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PION-PROTON ELASTIC SCATTERING:
LOW ENERGY EXPERIMENTS AND PHASE SHIFT ANALYSES

Claiborne Johnson and Herbert Steiner

December 1967

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Invited paper presented by H. Steiner
at the Conference on πN Scattering
held at the University of California,
Irvine, Dec. 1 and 2, 1967

UCRL-18001
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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I. INTRODUCTION

I guess the thought of the organizing committee was that before turning the theorists loose on you it might be useful to review briefly the experimental situation and the results of the various phase shift analyses. I will confine myself to $p_\pi \leq 2$ GeV/c . So as not to keep you in suspense let me summarize this summary with the comments that no startling new experimental results have been reported during the last year, although some further cross section and polarization measurements have been completed. The phase shifters on the other hand have been busy--so busy in fact that the number of reported πN resonances has doubled since the Berkeley Conference of last year.

What I would like to do then is first to bring you up-to-date on the present status of πP elastic scattering experiments below $p_\pi \sim 2$ GeV/c. Then I will discuss the phase shift situation--the methods used, the results obtained, and some of the problems encountered by the various groups engaged in this type of work.

II. EXPERIMENTS

A. Possible Experiments

All of the experiments that can possibly be done by elastically scattering pions on nucleons can be conveniently summarized by the equation:

$$I < \sigma_\mu >_f = I_o \sum_{\nu=0}^{3} D_{\mu\nu} < \sigma_\nu >_i$$

where I = the scattered intensity

I_o = the scattered intensity when the initial state nucleon is unpolarized

$\sigma_\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3)$ where σ_0 is the (2x2) unit matrix, 1 , and $\sigma_1, \sigma_2, \sigma_3$ are the three Pauli spin matrices.

the subscript f and i refer to the initial and final states respectively. $D_{\mu\nu}$ is the depolarization operator which acts on the initial state of polarization to produce the final polarization state. For example D_{jo} j = 1,2,3 refers to that experiment in which the 3 components of the vector polarization of the nucleon are determined when the initial state is unpolarized. Similarly $D_{k\ell}$ would describe the experiment where the target proton is polarized along the "ℓ" direction, and measurement is made of the final proton's polarization in the "k" direction.

In terms of the M-matrix which acts on the wave function describing the initial state to produce the final state wave function, D can be written

$$D_{\mu\nu} = \frac{1}{2}\,\mathrm{Tr}(M\sigma_\nu M^+\sigma_\mu)\Big/\frac{1}{2}\,\mathrm{Tr}(MM^+)$$

An explicit representation for D can be obtained if one choses a specific form for the M-matrix. For example let us use the oft-used form

$$M = G\mathbf{1} + iH\vec{\sigma}\cdot\hat{n}$$

Where G and H are functions of c.m. energy and angle, $\mathbf{1}$ is the (2×2) unit matrix, $\vec{\sigma}\cdot\hat{n}$ is the component of the spin operator in the direction normal to the scattering plane; i.e., $\hat{n} = \hat{k}_i \times \hat{k}_f$

$\hat{k}_i$ is the direction of the incident nucleon in the center-of-mass (along the +z axis)

$\hat{k}_f$ is the direction of the final nucleon in the center-of-mass (in x-z plane at angle θ with respect to the z-axis).

Then D can be written:

$$D = \begin{array}{c|cc|cc} f \backslash i & 0 & y & z & x \\ \hline 0 & 1 & \alpha & 0 & 0 \\ y & \alpha & 1 & 0 & 0 \\ \hline z & 0 & 0 & \gamma & \beta \\ x & 0 & 0 & -\beta & \gamma \end{array}$$

where $I_o\alpha = 2\mathrm{Im}\ GH^*$

$I_o\beta = 2\mathrm{Re}\ GH^*$

$I_o\gamma = |G|^2 - |H|^2$

$I_o = |G|^2 + |H|^2$

The z-x plane is the scattering plane, y is the normal to the scattering plane.

There are no elements of D connecting the (o,y) components with the (z,x) components. This is a consequence of parity conservation in the scattering process. (I chose the rather odd order of labeling the elements of the matrix written above to show its block diagonal nature when parity is conserved.) Note also that γ=1 means that there is no spin-flip whereas γ = -1 implies that there is only spin flip.

The various elements of the D-matrix can be directly related to the so-called Wolfenstein parameters.[1] For example

$$P = D_{oy} = D_{yo} = \alpha$$

$$D = D_{yy} = 1$$

A and R are related to β and γ as will be shown below. From the experimental point of view every time you see a non-zero element in any of the last three columns it means that that experiment involves the use of a polarized target. Everytime you see a non-zero element in any of the last three rows it means that the experiment involves an additional scattering to analyze the polarization of the final state nucleon. Thus only P and I_o can be determined by experiments involving a single scattering. This is the main reason that most of the experimental effort up to now has been devoted to measurements of these parameters.

One slight complication arises due to the fact that most high energy experiments up to now at least have been done not in the center-of-mass but in the laboratory system. The polarization components normal

to the scattering plane are unchanged under transformation from center-of-mass to laboratory frames of reference. On the other hand care must be taken in relating measurements in the lab involving polarization components in the plane of the scattering to the components of the depolarization tensor which is defined in the center-of-mass. In 1954 Wolfenstein[1] introduced the parameters A and R to describe the change of polarization in the plane of the scattering of the incident particle in the lab. In πN scattering where the incident projectile has spin zero it is convenient to introduce analogous parameters A_{recoil} and R_{recoil} which refer instead to the change of the target nucleon's polarization in the plane of the scattering. (See Fig. 1.)

$$A_{recoil} = -\beta \cos(\phi-\phi_L) + \gamma \sin(\phi-\phi_L)$$

$$R_{recoil} = +\beta \sin(\phi-\phi_L) + \gamma \cos(\phi-\phi_L)$$

Up to now only experiments involving measurements of total cross sections, differential cross sections, and Polarization normal to the scattering plane have been reported. However, A_{recoil} and R_{recoil} measurements are presently underway at CERN by a group from Saclay. These measurements are being made at energies higher than those considered in this report, but it seems likely that within the next few years we can expect to obtain information on β and γ also at lower energies. It should be kept in mind that A and R-type measurements involve both the use of a polarized target and the subsequent analysis of the recoil proton's polarization. Consequently experimental complications are likely to limit the accuracy and scope of these

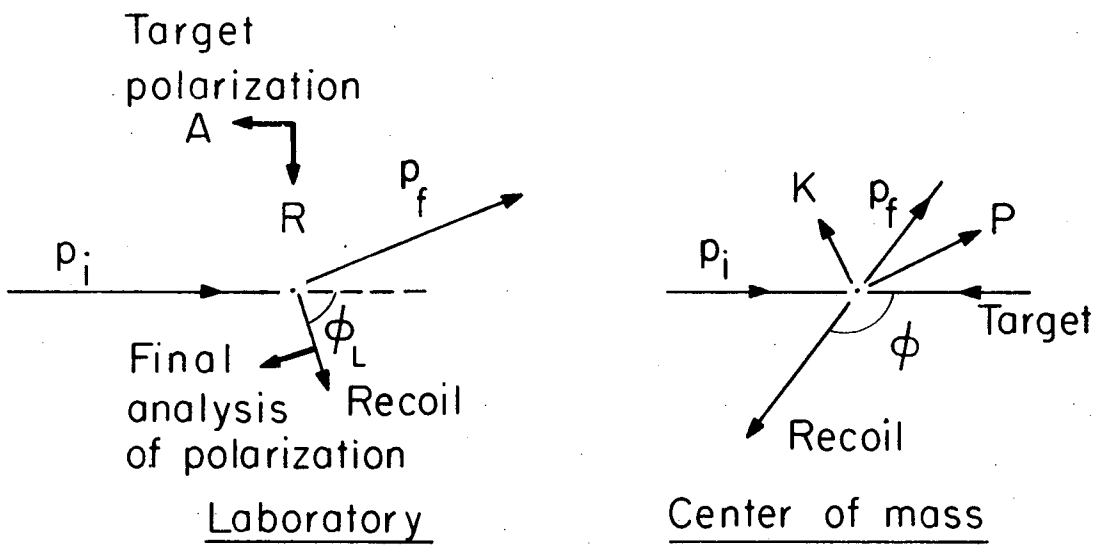

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Fig. 1. Geometry for measurement of depolarization parameters, **R** and A, using a polarized target.

measurements for a while at least.

B. New Results 1966-1967

There have been five experiments reported during the last year. A group at University College and Westfield College, London has measured $d\sigma/dt$ for π^-p scattering at 5 momenta between 1.72 and 2.46 GeV/c,[2] and for π^+p scattering at 10 momenta between 1.72 and 2.80 GeV/c.[3] In Fig. 2 we show some of these results. The purpose in showing these results here is to give you some idea of the quality of the data and the angular range covered. These results are typical or perhaps even slightly better than other measurements of the same type reported by other groups in the past at different energies.

A group at Brookhaven has reported[4] measurements of differential cross sections in backward directions for both π^+p and π^-p elastic scattering. These results were presented in one of the contributed papers to this conference.

New measurements of π^+p and π^-p total cross sections for momenta between .5 and 2.65 GeV/c have recently been reported. These results, which are of very high quality, were obtained in a collaborative effort between Cambridge, Rutherford, and Birmingham at Nimrod.[5]

Another group,[6] also at Nimrod, has made very detailed measurements of polarization in π^-p scattering at 50 different momenta between .64 and 2.14 GeV/c. As you will see in a moment, they did a very thorough job of it.

The last experiment I want to mention is also a measurement of the polarization in $\pi^\pm p$ scattering at 364, 425, 490 and 532 MeV/c at the

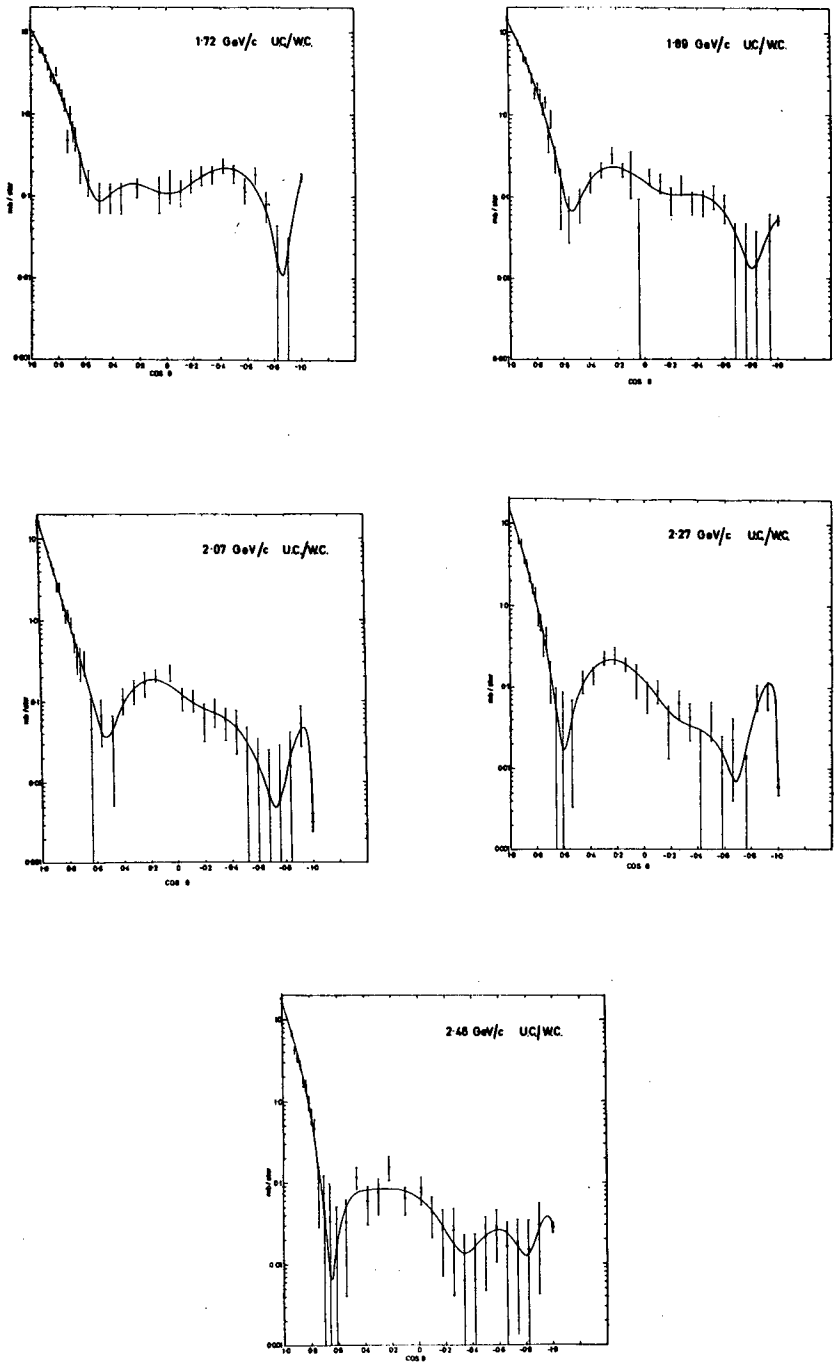

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Fig. 2.

$\pi^- p$ differential cross sections from Reference 2.

184-inch synchrocyclotron at Berkeley.[7] These results are not in final form but are of interest nevertheless because they disagree in places with the results of similar experiments reported earlier. In particular the polarizations in π^+p scattering at 532 and π^- at 427 MeV/c are shown in Figs. 3 and 4. Also shown are the results of previous experiments and the phase shift fits. It is interesting to point out that although the data have changed, the phase shifts needed to fit the new results differ only very minimally from those which fit the earlier data.

C. Summary of Current Status of πp Scattering Experiments

In Figs. 5 and 6 we have listed the momenta at which various measurements have been made. It is seen that the differential cross sections for π^+ and π^- elastic scattering are in pretty good shape, although I would add that experiments of higher accuracy would be very welcome indeed. This last statement applies even more to differential cross section data for the process $\pi^-p \rightarrow \pi^0n$. The results here are not as reliable as one could wish and further measurements would be very useful, especially at momenta of 1180, 1360, and 1440 MeV/c.

With respect to polarization measurements the greatest need obviously lies in π^-p charge exchange. In the energy region considered here a total of one measurement, at one angle at one energy (310 MeV) exists[7]--and even it is of rather low statistical accuracy. It is worth noting that charge exchange polarization results in the momentum region $2 \leq p_\pi \leq 5.5$ GeV/c have recently been reported by a group from Argonne.[8] These measurements, though far from easy, are feasible and are likely to become more extensive in the near future.

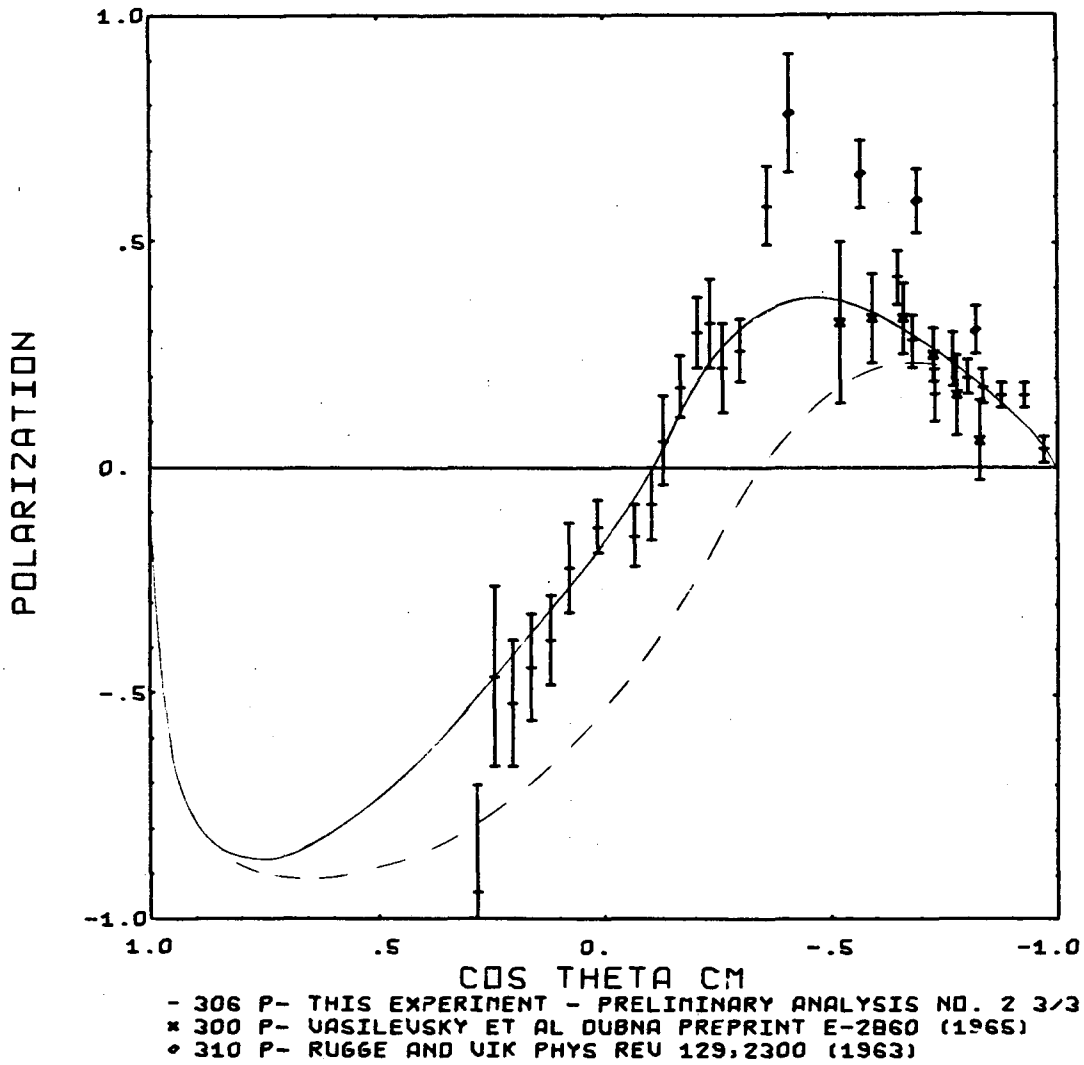

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Fig. 3.

Polarization in $\pi^- p$ scattering at 306 MeV as reported Gorn, et al.[7] The dashed and solid curves are phase-shift fits obtained before including and after including the new data.

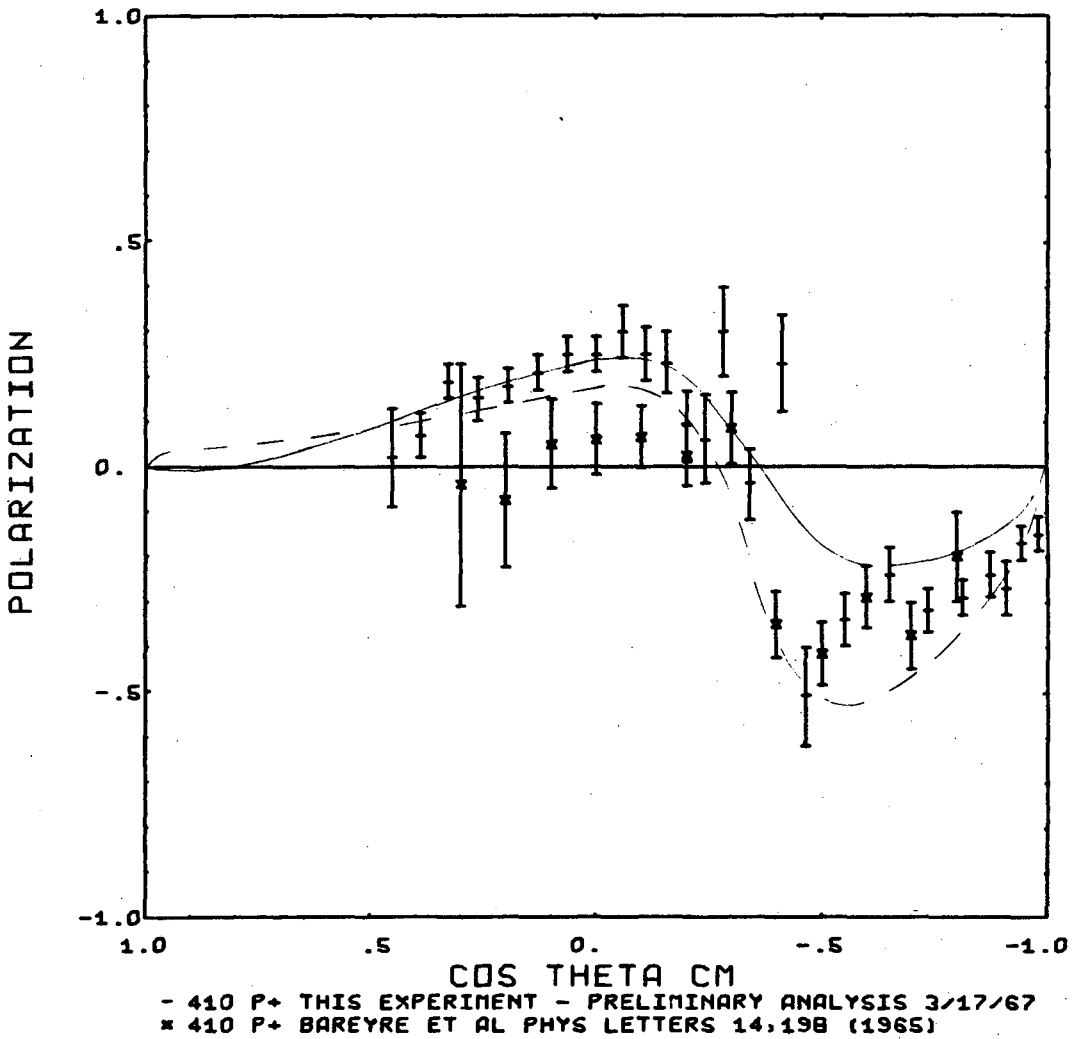

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Fig. 4. Polarization in π^+p scattering at 410 MeV as reported by Gorn, et al.[7] The dashed and solid curves are phase-shift fits obtained before including and after including the new data.

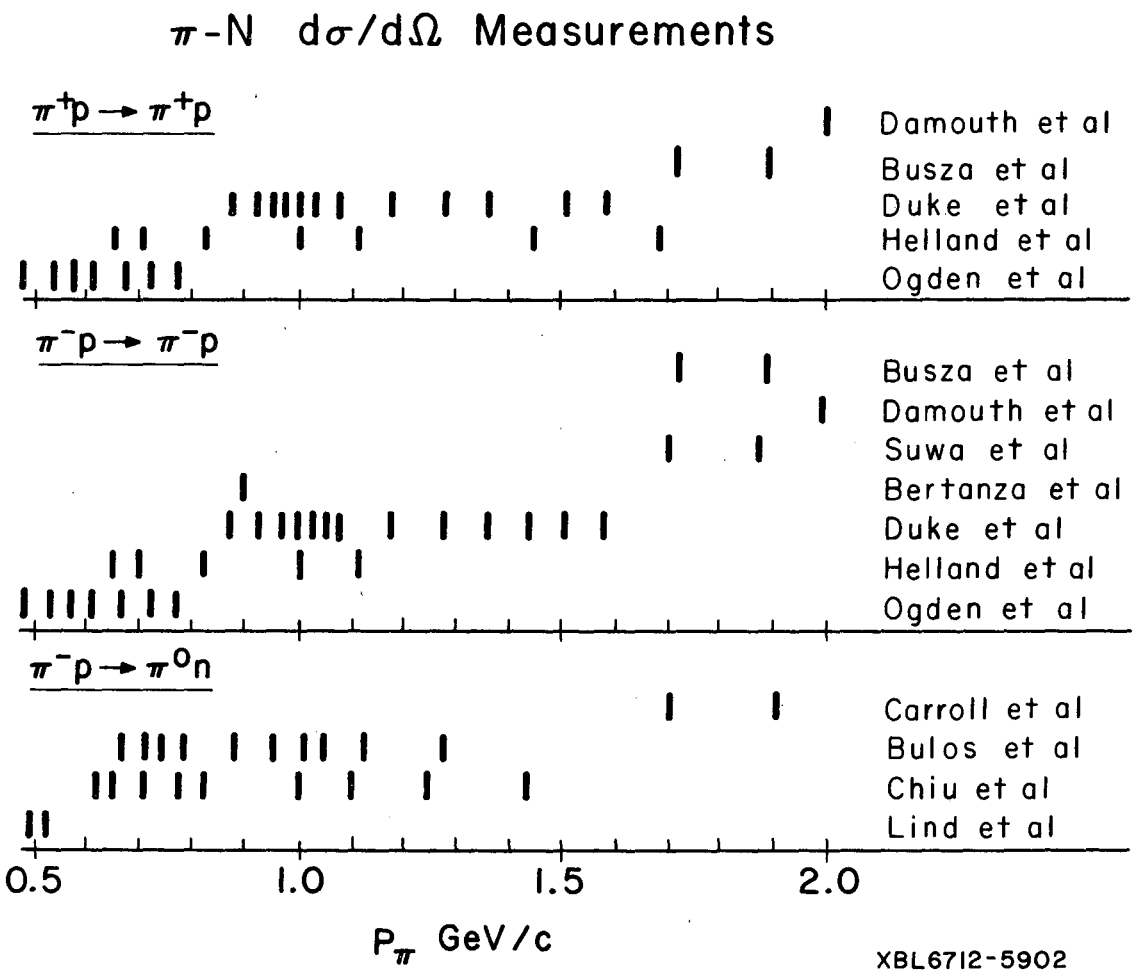
π-N dσ/dΩ Measurements
π⁺p → π⁺p
Damouth et al
Busza et al
Duke et al
Helland et al
Ogden et al
π⁻p → π⁻p
Busza et al
Damouth et al
Suwa et al
Bertanza et al
Duke et al
Helland et al
Ogden et al
π⁻p → π⁰n
Carroll et al
Bulos et al
Chiu et al
Lind et al
0.5
1.0
1.5
2.0
P_π GeV/c
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Fig. 5.

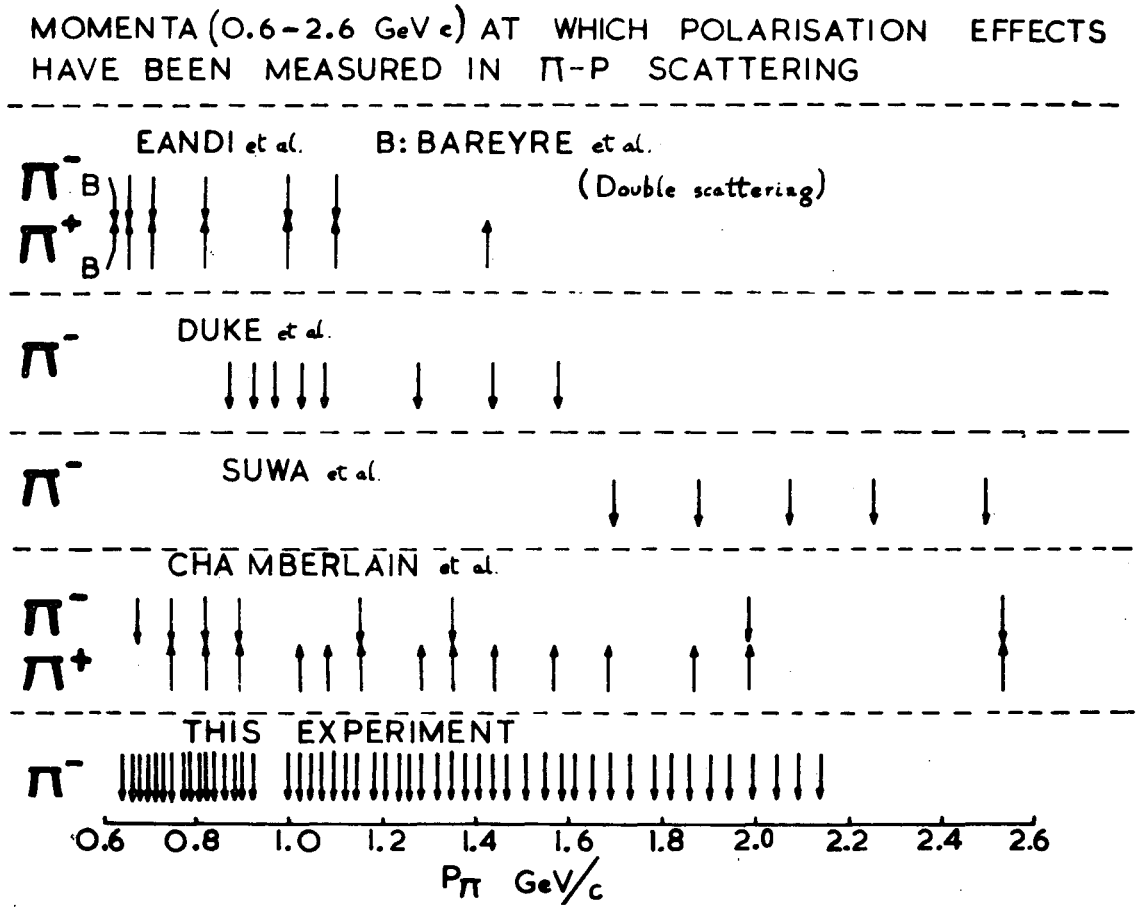

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Fig. 6.

I think most of the phase shifters here would agree that this type of experiment is probably of greatest interest at present. The polarization parameter in $\pi^- p$ elastic scattering has been very thoroughly measured--especially by the Nimrod Group[6] --so that for the time being at least no further measurements are called for. Polarization in $\pi^+ p$ scattering, on the other hand, has not been investigated as exhaustively and more detailed measurements would be welcome. To give you an idea of the quality of the existing polarization data, the results of the Berkeley group[10] at p_π=1.352 GeV/c are shown in Fig. 7. These are typical of the results obtained by other groups at different energies. A complete summary of references of the published πp experiments may be found in Reference 11.

It might be worthwhile to try to see if there are any trends or patterns to be observed in looking at these data. There are significant forward peaks in $d\sigma/dt$ for $\pi^+ p \to \pi^+ p$, $\pi^- p \to \pi^- p$, and even in $\pi^- p \to \pi^0 n$. Generally speaking the first minimum for $\pi^- p$ elastic scattering is at $t \simeq -.5\ (\mathrm{GeV})^2$ whereas the corresponding minimum for $\pi^+ p$ elastic scattering is somewhat wider, say $t \simeq -.6$ to $-.8\ (\mathrm{GeV})^2$. The polarization results near $t = 0$ usually show $P_{\pi^+} > 0$ whereas P_{π^-} is most often negative. The first zero in the polarization parameter for both π^+ and π^- almost always is closely correlated with the position of the minimum of the differential cross section. Typically it occurs somewhat before the cross section is at its first minimum.

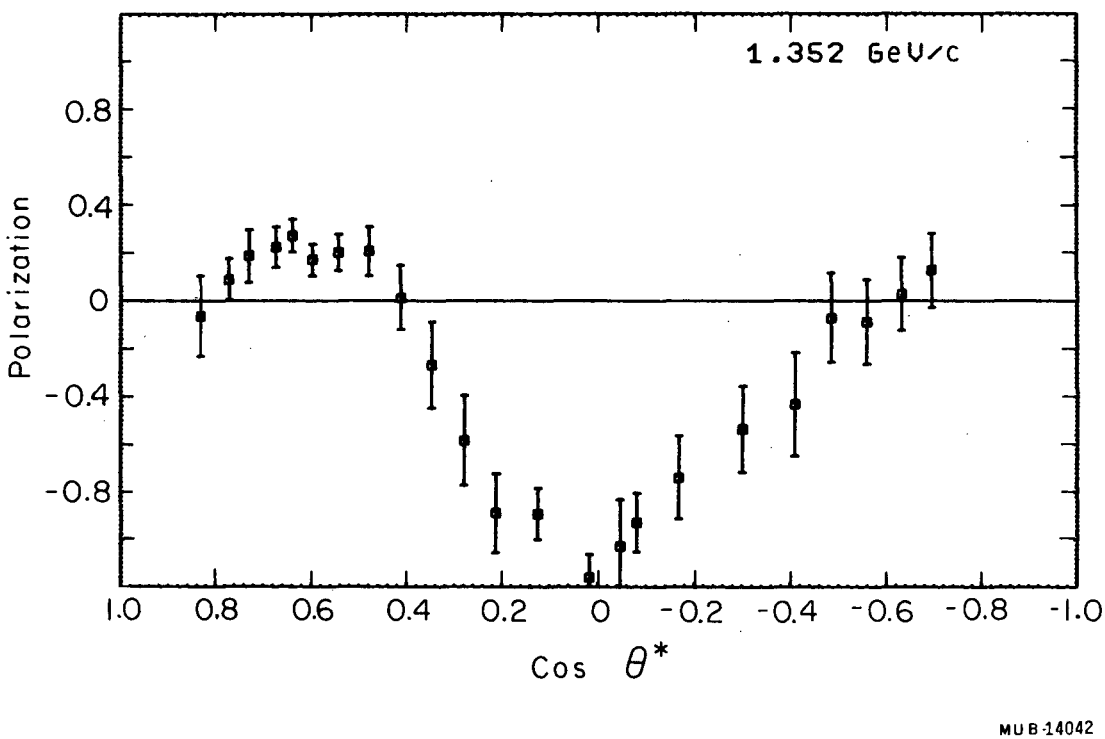


Fig. 7.

Polarization in $\pi^- p$ scattering at p_π=1.352 GeV/c as reported by Chamberlain, et al.[10]

An examination of the relevant Legendre coefficients in the expansions for I_o and I_oP, i.e.,

$$I_o = \lambda\!\!\!^{-2} \sum_{\ell=0}^{2\ell_{max}} A_\ell \, P_\ell(\cos\theta)$$

$$I_oP = \lambda\!\!\!^{-2} \sum_{\ell=0}^{2\ell_{max}-1} B \, P_\ell^1(\cos\theta)$$

is often useful in trying to get a rough idea of some of the main qualitative features of the data. The quantum number assignments for the $\Delta(1924)$ as F_{37}[12] and the N*(2190) as G_{17}[13] were initially based on a detailed study of these coefficients. Generally speaking, however, the very large number of partial waves involved in these scattering processes make it difficult, if not impossible, to untangle all of the various effects from a study of the Legendre coefficients alone. It is really only through detailed phase shift analyses of the data that the behavior of the partial wave amplitudes with energy can be established.

D. Experimental Outlook

Although this is primarily a theoretically oriented conference, a few words are in order about experimental developments. I will be brief. It is well known by now that the use of polarized targets has been responsible for much of the experimental progress in πN scattering. Two comments are in order. First, targets containing significantly more free protons are being developed and should supplant the so-called LMN targets in many of these experiments in the near future. The new targets contain about five times as much free hydrogen as the 3% contained in the LMN crystals. For example, Ethyl alcohol seems to be a very suitable material. Second, there are several groups trying to develop hydrogen-rich polarized targets[14] using the so-called "brute force" technique rather than the dynamic method used in existing targets. Here the object is to make the Boltzmann factor, $\mu_p H/kT$, as large as possible by using very high fields ($B \sim 105$ kgauss), and low temperatures ($T < 10^{-2}\ {}^{o}K$). These developments are a little further off in time, but if realized should find interesting applicability also in πN scattering.

As mentioned previously a target has been constructed[15] in which the protons can be polarized in the plane of the scattering. Consequently A_{recoil} and R_{recoil} results should be forthcoming in the foreseeable future.

One other technique deserves a brief comment: Wire Spark Chambers. With these detectors very precise position measurements are possible. The state of the technology here is advancing fast, and it seems likely that the very precise measurements of angles which can be made with these chambers will permit very detailed measurements of angular distributions. Automatic read-out systems are available which should make it possible to do high statistics experiments of very high quality for both elastic and inelastic scattering.

E. Other Experiments

Mainly because of time limitations I will not discuss here the very interesting and extensive results of the photoproduction experiments. It should be kept in mind, however, that these data are very relevant to any discussion of baryon resonances. There has perhaps not been enough effort devoted to making full use of all available data in trying to extract meaningful results from existing experiments. We have become too accustomed to analyze the elastic scattering experiments quite independently of the photoproduction results and vice versa.

The experimental situation with respect to inelastic πp scattering in this energy region will be discussed by Rosenfeld. Detailed information, especially at the lower energies, will provide useful bounds on some of the existing phase shift solutions. Partial wave analyses of these experiments have the drawback that they are all at least to some degree model dependent. Nevertheless these analyses can be used to check the existing phase shift solutions and vice versa. I will defer the discussion of this subject to Rosenfeld.

III. PHASE SHIFT ANALYSES

A. Formalism and Notation

I had hoped not to have to go through this again, but there are still some heretics around who refuse to accede to the dogma of the Basel Convention. They--and THEY know who THEY are--insist on defining $\hat{n}$ as $\hat{k}_f \times \hat{k}_i$. Burning at the stake is far too mild a punishment for such a heinous offense. THEY should be buried alive in ream after ream of the computer printout of their phase shift analysis. Then there are others who despite howls of anguish from their tortured brethren insist on expressing their phase shift results as a function of pion kinetic energy. It is difficult to conceive of a severe enough penalty for the perpetrators of such a sin. Perhaps we should bug their CDC 6600 so that it will ignore the middle 5 bits of all their computer words when they make their analysis. On second thought, this may not be such a good idea after all--they might get even better fits to their dispersion relations. At any rate let me make a plea for some standardization: Let's use total energy in the center-of-mass to specify the phases and the resonant states, and whenever possible also indicate the pion momentum in the lab. With respect to $\hat{n}$, can we all agree to use $\hat{n} = \hat{k}_i \times \hat{k}_f$ where $\hat{k}_i$ and $\hat{k}_f$ refer to the direction of the meson in the center-of-mass before and after the scattering?

The partial wave decomposition of the scattering amplitudes can be written

$$G = \frac{1}{k} \sum_{\ell=0}^{\infty} [(\ell+1)T_{\ell+} + \ell T_{\ell-}]P_\ell(\cos\theta)$$

$$H = \frac{1}{k} \sum_{\ell=1}^{\infty} [T_{\ell+} - T_{\ell-}]P_\ell^{1}(\cos\theta)$$

where $\ell\pm$ stands for $j=\ell \pm 1/2$ and

$$T_{\ell\pm} = \frac{\eta_{\ell\pm} e^{2i\delta_{\ell\pm}} - 1}{2i}$$ are the partial wave amplitudes,

$\eta_{\ell\pm}$ is the absorption parameter ($\eta_\ell = 1$ corresponds to no absorption, $\eta_\ell = 0$ corresponds to complete absorption), and

$\delta_{\ell\pm}$ is the phase shift for the state $j=\ell\pm 1/2$.

The partial wave amplitudes $T_{\ell\pm}$ are conveniently represented in graphical form by an Argand diagram (see Fig. 8).

A simple Breit-Wigner resonance can be written

$$T_e = \frac{x}{\epsilon - i}$$

with $x = \Gamma_{elastic}/\Gamma_{total}$ (the elasticity of the resonance which is not the same as absorption parameter η), and

$$\epsilon = \frac{E_{Res} - E}{\Gamma/2}$$ where

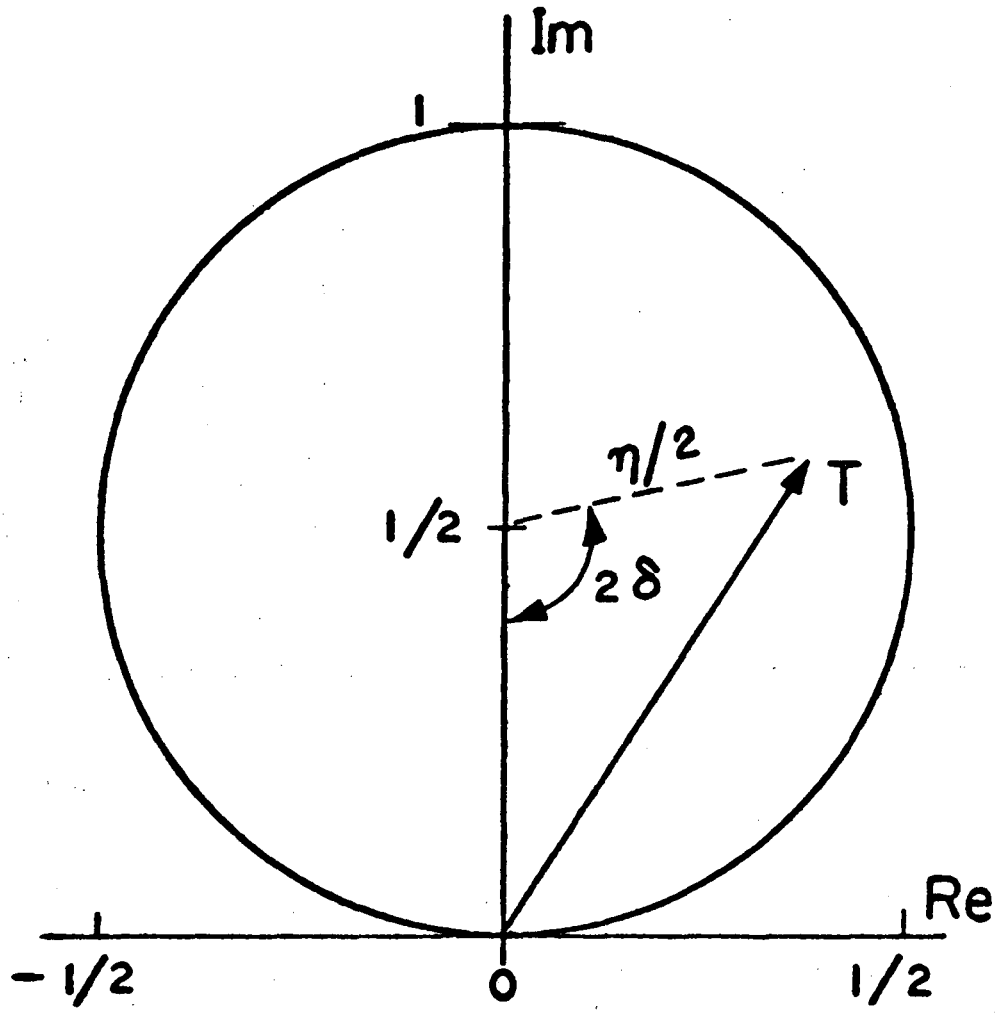
E_{Res} = the resonance energy,

E = the center-of-mass energy, and

Γ = the total width of the resonance.

In this representation such a resonant amplitude would describe a circle moving counterclockwise as the energy E increases. When $E=E_{Res}$ the resonant amplitude is pure imaginary. Thus, if there is no background, a resonant amplitude will have $\delta=0^\circ$ or 90° depending on whether $x < 1/2$ or $x > 1/2$ (see Fig. 9).

Often a resonance amplitude is superposed on some background. In that case the resonant circle will not originate at the origin but



$$T = \frac{\eta e^{2i\delta} - 1}{2i}$$

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Fig. 8.

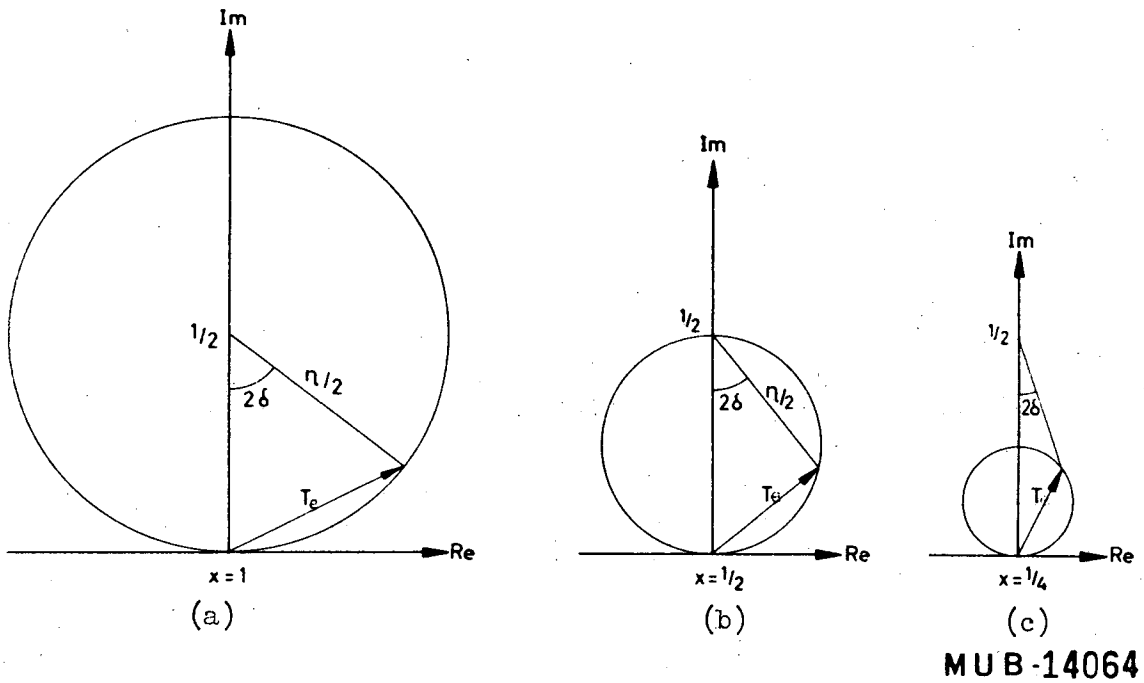

Fig. 9.

The elastic-scattering amplitude T_e in the complex plane (a) For pure elastic scattering (η=1), T_e lies on the unitary circle. If the amplitude is resonant, the circle represents a resonance with elasticity x=1. (b) Resonant amplitude for x=0.5. (c) Resonant amplitude for x < 0.5. Notice that at resonance $\delta=0^{o}$.

somewhere else within the unitary circle.[16] Other factors such as, for example, energy-dependent widths will further distort the picture of a smooth circle.

The problems of the phase shifter are thus two-fold. In the first place he must try to find a unique set of phase shifts which satisfactorily fit all the data at all energies. Then he must endeavor to extract the various resonance parameters from these results. For those partial wave amplitudes which describe a reasonable facsimile of a circle when plotted in an Argand diagram, this problem isn't too bad; but when the partial wave amplitudes wobble and stagger their way through the unitary circle, it is often very difficult to determine the resonance parameters, and sometimes even to know if the amplitude resonantes or not.

In presenting phase shift results we will use the usual notation for the partial wave amplitudes and phases: $L_{2T,2J}$. For example S_{31} refers to $\ell=0$, $T=3/2$, $J=1/2$.

B. Methods

Most of the groups involved in the phase shift business use slightly different methods to obtain their results. These differences concern mainly the extent to which assumptions about the variation of phase shifts with energy are put into the analysis *a priori*. The various methods can be divided into two broad classes--energy dependent and energy independent.

In the energy dependent approach one has some preconceived ideas of what nature must be like. One puts these ideas into what is usually a somewhat over-simplified mathematical form involving many free parameters

including the phase shifts. One analyzes all data at all energies simultaneously with the help of a computer to find the set of these parameters which best fit the data. If the fit is satisfactory one has a solution--if not one usually throws out the parametrization used (or sometimes even the theory) and tries something new and so on. The main proponents of this method are the Livermore[17] and Chilton (now Durham) groups.[18]

In the energy-independent phase shift searches only the experiments within a very narrow interval of energies are analyzed at any one time. The number of possible acceptable solutions at each energy depends on the quality and quantity of the experimental data as well as the number of partial waves used in the analysis. In general there are many solutions at the higher energies. For example at P_π=1.0 GeV/c the Berkeley group has at least 45 solutions which can be considered statistically acceptable fits to the data. At energies below about 400 MeV the solution obtained at each energy is thought to be unique. In this approach the sticky problem comes when one tries to make a continuation from one energy to the next. In trying to choose the "correct" solution at each energy each of the groups uses some sort of smoothness criterion to constrain the variation of the phase shifts with energy. I.e., the phase shifts or the amplitudes or the observables are expected to vary rather smoothly with energy--no <u>sharp</u> discontinuities are expected in any of these quantities. The CERN group[20] makes the energy continuation with the help of dispersion relations for the partial wave amplitudes. They go so far in fact that at a certain stage in their analysis the dispersion relation constraints become part of the "input data" (along with the experimental points) which

the phase shifts must fit. Consequently the CERN solution is very much smoother than any of the others--their dispersion relations force their solutions to behave smoothly with energy. I am sure Dr. Lovelace will elaborate on the methods used by the CERN group in his talk. The Saclay[19] and Berkeley[21] groups are less sophisticated (a carefully chosen word, in this case meaning that in reality we are probably still too stupid to fully appreciate the power of dispersion relations in this type of work), and consequently we use our eyeballs or the computer to select the "smoothest" combination of solutions. This latter approach has the advantage that solutions tend to be less biased by theory. As Lovelace has pointed out it is much better to be biased by a "good" theory than to be unbiased--nevertheless, the question which always lurks in the background concerns how valid the theory really is. Keep in mind that there is a bias in all of these approaches--the assumption of "smoothness" biases us against finding very narrow resonances or cusp-like phenomena. By hard experience we have learned that it is very difficult indeed to find a unique solution without some theoretical input. The Saclay and Berkeley solutions tend to be much less smooth than that of the CERN group. There is a lot of fine structure which is probably not significant but which neither the Saclay nor the Berkeley groups have been able to eliminate in their energy-independent, no-theory-input approach. It seems to me that the input of some theoretical assumptions cannot be avoided completely if one wants to get a "reasonable" solution to this problem. I think the approach of the CERN group is probably the closest approximation so far made to what should be done, although I would favor less reliance on the

partial wave dispersion relations for the low partial wave amplitudes than they used.

There is one other energy independent analysis which should be mentioned. It is the one of the Hawaii group.[22] Here an attempt was made to see if the existing data up to 700 MeV(P_π=830 MeV/c) could be fit with amplitudes which don't resonate. It was a sort of euthanasia campaign launched by Cence to see if he could curb the population explosion of πN resonances. It was an interesting exercise which indicated that a reasonably good fit could be obtained to all of the data with a set of amplitudes shifts which don't resonate (with the exception of $\Delta(1236)$). Closer examination of these results showed, however, that they were in contradiction with disperion relations for the spin-flip amplitudes, and I believe that the more complete experimental information now available cannot be satisfactorily fit by this solution.

In Table I, I have summarized the main features of the methods used and the results obtained in the various analyses which have so far been reported. I will eave it to Dr. Lovelace to give us his usual tempered assessment of the relative merits of each of these efforts.

D. New Results

In recent months three groups--Saclay, CERN, and Berkeley--have reported the latest versions of their phase shift analyses. The relevant Argand Diagrams for each of these solutions as well as a table summarizing the present status of πN resonance parameters is shown in Table II and Figs. 10 and 11 . For the true aficionados we have even prepared a

Table I. Summary of main features of various phase shift analyses.

Group	Method	Input Assumptions	Energy Range	Comments
Livermore[a]	Energy dependent	$\delta_\ell = k^{2\ell+1} \sum_{n=0}^{n_{max}} A_n k^n$ + Breit-Wigner Resonances	Up to T_π=800 MeV (P_π=930 MeV/c, M*=1630 MeV)	First found P_{11}(1470) "Roper" resonance Assumptions about energy dependence of phases tends to bias against finding new resonances. Results published.
Saclay[b]	Energy independent	none	Up to T_π=1.6 GeV (P_π=1.7 GeV/c, M*=2025 MeV)	Many solutions at each energy. Energy Continuation made by making smooth connection between phase shifts at different energies. First found complex resonant structure in regions of (1512) and (1688) resonances. Work completed.
Chilton[c]	Energy dependent	Energy Continuation made with help of dispersion relations for inverse amplitudes.	Up to T_π=1.1 GeV (P_π=1.23 GeV/c, M*=1795 MeV)	Some results published. Work in progress.
Hawaii[d]	Energy independent	Assumed no resonances	Up to T_π=700 MeV (P_π=830 MeV/c, M*=1575 MeV)	Wanted to see if existing data could be satisfactorily fit with non-resonant amplitudes. Found solution which is in reasonable agreement with experimental observations. However, results disagree with spin-flip dispersion relations and with newer, more extensive data.

Table I. Summary of main features of various phase shift analyses. (Continued)

Group	Method	Input Assumptions	Energy Range	Comments
CERN[e]	Essentially energy independent analysis but with dispersion relation input.	Energy Continuation made with help of dispersion relations for partial wave amplitudes.	Up to T_π=1.94 GeV (P_π=2.07 GeV/c, M*=2190 MeV)	Most sophisticated analysis. They check self-consistency of dispersion relation input. Results could be slightly biased because solutions are forced to be in accord with dispersion relation input. Have found 18 resonant states. Results published
Berkeley[f]	Energy independent	none	Up to T_π=1.6 GeV (P_π=1.7 GeV/c, M*=2025 MeV)	Many solutions at each energy. Energy continuation based on smooth variation of amplitudes made with help of computer. Uniqueness of solutions not established. Work in progress.

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(c) B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, Proc. Roy. Soc. 289A (538) 1966 (300-700 MeV); Phys. Letters 19, (420) 1965 (700-1100 MeV).

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(e) A. Donnachie, R. G. Kirsopp, C. Lovelace, CERN Internal Report TH.838.

(f) C. H. Johnson, thesis, UCRL-17683 (1967).

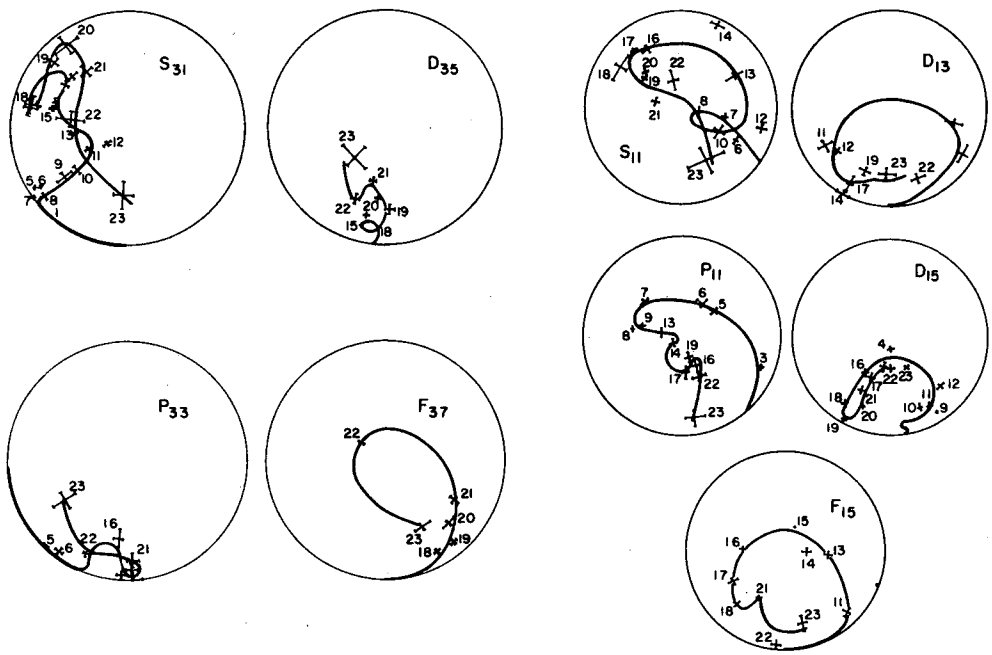
Table II. π-N Resonance parameters (in MeV).

wave	mass	Γ_{tot}	Γ_{el}/Γ_{tot}
P_{33}	1235.8	125.1	1.0
P_{11}	1470	210	0.65
D_{13}	~ 1520	115	0.55
S_{11}	1535	120	0.35
S_{31}	1640 ?	180	0.30
D_{15}	1680	170	0.40
F_{15}	1690	130	0.65
$*P_{33}$	1688	280	0.10
$*D_{33}$	1691	270	0.14
S_{11}	1710	300	0.80
$*P_{11}$	~ 1751	330	0.32
$*P_{13}$	~ 1863	~ 300	~ 0.21
$*F_{35}$	1913	350	0.16
$*P_{31}$	1934	340	0.30
F_{37}	1950	220	0.40
$*D_{35}$	~ 1954	~ 310	~ 0.15
$*F_{17}$	1983	225	0.13
$*D_{13}$	2057	290	0.26
G_{17}	~ 2200	300	0.35

* means new resonance.

The parameters of the new resonances are taken from A. Donnachie, R. G. Kirsopp, C. Lovelace, CERN TH.838. Those of the more established resonances are obtained from weighing the results of the Berkeley, Saclay, and CERN groups.

PARTIAL WAVE AMPLITUDES OBTAINED BY THE SACLAY PHASE SHIFT ANALYSIS (BAREYRE et al)

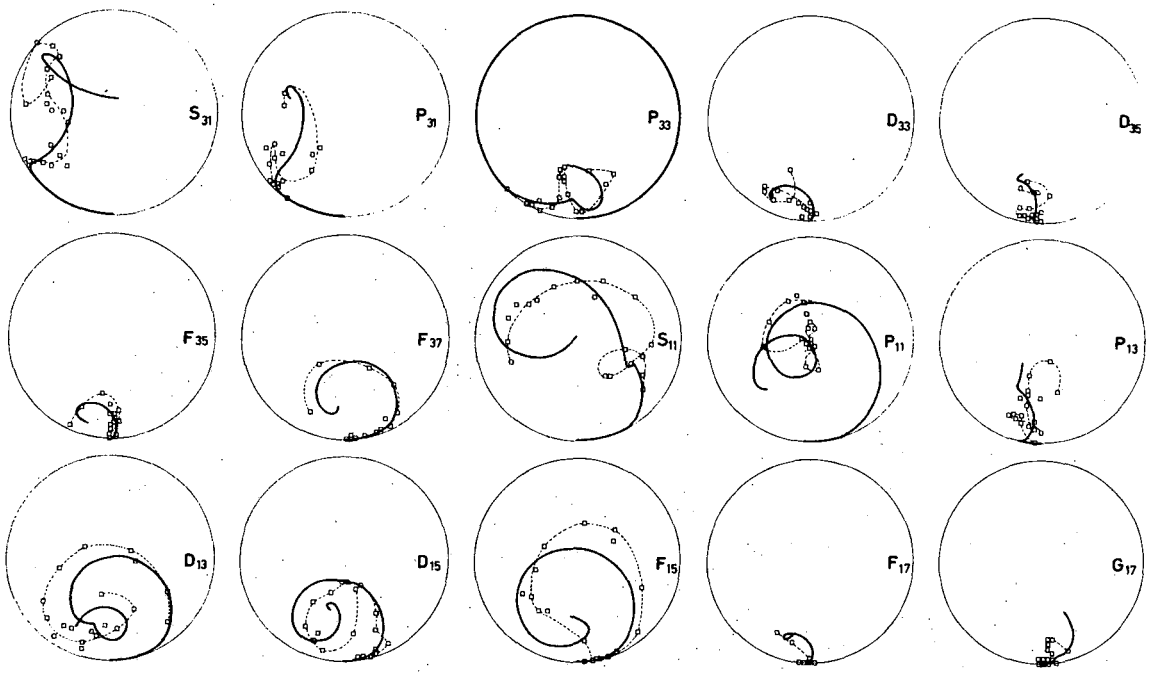


	E_{cm}
1.	1320
2.	1362
3.	1390
4.	1443
5.	1470
6.	1501
7.	1524
8.	1543
9.	1573
10.	1603
11.	1617
12.	1629
13.	1658
14.	1673
15.	1688
16.	1716
17.	1738
18.	1769
19.	1822
20.	1862
21.	1896
22.	1968
23.	2021

XBL 6712-6296

Fig. 10.

Taken from CERN report TH.838. A. Donnachie, R. G. Kirsopp, C. Lovelace, Evidence from πp Phase Shift Analysis for Nine More Possible Nucleon Resonances (October 1967).



Solid line - Partial wave amplitudes obtained from the dispersion relation results of the CERN group.
Dashed line - Partial wave amplitudes obtained from the phase shifts of the Berkeley group.

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Fig. 11.

limited number of copies of the detailed results of each of these groups. These are available from the authors while the supply lasts. Time does not permit me to discuss each of these analyses in detail. What I would like to do, instead, is to point out some of the main characteristics of each, and then to discuss some of their similarities and differences.

Let's begin with Saclay. Their new results differ from their earlier work[23] in that they are based on more reliable experimental data and new polarization measurements. The new analysis goes up to 2025 MeV total mass (P_π=1700 MeV/c). The maximum orbital angular momentum used in ℓ_{max}=4 (G waves). They obtained χ^2=1633 for 1433 degrees of freedom.[24] The variation of the phase shifts with energy is quite bumpy for many of the partial waves, i.e., there seems to be some fine structure which they are unable to get rid of. It is not clear at present what, if any, significance should be attached to this fine structure. What is clear, is that up to now they have not been able to get rid of it even though they have tried pretty hard. Their results, which have been submitted to the Physical Review, represent the final version of this stage of the analysis. No further work is contemplated in the immediate future.

The Berkeley results are not finished. They represent the present status of an analysis which is in progress. The experimental data used are substantially the same as those used by the Saclay group. Although ℓ_{max}=4 also in the case of this work, the actual number of parameters used to fit the data was slightly larger because 5 additional parameters were used to normalize the 5 different types of experiments at different energies. In this way systematic errors resulting from improper normali-

zation of the reported experimental measurements could be taken into account. At present this analysis, like that of Saclay, extends to a total mass of 2025 MeV(P_π=1700 MeV/c). Preliminary results have been reported.[21] χ^2=1790 for 1466 degrees of freedom.[24] The variation of the phase shifts with energy is comparable in "bumpiness" with that of Saclay, although there are noticeable differences in the behavior of several of the partial wave amplitudes between the Berkeley and Saclay results. Although the procedure used to determine the continuation of the partial wave amplitudes with energy was quite different in the case of Berkeley and Saclay (i.e. Berkeley used the computer to impose a certain smoothness condition while Saclay imposed quite a different condition and did it more or less by hand), most of the results are very similar.

The CERN analysis is certainly the most impressive of the lot. Their results extend to a mass of 2190 MeV (P_π=2070 MeV/c). They go to ℓ_{max}=4 except at their highest energies where H-waves are used. Some care should be exercised in using these results. There are two sets of phase shifts quoted. One is the "experimental" set, which is based on the fits to the experimental data plus the additional constraints imposed by the dispersion relations. The second is the "dispersion relation" set, which is simply the set calculated directly from the dispersion relations themselves. This latter set by its very nature is exactly as smooth as the dispersion relations specify. The "experimental" phases are somewhat bumpier, but even they seem to be strongly enough constrained by the dispersion relations in most cases so as to be much, much smoother than anything Saclay or Berkeley has to offer. The quoted χ^2 is 4421 for 4102 degrees of freedom.[24] The

disparity in the χ^2 values and the number of degrees of freedom[24] between the various results comes mainly from the fact that the CERN group includes much more low-energy data in their analysis than Berkeley and Saclay. The CERN results are published in preprint form[20] and have been reported at the Heidelberg Conference.

As a result of their phase shift analysis the CERN group has proposed a total of 18 resonances including 9 previously undiscovered ones in the mass region below about 2300 MeV. The resonances in Table II which are not preceded by an asterik are considered to be well established. Some of these, like the $P_{33}(1238)$, $D_{13}(1520)$, $D_{15}(1680)$, $F_{15}(1690)$, $F_{37}(1950)$, $G_{17}(2200)$, $S_{11}(1710)$, $P_{11}(1470)$ are beyond all doubt, although the exact resonance parameters may well change slightly. Where the energy regions of all 3 analyses overlap these resonances are present in all of them. Sometimes the exact behavior of the amplitudes in the Argand plot is somewhat different, as for example in the case of F_{37} where CERN and Berkeley are very similar, but Saclay is noticeably different. There is also little doubt that S_{31} resonates at about 1640 MeV, although the behavior of this amplitude is rather strange. The first S_{11} resonance seems to be present in the Saclay and Berkeley results, but missing in the case of CERN. Interestingly enough, when the CERN results are used as input to an unconstrained analysis(i.e. no dispersion relations) in which the Berkeley data set is fitted, the S_{11} resonance near 1540 MeV reappears. Although this may not be a completely fair way to play the game it is worth pointing out that the very smooth CERN phases all readjust themselves, and in so doing they become just as bumpy as the Berkeley and Saclay results. It seems that the dispersion

relations are powerful medicine indeed against such hiccups.

The resonances which are preceded by an asterisk are the new resonances uncovered by the CERN group.[20] It is premature to consider all of them as well established. Certainly the general behavior of several of these amplitudes such as, for example, F_{35}, D_{33}, P_{31}, and $P_{33}(1688)$ have common resonance-like features in the CERN and Berkeley analyses. I believe that the chances are good that these resonances are real. On the other hand, it takes a lot of imagination and probably some wishful thinking to see resonance-like behavior in the Argand plots of D_{35} and P_{13}. Even if all of these resonances were to survive it seems likely that the quoted resonance parameters may well undergo significant changes as more information becomes available. <u>These remarks are intended as a word of caution to those who would take these results too seriously</u>. One of the reasons that it is difficult to draw firm conclusions about many of these new resonances is that they tend to be quite inelastic; i.e., their coupling to the elastic channel is small. Consequently they don't manifest themselves as obviously as one might like in an analysis of elastic scattering experiments. The detailed inelastic scattering experiments now in progress should be of great help in resolving these questions.

A question which often comes up in a discussion like this is: "Are these phase shift solutions really unique, or are there other equally good solutions lying undiscovered somewhere in the bowels of the 6600?" The CERN and Saclay groups believe that their solutions <u>are</u> unique. That does not mean that the existing solutions cannot undergo some readjustments. It does mean, however, that these groups expect the general features

of their results not to change significantly. The Berkeley group is much less sure that there are not other solutions in the energy region spanned by their analysis. For example, only a few days ago they obtained another solution up to ~1120 MeV/c (center-of-mass energy = 1740 MeV) which fits the data even slightly better than the solution quoted here and whose behavior differs in several respects from the usual solution. These differences concern mainly the elasticities of the T=1/2 resonances and the behavior of P_{11} above the 1470 MeV resonance. It is too early to tell if this new solution will ultimately also fit the data at higher energies. It is possible that it is only a variant of the existing solution. So far as the Berkeley group is concerned the question of uniqueness is still open.

In discussing the question of uniqueness one must specify some conditions which determine when two solutions are really different and when they are the same. Generally speaking the errors associated with the phase shifts are smaller than most of us feel they have any right to be. That is, there are often solutions which differ in several of the phases by many standard deviations--and still we think the solutions are essentially the same. It is often as though the hypersurface near the χ^2 minimum in the phase shift space is covered with little "pot holes." Perhaps the assumption of hyperparabolic behavior of χ^2 as a function of the phase shifts near χ^2_{min} is questionable in such a case. At any rate two solutions are usually thought to be the same unless the general behavior of at least one of the phases is markedly different from one solution to the other. Again here I would caution the uninitiated to consider the quoted phase shift errors to be smaller in general than the actual uncertainty associated

with our knowledge of these parameters. As an example of this fact, consider the "fine structure" observed in the Saclay and Berkeley analyses. This fine structure is usually much larger than the errors, and yet we don't know yet if such fine structure is really significant. Perhaps we just have to try a little harder if we want it to go away.

Let me conclude this survey with the most spectacular result of all. Most of you are familiar with the view espoused in certain quarters on the fifth floor of Building 50B in Berkeley, that the bubble chamber represents the ultimate tool of the research physicist. It took a while, but I have finally become a believer. And many of you will, too, once you have seen what I am about to show you. Recall the beautiful results of the CERN group such as those shown in the Argand Diagram for P_{11}. (Fig. 12) With due apologies to Dr. Lovelace, I must state that his results pale into insignificance compared to the result I am about to report. It is an Argand plot of P_{11} from threshold to infinity taken in a bubble chamber. (You can just hear the mouths of the Reggeists in the audience water at the thought.) The 9 new P_{11} resonances are shown in Fig. 13.

This work was done under auspices of the U. S. Atomic Energy Commission.

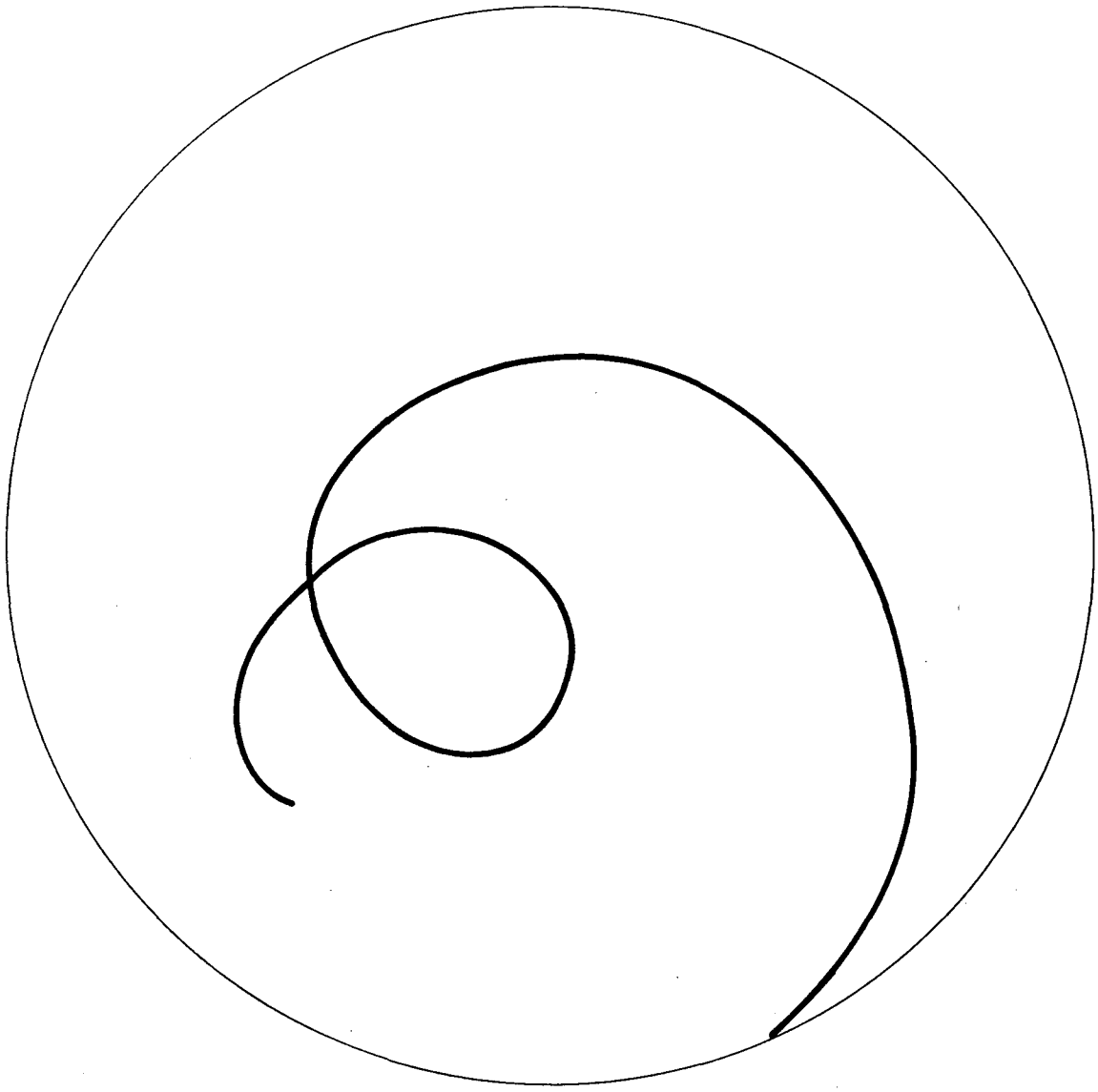

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Fig. 12.

Argand Diagram for partial wave amplitude P_{11} obtained by the CERN group.[20]

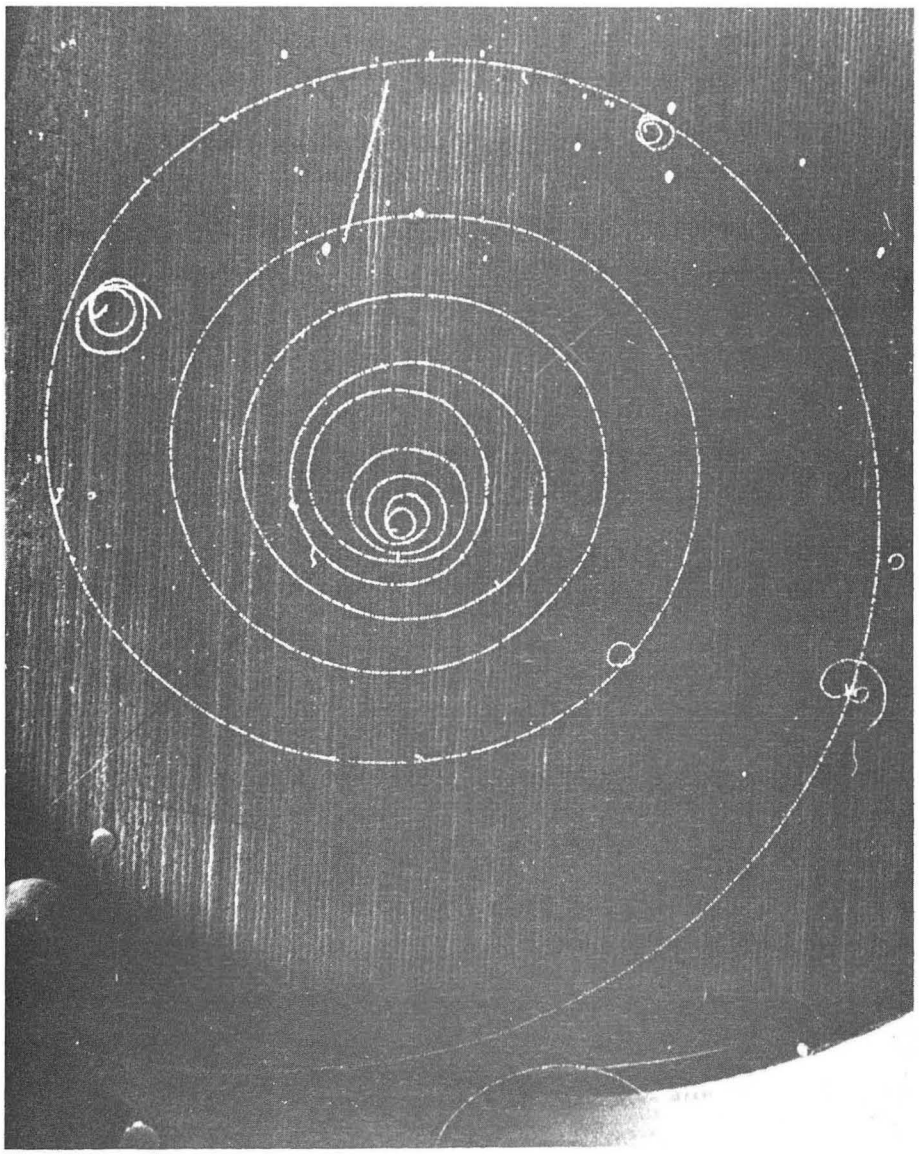

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Fig. 13

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24. In defining the total number of degrees of freedom here we have simply added together the number of degrees of freedom (i.e., number of measurements minus number of phase shift parameters) at each of the energies used in the analysis. This number is certainly an underestimate for the total number of degrees of freedom because the phase shift parameters at different energies are correlated. Consequently conclusions about the goodness of these fits based on statistical considerations alone tend

to be misleading. It is very difficult to make a precise statement about the statistical significance of the various analyses described here.

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