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Phase contours are curves along which the phase of a collision amplitude is a real constant. In their simplest form they describe the phase as a function of real energy and momentum transfer and provide a convenient summary of experimental results for a scattering process. More generally they can be used to provide consistency tests in the dynamics of strong interactions of elementary particles.

We will give a brief discussion of the following topics:

(1) Properties of phase contours.

(2) Phase contours for pion-nucleon scattering below 1.4 GeV.

(3) Phase contours in a Regge model for pion-nucleon scattering.

(4) The use of phase contours for studying consistency between resonance poles, zeros and high energy behavior in a crossing symmetric model.

1. PROPERTIES OF PHASE CONTOURS

The phase $\phi(s,t)$ of an invariant scattering amplitude $F(s,t)$ is defined by

$$\phi(s,t) = \mathrm{Im}[\log F(s,t)], \tag{1}$$

where Im denotes the imaginary part, and s,t denote the invariant energy and momentum transfer variables. It is also necessary to define the phase at an initial point (s_0,t_0). When the amplitude has zeros or poles on the physical sheet, the phase depends on the route taken from (s_0,t_0) to (s,t), so we must always specify the route.

A phase contour is defined by

$$\phi(s,t) = C, \tag{2}$$

where C is a real constant. It is useful to study phase contours both for real s and t, and for complex s and fixed t, etc. Their properties include:

(a) Phase contours, for different values of C, do not meet each other except at zeros and poles of the amplitude and at certain other singularities.

(b) The phase change clockwise in the complex plane round a zero is -2π, and round a pole is 2π.

(c) For fixed t, and $s = s_1 + is_2$, the phase is an harmonic function of s_1 and s_2, and the phase contours are orthogonal to the modulus contours in this complex s plane.

(d) From the optical theorem,

$$\mathrm{Im}F(s,0) > 0, \qquad \text{for} \quad s > (m + M)^2. \tag{3}$$

Assuming an asymptotically constant total cross section and the Pomeranchuk theorem,

$$F(s,t) \sim isB, \qquad \text{as} \quad s \to +\infty. \tag{4}$$

We choose our initial point (s_0, t_0) as the limit along $(s + i0,\ t = 0)$ as $s \to +\infty$, and take

$$\phi(s \to +\infty,\ 0) = \frac{1}{2}\pi. \tag{5}$$

(e) The phase contours $\phi(s,t) = 0$, or π, cannot enter the region $0 \leqslant t < 4m^2$, when $s > 4m^2$.

(f) If the scattering amplitude has power behavior as $s \to \infty$, the phase is asymptotically constant. In the particular case of a symmetric amplitude, or a Regge term of even signature, we can write

$$F(s,t) \sim \beta(t)\, s^{\alpha(t)} \exp[i\pi\{1 - \tfrac{1}{2}\alpha(t)\}]. \tag{6}$$

If $\beta(t)$ is non-zero in the range $[0,t]$, and $\alpha(t)$ is real, then the asymptotic phase is given by

$$\phi(s,t) \sim \pi[1 - \tfrac{1}{2}\alpha(t)], \tag{7}$$

so the asymptotic phase contours are parallel to $t = \text{const}$.

(g) Assuming asymptotic power behavior s^{α} of $|F|$, the power $\alpha(t)$ can be determined from the phase contours along the real s axis and the zeros of F in the complex s plane (and poles if there are bound states). The location of zeros is of great importance. They are indicated by intersections of phase contours, and their complex locations can be studied by means of phase contour surfaces when both s and t are complex variables in Eq. (1) and (2).

2. PHASE CONTOURS IN PION-NUCLEON SCATTERING

We have used the results of phase shift analysis of pion-nucleon scattering experiments to obtain phase contours for invariant amplitudes,[1] in the energy range

$$0 < T_{\pi} < 1.4 \text{ GeV}, \tag{8}$$

where T_{π} denotes the pion kinetic energy. The amplitudes used are the invariant combinations of $\pi^- p$ and $\pi^+ p$ scattering amplitudes[2]

$$A'^{(+)},\ B^{(+)},\ A'^{(-)},\ B^{(-)}. \tag{9}$$

The resulting phase contour diagrams provide a useful visual aid for comparing different phase shift solutions and for indicating regions where the resulting scattering amplitude is complicated. In these regions one can conclude that more experiments are desirable, especially in the region around $T_{\pi} = 0.8$ GeV, where there are several resonances and the phase contours for the last three amplitudes listed in (9) indicate unusually rapid phase variations.[1]

Phase contours for $A'^{(+)}$, derived from the Lovelace phase shifts,[3] are shown in Fig.1. There are two real zeros of this amplitude, indicated by intersections of phase contours, and it is evident that there are some nearby complex zeros. Complex zeros are indicated by a bunching of phase contours, for example in Fig. 1 between 0.2 and 0.6 GeV. The modulus contours[1] for $A'^{(+)}$ show a valley in this region, where the amplitude is very small.

3. PHASE CONTOURS IN A REGGE MODEL FOR PION-NUCLEON SCATTERING

The method of phase contours is useful in studying the relation between high energy and low energy scattering. It can be applied either as a consistency method, or as an aid in comparing phenomenological solutions for a given scattering process. We illustrate the latter by giving in Fig. 2 the phase contours for $A'^{(+)}$ that are obtained by extrapolating from a Regge model for pion-nucleon scattering. Our model is not meant to be realistic at this stage, but it gives surprisingly good agreement with the phase contours shown in Fig. 1 out to 90 deg for $T_\pi > 1$ GeV.

Our Regge model is the sum of a Regge solution[4] for near-forward πN scattering based on P, P′ and ρ exchange, and a Regge solution[5] for near-backward πN scattering based on N and N* exchange. We have also assumed that the trajectories fall linearly as t decreases (or as u decreases), and have included effects from zeros of residues.

The modulus contours for $A'^{(+)}$, that correspond to Fig. 1 and Fig. 2, also show a striking resemblance in the low energy region.[1] However, the Regge solutions for the other three amplitudes do not resemble the solutions obtained from phase shift analysis. A comparison of their phase contour diagrams[1] indicates that a more complicated Regge model is required in this low energy region.

4. RESONANCE POLES, ZEROS AND HIGH ENERGY BEHAVIOR

Phase contours provide a method for studying part of the consistency problem in strong interactions, and may help towards understanding whether the bootstrap problem can be formulated in a meaningful approximation. We do not introduce unitarity at this stage, but consider only the consistency that is required by crossing symmetry for a given high energy behavior, taking into account the associated resonance poles (Regge poles). We consider[6] a symmetric scattering amplitude for equal mass spinless bosons, and construct a class of models that is consistent with a high energy behavior based on Regge terms of the type indicated in Eq. (6). Our model includes effects of zeros of residues below threshold, and resonance poles above threshold on unphysical sheets. We also assume dominance by a leading Regge pole with a continuously rising trajectory.

We find zeros of the scattering amplitude from three sources;[6] (1) from symmetry requirements, there are zeros along $s = u$ in $t < 0$, for s real $(s + i0)$ and u real $(u - i0)$ above and below their branch cuts respectively; (2) from zeros of residues, there are

zeros of the amplitude that move in from infinity along $Re(s) = Re(u)$ as t varies in $t < 0$; (3) from interference between resonance poles. By studying the phase contour surfaces for complex s and t, we find that these three types of zero may be identified as lying on different parts of two dimensional complex surfaces of zeros in the four dimensional space.[6]

Further analysis of phase contours is in progress and further details of the above work will be given elsewhere.[1,6]

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FOOTNOTES AND REFERENCES

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† At the Cavendish Laboratory, Cambridge, England, after January 1, 1968.

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FIGURE CAPTIONS

Fig. 1. Phase contours for the $A'^{(+)}$ amplitude in pion-nucleon scattering derived from the (1966) Lovelace phase shift solutions.

Fig. 2. Phase contours for $A'^{(+)}$ derived from a Regge model for pion-nucleon scattering.

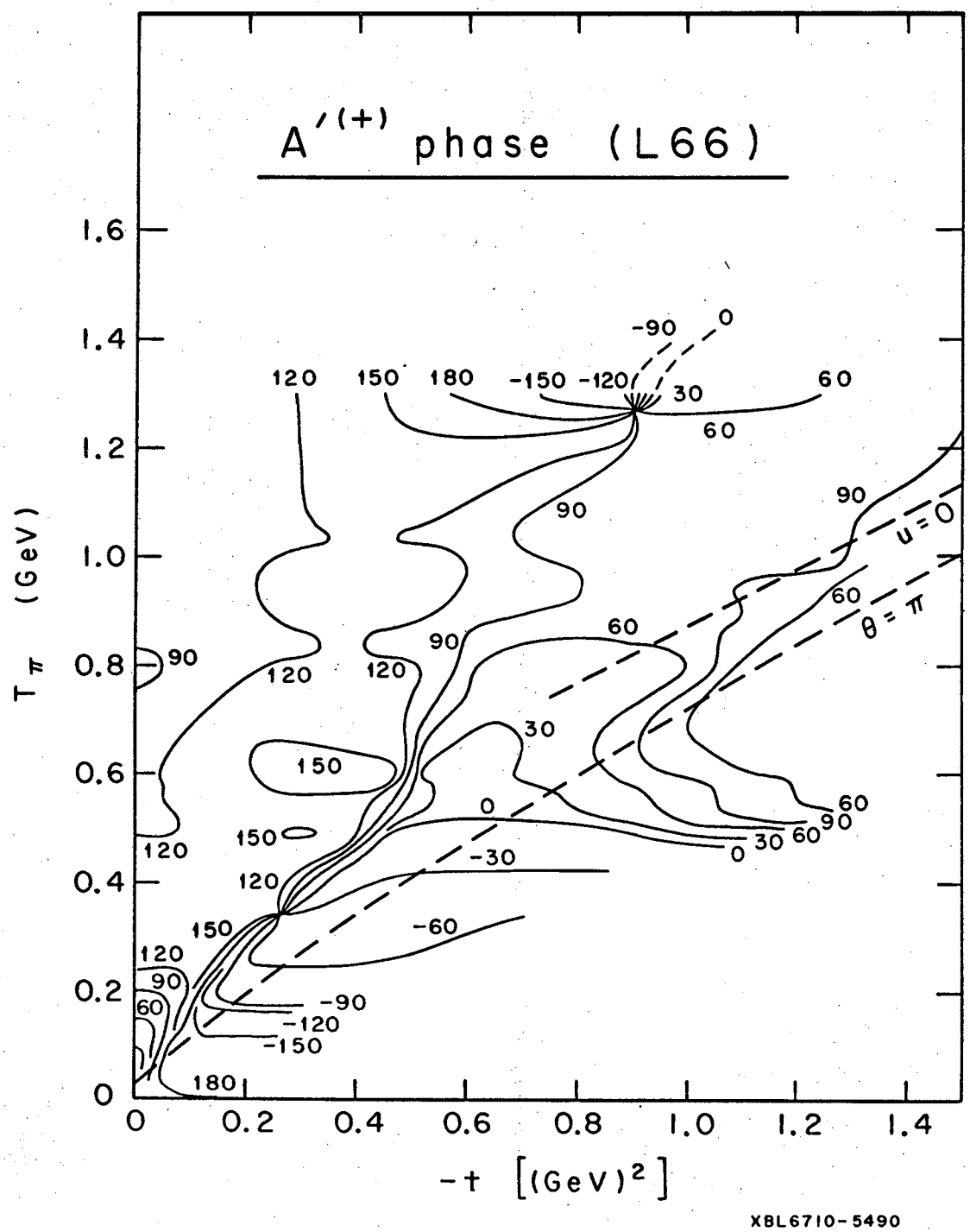

Fig. 1

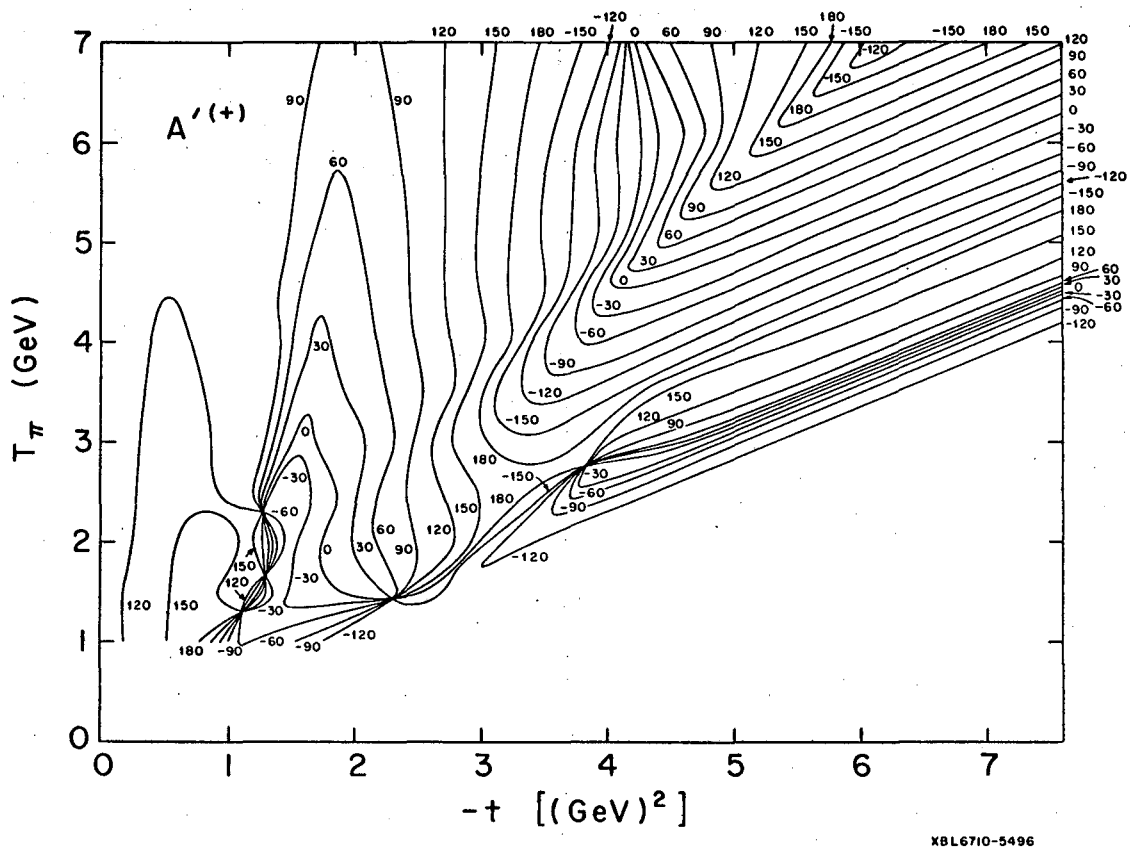

Fig. 2

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