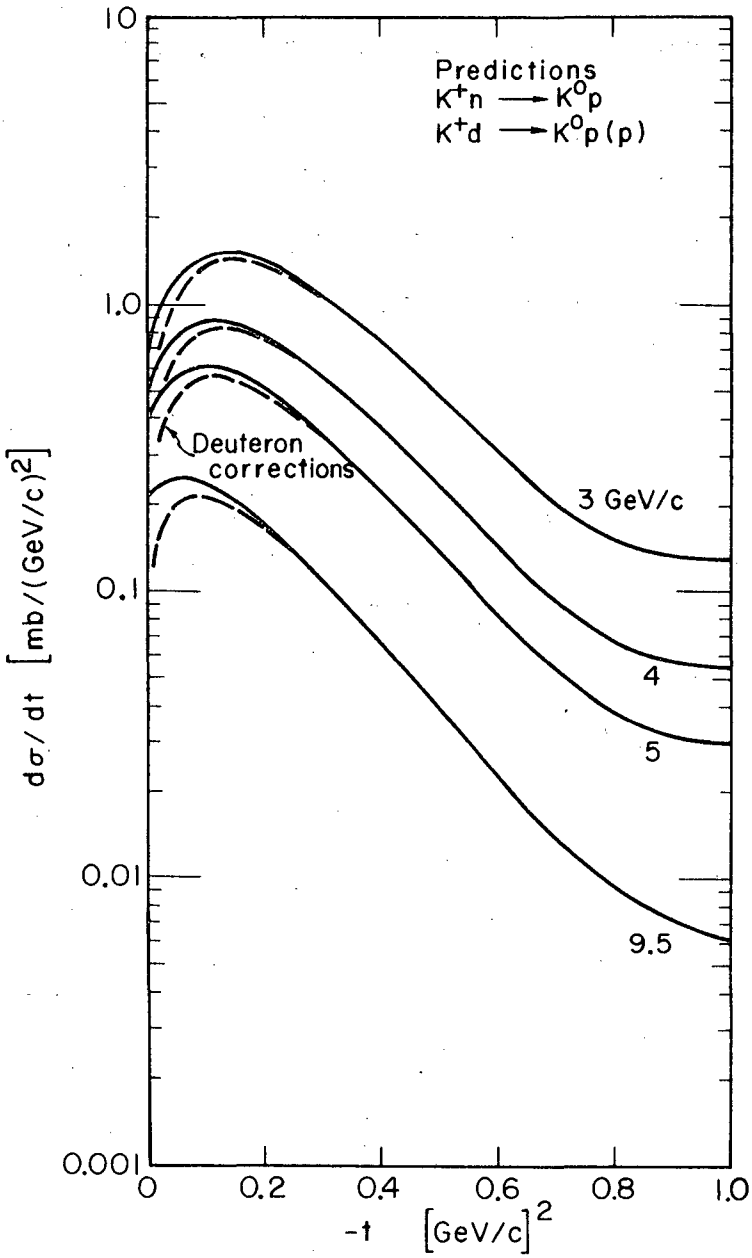

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Fig. 10. Predictions of higher energy K^+n charge exchange, from the fitted parameters, Table II. Up to 5 GeV/c we use here the exact Legendre functions. The dashed curves near $t = 0$ show the predicted $K^+d \rightarrow K^0p(p)$, i.e., the deuteron correction.

ACKNOWLEDGMENTS

I wish to thank Professor William Rarita for his close collaboration and indispensible guidance in the Regge theoretical analysis. My thanks also to Dr. Ian Butterworth and Mr. Allan Hirata, my collaborators in the preparation of the K^+n charge-exchange data. To Professor Gerson Goldhaber, who together with his late wife Shulamith gave guidance and forbearance in generous measure during my graduate studies, my special gratitude.

To the many incomparable persons and institutions of our beloved Berkeley, but for whom this work might have been finished sooner but with less joy in the making, and whose hopes for humanity I share, my thanks.

The encouragement of my mother and of my family in Berkeley, which served so well in loco parentis, I gratefully acknowledge.

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Footnotes and References

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APPENDICES

Appendix A

K^+n CHARGE-EXCHANGE SCATTERING AT 2.3 BeV/c*

Ian Butterworth, John L. Brown,† Gerson Goldhaber, Sulamith Goldhaber, Allan A. Hirata, John A. Kadyk, Bertram M. Schwarzschild, and George H. Trilling

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 20 September 1965)

We present here the differential cross section for the reaction $K^+ + n \to K^0 + p$ at 2.3 BeV/c. On comparing the differential scattering cross section in the forward direction with the value derived from the optical theorem, we find the experimental value to be considerably larger than the optical-theorem point. This indicates that the charge-exchange amplitude is predominantly real.

This study is based on an analysis of 297 events of the type

$$K^+ + d \to K^0 + p + p, \qquad (1)$$

with a visible K^0 decay. The events were obtained in 100 000 photographs taken with the Brookhaven National Laboratory 20-in. bubble chamber filled with deuterium and exposed to a K^+ beam at the AGS.[1] In this sample we

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find 53% of the events with two visible protons and 47% with only one visible proton. For the latter we fitted the events as one-constraint fits to Reaction (1).

In Reaction (1) the choice as to which of the two protons is the recoil proton and which the spectator has been made on the basis of their respective momenta. If the slower proton is chosen as the supposed spectator, it is found to have a momentum distribution which agrees well with that expected from the Hulthén wave function, provided that its momentum does not exceed 300 MeV/c. With the same momentum limitation, the angular distribution of the spectator in the laboratory is isotropic. We find that in 14% of the events, both protons have momenta greater than 300 MeV/c, whereas the expected number consistent with the Hulthén wave function is 1 to 2%. We attribute this discrepancy to double scattering in the deuteron. In the subsequent analysis we have included only events with spectator momenta below 300 MeV/c. There are 257 such events. All events were weighted according to the probability that a K^0 of the observed momentum decays within the chosen fiducial volume. Cross sections have been corrected to allow for K_2^0 and neutral K_1^0 decays. By this procedure we find the cross section for charge exchange to be 1.50 ± 0.15 mb.

Figure 1 shows the observed angular distribution of the K^0 from Reaction (1) in the laboratory system. Figure 2 shows (solid bars) the observed values of $d\sigma/d\Omega$ as a function of the K scattering angle Θ^* in the Kn center-of-mass system (or $\partial\sigma/\partial t$ as a function of momentum transfer, t).

If the charge-exchange scattering amplitude on a free neutron is given by $f = a + b(\hat{\sigma}\cdot\hat{n})$, where $\hat{n}$ is the unit vector perpendicular to the scattering plane, then the differential cross section for a neutron bound in a deuteron is given by

$$d\sigma/d\Omega = [\,|a|^2 + (\tfrac{2}{3})|b|^2][1 - H(q)] + \tfrac{1}{3}|b|^2[1 + H(q)]. \qquad (2)$$

Here the cross section is given in the K^+n center-of-mass system, $H = \int \Psi^*(\vec{r}) \exp(-i\vec{q}\cdot\vec{r}) \times \Psi(\vec{r}) d\vec{r}$ is a real and positive quantity, Ψ is the deuteron spatial wave function, and $\vec{q}$ is the difference between the initial and final K momenta in the laboratory system. Final-state interaction and double-scattering effects are ignored in this expression.[2]

Equation (2) may be rewritten as

$$d\sigma/d\Omega = [1 - H(q)](d\sigma/d\Omega)_{nf} + [1 - \tfrac{1}{3}H(q)](d\sigma/d\Omega)_f, \qquad (3)$$

where $(d\sigma/d\Omega)_{nf}$ and $(d\sigma/d\Omega)_f$ are the free-neutron cross sections for spin nonflip and spin flip, respectively.

For events produced with $\cos\Theta^* < 0.92$, we obtain $H(q) \leq 0.1$, and effects due to the deuteron are thus negligible. For the remaining events the value of $H(q)$ becomes significant, and corrections implied by Eq. (3) must be included. To apply these corrections, one must know the relative size of spin-flip and spin-nonflip contributions. The relative importance of these two terms is not known. However, since all

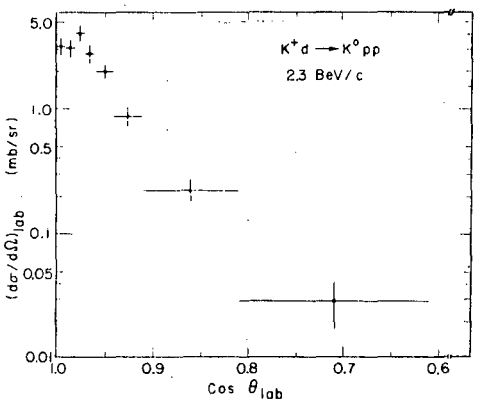

FIG. 1. Observed laboratory differential cross section for the reaction $K^+ + d \to K^0 + p + p$ as a function of the K^0 production angle.

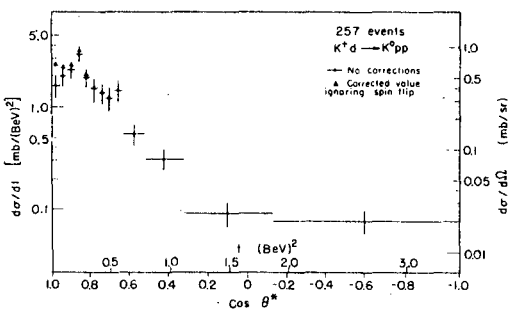

FIG. 2. Differential cross section for the reaction $K^+ + d \to K^0 + p + p$ as a function of the K^0 production angle in the K^+n center-of-mass system (or momentum transfer t). Solid bars show experimental data; triangles show the conversion from K^+d scattering to K^+n scattering if spin-flip contributions to the scattering are ignored.

corrections apply primarily to forward-scattering angles we have neglected the spin-flip term. The triangles in Fig. 2 show the result of correction where only the spin-nonflip term has been included.

We now compare the forward-scattering cross section with that derived from the optical theorem. In terms of isotopic-spin amplitudes, the amplitudes for K^+p scattering, K^+n scattering, and K^+n charge exchange (ce) are given, respectively, by

$$f(K^+ + p \to K^+ + p) = f_1, \quad f(K^+ + n \to K^+ + n) = \tfrac{1}{2}(f_1 + f_0),$$
$$\text{and } f_{ce} = \tfrac{1}{2}(f_1 - f_0);$$

hence

$$f_{ce} = f(K^+ + p \to K^+ + p) - f(K^+ + n \to K^+ + n).$$

From the optical theorem we thus obtain

$$(\mathrm{Im} f_{ce})_{t=0} = k/4\pi[\sigma_t(K^+p) - \sigma_t(K^+n)], \tag{4}$$

which yields the inequality

$$(d\sigma_{ce}/d\Omega)_{t=0} \geq \{k/4\pi[\sigma_t(K^+p) - \sigma_t(K^+n)]\}^2. \tag{5}$$

From the uncorrected data in Fig. 2, which is a lower limit to the K^+n charge-exchange cross section, we would predict a difference of ~5 mb between the K^+p and K^+n cross sections if Eq. (5) is taken to be an equality. The measured cross-section difference $\sigma_t(K^+p) - \sigma_t(K^+n)$ at this energy was given by Cook et al. as -0.6 ± 1.0 mb.[3] These two results are clearly incompatible, which implies that Eq. (5) must be considered as an inequality. Thus the real part of the forward K^+n charge-exchange amplitude, f_{ce}, must be considerably greater than the imaginary part. It is noteworthy that this is in contrast with high-energy K^-p charge exchange, which has a predominantly imaginary amplitude.[4] On the basis of a Regge-pole model of KN scattering invoking only ρ and A_2 trajectories, Phillips and Rarita have predicted that K^+n charge exchange should have a predominantly real amplitude and K^-p charge exchange a predominantly imaginary amplitude.[5] The possible validity of such a Regge approach at an energy as low as 2.3 BeV/c is supported by the fact that K^+n scattering is free of resonances in the direct channel, and that the only other trajectory that might have to be considered would be an $I, J^P = 1, 0^+$ exchange.

We are grateful to R. Shutt and his co-workers at the Brookhaven National Laboratory for making available the 20-in. bubble chamber, to the AGS crew, the 20-in. bubble chamber crew, and particularly to H. Brown for help with the exposure at the AGS. We thank the many people at Berkeley who helped with scanning, measuring, and computing. We would like to acknowledge helpful discussions with Professor G. Chew and Dr. W. Rarita.

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†Present address: Stanford Linear Accelerator Center, Stanford, California.

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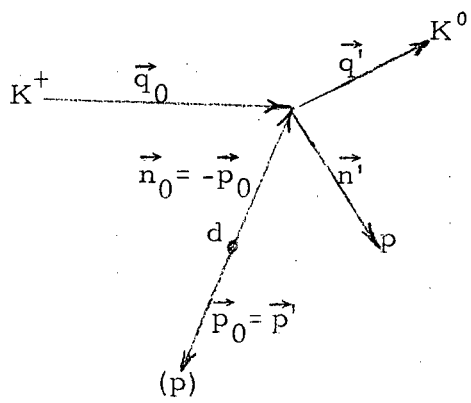
B. Deuteron Correction–Impulse and Closure Approximation

We will here derive the deuteron correction formula (Eq. 26). Consider the scattering matrix for the process $K^+d \to K^0p(p)$, where the slower (spectator) proton (p) has lab momentum < 300 MeV.

$$T = i(1 - S) \equiv \delta^4(P' - P_0)r \tag{B1}$$

where $P = (E, \vec{P})$ is the total four momentum, and sub(super)script zero (prime) will generally denote initial (final) momenta. The diagram below defines the several lab momenta in the spirit of the impulse approximation.



Lab momenta

$K^+d \to K\ p(p)$

We define $\vec{k} = 1/2(\vec{p} - \vec{n})$ and $\vec{K} = (\vec{p} + \vec{n})$, and denote the spin state of the two-nucleon system by X_s^m.

For convenience we consider first only the singlet final state $|f\rangle = |\vec{k}', \vec{q}', \vec{n}', X_0^0\rangle$. Expanding the initial state in plane waves of the relative momentum $\vec{k}$ inside the deuteron, we have

$$|i\rangle = |\vec{q}_0, D, \vec{K}_0 = 0, X_1^m\rangle = \sum_{\vec{k}} |\vec{q}_0, \vec{k}, \vec{n} = -\vec{k}, X_1^m\rangle\langle\vec{k}|D\rangle. \tag{B2}$$

We ignore the d wave in the deuteron wave function D, thus making

the inner product $\langle \vec{k} | D \rangle$ independent of the spin orientation m. We take the normalization

$$\sum_{\text{all } \vec{k}} |\vec{k}\rangle\langle\vec{k}| = I .$$

Averaging over the initial spin states, we have for the cross section

$$\sigma_s = \frac{1}{v_0} \int \delta(E' - E_0) \frac{1}{3} \sum_m |r|^2 \delta^3(\vec{k}' + \vec{n}' - \vec{\Delta}) d^3\vec{k}' d^3\vec{n}' d^3\vec{q}' , \quad \text{(B3)}$$

where $\vec{\Delta} = 1/2(\vec{q}_0 - \vec{q}')$, and v_0 is the initial lab velocity. Note that

$$d^3\vec{p}' \delta^3(\vec{P}' - \vec{P}_0) = \delta^3(\vec{p}' + \vec{n}' - 2\vec{\Delta}) d^3\vec{p}' = \delta^3(\vec{k}' + \vec{n}' - \vec{\Delta}) d^3\vec{k}' .$$

From Eq. (B2) we have

$$\langle f | r | i \rangle = \sum_{\vec{k}} \langle \vec{k}', \vec{q}', \vec{n}', X_0^0 | r | \vec{q}_0, \vec{k}, \vec{n} = -\vec{k}, X_1^m \rangle \langle \vec{k} | D \rangle . \quad \text{(B4)}$$

Now we make the approximation

$$\langle f | r | i \rangle = \sum_{\vec{k}} \langle \vec{q}', \vec{n}', X_0^0 | r_n | \vec{q}_0, \vec{n} = -\vec{k}, X_1^m \rangle \delta^3[\vec{k} - (\vec{k}' + \vec{\Delta})] \langle \vec{k} | D \rangle , \quad \text{(B5)}$$

where $r_n \delta^3[\vec{k}_0 - (\vec{k}' + \vec{\Delta})]$ describes K^+ charge exchange on a free neutron (the propagator of the diagram above, taken to be on the mass shell) with a delta function conserving momentum at the charge-exchange vertex. This will be defined more clearly later by relating r_n to $\sigma(K^+ n \to K^0 p)$. Integrating over all $\vec{k}$ with this delta function gives

$$\langle f | r | i \rangle = \langle \vec{q}', \vec{n}', X_0^0 | r_n | \vec{q}_0, \vec{n}_0 = -(\vec{k}' + \vec{\Delta}), X_1^m \rangle \langle \vec{k}' + \vec{\Delta} | D \rangle . \quad \text{(B6)}$$

This singlet final spin state X_0^0 being antisymmetric with respect to exchange of the two protons, Fermi statistics require that we symmetrize the amplitude to read

$$\langle f|r|i\rangle = \langle \vec{q}', \vec{n}', X_0^0 | r_n | \vec{q}_0, \vec{n}_0 = -(\vec{k}' + \vec{\Delta}), X_1^m \rangle \sqrt{\tfrac{1}{2}} (\langle \vec{k}' + \vec{\Delta}| + \langle -\vec{k}' + \vec{\Delta}|)D\rangle. \quad \text{(B7)}$$

Writing r_n in terms of spin flip and nonflip operators, $r_n = r_1 + \vec{\sigma} \cdot \hat{n} r_2$, we have the general theorem

$$\sum_m |\langle \vec{q}', \vec{n}', X_0^0 | r_1 + \vec{\sigma} \cdot \hat{n} r_2 | \vec{q}_0, \vec{n}_0, X_1^m \rangle|^2 = |\langle \vec{q}', \vec{n}' | r_2 | \vec{q}_0, \vec{n}_0 \rangle|^2. \quad \text{(B8)}$$

Therefore, for the singlet final state the cross section is

$$\sigma_s = \frac{1}{3v_0} \int \delta[E'(q', n', p' = |2\vec{k}' + \vec{n}'|) - E_0] \delta^3(\vec{k}' + \vec{n}' - \vec{\Delta}) \quad \text{(B9)}$$

$$\times |\langle \vec{q}', \vec{n}' | r_2 | \vec{q}_0, \vec{n}_0 = -(\vec{k}' + \vec{\Delta})\rangle|^2 \tfrac{1}{2} |(\langle \vec{k}' + \vec{\Delta}| + \langle -\vec{k}' + \vec{\Delta}|)D\rangle|^2 d^3\vec{n}' d^3\vec{k}' d^3\vec{q}'.$$

Integrating over the final target nucleon momentum $\vec{n}'$, consuming the total momentum delta function, we get for the lab distribution

$$\frac{d\sigma_s}{d\Omega}(\hat{q}') = \frac{1}{3v_0} \int \delta[E'(q', n' = |\vec{\Delta} - \vec{k}'|, p = |\vec{\Delta} + \vec{k}'|) - E_0] \tfrac{1}{2} |(\langle \vec{k}' + \vec{\Delta}| + \langle -\vec{k}' + \vec{\Delta}|)D\rangle|^2 \quad \text{(B10)}$$

$$\times |\langle \vec{q}', \vec{n}' = \vec{\Delta} - \vec{k}' | r_2 | \vec{q}_0, \vec{n}_0 = -(\vec{k}' + \vec{\Delta})\rangle|^2 d^3\vec{k}' q'^2 dq',$$

where $\hat{q}'$ is the unit vector for the K^0 lab momentum, with differential solid angle $d\Omega$.

One now argues that setting $\vec{k}' = -\vec{\Delta}$ (i.e., $n_0 = p_0 = p' = 0$) in $E'(\ )$ and $\langle |r_2| \rangle$ approximates the integral (B10) well, thus achieving the desired result of suppressing their $\vec{k}'$ dependence. We then have

$$\frac{d\sigma_s}{d\Omega}(\hat{q}') = \frac{1}{3v_0}\int q'^2\,\delta[E'(q', n' \approx 2\Delta, p' \approx 0) - E_0]\,|\langle \vec{q}', \vec{n}' \approx 2\vec{\Delta}|\, r_2\, |\vec{q}_0, n_0 \approx 0\rangle|^2$$

$$\times\left[\tfrac{1}{2}\int d^3\vec{k}'\,|(\langle \vec{k}' + \vec{\Delta}| + \langle -\vec{k}' + \vec{\Delta}|)D\rangle|^2\right] dq'. \tag{B11}$$

This facilitates matters greatly. Representing the inner product $\langle\,|D\rangle$ in configuration space, we have

$$\tfrac{1}{2}\int d^3\vec{k}'\,||^2 = \tfrac{1}{2}\,\frac{1}{(2\pi)^3}\int d^3\vec{k}'\,d^3\vec{x}\,d^3\vec{y}\,\{\exp[-i(\vec{k}' + \vec{\Delta})\cdot\vec{x}] + \exp[-i(-\vec{k}' + \vec{\Delta})\cdot\vec{x}]\}\Psi_D(\vec{x})$$

$$\times\{\exp[i(\vec{k}' + \vec{\Delta})\cdot\vec{y}] + \exp[i(-\vec{k}' + \vec{\Delta})\cdot\vec{y}]\}\Psi_D^*(\vec{y}) \tag{B12}$$

$$= \tfrac{1}{2}\,\frac{1}{(2\pi)^3}\int d^3\vec{x}\,d^3\vec{y}\,\Psi_D(\vec{x})\Psi_D^*(\vec{y})\,e^{i\vec{\Delta}\cdot(\vec{y}-\vec{x})}\int d^3\vec{k}'\begin{cases}\left.\begin{matrix}\exp[i\vec{k}'\cdot(\vec{y}-\vec{x})]\\ +\exp[i\vec{k}'\cdot(\vec{x}-\vec{y})]\end{matrix}\right\} \to 2(2\pi)^3\,\delta^3(\vec{x}-\vec{y})\\ \left.\begin{matrix}+\exp[-i\vec{k}'\cdot(\vec{x}+\vec{y})]\\ +\exp[i\vec{k}'\cdot(\vec{x}+\vec{y})]\end{matrix}\right\} \to 2(2\pi)^3\,\delta^3(\vec{x}+\vec{y})\end{cases}$$

$$= \int d^3\vec{x}\,[|\Psi_D(\vec{x})|^2 + \Psi_D(\vec{x})\Psi_D^*(-\vec{x})\exp(-2i\vec{\Delta}\cdot\vec{x})] \tag{B14}$$

$$= 1 + \int d^3\vec{x}\,|\Psi_D(\vec{x})|^2\exp(-2i\vec{\Delta}\cdot\vec{x}) \equiv 1 + H(\Delta). \tag{B15}$$

Therefore

$$\frac{d\sigma_s}{d\Omega}(\hat{q}') = \frac{1}{3v_0}\int q'^2 dq'\Big(\delta[E_0 - E'(q', n' = 2\Delta, p' = 0)]$$

$$\times\ |\langle \vec{q}', \vec{n}' = 2\vec{\Delta}|\, r_2\, |\vec{q}_0, n_0 = 0\rangle|^2[1 + H(\Delta)]\Big).$$

Invoking the delta function, we pull $(1+H)$ outside the integral and get finally

$$\frac{d\sigma_s}{d\Omega}(\hat{q}') = \frac{1}{3v_0}\{1+H[\Delta^*(\hat{q}')]\}\int q'^2 dq' \Big(\delta[E_0 - E'(q', n'=2\Delta, p'=0)] \times |\langle \vec{q}', \vec{n}' = 2\vec{\Delta}|r_2|\vec{q}_0, n_0 = 0\rangle|^2\Big) \tag{B17}$$

where

$$\Delta^*(\hat{q}') = |\tfrac{1}{2}(\vec{q}_0 - \hat{q}' q^*)| \quad \text{and} \quad \sqrt{q^{*2}+m^2} + \sqrt{(2\Delta^*)^2+M^2} + M = E_0. \tag{B18}$$

To get the triplet cross section σ_T, we antisymmetrize $\langle \vec{k}' + \vec{\Delta}|D\rangle$ and note the theorem corresponding to (B8), namely

$$\frac{1}{3}\sum_{mn} |\langle \vec{q}', \vec{n}', X_1^n | r_1 + \vec{\sigma}\cdot\hat{n} r_2 | \vec{q}_0, \vec{n}_0, X_1^m\rangle|^2$$
$$= |\langle \vec{q}', \vec{n}' | r_1 | \vec{q}_0, \vec{n}_0\rangle|^2 + \frac{2}{3}|\langle \vec{q}', \vec{n}' | r_2 | \vec{q}_0, \vec{n}_0\rangle|^2.$$

Then the previous arguments give the correspondent of (B17)

$$\frac{d\sigma_T}{d\Omega} = \frac{(1-H)}{v_0}\int q'^2 dq' \delta(E_0 - E')\left(|\langle |r_1|\rangle|^2 + \frac{2}{3}|\langle |r_2|\rangle|^2\right). \tag{B20}$$

Consider now K^+ charge exchange on a free neutron, stationary in the lab, with $\vec{q}_0$ and $\hat{q}'$ the same as for the deuterium scattering under consideration. The lab distribution is given by

$$\left(\frac{d\sigma}{d\Omega}\frac{\text{spinflip}}{\text{nonflip}}(\hat{q}')\right)_{\text{neutron}}$$

$$= \frac{1}{v_0}\int \delta[\mathcal{E}'(q',n') - \mathcal{E}_0]\,\delta^3(\vec{q}'+\vec{n}'-\vec{q}_0)\,|\langle \vec{q}',\vec{n}'|r_{\substack{2\\1}}|\vec{q}_0, n_0=0\rangle|^2 q'^2 dq' d^3\vec{n}'$$

$$= \frac{1}{v_0}\int \delta[\mathcal{E}'(q',n'=2\Delta) - \mathcal{E}_0]\,|\langle \vec{q}',\vec{n}'=2\vec{\Delta}|r_{\substack{2\\1}}|\vec{q}_0, n_0=0\rangle|^2 q'^2 dq'. \qquad \text{(B21)}$$

Now the energy delta function in (B21) differs from those in (B17, B20) only by the deuteron binding energy, which we are ignoring. Summing over final spin states for the deuterium cross section, we write

$$\left(\frac{d\sigma}{d\Omega}(\hat{q}')\right)_{\text{deut}} = \frac{d\sigma}{d\Omega}S(\hat{q}') + \frac{d\sigma}{d\Omega}T(\hat{q}')$$

which from (B17, B20, B21) becomes

$$\left(\frac{d\sigma}{d\Omega}(\hat{q}')\right)_d = \frac{1}{3}[1+H(\Delta^*)]\left(\frac{d\sigma}{d\Omega}\,\text{sf}(\hat{q}')\right)_n + [1-H(\Delta^*)]\left[\left(\frac{d\sigma}{d\Omega}\,\text{nf}(\hat{q}')\right)_n + \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\,\text{sf}(\hat{q}')\right)_n\right].$$

Finally, therefore, if t for each event is taken to be the four-momentum transfer corresponding to the observed K-meson lab scattering angle imagined to be off a free neutron at rest in the lab (cf. B18), we have

$$\left(\frac{d\sigma}{dt}\right)_d = \left(\frac{d\sigma}{dt}\right)_n \frac{1 - H(t) + R(t)[1 - H(t)/3]}{1 + R(t)} \qquad \text{(B26)}$$

Q. E. D.

The vanishing of $(d\sigma/dt)_d$ at $t = 0$ may be understood in terms of Fermi statistics. In the impulse approximation, forward scattering leaves the two nucleons kinematically undisturbed (although in fact they must be nudged onto the mass shell) except that the isotopic states goes from odd to even. This is clearly forbidden.

תושלב"ל

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