UCRL-1803

UNCLASSIFIED

UNIVERSITY OF CALIFORNIA — BERKELEY

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

RADIATION LABORATORY

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE VII
April 14, 1952
E. Martinelli

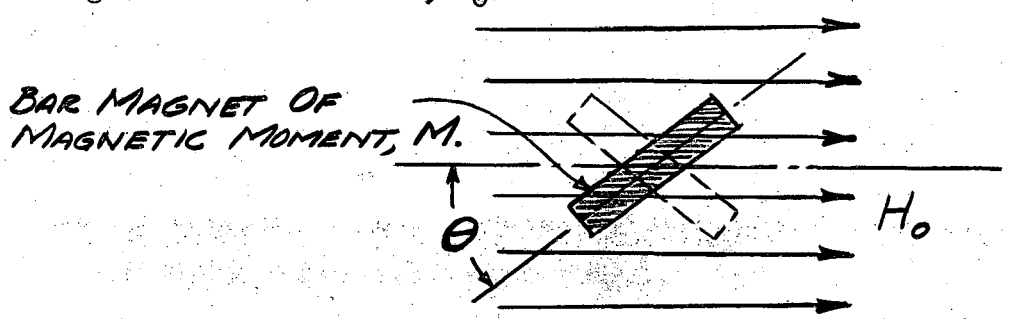
(Notes by: L. Lucas and R. Marker)

ELECTROMAGNETIC FIELDS

1. In visualizing a magnetic field it should be noted that there is no "free magnetic charge" as in electrostatics; that is, one cannot isolate a piece of material small enough to consist of, say, only a single north pole. The smallest magnetic element must be considered to be a small magnetic dipole or a small current carrying loop. This means that the intensity of a magnetic field cannot be measured with a small charged proof sphere as was done for electrostatic fields.

2. Magnetic fields are measured by making two independent tests.

2a. If a bar magnet, small in comparison with the dimensions of the field to be measured, is placed in the unknown field, H_o, and suspended so that it may swing like a compass needle, it will oscillate if displaced from its position of natural allignment with the field, H_o.



At displacement θ the restoring couple is $T = |M| \times |H_o| \sin\theta$ (1)

M is the "magnetic moment" of the permanent magnet and is defined by equation (2):

$$M = \int I \, dV \tag{2}$$

Where I = magnetic moment per unit volume or the intensity of magnetization in the bar magnet.

If J = polar moment of inertia of the bar magnet, the magnet will oscillate at frequency, $\omega = \sqrt{\frac{M H_o}{J}}$ (3)

Measuring the frequency of oscillation and solving equation (3) will give a value for the product $M H_o$.

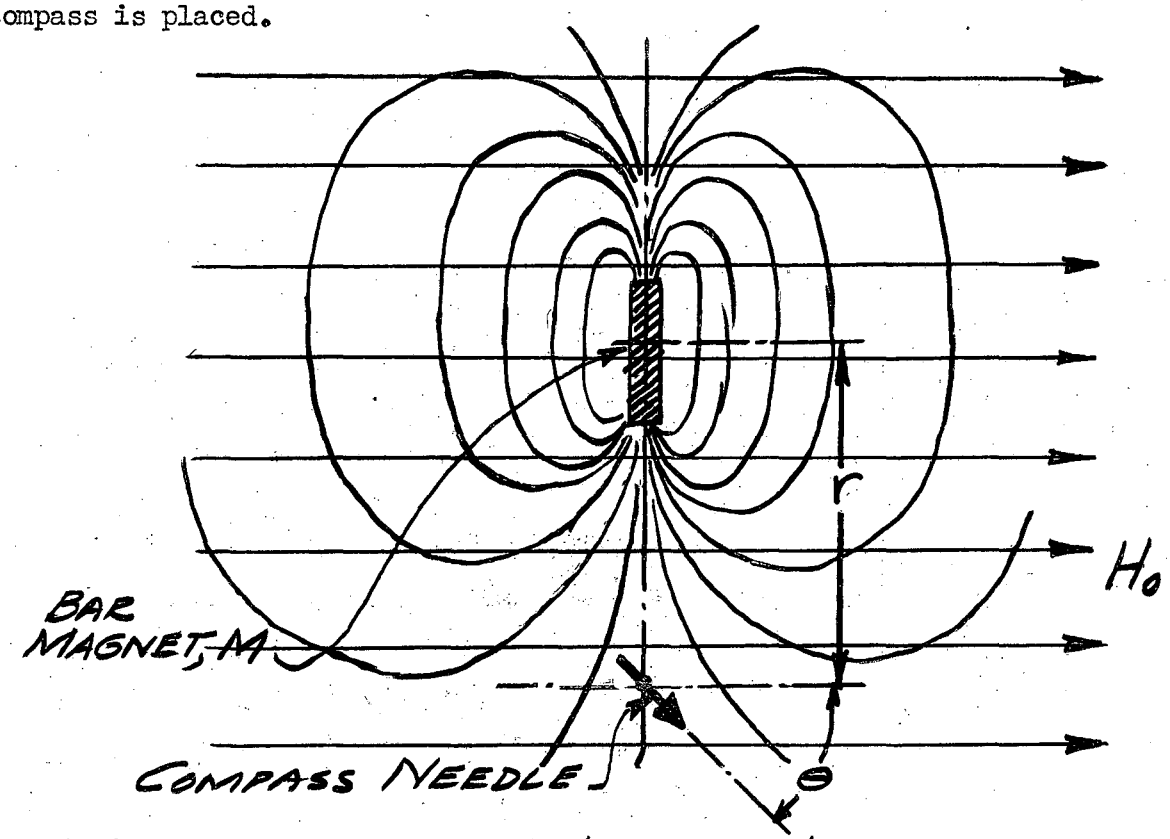
2b. The second test consists of placing the same bar magnet, M, at right angles to the field H_o. A small compass needle placed in the fields of M and H_o at distance r from the center of and in line with the axis of the bar magnet, M, will align itself with the resultant field according to equation(4):

$$\tan \theta = \frac{H_r}{H_o} \tag{4}$$

Where H_r is the radial component of the field due to the magnet, M, at a distance r from the magnet center. It can be shown that H_r varies with r per equation (5):

$$H_r = \frac{2\ M}{r^3} \cos \phi \tag{5}$$

ϕ is the angle between the axis of the magnet and the line along which the compass is placed.



In this particular arrangement, $\phi = 0$ and $\cos \phi = 1$; combining equations (4) and (5), we see that

$$\tan \theta = \frac{2\ M}{H_o\ r^3} \tag{6}$$

2c. Measuring θ and r will give us the quotient $\frac{M}{H_o}$. Knowing the value of $M H_o$ and $\frac{M}{H_o}$ permits us to calculate M and H_o. Thus we have a calibrated bar magnet with which we may determine other unknown fields by performing either one of the above tests. These methods of field measurement are practical only for relatively weak fields (10 gauss or less). Measurement of strong magnetic fields require refined tests employing pick-up coils, magneto-strictive elements, etc.

3. A bar magnet produces a field H. The source of the H field is the intensity of magnetization, I, of the bar magnet. Mathematically, this may be expressed by equation (7):

$$\text{div}\,\vec{H} = -4\pi\,\text{div}\,\vec{I} \qquad (7)$$

Equation (7) happens to coicide with the equation for dielectric polarization:

$$\text{div}\,\vec{E} = -4\pi\,\text{div}\,\vec{P} \qquad (8)$$

This analogy will be of use in the solution of magnet problems to be covered in future lectures.

Another property of magnetostatic fields (not varying with time) is expressed by equation (9):

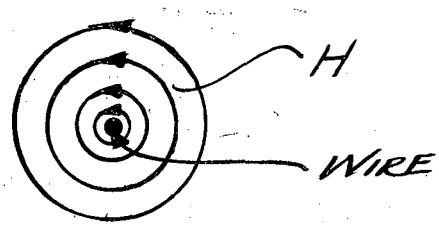
$$H = -\text{grad}\,\phi_M \qquad (9)$$

ϕ_M is the potential due to a magnet which acts as a "doublet" or two point source of field. It can be shown that $\phi_M = \frac{M}{r^2}$ (10)

This type of field created by the magnetic moment, M, of a permanent magnet is irrotational. At the sources of the field (poles of the magnet),

$$\text{div}\,M = 0. \qquad (11)$$

4. A wire carrying a steady current will have a magnetic field surrounding the wire:

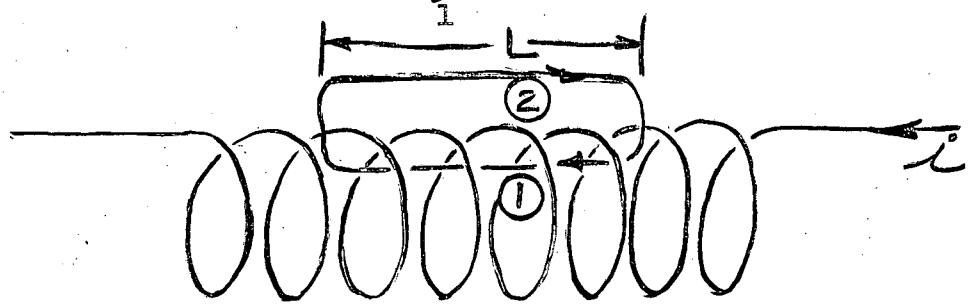


The line integral of H around a closed loop equals the total current threading the closed loop:

$$\oint H\,dl = i \qquad (12)$$

Integrating around a circle of radius, r, we find that $i = 2\pi rH$ (13)
or $H = \frac{i}{2\pi r}$, ampere turns/meter. (14)

For an air core solenoid the integral $\oint_1^2 H\, dl = H_1 L - H_2 L$ (15)



Since H_2 is much less than H_1, $i \approx H_1 L$ (16)

Where i is the total current flowing over the length L, and is numerically equal to (ni) where n is the number of turns in length L and i is the current flowing through the wires of the coil. Therefore, we can see that the field inside a solenoid is

$$H = \frac{ni}{L}, \text{ ampere turns/meter.} \quad (17)$$

Actually, for a straight solenoid, the effective length of the field is longer than the coil windings by one or two coil diameters due to the bunched field leaving the coild end.

5. Thus it can be seen that there are two ways of producing magnetic fields: One is by passing current through a conductor. In this case the mathematical equations describing the conditions are:

$I = 0$ (implying that no magnetic materials are present) (18)

$\text{curl } \vec{H} = i$ (19)

$\text{div } \vec{H} = 0$ (20)

The other method is to have a permanent magnet. In this case the equations are:

$i = 0$ (implying no currents are flowing) (21)

$\text{div } \vec{H} = -4\pi \text{ div } \vec{I}$ (see equation 7) (22)

$\text{curl } \vec{H} = 0$ (23)

6. When these two cases are combined; that is, currents flowing with magnetic materials present, the following general equations hold:

$\text{div } \vec{H} = -4\pi \text{ div } \vec{I}$ (24)

$\text{curl } \vec{H} = \vec{i}$ (25)

However, these are difficult to use in practice since H and I are interdependent and in turn depend on the magnetic properties of the material in the field.

7. Several classes of materials affect the magnetic field:

7a. Diamagnetic materials are only slightly affected by the field and then in a direction that reduces the field strength. Examples of diamagnetic materials are:

gold, $\chi = -3 \times 10^{-6}$

water, $\chi = -.77 \times 10^{-6}$

Hydrogen, $\chi = -.50 \times 10^{-9}$

7b. Paramagnetic materials are also only slightly affected by the magnetic field. Examples of paramagnetic materials are:

oxygen, $\chi = + .14 \times 10^{-6}$

Manganese, $\chi = + 300 \times 10^{-6}$

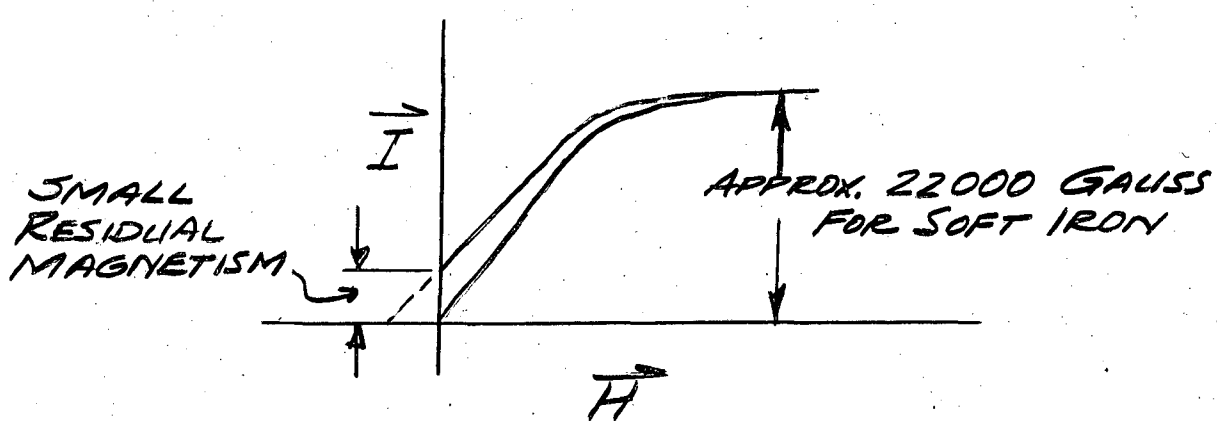
χ is called the magnetic susceptability per unit volume, and affect the field per equation (26):

$$\vec{I} = \chi \vec{H} \tag{26}$$

7c. Ferromagnetic materials are greatly affected by magnetic fields; but as the field is increased, the increase of magnetization becomes successively smaller until a point of "saturation" is reached. Any increase in field, H, beyond this point does not increase the intensity of magnetization, I. χ, then, is a variable and equation (26) does not apply.

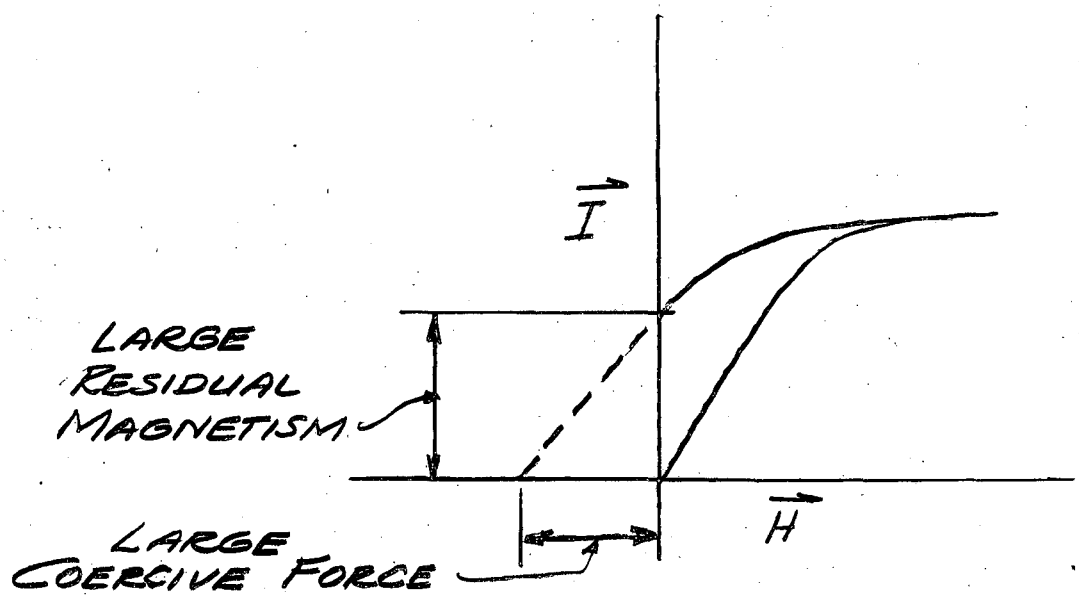
Further, this characteristic cannot in general be expressed analytically.

1. Soft ferro-magnetic materials have a typical magnetization curve as shown below.



After saturation is reached, the magnetization does not increase regardless of how large H becomes. As H is reduced I decreases at a slightly different rate so that when $H = 0$, there remains some residual magnetism. This type of material makes a poor permanent magnet but is desirable for transformer cores and other alternating current machines.

2. Hard or permanent magnetic materials have the following typical magnetization curve:



Here, saturation is also present but the amount of residual magnetism is much greater, and the reversed field required to reduce I to zero is much greater than for typically soft materials.

8. Since equations (24) and (25) are difficult to apply, another equation is defined to describe magnetic induction phenomena:

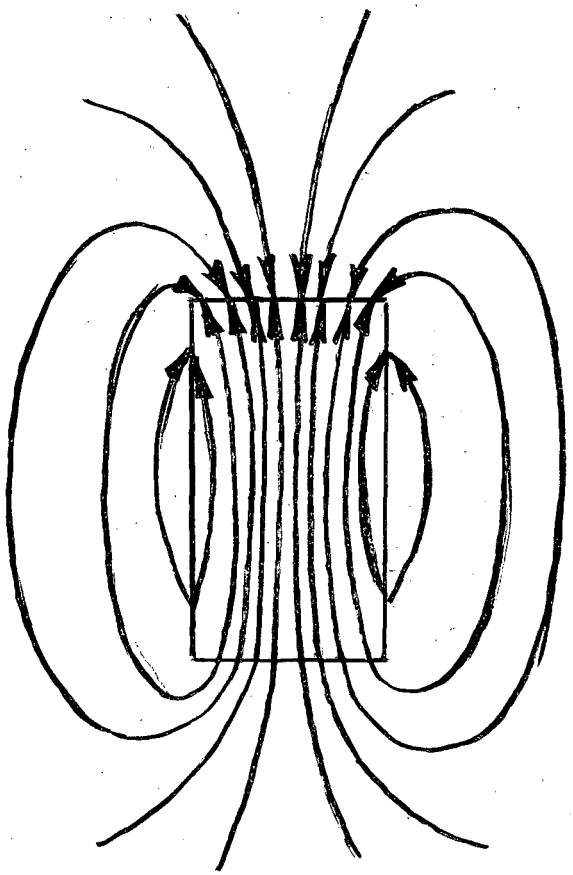
$$\vec{B} = \vec{H} + 4\pi \vec{I} \tag{27}$$

$\vec{B}$ is called "Magnetic induction". The $\vec{B}$ field is solenoidal; that is, it has no source or sink and hence forms a closed loop. Mathematically, these characteristics are expressed as:

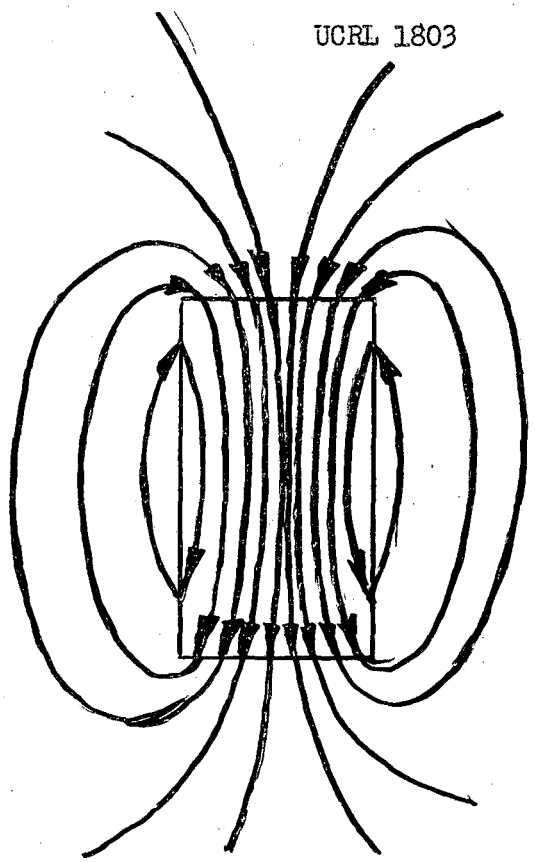
$$\text{div } \vec{B} = 0 \tag{28}$$

$$\text{curl } \vec{B} = \vec{i} + 4\pi \text{ curl } \vec{I} \tag{29}$$

Graphically, a bar magnet and its associated field might look like either of the sketches on the following page, depending on the concept employed to explain the phenomena of magnetic induction.



H FIELD



B FIELD

If a magnetized material is assumed to consist of many small magnets oriented end to end, then inside the magnet the field will appear as in the "H field" sketch. If the small current-carrying loops are assumed to be oriented with axes parallel and all acting in the same direction so that the net current exists only on the external surface of the magnet, then the field inside the magnet will appear as in the "B field" sketch.

Interpretation of equations (24) and (25) will suggest a field direction as shown in "H field" sketch. Interpretation of equations (27) and (28) suggest a field direction inside the magnet as shown in "B field" sketch.

Note that for field external to the magnet both sets of equations give the same results for field direction and strength.

To determine which concept is correct, charged particles were sent through the magnet at right angles to the axis of the magnet. By noting the direction of the particles, it has been established that a field representation as shown in "B field" sketch best represents nature.

plc: 5/10/52