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Abstract

Predicted values of the branching ratio $R = \Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ anywhere between 1.6 and 0.7 are perfectly consistent with the Dalitz plot of 640 background-free η decays $\eta \to \pi^+\pi^-\pi^0$. These events were analyzed under the assumption of pure T = 1 final states with a complex matrix element that includes terms cubic in pion energies. To obtain low values of R final-state interactions are needed. T=3 is not needed.

In the decays

$$\eta \rightarrow \pi^+\pi^-\pi^0 \tag{1}$$

and

$$\eta \rightarrow 3\pi^0 \tag{2}$$

the three pions must have isotopic spin T = 1 or 3. (We assume C invariance.) If the decay is electromagnetic and if the photon can carry only isospin 0 or 1, then the T=3 amplitude carries one more power of α than the T=1 amplitude. In that case T=1 may dominate.[1]

On the basis of the similarity between the τ, τ', and η Dalitz plots,[2] it has long been conjectured[3] that the three pions are mostly in the totally symmetric T=1 state, with a small admixture of the other two (nonsymmetric) T=1 states. For the totally symmetric T=1 state the branching ratio

$$R \equiv \Gamma(\eta\rightarrow 3\pi^0)/\Gamma(\eta\rightarrow\pi^+\pi^-\pi^0)$$

is given by R = (1.15)(3/2) = 1.7.

The experimental Dalitz plot for Reaction (1) is not totally symmetric. However, it is well fitted by a real linear matrix element[4]

$$M(+-0) = 1 + b\, y_3, \tag{3}$$

where $y_3 = (3T_0/Q) - 1$, and b is real. The fit gives typically b = -0.45±0.05, which predicts R = 1.63±0.03.[5,6]

Two published direct measurements of R give the results R = 0.83±0.32 (Ref. 7) and R = 0.90±0.24 (Ref. 6), under the assumption that there are no decays $\eta\rightarrow\pi^0\gamma\gamma$. The existence of $\eta\rightarrow\pi^0\gamma\gamma$ (Ref. 8) would give corrections which would reduce these two results to values as low as 0.5.[9] Other experiments have been combined[10] to give the indirect

result $R = 0.93 \pm 0.16$. Thus several experiments are in poor agreement with the value of R predicted by the real linear matrix element model with T=1.[11] If one fits the Dalitz plot to other models having T=1, such as that involving the hypothetical σ meson,[12] or involving ρ dominance,[13] one can obtain predicted values of R as low as about 1.2.[5,6]

By including contributions from both ρ and σ one can reach predicted values of R as low as unity.[14] However, the existence of the σ meson is not experimentally established.

The poor agreement has led Veltman and Yellin[15] and Woo[16] to conclude that T=3 must be present, since, as is well known,[1] the (totally symmetric) T=3 amplitude can interfere with the totally symmetric part of the T = 1 amplitude to give R as small as one pleases, for example between 0.5 and 1.0. The possible existence of a large T=3 amplitude has led Adler[17] to suggest that there may be an isotensor part to the electromagnetic interaction.

What we wish to demonstrate is that, <u>assuming only T=1</u>, we can find excellent fits to the Dalitz plot of a sample of 640 η decays of Type (1), giving <u>predicted values of R anywhere between 1.6 and 0.7</u>. We emphasize that those fits that correspond to predicted low values of R are <u>allowed</u>, not demanded, by the data. (We also find good fits with predictions as high as R=1.6. For example, the real linear matrix element gives a good fit.)

We assume that only T=1 is present, and expand the matrix elements for Reactions (1) and (2) through third order in y_1, y_2, and y_3, where $y_i = (3T_i/Q) - 1$. The most general C-invariant matrix elements are then[18]

$$M(+-0) = 1 + by_3 + cy_3^2 + dy_1y_2 + ey_3^3 + fy_1y_2y_3 \tag{4}$$

and

$$M(000) = 6^{-\frac{1}{2}}[3+(c-\tfrac{1}{2}d)(y_1^2+y_2^2+y_3^2)+3(e+f)y_1y_2y_3]. \tag{5}$$

All parameters are complex. Thus there are ten real parameters.

We fit the matrix element M(+-0) to the Dalitz plot of 640 low-background η decays of Type (1), produced by pions of order 1 BeV/c in the reactions

$$\pi^{\pm}p \rightarrow \pi^{\pm}p\eta.$$

Of these events 488 were produced in the Alvarez 72-inch hydrogen bubble chamber,[5, 6] and 152 were produced in the Brookhaven National Laboratory 20-inch hydrogen bubble chamber.[19] The non-η background is less than 3%. The decays $\eta \rightarrow \pi^+\pi^-\gamma$ have been removed. The Dalitz plot is given in Ref. 20.

In fitting these events to M(+-0) we find that d and f are small and consistent with zero. Therefore we revise our model and set these four real parameters identically equal to zero. Then M(+-0) depends only upon y_3. We therefore divide the sample into 20 equal bins in y_3.[21]

The predicted value of R turns out to be sensitive to the real part of c, Re(c), whereas it is relatively insensitive to the other parameters.[22] In Fig. 1 we plot R and χ^2 versus Re(c), where χ^2 corresponds to the best fit to the Dalitz plot that can be obtained by varying all parameters except Re(c). The horizontal line represents the "expected" χ^2 of 13. All values of Re(c) for which χ^2 is below the line correspond to excellent fits to the data. We see that <u>any value of R from 1.6 to 0.7 corresponds to an excellent fit</u>.

In Fig. 2 we show the π^0 energy spectrum and the fitted curves corresponding to predicted values R = 1.6 and 0.7 for our complex cubic matrix element model.

Similarly, if we use a complex quadratic matrix element model we obtain excellent fits with predicted values of R anywhere between 1.6 and 1.1. For a complex linear matrix element model we obtain excellent fits for R between 1.6 and 1.4.

In Table I we give best-fit (minimum χ^2) parameters for complex linear, quadratic, and cubic matrix element models, and also the parameters for the complex cubic model that give R = 0.7. Notice that in order to obtain a small predicted value of R the imaginary parts of the parameters must be comparable in magnitude to the real parts. In fact we find that, using a real cubic matrix element, we cannot obtain any good fit that gives a predicted value of R less than 1.5. Thus the imaginary parts are needed to obtain predicted values of R less than 1.5. That in turn implies that final-state interactions are important[23] to obtain R<1.5. Just how important cannot be told directly from our parameterization.

We now discuss a matrix element more general than that given by Eq. (4), namely, the matrix element obtained by multiplying M(+-0) of Eq. (4) by a phase factor. Then, for example, Eq. (3) becomes

$$M'(+-0) = M(+-0)e^{i\phi_3} = (1+by_3)e^{i\phi_3} . \qquad (6)$$

This matrix element was introduced by Foster et al.[24], who fitted ϕ to the branching ratio R. Notice that this phase factor can have no effect on the $\pi^+\pi^-\pi^0$ Dalitz plot, since

$$|M'(+-0)|^2 = |M(+-0)|^2 .$$

Thus if such a phase exists, it is impossible to predict R from the $\pi^+\pi^-\pi^0$ Dalitz plot alone.[25]

The simplest nontrivial[25] expansion of ϕ is[24]

$$\phi_i = \gamma\, y_i .$$

Symmetrizing Eq. (6), we have

$$M'(000) = 6^{-\frac{1}{2}}[(1+by_3)e^{i\phi_3} + (1+by_2)e^{i\phi_2} + (1+by_1)e^{i\phi_1}]. \qquad (7)$$

To illustrate the dependence of R upon γ, we note that if $b=0$, then as γ varies from 0 to ∞, R goes from $1.5\times1.15 = 1.7$ down to $0.5\times1.15 = 0.57$. If $b\neq 0$, these limits are of course different. However, it is clear that R is strongly dependent upon γ.

As for its physical meaning, we notice that since ϕ does not affect the $\pi^+\pi^-\pi^0$ Dalitz plot, it can have nothing to do with final-state interactions. Therefore, ϕ must arise from the "intrinsic η decay" process itself. The angle ϕ corresponds to changing the relative amounts of the symmetric and nonsymmetric states in a manner that changes the branching ratio without changing the $\pi^+\pi^-\pi^0$ Dalitz plot.

A Dalitz plot for $\eta\to3\pi^0$ will be necessary in order to answer the questions:

(a) Are there quadratic and cubic terms?

(b) Is there a phase factor, ϕ?

(c) Is there some T=3?

One way to answer the last question is to measure R for only those events lying very close to the center of the Dalitz plots. Call this R_0. At the very center, $R_0 = 3/2$ for the T=1 state <u>regardless</u> of the amount of nonsymmetric T=1 state present, and regardless of the energy dependence of the T=1 matrix element. Similarly, for T=3, $R_0 = 2/3$

regardless of energy dependence of the matrix element. Now, the T = 1, and T=3 states must be relatively real (in the absence of final-state interactions).[23] In that case the branching ratio at the center of the Dalitz plots gives uniquely the relative amounts of T=1 and T=3. Even if there are final-state interactions, any deviation from $R_0 = 3/2$ will show that T=3 is present.

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The Dalitz plot follows: angular zone, radial zone: counts; 1, 3:8; 1, 2:11; 1, 1:21; 2, 3:12; 2, 2:12; 2, 1:22; 3, 3:12; 3, 2:14; 3, 1:20; 4, 3:13; 4, 2:16; 4, 1:29; 5, 3:22; 5, 2:18; 5, 1:26; 6, 3:17; 6, 2:21; 6, 1:28; 7, 3:23; 7, 2:24; 7, 1:38; 8, 3:32; 8, 2:43; 8, 1:30; 9, 3:42; 9, 2:40; 9, 1:46.

21. Sample of 640 events divided into 20 equal bins in y_3. Increasing bin number corresponds to increasing y_3. We have bin: counts; 1:16; 2:26; 3:36; 4:43; 5:52; 6:43; 7:45; 8:51; 9:41; 10:40; 11:40; 12:42; 13:30; 14:25; 15:32; 16:18; 17:21; 18:19; 19:14; 20:6. This spectrum is in good agreement with the entire sample of events given in Table VIII, Ref. 20.

22. This is understood qualitatively as follows: Square the matrix elements given by Eqs. (4) and (5). Average them over the Dalitz plot. Neglect the averages of all terms except the constant and quadratic terms. Set $\langle y_i^2 \rangle = 1/4$, its nonrelativistic value. Then $R \approx (3/2)[1 + 1/2\ \mathrm{Re}(c)]/[1 + 1/2\ \mathrm{Re}(c) + 1/4|b|^2]$. We see that $R \approx 0$ for $\mathrm{Re}(c) \approx -2$. When the terms neglected here are included we get the results shown in Fig. 1 for this sample of 640 decays.

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25. If ϕ were a completely symmetric function of y_1, y_2, and y_3, it would also factor out of $M'(000)$, and thus would have no observable effects.

Table I. The best-fit parameters for our first-, second-, and third-order fits and also the parameters for the complex cubic model that give $\chi^2 = \langle \chi^2 \rangle$. All errors correspond to increasing χ^2 by 1. We arbitrarily take Im(b) to be positive.

Parameter	Order of fit: First	Second	Third	Third order at $\chi^2 = \langle \chi^2 \rangle$ (a)
$\langle \chi^2 \rangle$	17	15	13	13
χ^2	8.9	7.9	7.4	13.0
R	1.63	1.55	1.61	.72
Re(b)	$-0.43^{+0.04}_{-0.05}$	$-0.50^{+0.09}_{-0.09}$	$-0.55^{+0.13}_{-0.13}$	-0.52
Im(b)	$0.20^{+0.27}_{-0.27}$	$0.57^{+0.40}_{-0.60}$	$0.4^{+1.5}_{-1.2}$	2.56
Re(c)	---	$-0.20^{+0.40}_{-0.30}$	$0.03^{+0.10}_{-0.55}$	-2.53
Im(c)	---	$0.15^{+0.50}_{-0.75}$	$-0.03^{+0.7}_{-0.7}$	-0.68
Re(e)	---	---	$0.32^{+0.25}_{-0.25}$	0.35
Im(e)	---	---	$-0.62^{+1.3}_{-0.6}$	-2.49

(a). In the vicinity of Re(c) = 2.5 (see Fig. 1).

Figure Captions

Fig. 1. Plot of the branching ratio, R (solid line), and χ^2 for the fit to the $\pi^+\pi^-\pi^0$ Dalitz plot (dashed line) vs the parameter Re(c) for the third-order fit. The horizontal line is the "expected" χ^2, $\langle\chi^2\rangle$.

Fig. 2. Energy distribution of π^0 in $\eta \to \pi^+\pi^-\pi^0$. The 640 events are divided into 20 equal bins in T_{π^0}. Increasing bin number corresponds to increasing T_{π^0}. The dotted curve is our best fit to the third-order matrix element, which gives R = 1.6. The solid curve is our third-order fit with χ^2 = "expected" χ^2 (see Table I), which gives R = 0.7.

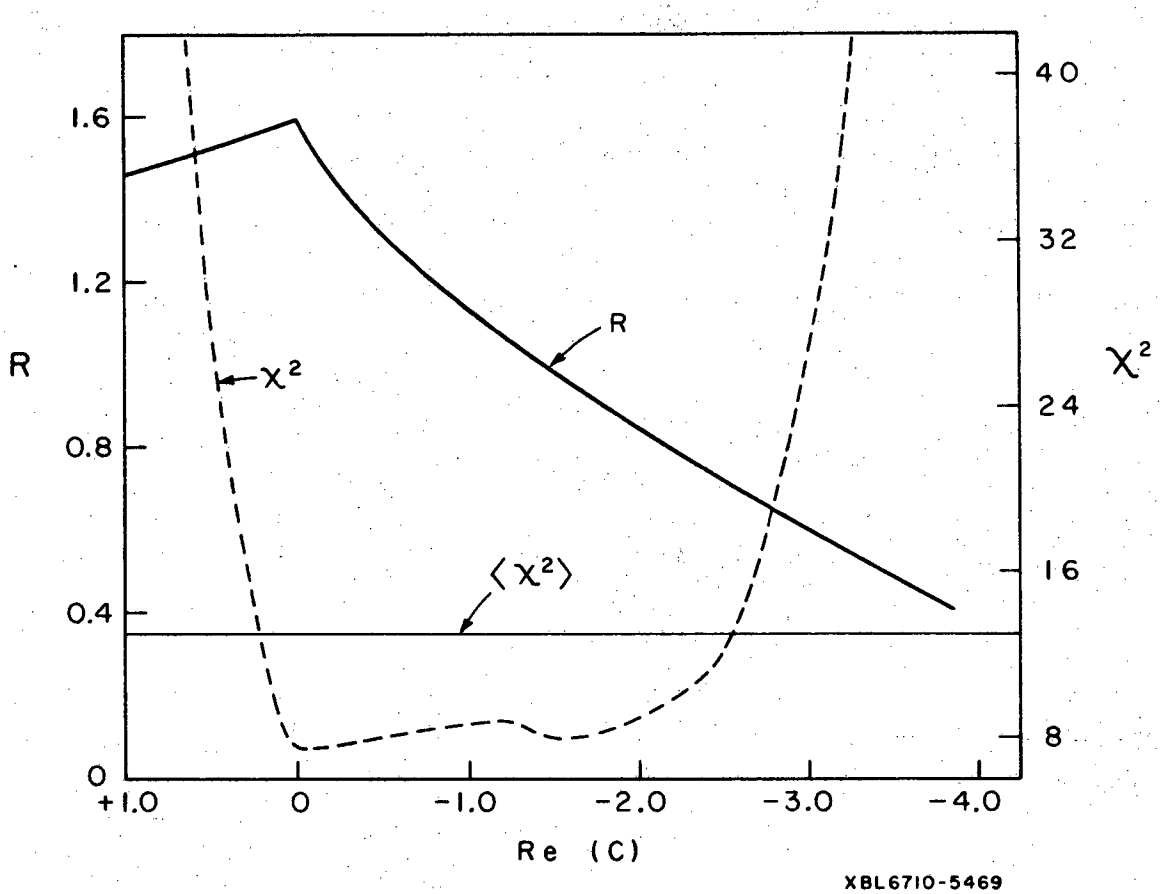
1.6
1.2
0.8
0.4
0
R
40
32
24
16
8
χ^2
R
χ^2
$\langle \chi^2 \rangle$
+1.0
0
-1.0
-2.0
-3.0
-4.0
Re (C)
XBL6710-5469

Fig. 1

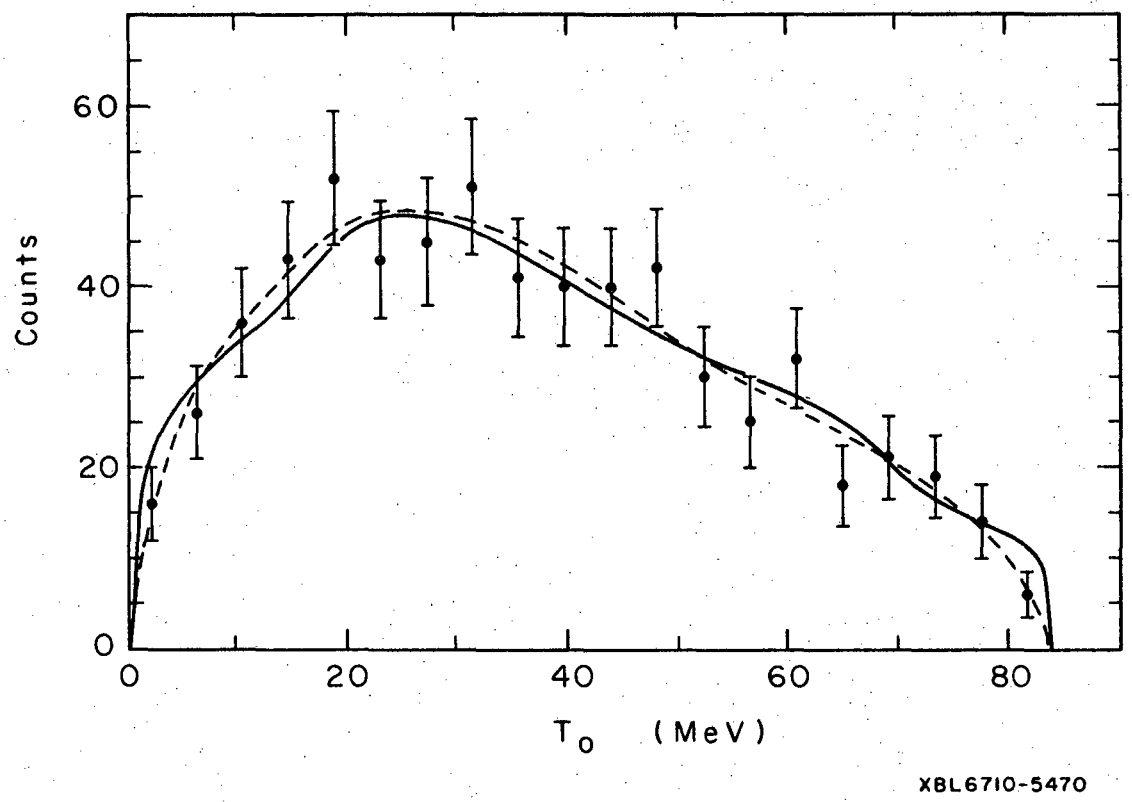

Fig. 2

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