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VECTOR CURRENT SUM RULE
AND THE BROKEN MAGNETIC MOMENT OF THE Σ^+

Thomas F. Walsh

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Summary. A familiar vector current sum rule of the Fubini-Furlan-Rossetti type is presented, and a model for the continuum corrections evolved. The symmetry breaking is introduced in a simple way by coupling the spurion in the sum rule to the baryon line in the graphs which contribute to the continuum terms. The decuplet does not contribute in this low-energy model. The correction terms turn out to be typically a few tenths of a magneton. The result for the Σ^+ moment, which depends on the D/F ratio, is, for the usual choice of this ratio, $\mu^T(\Sigma^+) = 2.6$ nuclear magnetons.

1. Introduction.

Recently, the existence of improving experimental[1] determinations of the baryon magnetic moments has increased interest in them and the question of symmetry breaking. There now exists a body of knowledge concerning the calculation of magnetic moments with the use of current algebra and dispersion relations. We will discuss the question of the broken Σ^+ moment by the method of Fubini and Furlan;[2] Fubini, Furlan and Rossetti;[3] and Furlan, Lannoy, Rossetti and Segré.[4] The actual formulation will be similar to that of Gasiorowicz,[5] although in a different context. There exists a calculation of the broken moments by Donini et al.[6] with no clear relation to this one. The closest analogous work is that of Pagels[7] who discussed the baryon moments with a sidewise dispersion relation.

In Section II we present the model through consideration of the commutator of a vector SU_3 current and the electromagnetic current. We point out the tenuous point of correspondence between the soft spurion limit of Fubini, Furlan and Rossetti and the $\nu \to \infty$ limit of Gasiorowicz. We then write the sum rule, pick out the one-particle intermediate states (which give the symmetry limit), and then obtain the amplitudes for the two-particle continuuum states from the reduction formalism. We consider only the pion-baryon intermediate states in the dispersion integral which breaks the symmetry, since they have thresholds nearest to the point where the dispersion relation is sampled ($k \to 0$). We then obtain the continuum contribution in a low-energy approximation. In the last section we present the value for the Σ^+ moment and discuss it briefly.

2. Calculation.

Consider the matrix element of the electromagnetic current j_μ and a vector current J^c_μ with SU_3 index c,[5]

$$T_{\nu\mu} = \int d^4x\, e^{ik\cdot x}\, \theta(x_0)\, \langle p's'b \mid [J^c_\nu(x),\, j_\mu(0)] \mid psa\rangle\,, \tag{1}$$

where a and b are the SU_3 indices of the respective states. Then

$$k^\nu T_{\nu\mu} = \int d^4x\, \left(\frac{1}{i}\frac{\partial}{\partial x_\nu} e^{ik\cdot x}\right) \theta(x_0)\, \langle p's'b \mid [J^c_\nu(x),\, j_\mu(0)] \mid psa\rangle\,,$$

and there follows

$$\begin{aligned} k^\nu T_{\nu\mu} &+ i\int d^3x\, e^{ik\cdot x}\, \langle p's'b \mid [J^c_0(x),\, j_\mu(0)] \mid psa\rangle\Big|_{x_0 = +\infty} \\ &= i\int d^3x\, e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}\, \langle p's'b \mid [J^c_0(\underset{\sim}{x},0),\, j_\mu(0)] \mid psa\rangle \\ &+ i\int d^4x\, e^{ik\cdot x}\, \theta(x_0)\, \langle p's'b \mid [D^c(x),\, j_\mu(0)] \mid psa\rangle \end{aligned} \tag{2}$$

where $D^c(x) \equiv \frac{\partial J^c_\nu}{\partial x_\nu}$. If, in the limit $k \to 0$, the LHS of (2) vanishes, we have for the RHS equal to zero,

$$\begin{aligned} &\lim_{k\to 0} \int d^4x\, e^{ik\cdot x}\, \theta(x_0)\, \langle p's'b \mid [D^c(x),\, j_\mu(0)] \mid psa\rangle \\ &= -\langle p's'b \mid [\int d^3x\, J^c_0(\underset{\sim}{x},0),\, j_\mu(0)] \mid psa\rangle\,, \end{aligned} \tag{3}$$

the familiar relation of Fubini, Furlan and Rossetti.

We turn to the LHS of (2). The second term is, at $k = 0$, just

$$\begin{aligned} &i(2\pi)^3 \sum_n e^{i(p'_0 - P_{n0})x_0}\, \delta(\underset{\sim}{p}' - \underset{\sim}{P}_n)\, \langle p's'b|J^c_0|n\rangle \\ &\times \langle n|j_\mu|psa\rangle\Big|_{x_0 = +\infty} - \text{crossed term.} \end{aligned} \tag{4}$$

This vanishes unless $P_n = p'$ for the direct term or $P_{n'} = p$ for the crossed term. If the masses of the external states and a one-particle (OP) intermediate state satisfy $M_n = M'$ (direct or $M_{n'} = M$ (crossed), then this term will not vanish. It can be made to do so by choosing the external masses so that $M' \neq M_n$, $M \neq M_{n'}$ and taking the limit of equal masses at the end of any calculation. It may occur that these masses are broken by some interaction, so that the limit is physically realistic. If the mass splitting between the intermediate and external particle is sufficiently large, it may happen that continuum states satisfy the condition for (4) not to vanish.

For small mass splitting between external and internal masses, we are assured that the second term on the LHS of (2) vanishes. The first term will likewise vanish provided there is no pole of $T_{\nu\mu}$ at $k = 0$. If the conditions for the second term to vanish hold, then $T_{\nu\mu}$ will have no pole at $k = 0$ and the entire LHS of (2) vanishes. In the limit of degenerate masses, the two contributions exactly cancel one another. These matters are discussed briefly in Appendix A.

The second term on the LHS of (2) can be compelled to vanish another way, pointed out by Gasiorowicz.[5] If k is not taken to zero, the offending term is zero unless $(p' + k)^2 = M_n^2$ for the direct term and $(p - k)^2 = M_{n'}^2$ for the crossed term. When the ancient variables

$$\nu = P\cdot Q \qquad P = \frac{1}{2}(p' + p) \qquad Q = \frac{1}{2}(\ell + k)$$

$$t = (p' - p)^2 = \Delta^2 \qquad \Delta = p' - p \qquad \ell = p' + k - p$$

and used, the condition is that

$$M_n^2 = \frac{1}{2}(M^2 + M'^2 + \ell^2 + k^2) - t + 2\nu ,$$

$$M_{n'}^2 = \frac{1}{2}(M^2 + M'^2 + \ell^2 + k^2) - t + 2\nu$$

(where $p'^2 = M'^2$, $p^2 = M^2$, and ℓ^2 and k^2 are customarily taken to be zero). Then if for $\nu \to \infty$ no infinite-mass states contribute to the $x_0 = \infty$ term, it will again vanish. If j_μ were a scalar current and the first term on the RHS of (2) obeyed an unsubtracted dispersion relation, we would get the sum rule of Gasiorowicz

$$\lim_{\nu\to\infty} k^\nu T_{\nu\mu} = i \langle p's'b | [Q^c(0), j(0)] | psa \rangle , \tag{5}$$

at least in the absence of Schwinger terms in the equal-time commutator.

We proceed to discuss the $k = 0$ sum rule, (3). With our variables, $k = 0$ corresponds to $\nu = \nu_0 \equiv \frac{1}{2}(M'^2 - M^2)$, and we will take the photon on shell and we set $k^2 = 0$. When the LHS of (3) is multiplied by a photon polarization vector ϵ^μ (satisfying $\epsilon\cdot\ell = 0$), it becomes

$$\begin{aligned} \bar{u}(p',s')\, A\, u(p,s) = \bar{u}(p',s')\, [a(\nu,t)\, I_a &+ b(\nu,t)\, I_b \\ &+ c(\nu,t)\, I_c + d(\nu,t)\, I_d]\, u(p,s) \end{aligned} \tag{6}$$

where $F^{\mu\nu} = \epsilon^\mu \ell^\nu - \epsilon^\nu \ell^\mu$, and the standard choice for the I is

$$\begin{aligned} I_a &= -\frac{1}{2}\gamma_\mu\gamma_\nu F^{\mu\nu} & I_c &= \gamma_\mu k_\nu F^{\mu\nu} \\ I_b &= [\gamma_\mu P_\nu + \frac{1}{4}(M' - M)\gamma_\mu\gamma_\nu]F^{\mu\nu} & I_d &= P_\mu k_\nu F^{\mu\nu} . \end{aligned} \tag{7}$$

We write an unsubtracted dispersion relation (taking $t = (p' - p)^2 = 0$) for the first invariant

$$a(\nu,0) = \frac{1}{\pi}\int \frac{\mathrm{abs}[a(\nu',0)]}{\nu' - \nu}\, d\nu' \quad , \tag{8}$$

with absorptive part

$$\begin{aligned} \mathrm{abs}[a(\nu,0)] = -\frac{i}{2}\sum_{s,s'} (2\pi)^4 \Bigg\{ & \sum_n \delta^4(p' + k - P_n) \\ & \times \langle p's'b|D^c|n\rangle \langle n|\epsilon^\mu j_\mu \mid psa\rangle\, O_{ss'} \\ & - \sum_{n'} \delta^4(p - k - P_{n'}) \langle p's'b|\epsilon^\mu j_\mu|n'\rangle \langle n'|D^c|psa\rangle\, O_{ss'} \Bigg\} \end{aligned} \tag{9}$$

where $O_{ss'}$ is a suitable projection operator, and the remaining invariants I_b, I_c, I_d vanish at $k = 0$. To satisfy (3) we must then have

$$a(\nu_0,0) = \frac{1}{\pi}\int \frac{\mathrm{abs}\,[a(\nu',0)]}{\nu' - \nu_0}\, d\nu' = 0 \ , \tag{10}$$

which is our sum rule.

The spin projection operator satisfying

$$\sum_{ss'} \bar{u}(p',s')\, A\, u(p,s)\, O_{ss'} = a(\nu,0)$$

is

$$\frac{1}{2(W^2 - M^2)^2}\, \bar{u}(p\ s)\, \gamma\cdot\epsilon\ \gamma\cdot\ell\ u(p's') \equiv N\, \bar{u}\, O\, u' \ . \tag{11}$$

In order to calculate the projections, we may use for $t = 0$: $p\cdot\ell = p'\cdot\ell$, $p\cdot k = p'\cdot k$, $k\cdot\ell = 0$, and $\epsilon\cdot p = \epsilon\cdot p' = \epsilon\cdot k = \epsilon\cdot\ell = 0$. The kinematics are then collinear and the projections are easily evaluated from

$$\sum_{ss'} \bar{u}(p's')\, I_r u(p\ s)\, O_{ss'} = N \operatorname{tr}(I_r\, O_+)$$

$$O_+ \equiv \Lambda_+(p)\, O\, \Lambda_+(p') = O_e + O_O\ ,$$

where

$$O_e = \gamma\cdot\epsilon\gamma\cdot\ell\,[MM' + \gamma - p\gamma\cdot p'] - 2p\cdot\ell\gamma\cdot\epsilon\gamma\cdot p'$$

and

$$O_O = O_e\,\frac{\gamma\cdot p'}{M'}\ .$$

The OP term in the absorptive part is

$$\operatorname{abs}[a(\nu,0)] = -\frac{i}{2}\sum_{ss'} 2\pi\Big\{\delta[(p'+k)^2 - P_n^2] \tag{12}$$

$$\times\,\bar{u}(p's')[i(M'-M_n)G^{bcn}(0)][M_n+M'+\gamma\cdot k]\,\epsilon^{\mu} j_{\mu}^{na}\,u(ps)\,O_{ss'} + \text{crossed}\Big\}\ ,$$

where

$$\langle p's'b|D^c(0)|p_n s_n n\rangle = i(p'-P_n)^{\mu}\,\bar{u}(p's')[\gamma_{\mu}G^{bcn}(k^2) + \cdots]\,u(ps)$$

and

$$j_{\mu}^{na} \equiv \epsilon_{\mu}F_1^{na} + i\sigma_{\mu\nu}(P_n - p)^{\nu}F_2^{na}\ ,$$

since $k^2 = 0$; $G^{bcn}(k^2{=}0)$ is just the appropriate SU_3 Wigner coefficient because of the Ademollo-Gatto theorem and the fact that we are interested in only the lowest order symmetry-breaking terms. It is important that the coefficients of the second-order terms we have ignored in G^{bcn} be of order 1 and not, for example, π^2, and that "second order" not mean, for example, $(M_K^2 - m_\pi^2)^2/M_K^4$. The model we will

develop will satisfy both requirements, though for the particular sum rule we study, the first is not essential.

The sum rule now becomes

$$- G^{bcn} F_2^{na} + G^{n'ca} F_2^{bn'} + \frac{1}{\pi} \int_{\nu_D}^{+\infty} d\nu' \frac{\mathrm{abs}[a(\nu',0)]_{\mathrm{direct}}}{\nu' - \nu_0}$$

$$+ \frac{1}{\pi} \int_{\nu_C}^{-\infty} d\nu' \frac{\mathrm{abs}[a(\nu',0)]_{\mathrm{crossed}}}{\nu' - \nu_0} = - \sum_{ss'} \langle p's'b|[Q^c, \epsilon^\mu j_\mu]|psa\rangle O_{ss'} , \tag{13}$$

where we assume that such combinations of indices are chosen as will give initial and final states definite (broken) masses and charges, and that more than one state n may appear. We will study only one case, for clarity: $b \to \Sigma^+$, $a \to p$. The dispersion relation is sampled at the point ν_0, the OP pole is at $-\nu_0$, and the direct cut begins, for a baryon + pion intermediate state, at $\nu_D = -\nu_0 + Mm + \frac{1}{2}m_\pi^2$ and goes to $+\infty$. The crossed cut goes from $-\nu_0 - M'm - \frac{1}{2}m^2$ to $-\infty$. For $M = M_N$ and $M' = M_\Sigma$, $\nu_D < \nu_0$ and the integral is to be interpreted as a principal part.

For a two-particle (8 baryon + 8 PS meson) intermediate state

$$\mathrm{abs}[a(\nu,0)]_D = - \frac{i}{2}(2\pi)^4 \sum_{ss'} \sum_n \delta^4(p' + k - p_1 - q_1)$$

$$\times \langle p's'b|D^c|p_1 s_1 d;\ q_1\ e\rangle\ \langle q_1 e;\ p_1 s_1 d|\epsilon^\mu j_\mu|psa\rangle\ O_{ss'} \tag{14a}$$

and

$$\mathrm{abs}[a(\nu,0)]_C = -\frac{i}{2}(2\pi)^4 \sum_{ss'} \sum_{n'} \delta^4(p - k - p_2 - q_2)$$

$$\times \langle p's'b|\epsilon^\mu j_\mu|p_2 s_2 d;\ q_2 e\rangle\ \langle q_2 e;\ p_2 s_2 d|D^c|psa\rangle\ 0_{ss'}\ . \tag{14b}$$

If we write a dispersion relation for the $a + (\text{photon}) \to d + e$ amplitude with absorptive part

$$\frac{1}{2}(2\pi)^4 \left\{ \sum_n \delta^4(p_1 + q_1 - P_n)\ \langle p_2 s_2 d|j^e(0)|n\rangle\ \langle n|\epsilon^\mu j_\mu|psa\rangle \right.$$

$$\left. - \sum_{n'} \delta^4(p_1 - q_1 - P_{n'})\ \langle p_1 d|\epsilon^\mu j_\mu(0)|n'\rangle\ \langle n'|j^e(0)|psa\rangle \right.$$

obtained by use of the reduction formalism, we may extract the usual Feynman amplitude by writing

$$\langle p_1 s_1 d|\epsilon^\mu j_\mu|p_n s_n n\rangle = \epsilon^\mu j_\mu^{dn}$$

and keeping only the OP states in n, n' with

$$\langle p_1 s_1 d|j^e(0)|p_n s_n n\rangle = 2ig\bar{u}(p_2 s_2)\ \Lambda^e\ u(p_n s_n),$$

where Λ is an 8×8 matrix defined by

$$(\Lambda^e)_{dn} = \alpha\ d_{den} + (1 - \alpha)(-if_{den})\ . \tag{15}$$

The Feynman amplitude is then

$$2ig\ \bar{u}_d(p_1 s_1) \left\{ - \frac{(\Lambda^e)_{dn'}\ \gamma_5\ \gamma\cdot q_1\ [\gamma\cdot\epsilon\ F_1^{n'a} + \cdots]}{(p_1 + q_1)^2 - M^2} \right.$$

$$- \frac{[\gamma\cdot\epsilon\ F^{dn'} + \cdots]\ (-\gamma\cdot q_1)\ \gamma_5\ (\Lambda^e)_{na}}{(p_1 - q_1)^2 - M^2} + \tag{16}$$

$$+ e_\pi \epsilon^\mu \left(q_{1\mu} + (p - p_1)_\mu\right) \frac{1}{m_\pi^2 - (p - p_1)^2} \gamma_5 (\Lambda^e)_{da} \Bigg\} u_a(ps)$$

where we have included the t-channel contribution of the mesonic current required by gauge invariance and obtained either by a tedious calculation of the equal-time term left out of (16), or by selecting the t-channel part out of a dispersion relation obtained by using the reduction formalism to reduce out the final-state baryon in the $a + \text{photon} \to d + e$ amplitude. The higher resonances in the s, u, and t-channels have been left out of this amplitude in the expectation that near threshold the amplitude is adequately given by the above. The decuplet resonances may later be inserted into the sum rule in the zero-width approximation.

The Feynman amplitude for $e + d \to b + \text{spurion}$ calculated in the same way is

$$\langle p's'b|D^c|p_1 s_1 d;\ q_1,\ \dot{e}\rangle \Big|_{\text{Feynman}}$$

$$= i \int dx e^{-iq_1 \cdot x} \langle p's'b|\theta(x_0)\ [j^e(x),\ D^c(0)]\ |\ p_1 s_1 d\rangle \Big|_{OP}$$

$$= 2ig\ \bar{u}(p's') \Bigg\{ \frac{(\Lambda^e)_{bn}\ \gamma_5 \left(M' + \gamma\cdot(p' - q_1)\right) i(M' - M_1)\ G^{ncd}}{(p' - q_1)^2 - M'} \tag{17}$$

$$+ \frac{i(M' - M_1)\ G^{bcn'} \left(M_1 + \gamma\cdot(p_1 + q)\right) (\Lambda^e)_{n'd}\ \gamma_5}{(p_1 + q_1)^2 - M_1^2} \Bigg\} u(p_1 s_1) \quad .$$

It is amusing to note that the Feynman amplitudes may be quickly extracted from the reduction formalism by use of the Fubini-Furlan $P \to \infty$ trick without the intervention of a dispersion relation.

An equal-time term has been left out of (17). The form of this term can be obtained by the use of PCAC in the reduction formalism

and the zero-momentum pion limit. The baryon pole terms, of first order in the symmetry breaking, vanish in this limit. The equal-time term of interest is finite for $q \to 0$ and does not involve the symmetry breaking, and even factors m_π^2/M_K^2 cancel out. Because the presence of such a term in (17) would destroy the symmetry limit by enforcing a symmetry-breaking independent term in (17), so this term has been excluded. Moreover, we will use (17) as the entire baryon-pion → baryon-spurion amplitude. Our model for the symmetry breaking is then essentially one in which a soft spurion couples to the baryon line in the graphs contributing to the continuum corrections. This is the most obvious extension of the way in which the symmetry limit is generated by a $k \to 0$ spurion with a coupling proportional to the baryon mass differences hooking onto the baryon line in the process baryon → baryon + photon, with the mass difference in the coupling cancelled by a similar term in the propagator introduced by the spurion. In the continuum terms this propagator is absent, hence the finite first-order correction terms. Near threshold we find that the absorptive part coming from the above Feynman amplitudes consists of two pieces. The baryon pole amplitudes in the photoproduction amplitude contribute a part which vanishes at threshold and rises only very slowly above it, as first and second powers of $(W^2 - M^2)/M^2$. The pion-current graph contributes a part which also vanishes at threshold, but rises quickly to a value from which it falls off rather slowly for $W \gtrsim M$. If we take zero pion mass this contribution is finite at threshold, and decreases slowly. The denominator in the dispersion relation enhances such a low-energy contribution. The situation is somewhat like that for the sidewise dispersion relation calculation of the baryon moments by

Pagels,[7] except that the relevant amplitude here contains the spurion and contributes to the symmetry-breaking term. The threshold enhancement comes directly from the singularity of the "extra" propagator introduced in the baryon line by the spurion. We will use only the pion-current part of the amplitude in what follows.

We may extract the absorptive part with the help of

$$\mathrm{tr}\,(\gamma_\mu\, O_+) = -4\,\epsilon_\mu\, p\cdot\ell\,(M' - M)$$

$$\mathrm{tr}\,(\gamma_\mu\gamma_\nu O_+) = -2(M' + M)^2\,(\epsilon_\mu \ell_\nu - \epsilon_\nu \ell_\mu)$$

$$+ 4p\cdot\ell\,[\epsilon_\mu (p' + p)_\nu - \epsilon_\nu (p' + p)_\mu]$$

finding for the direct term in the zero pion mass limit and with equal baryon masses in the expression multiplying $(M' - M)$ and with SU_3 indices suppressed,

$$-i\,\frac{1}{(2\pi)^2}\,\frac{M}{4}\,\frac{|q|}{W}\int d\Omega_q \left\{ -(M' - M)\, e_\pi\, \frac{i}{2M_1}\,(2ig)^2 \right. \tag{18}$$

$$\times \left(\frac{\Lambda G|_{\text{crossed}}}{(p'-q)^2 - M'^2} + \frac{\Lambda G|_{\text{direct}}}{(p+\ell)^2 - M^2} \right) \frac{(\epsilon\cdot q)^2}{q\cdot\ell}\quad \Lambda\Big|_{\pi\ \text{current}}$$

where

$$W^2 - M^2 = 2p\cdot\ell; \qquad W^2 = s; \qquad \underset{\sim}{q} = \text{c.m. pion momentum}\ .$$

Then the absorptive part becomes $(\Delta M \equiv M' - M)$

$$-\frac{1}{16\pi}\,\frac{e_\pi}{2M}\,\frac{\Delta M}{M}\,\left(\frac{M}{W}\right)^2 (2ig)^2 \left(\Lambda G\big|_D - \frac{2W^2}{W^2+M^2}\, f(v)\, \Lambda G\big|_C \right) \Lambda\Big|_\pi\ , \tag{19}$$

where

$$f(v) = \frac{1}{v} - \frac{1}{2v^2}(1 - v)\ \ln\left(\frac{1+v}{1-v}\right) \quad v = |q|/p_{10} \ .$$

The term $[2W^2/(W^2 + M^2)]\ f(v)$ is unity at $W = M$ and varies slowly. We will approximate it by unity. The contribution of the direct term to the sum rule becomes, when the dispersion relation is integrated to the physical threshold,

$$\frac{1}{16\pi^2}\ \frac{e_\pi}{2M}\ \frac{\Delta M}{M}\ (2ig)^2\ \left(\Lambda G\big|_D - \Lambda G\big|_C\right)\Lambda\big|_\pi \times \ln\left|\frac{M'^2 - (M + m_\pi)^2}{(M + m_\pi)^2}\right| , \tag{20}$$

and the crossed term is

$$+\frac{1}{16\pi^2}\ \frac{e_\pi}{2M}\ \frac{\Delta M}{M}\ (2ig)^2\ \left(\Lambda G\big|_D - \Lambda G\big|_C\right)\Lambda\big|_\pi \times \ln\left|\frac{(M' + m_\pi)^2 - M^2}{(M' + m_\pi)^2}\right| . \tag{21}$$

Putting in the Σ^+ and p as final and initial states, and taking the baryon-baryon-spurion couplings from

$$G^{abc} = \frac{1}{\sqrt{2}}\ (-i\ f_{abc})$$

(since they are pure F-type), we find for the sum rule

$$\frac{K_p}{2M_N} - \frac{K_{\Sigma^+}}{2M_\Sigma} + \frac{M_\Sigma - M_N}{M}\ \frac{1}{4\pi}\ \frac{g^2}{4\pi}\ \frac{|e|}{2M} \times [(0.70)(-\frac{16}{3}\alpha^2 + 8\alpha - 2)] = 0 , \tag{22}$$

since the commutator on the RHS of (3) is just proportional to

$[U^+, j_\mu(0)] = 0$ if j_μ is assumed to transform as the $3 + 8$ member of an octet.

In this particular model, there is no resonant intermediate state in the baryon + pion → baryon + spurion amplitude. If the model is used in the calculation of the baryon-spurion-decuplet coupling, it turns out to be very small (Appendix B). The model is thus consistent in that it does not of itself generate such a coupling. Also, we can leave the (zero width) decuplet states out of the sum rule on account of the small coupling.

3. Discussion.

For the conventional D/F ratio, $\alpha \approx 0.35$, and the correction term in (22) is practically zero (0.03 nuclear magnetons). The major part of the symmetry breaking then comes from the appearance of the Σ^+ mass in the sum rule, and we find for the total moment.

$$\mu^{TOT}(\Sigma^+) \approx 2.6 \text{ nm.}$$

The present experimental value[1] is 3.0 ± 1 nm. The experimental value has declined since the first experiment and it is unclear whether it will continue to do so or not. There now exist theoretical calculations with a spread as great as the experimental value. In this particular model the moment can be increased by 0.2 nm. or decreased by 0.4 nm. by a choice of pure D or pure F coupling, respectively. The other octet moments are also obviously broken by rather small amounts. A more systematic treatment of this vector-current symmetry breaking model will have to wait on better Σ^+ moment experiments, since this is the most unambiguous prediction of the model and should decide whether it is to be taken seriously or not.

The corrections have also been calculated by Donini et al,[6] who assumed that the decuplet saturates the corrections, the decuplet-baryon-spurion coupling being given by a unitarity argument.[8] These two models are not necessarily contradictory. It is possible that the low-energy contributions can be mimicked by a pole in the continuum integral. Ideally, the calculations of the moments by vector and axial vector commutators should agree. That they do not is at present an unexplained

fact. The axial vector calculation gives 3.5 nm.,[9] and the vector calculations 0.5 to 0.9 nm. less.

We have presented a simple model, evolved with respect to a specific example, of the breaking of SU_3 symmetry for the algebra of vector current commutators. The model is a natural application to the symmetry breaking continuum terms in the relevant dispersion sum rules of the type of spurion-baryon coupling which generates the symmetry limit predictions. The main contribution to the symmetry breaking is nonresonant and low energy. The model is consistent in the sense that it does not generate a large decuplet-baryon-spurion coupling. It predicts rather small symmetry breaking and is consistent, though not dramatically so, with experiment.

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APPENDIX A - THE $x_0 = +\infty$ SURFACE TERM

The $x_0 = +\infty$ term in (2) is

$$i \int_{x_0=+\infty} d^3x\, e^{ik\cdot x} \langle p's'b|\, [J_0{}^c(x), j_\mu(0)]\, |psa\rangle$$

$$= i(2\pi)^3 \left\{ \sum_n \delta(\underset{\sim}{P}_n - \underset{\sim}{k} - \underset{\sim}{p}')\exp[i(p_0' + k_0 - P_{n0})x_0] \right.$$

$$\left. \langle p's'b|J_0{}^c|n\rangle\langle n|j_\mu|psa\rangle + \text{crossed} \right\}. \qquad \text{(A1)}$$

If the continuum is well separated from p_0' at $k = 0$ it is sufficient to consider only the OP states in n and n', and ignore the seemingly noncovariant character of (A1), since pair states in the sum will not contribute. Then, if $M' \neq M_n$ and $M \neq M_{n'}$ this term oscillates at $k = 0$ and may be taken to be zero. If $M' = M_n$ and $M = M_{n'}$, then $p_0' + k_0 - P_{n0} \to k_0 - (\underset{\sim}{k}\cdot\underset{\sim}{p}')/p_0'$ as $k \to 0$; and if x_0 is taken finite $(=X_0)$, (A1) is just

$$i\,\frac{M'}{p_0'} \left\{ \sum_{s_n} \langle p's'b|J_0{}^c|P_n\, s_n\rangle\langle P_n\, s_n|j_\mu|psa\rangle + \text{crossed} \right\}$$

$$= i\,\bar{u}(p's')\left\{ G^{bcn}\, j_\mu{}^{na} - G^{nca}\, j_\mu{}^{bn} \right\} u(ps), \qquad \text{(A2)}$$

since at $k = 0$

$$\langle p's'b|J_0{}^c|n\rangle = G^{bcn}\, \frac{p_0'}{M'}\, \bar{u}(p's')\, u(p', s_n)$$

and

$$j_\mu{}^{bn} \equiv \gamma_\mu\, F_1{}^{bn} + i\,\sigma_{\mu\nu}(p' - P_n)^\nu\, F_2{}^{bn}\,,$$

and G and $F_{1,2}$ are both taken at $t = 0$.

In the unequal mass case $T_{\nu\mu}$ clearly has no pole at $k = 0$ since the direct term, for example, is proportional to

$$\text{direct term} \propto \frac{1}{(p' + k)^2 - M_n^{\ 2}} = \frac{1}{2p'\cdot k - M_n^{\ 2} + M'^2}$$

and (with $x_0 = \infty$ for this case) so $\lim_{k\to 0} k^\nu T_{\nu\mu} = 0$ for the unequal-mass case. For equal masses we again take $x_0 = X_0$, and find for the leading term in k that

$$\lim_{k\to 0} k^\nu \left\{ \frac{M'}{p_0'} p_\nu' \, \bar{u}(p's')G^{bcn} \, j_\mu^{\ na} \, u(ps) + \text{crossed} \right\} = 0.$$

If we consider both terms on the LHS of (2) for the equal-mass case and $x_0 = \infty$, the $x_0 = \infty$ itself will oscillate for all finite k, and we take it to be zero. In the second term, $T_{\nu\mu}$ now has a pole at $k = 0$ and, in fact,

$$\begin{aligned} k^\nu T_{\nu\mu}\Big|_{\text{pole}} &= -\frac{i}{2}\left\{ \frac{2p'\cdot k}{-p'\cdot k} \, \bar{u}'G^{bcn} \, j_\mu^{\ na} \, u + \text{crossed} \right\} \\ &= i\,\bar{u}(p's')\left\{ G^{bcn} \, j_\mu^{\ na} - G^{n'ca} \, j_\mu^{\ an'} \right\} u(p,s). \end{aligned} \tag{A3}$$

This is just the same as (A2) wherein we took $k \to 0$ before $X_0 \to \infty$. The LHS of (2) then vanishes only for unequal masses. For the unequal mass case, the divergence term on the RHS of (2) is just (cf. (13) of the text)

$$\begin{aligned} & i \int d^4x \; e^{ik\cdot x} \, \theta(x_0) \langle p's' | [D^c(x), \, j_\mu(0)] | psa \rangle \Big|_{\text{pole}} \\ &= -i\,\bar{u}(p's') \left\{ G^{bcn} \, j_\mu^{\ na} - G^{n'ca} \, j_\mu^{\ bn'} \right\} u(ps), \end{aligned} \tag{A4}$$

whereas for the equal-mass case it vanishes because of the factor $D^c(x)$. For this case the missing OP term is supplied by the LHS of (2). Whatever the choice of limits in the evaluation of (2) for the degenerate-mass case, the result is the same as if the limit of degenerate masses is approached through nondegenerate masses at the end of a calculation, and there is no ambiguity in the choice.

When the continuum overlaps the point where $k = 0$, the apparent difficulty that the LHS of (2) might not vanish may be easily overcome. We may choose indices so as to put the lighter of the external baryons in the final state, provided the initial baryon mass + pion mass is less than the final baryon mass. Alternatively, the masses may be continued to their correct values after a calculation in which they are held so as to keep the cut away from $k = 0$. Also, the integral in the sum rule can be interpreted as a principal part. This is so since for the continuum the $x_0 = +\infty$ term will vanish even if it overlaps $k = 0$ because of the integral over internal momenta implied in (A1) and the oscillating exponential factor. The $T_{\nu\mu}$ term is finite if the limit is taken as

$$\frac{1}{2}[T_{\nu\mu}(k + i\epsilon) + T_{\nu\mu}(k - i\epsilon)] ,$$

so $\lim_{k\to 0} k^\nu T_{\nu\mu} = 0$. The limit on the RHS of (2) automatically becomes a principal part, so that the dispersion integral in the sum rule is likewise interpreted in the same way.

APPENDIX B

In order to evaluate the decuplet contribution to the sum rule, we need the coupling of the $j = 3/2$ intermediate-state particle to the final baryon + spurion state. To obtain this we will use a reduction formalism for the $j = 3/2$ particle. The Rarita-Schwinger field satisfies

$$(\gamma\cdot p - M_R)\, u_\mu(p_R\ s_R) = 0$$

$$\gamma^\mu\, u_\mu(p_R,\ s_R) = p_R{}^\mu\, u_\mu(p_R,\ s_R) = 0$$

$$\bar{u}_\mu(p_R,\ s_R)\, u_\nu(p_R,\ s_R) g^{\mu\nu} = 1$$

and the reduction formula is

$$\langle p's'b|D^c|p_R\ s_R\ a\rangle$$

$$= -i\int d^4x\ e^{-ip_R\cdot x}\langle p's'b|\theta(-x_0)[D^c(0),\ \bar{\eta}_\nu(x)]|0\rangle\ u^\nu(p_R\ s_R), \tag{B1}$$

where $\eta_\nu(x) \equiv (i\gamma\cdot\partial - M_R)\psi_\nu(x)$. There is an equal-time term involving $\delta(x_0)[\partial^\lambda J_\lambda(0),\ \psi_\mu{}^+(x)]$ which we take to be ignorable, in the absence of any means to calculate it. Then

$$\langle p's'b|D^c|p_R\ s_R\ a\rangle$$

$$= G^{bca}(p'^2{=}M'^2,\ \ p_R{}^2 = M_R{}^2,\ \ k^2{=}0)\ \bar{u}(p's')\gamma_5\ p'^\mu\ u_\mu(p_R\ s_R), \tag{B2}$$

and we will write an unsubtracted dispersion relation in the $j = 3/2$ mass for the quantity $G(p'^2,\ p^2,\ k^2{=}0)$ with $p'^2 = M'^2$ fixed. Then,

following Bincer[10] we write (B2) as $\bar{u}(p's')\Gamma^{\mu} u_{\mu}(p_R\ s_R)$ and put

$$\bar{u}(p's')\Gamma^{\mu} = \bar{u}(p's')[G(W)\gamma_5\ p'_{\nu}\ \Lambda^{\nu\mu}(W) + G(-W)\gamma_5\ p'_{\nu}\ \Lambda^{\nu\mu}(-W)]$$

$$p'^2 = M'^2 \qquad (B3)$$

$$p^2 = W^2.$$

This differs from the spin 1/2 form by the presence of the spin 3/2 projection operators

$$\Lambda^{\nu\mu}(p_R) = \frac{W + \gamma\cdot p_R}{2W}\ [g^{\nu\mu} - \frac{2}{3W^2}\ p_R{}^{\nu}\ p_R{}^{\mu} - \frac{1}{3}\gamma^{\nu}\ \gamma^{\mu}$$

$$- \frac{1}{3W}(\gamma^{\nu}\ p_R{}^{\mu} - \gamma^{\mu}\ p_R{}^{\nu})] \qquad (B4)$$

on the right. Since it is in general impossible to apply the subsidiary conditions for the Rarita-Schwinger field off the mass shell, we can only hope that the appearance of the $\Lambda^{\nu\mu}$ in (B3) is correct. In principle there might be other, physically meaningless, form factors in Γ_{μ}. We now take G to satisfy

$$G(p'=M'^2,\ p^2=M_R{}^2,\ k^2=0)$$

$$= \frac{P}{\pi}\int_{M''+m_{\pi}} dW'\ \left[\frac{\text{abs } G(W)}{W - M} + \frac{\text{abs } G(-W)}{W + M}\right]$$

with threshold at $(M'' + m_{\pi})$, and we will take only the lowest mass (baryon + pion) intermediate state into account. The absorptive part is then

$$\mathrm{abs}[G(\pm W)] = -\frac{1}{2} \sum_{s'} \sum_{n} (2\pi)^4 \, \delta(p_R - P_n)$$

$$\times \langle p's'b|D^c|n\rangle\langle n|\bar{\eta}_\mu|0\rangle \nu^\mu(\pm W)\, u(p's'), \qquad \text{(B5)}$$

with a projection operator $\nu^\mu(\pm W)$ which picks $G(\pm W)$ out of (B3):

$$\nu^\mu(\pm W) = \pm\, 12 \, \frac{W^3 \, M'}{(W \pm M')^4 \, (W + M')^2} \; \Lambda^{\mu\sigma} \, (\pm W) p'_\sigma \, \gamma_5 \, .$$

We now use our model for the baryon + pion → baryon + spurion amplitude,

$$\langle p's'b|D\,|p'',s''d;\, qe\rangle = F \; \frac{\bar{u}(p's')\gamma\cdot q \; \gamma_5 \; u(p''\; s'')}{W^2 - M^2}$$

with

$$F \approx (2ig)i(M' - M'')\,(-\Lambda^e{}_{bn}\, G^{ncd} + \Lambda^e{}_{nd}\, G^{ncd}).$$

Moreover,

$$\langle p''\; s'';q|\,\bar{\eta}_\mu\,|0\rangle\nu^\mu = K\,\bar{u}(p''\; s'')\; p''_\mu\; \nu^\mu$$

with $K = -\sqrt{2}\,\frac{\lambda}{m_\pi}\, C^{bna}$, where C^{bna} is an appropriate SU_3 Wigner coefficient[11] and $\lambda \approx 2.2$.

To get a rough value for G we will use a threshold approximation, putting $M_R = M' = M'' = M$ and keeping only the lowest order term in the pion mass (apart from that in K), and the external factor $(M' - M'')$. After a trace calculation and integration over the variables in the intermediate state, we find

$$\mathrm{abs}[G(+W)] = \frac{1}{16\pi} F K \frac{W + M}{W(W - M)^2} m_\pi^{\ 2} . \tag{B6}$$

Then only the first $[\mathrm{abs}\big(G(+W)\big)]$ term gives a contribution of order $m_\pi^{\ -2}$, and to lowest order in the pion mass, and suppressing SU_3 indices,

$$G^{bna} \approx \frac{1}{16\pi^2}(2ig)i \frac{M_\Sigma - M_N}{m_\pi} \left[\Lambda G|_D - \Lambda G|_C\right] (-\sqrt{2}\,\lambda) ,$$

a very small result indeed. The coupling G is typically of order 10^{-2} or so in this model.

FOOTNOTE AND REFERENCES

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