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HEAT TRANSFER THROUGH A THIN ROTATING NUCLEAR TARGET

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January 10, 1968

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## Abstract

The approximate time-dependent steady-state temperature of a thin rotating nuclear target system is found by solving the differential equation for a corresponding one-space-dimensional heat transfer problem in terms of an eigenfunction series. The series obtained can be evaluated numerically to necessary design accuracy with use of only a small amount of computer time. The results are given for a typical problem, for which the solution is depicted graphically.

## Introduction

The advent of large beam energies in particle accelerators requires that some scheme be used to prevent the melting of solid beam targets. One scheme is rotating the target, so that the portions of it exposed to the beam are bombarded for only a fraction of each rotation. In this paper such a nuclear target system is considered; it consists of a plane target of thickness $a$ attached to a heat-conducting backing of thickness $b$ that dissipates heat to a coolant (fig. 1). The coolant either is maintained at a constant fixed temperature $\overline{T}_0$, corresponding to boiling heat transfer, or has a film coefficient $h$ with a constant fixed sink temperature $\overline{T}_0$, corresponding to nonboiling heat transfer. The target is rotated with period $\overline{\tau}$ at constant speed, and a uniform beam, parallel to the axis of rotation, of width $\epsilon$ impinges on the target at a distance $R$ from the axis to the center of the beam. For design purposes it is of interest to know the steady-state temperature of the target and backing as a function of time, and, in particular, to know if the maximum temperatures lie below certain upper bounds. Rough estimates for the peak temperatures for a similar target system have been given elsewhere as obtained by bulk time-averaged heat-transfer considerations[1]. Such estimates, however, may provide insufficient information for the proper design of an optimal rotation speed, for which more detailed estimates, especially of the time dependence, are required.

In this study, temperature estimates are obtained by solving the differential equation for a one-space-dimensional time-dependent problem approximating the original one. In the approximating problem only the temperature on the surface of the circular cylinder traced out by the center line of the beam is considered, since it is there that the maximum temperature occurs. The temperature is taken to be a function only of the distance along the beam direction; that is, conduction perpendicular to the axis of the beam is neglected compared with conduction along the axis. If the target and backing are thin compared with the width of the beam, then this is a good approximation. In any case, neglecting the transverse conduction results in temperatures that err in being too large, so that designs based on them will also satisfy the upper temperature bounds of the original higher-dimensional problem. The advantage of this one-space-dimensional approximation over the original problem is that it is easily and simply solved in terms of an eigenfunction series with only a small amount of computational effort.

## Mathematical Formulation

The approximating problem is depicted in Fig. 2.

The target occupies the region $-a \leq x \leq 0$ and has thermal conductivity $K_1$, density $\rho_1$, specific heat $C_1$, and diffusivity $D_1 = K_1/\rho_1 C_1$; the conducting backing occupies the region $0 < x \leq b$ and has thermal conductivity $K_2$, density $\rho_2$, specific heat $C_2$, and diffusivity $D_2 = K_2/\rho_2 C_2$. The beam travels in the positive $x$ direction, impinging on the target at $x = -a$. The fraction of the heat energy absorbed by the target (and possibly backing) in the length $(x, x+dx)$ is $\overline{f}(x)dx$, where $\overline{f}(x)$ is a given function of $x$, and $\int_{-a}^{b} \overline{f}(x)dx = 1$. It is assumed at $x = -a$ that no heat is conducted away from the target and at $x = b$ that the coolant either is maintained at a constant temperature $\overline{T}_0$ $(\delta = 0)$, or has a film coefficient $h$ with a constant sink temperature $\overline{T}_0$ $(\delta = 1)$. The time scale is defined so that when the time $\overline{t}$ is zero the beam is just beginning to pass the chosen one-dimensional element. The total rate of heat energy absorption by the element per unit time during one period is then

$$Q(\overline{t}) = \begin{cases} \dfrac{P\,\overline{\tau}}{2\pi R \epsilon \overline{\tau}_p}, & 0 < \overline{t} \leq \tau_p, \\ 0, & \overline{\tau}_p < \overline{t} \leq \overline{\tau}, \end{cases}$$

where $\overline{\tau}_p = \dfrac{\epsilon \overline{\tau}}{2\pi R}$ is the length of time it takes the beam to pass by the element, and $P$ is the power of the beam absorbed by the target and backing.

The governing steady-state equations for the temperature $\overline{T}$ for one period are

$$\left.\begin{aligned} \frac{\partial \overline{T}(\overline{t},x)}{\partial \overline{t}} &= D_1 \frac{\partial^2 \overline{T}(\overline{t},x)}{\partial x^2} + \frac{Q(\overline{t})\,\overline{f}(x)}{C_1\rho_1}, \quad -a<x<0 \\ \frac{\partial \overline{T}(\overline{t},x)}{\partial \overline{t}} &= D_2 \frac{\partial^2 \overline{T}(\overline{t},x)}{\partial x^2} + \frac{Q(\overline{t})\,\overline{f}(x)}{C_2\rho_2}, \quad 0<x<b \end{aligned}\right\}, \quad 0<\overline{t}<\overline{\tau}$$

with boundary conditions

$$\frac{\partial \overline{T}}{\partial x}(\overline{t}, -a) = 0, \qquad \delta K_2 \frac{\partial \overline{T}}{\partial x}(\overline{t}, b) + h\left[\overline{T}(\overline{t}, b) - \overline{T}_0\right] = 0, \ \delta = 0 \text{ or } 1,$$

interface conditions

$$\overline{T}(\overline{t}, 0^-) = \overline{T}(\overline{t}, 0^+),$$

$$K_1 \frac{\partial \overline{T}}{\partial x}(\overline{t}, 0^-) = K_2 \frac{\partial \overline{T}}{\partial x}(\overline{t}, 0^+),$$

and periodicity condition

$$\overline{T}(0,x) = \overline{T}(\overline{\tau}, x).$$

The problem can be written in dimensionless form by letting

$$t = \frac{D_1 \overline{t}}{a^2},$$

$$\tau = \frac{D_1 \overline{\tau}}{a^2}, \quad \tau_p = \frac{D_1 \overline{\tau}_p}{a^2},$$

$$T = \frac{\overline{T} - \overline{T}_0}{\overline{T}_r} = \frac{2\pi\, a \epsilon R \rho_1 C_1}{P\overline{\tau}} (\overline{T} - \overline{T}_0),$$

$$H = \frac{h\,b}{K_2 \alpha},$$

$$\xi = \begin{cases} \dfrac{x}{a}, & -a \leq x \leq 0 \\ \dfrac{x}{b}\alpha, & 0 < x \leq b \end{cases}, \qquad f = \begin{cases} a\overline{f}, & -a \leq x \leq 0 \\ \dfrac{b}{M}\overline{f}, & 0 < x \leq b, \end{cases}$$

where the dimensionless parameters $\alpha$ and M are

$$\alpha = \frac{b}{a}\sqrt{\frac{D_1}{D_2}}\ , \qquad M = \frac{b}{a}\,\frac{\rho_2 C_2}{\rho_1 C_1}\ .$$

The reference temperature $\overline{T}_r$ chosen here for the nondimensionalization is $\overline{T}_r = \frac{P\overline{\tau}}{2\pi a \epsilon R \rho_1 C_1}$, the temperature increment in the target that would result were all the heat energy per period transferred instantaneously ($\overline{\tau}_p \to 0$) and uniformly $[\overline{f}(x) = \frac{1}{a},\ -a \leq x \leq 0;\ \overline{f}(x) = 0,\ 0 < x \leq b]$ to it from the beam.

The equations then become

$$\left.\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial^2 T}{\partial \xi^2} + \frac{f}{\tau_p}\ , \quad 0 < t < \tau_p \\ \frac{\partial T}{\partial t} &= \frac{\partial^2 T}{\partial \xi^2}\ , \quad \tau_p < t < \tau \end{aligned}\right\} \begin{aligned} -1 < \xi < 0, \\ 0 < \xi < \alpha, \end{aligned} \tag{1}$$

with boundary and interface conditions

$$\frac{\partial T}{\partial \xi}(t, -1) = 0, \qquad \delta\,\frac{\partial T}{\partial \xi}(t, \alpha) + H\,T(t, \alpha) = 0,\ \delta = 0 \text{ or } 1,$$

$$T(t, 0^-) = T(t, 0^+), \qquad \frac{\partial T}{\partial \xi}(t, 0^-) = \frac{M}{\alpha}\,\frac{\partial T}{\partial \xi}(t, 0^+), \tag{2}$$

and periodicity and continuity conditions

$$T(0, \xi) = T(\tau, \xi), \qquad T(\tau_p^-, \xi) = T(\tau_p^+, \xi). \tag{3}$$

## Solution

The solution in terms of the associated eigenfunctions is found in a manner similar to that in Ref. 2. Consider the associated eigenvalue problem for $T_k(\xi)$,

$$\frac{d^2 T_k}{d\xi^2} + \lambda_k^2 T_k = 0, \qquad \begin{array}{l} -1 < \xi < 0, \\ 0 < \xi < \alpha, \end{array}$$

with $T_k(0^-) = T_k(0^+)$, $\frac{dT_k}{d\xi}(0^-) = \frac{M}{\alpha}\frac{dT_k}{d\xi}(0^+)$,

and $\frac{dT_k}{d\xi}(-1) = 0$ $\quad \delta \frac{dT_k}{d\xi}(\alpha) + H\, T_k(\alpha) = 0, \quad \delta = 0$ or $1$.

Its solution is

$$T_k(\xi) = A_k \begin{cases} \cos\lambda_k \cos\lambda_k\xi - \sin\lambda_k \sin\lambda_k\xi, & -1 \leq \xi \leq 0 \\ \cos\lambda_k \cos\lambda_k\xi - \frac{\alpha}{M}\sin\lambda_k \sin\lambda_k\xi, & 0 < \xi \leq \alpha, \end{cases} \tag{4}$$

with the eigenvalues given by

$$\tan\lambda_k = \frac{M}{\alpha}\,\frac{H - \delta\lambda_k \tan\alpha\lambda_k}{H\tan\alpha\lambda_k + \delta\lambda_k}, \qquad k = 1, 2, 3, \cdots. \tag{5}$$

Only the positive eigenvalues need be considered; let them be ordered so that $0 < \lambda_1 < \lambda_2 < \lambda_3 < \cdots$.

The eigenfunctions $\{T_k(\xi)\}$ do not form an orthogonal set over $(-1, \alpha)$, but they are biorthogonal with the (discontinuous) functions

$$S_k(\xi) = \begin{cases} T_k(\xi), & -1 \leq \xi \leq 0 \\ \frac{M}{\alpha} T_k(\xi), & 0 < \xi \leq \alpha; \end{cases}$$

that is,

$$\int_{-1}^{\alpha} T_k(\xi)\, S_j(\xi)\, d\xi \quad \begin{cases} = 0 & \text{if } k \neq j \\ \neq 0 & \text{if } k = j. \end{cases}$$

If $$A_k = \left[ \frac{2}{1 + (M + \delta \frac{H\,M}{\alpha(H^2+\delta^2\lambda_k^2)})(\frac{\alpha^2}{M^2}\sin^2\lambda_k + \cos^2\lambda_k)} \right]^{1/2}, \quad k = 1, 2, 3, \cdots,$$

then the eigenfunctions are normalized so that

$$\int_{-1}^{\alpha} T_k(\xi)\, S_k(\xi)\, d\xi = 1.$$

The expansion of $f(\xi)$ in terms of these eigenfunctions is given by

$$f(\xi) = \sum_{k=1}^{\infty} B_k\, T_k(\xi), \qquad -1 \leq \xi \leq \alpha\,, \tag{6}$$

where

$$B_k = \int_{-1}^{\alpha} f(\xi)\, S_k(\xi)\, d\xi\,. \tag{7}$$

The solution of the original problem for one period is then found in the form

$$T(t, \xi) = \sum_{k=1}^{\infty} C_k(t)\, T_k(\xi) \qquad 0 \leq t \leq \tau\,, \quad -1 \leq \xi \leq \alpha \tag{8}$$

by substituting Eqs. (6) and (8) into Eq. (1), which yields the equations for $C_k$,

$$\frac{d\,C_k}{dt} = -\lambda_k^2\,C_k + \frac{B_k}{\tau_p}\,, \qquad 0 < t < \tau_p$$

$$\frac{d\,C_k}{dt} = -\lambda_k^2\,C_k\,, \qquad \tau_p < t < \tau\,.$$

The solution satisfying the periodicity and continuity conditions, Eq. (3), is

$$C_k(t) = \frac{B_k}{\lambda_k^2\,\tau_p}\begin{cases} 1 - \dfrac{1-e^{-\lambda_k^2(\tau-\tau_p)}}{1-e^{-\lambda_k^2\tau}}\,e^{-\lambda_k^2 t}\,, & 0 < t \leq \tau_p \\[2ex] \dfrac{1-e^{-\lambda_k^2\tau_p}}{1-e^{-\lambda_k^2\tau}}\,e^{-\lambda_k^2(t-\tau_p)}, & \tau_p < t \leq \tau\,. \end{cases} \tag{9}$$

It is of interest also to have expressions for the time-average of the temperature at any point $\xi$ and for the heat flux into the coolant at $\xi = \alpha$. The average temperature can be found by taking the average of $C_k(t)$ over one period, and is

$$T_{av}(\xi) = \frac{1}{\tau}\int_0^{\tau} T(t,\,\xi)\,dt = \sum_{k=1}^{\infty} \frac{B_k}{\lambda_k^2\,\tau}\;T_k(\xi). \tag{10}$$

The (dimensional) heat flux at $\xi = \alpha$ is

$$-K_2\,\frac{\partial\,\overline{T}}{\partial\,x}(t,\alpha) = \frac{\alpha\,K_2\,\overline{T}_r}{b}\sum_{k=1}^{\infty}\lambda_k\,A_k\,C_k(t)\left(\cos\lambda_k\,\sin\alpha\lambda_k + \frac{\alpha}{M}\sin\lambda_k\,\cos\alpha\lambda_k\right).$$

## Numerical Computations

Computations were carried out for the case in which the beam energy is totally absorbed by the target in a manner that decreases linearly with penetration depth. That is,

$$\bar{f}(x) = \begin{cases} -2\dfrac{x}{a^2} & , \quad -a \leq x \leq 0 \\ 0 & , \quad 0 < x \leq b \end{cases}$$

or

$$f(\xi) = \begin{cases} -2\xi & , \quad -1 \leq \xi \leq 0 \\ 0 & , \quad 0 < \xi \leq \alpha . \end{cases}$$

The corresponding expansion coefficients for $f(\xi)$, Eq. (7), are given by

$$B_k = \frac{2A_k}{\lambda_k^2}(1 - \cos\lambda_k) \quad , \quad k = 1, 2, 3, \cdots .$$

For this $f(\xi)$ the average temperature, Eq. (10), can be expressed in closed form as

$$T_{av}(\xi) = \frac{1}{\tau} \begin{cases} \dfrac{\xi^3}{3} - \xi + \dfrac{\alpha^2}{M} + \delta\dfrac{\alpha}{MH} , & -1 \leq \xi \leq 0 \\ -\dfrac{\alpha}{M}\xi + \dfrac{\alpha^2}{M} + \delta\dfrac{\alpha}{MH} , & 0 < \xi \leq \alpha . \end{cases} \tag{11}$$

A Chippewa FORTRAN program was written to sum the series, Eq. (8), for the temperatures at desired values of x and $\bar{t}$; the necessary eigenvalues, Eq. (5), were calculated by using a revised version

of the program described in Ref. 3 for calculating similar eigenvalues. A typical case run with the program was one for a 0.01-cm platinum target and a 0.3-cm copper backing (for which $\alpha = 16.3$ and $M = 39.3$ to three figures) with $P = 10$ kW, $\tau = 0.06$ sec, $R = 10$ cm, $\epsilon = 1.0$ cm, and $\overline{T}_0 = 160°C$ with $\delta = 0$ (fixed coolant temperature). About 30 terms in Eq. (8) were required to give an error of less than about 0.5°C in $\overline{T}$ for small $t$ (the order of $\tau_p$), but far fewer terms were required to give this accuracy for larger $t$. Calculations of $T_{av}$ from Eqs. (10) and (11) agreed to within the expected accuracy. The time required to calculate $\overline{T}$ for a grid of ten different $x$ and eleven different $t$ values, including the eigenvalue computations, was less than half a second on the CDC 6600 computer. (For $\delta = 1$, slightly more time was required.) Figures 3, 4, and 5 show the behavior of the solution. The results of other calculations and engineering design conclusions are presented elsewhere[4].

It should be noted that the present study requires that $\delta$ be constant at either 0 or 1 in the boundary condition at $\xi = \alpha$, Eq. (2). The case in which $\delta$ switches between 0 and 1 as a function of $T(t, \alpha)$ and $\frac{dT}{d\xi}(t, \alpha)$ (intermittent coolant boiling) is not included here, and requires additional formulation.

This work was performed in conjunction with Finn Reinath, who suggested the original problem, and Raymond Schwarz, who wrote portions of the computer program.

## Footnote and References

*Work done under auspices of the U. S. Atomic Energy Commission.

1) Rex Booth, Rotating Neutron Target System, IEEE Trans. Nucl. Sci., Vol. NS-14 (1967) 938-941.

2) P. Concus and D. R. Olander, Transient Diffusion in a Composite Slab, Intern. J. Heat and Mass Transfer Vol. 11 (to appear).

3) Paul Concus, Table of the Solutions of a $\tan(\pi x) = -b \tan(a\pi x)$, Lawrence Radiation Laboratory Report UCRL-17608, June 1967.

4) Finn Reinath, Mech. Eng. Note, Lawrence Radiation Laboratory, Berkeley (in preparation).

## Figure Captions

Fig. 1. Target system.

Fig. 2. One-dimensional problem.

Fig. 3. Temperature vs time at the target interfaces, $x = -a$ and $x = 0$.

Fig. 4. Temperature at the beginning and end of the heating pulse and the time-average temperature vs distance.

Fig. 5. Heat flux into coolant at $x = b$ vs time.

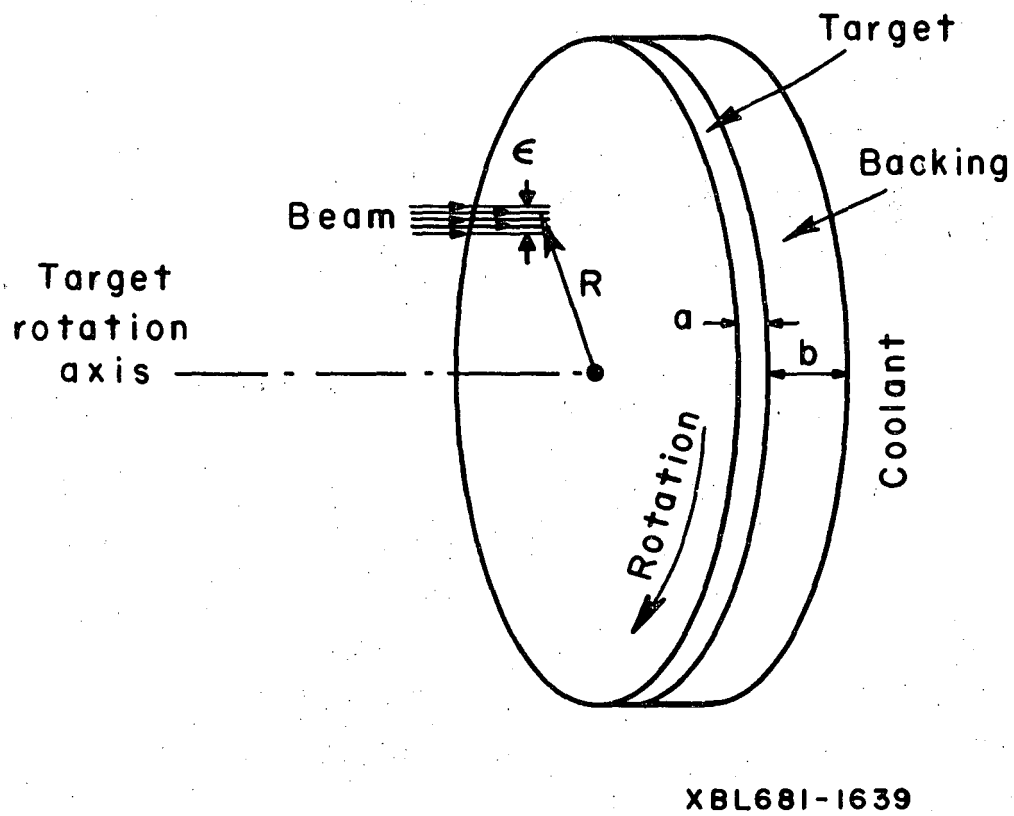
Target
Backing
ϵ
Beam
R
Target
rotation
axis
a
b
Coolant
Rotation
XBL681-1639

Fig. 1

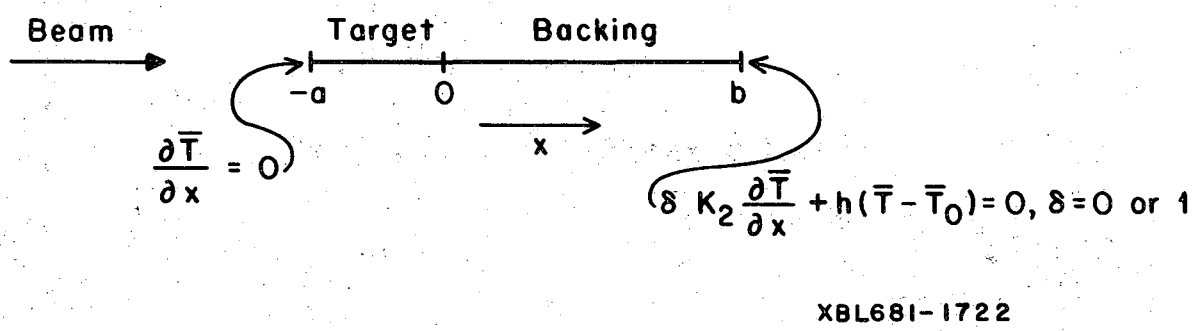
Beam
Target
Backing
−a
0
b
x
$\frac{\partial \overline{T}}{\partial x} = 0$
$\delta K_2 \frac{\partial \overline{T}}{\partial x} + h(\overline{T} - \overline{T}_0) = 0, \delta = 0$ or 1
XBL681-1722

Fig. 2

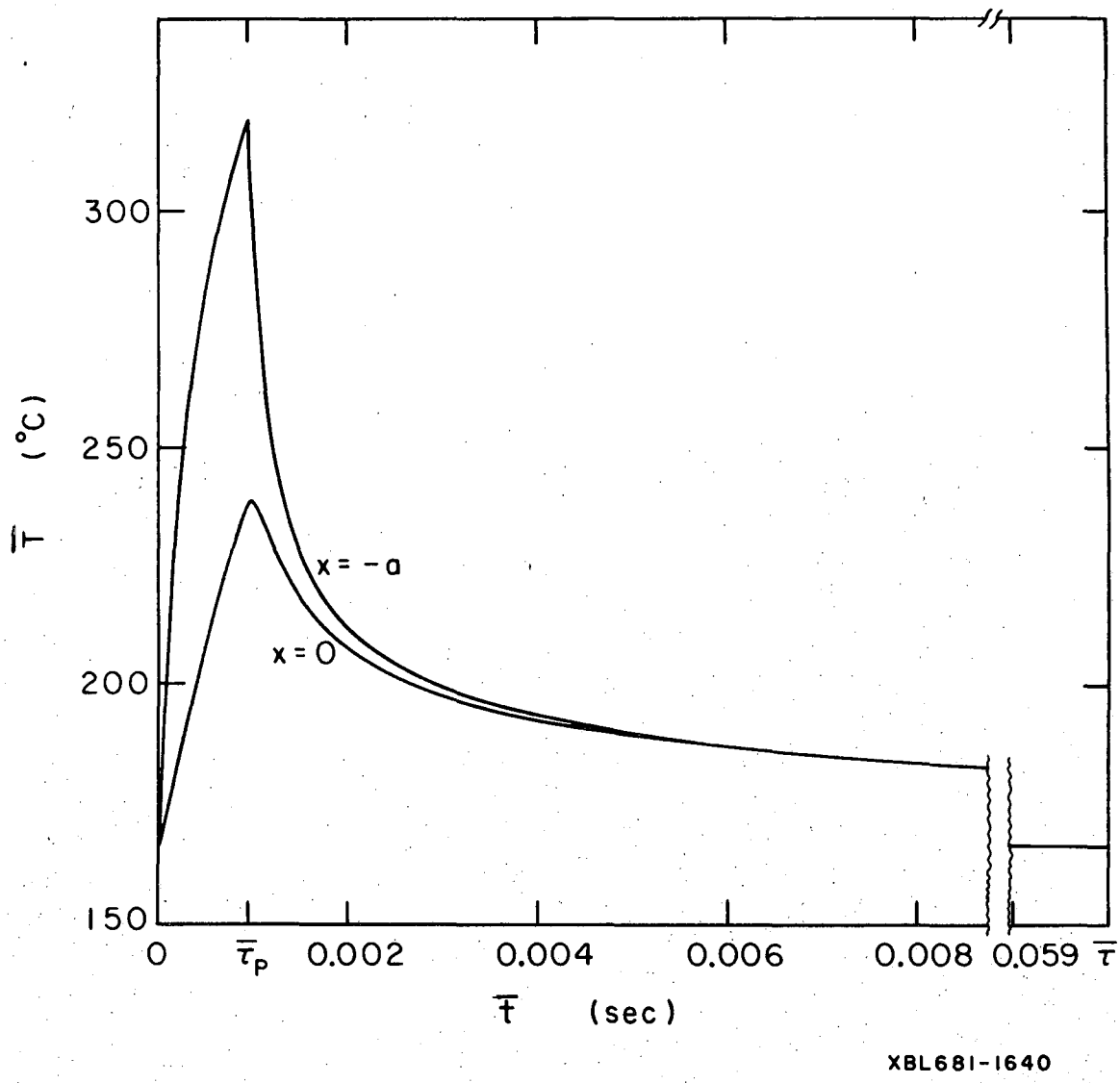
300
250
200
150
$\bar{T}$ (°C)
x = -a
x = 0
0
$\bar{\tau}_p$
0.002
0.004
0.006
0.008
0.059
$\bar{\tau}$
$\bar{t}$ (sec)
XBL681-1640

Fig. 3

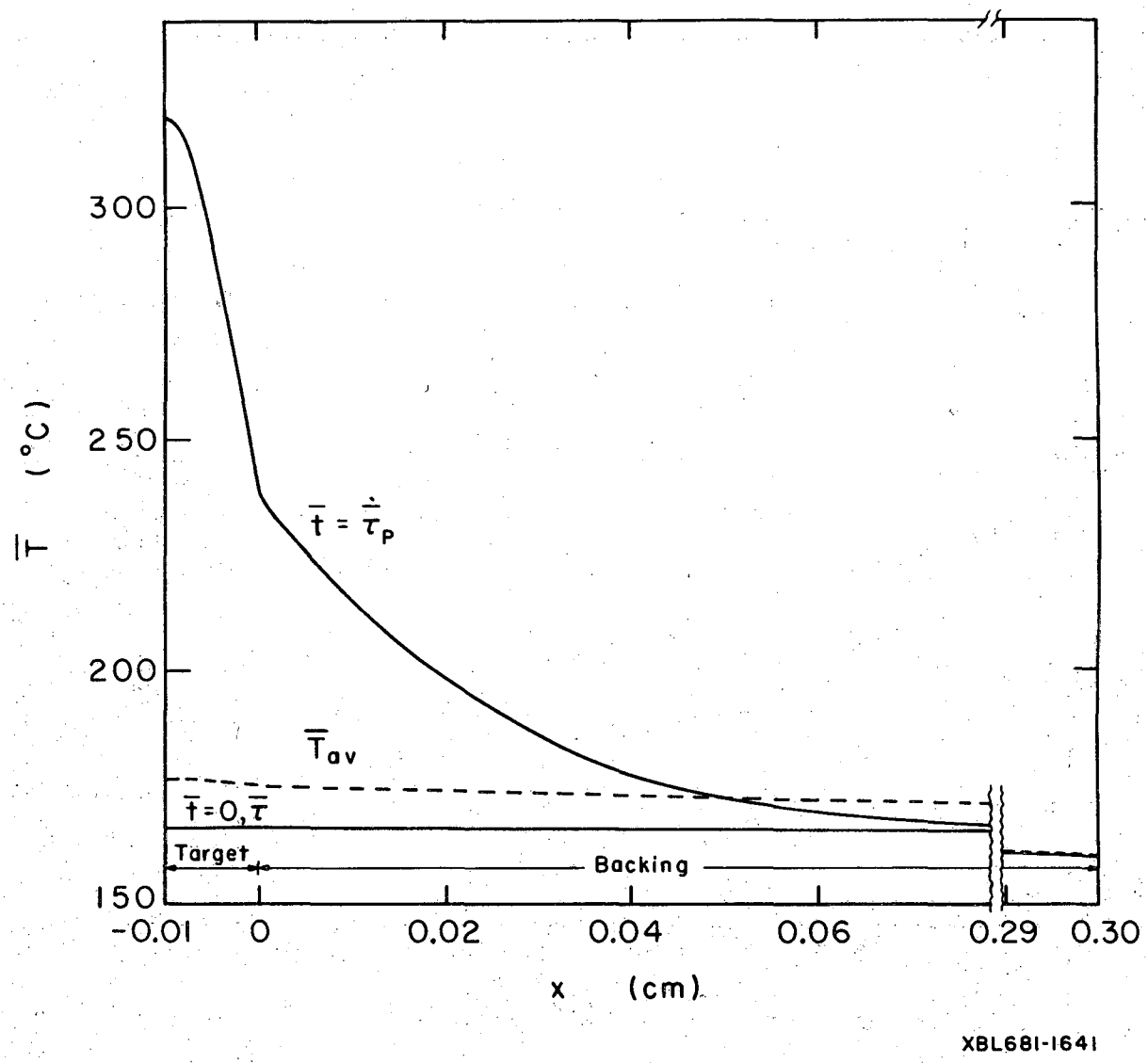
300
250
200
150
$\bar{T}$ (°C)
$\bar{t} = \bar{\tau}_p$
$\bar{T}_{av}$
$\bar{t} = 0, \bar{\tau}$
Target
Backing
-0.01
0
0.02
0.04
0.06
0.29
0.30
x (cm)
XBL681-1641

Fig. 4

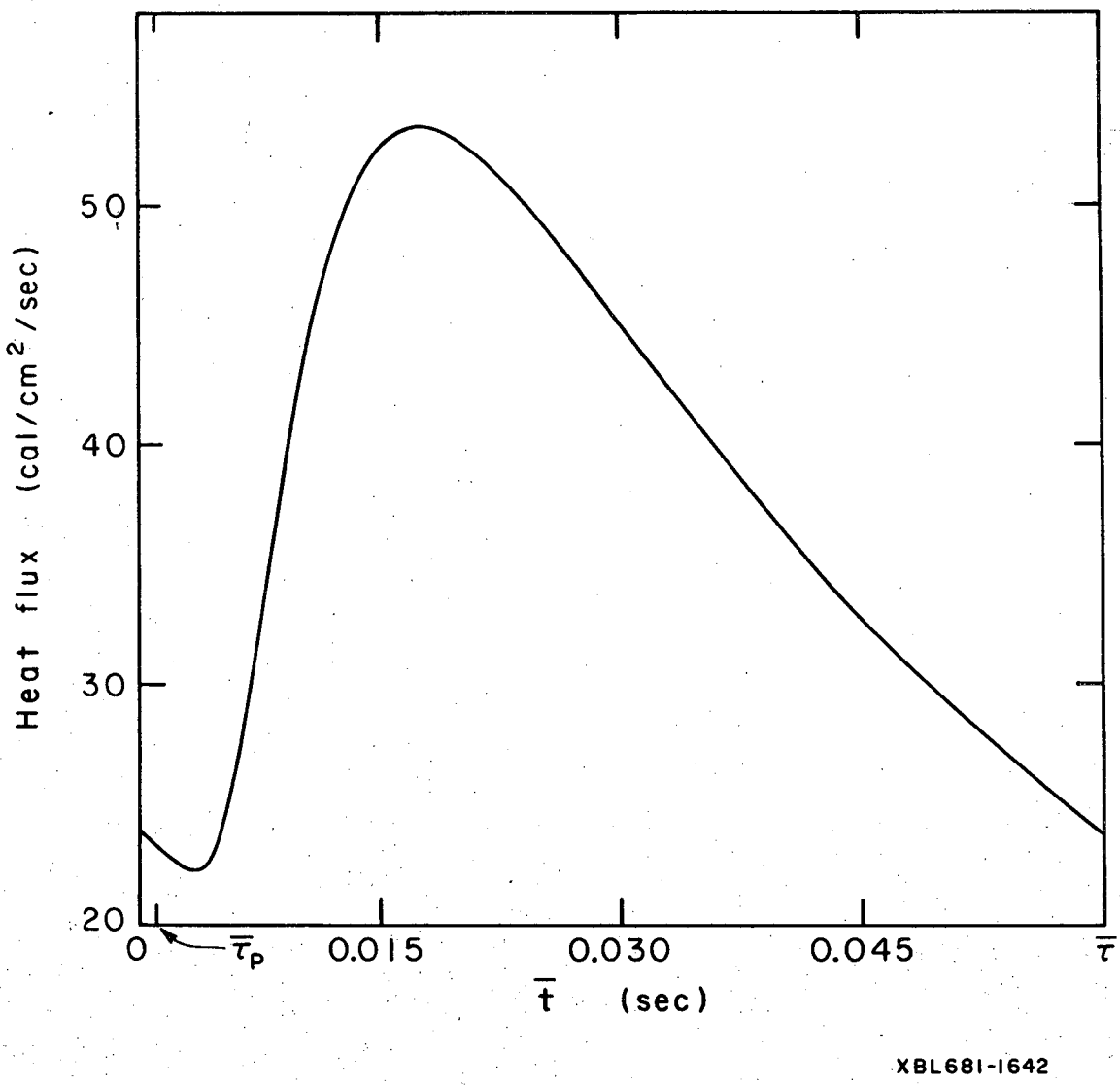
Heat flux (cal/cm$^2$/sec)
50
40
30
20
0
$\bar{\tau}_p$
0.015
0.030
0.045
$\bar{\tau}$
$\bar{t}$ (sec)
XBL681-1642

Fig. 5

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