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A Method of Calculating the Axial Vector Current Coupling Constants for Leptonic Decays of Baryons * +

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Abstract

Based on the usual PCAC and current algebra assumptions together with the assumption that the BBM vertex function vanishes as the four momentum squared of one of the baryons is "dispersed" to infinity, it is shown that the classic SU(6) and current algebra results for the axial vector current coupling constant--$G_A=5/3$ and $F/D=2/3$--are the unrenormalized values. The renormalized values are then expressed in terms of the unrenormalized values and certain side-wise dispersion integrals analogous to those introduced by Bincer, Drell and Pagels. A rough estimation of these integrals gives $G_A=1.22$, which is improved to 1.20 if the experimental pion-nucleon scattering lengths are used in the calculation. The method is general and can be applied to other leptonic decays of baryons, two of which are also considered and the results agree well with experiment.

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I. Introduction

Not long ago, K. Bardakci [1] made use of a sidewise dispersive method to calculate the axial vector current coupling constant G_A and obtained a value of 1.04 . His method is purely theoretical and is easily applicable to other leptonic decays of baryons where direct application of Adler and Weisberger's method fails because the cross sections for general baryon-meson scattering have not been measured accurately.

In the following Bardakci's method will be modified and generalized to the calculation of the axial vector current coupling constant $G_A^{fi\alpha}$ for the leptonic decay of baryon B_i to baryon B_f via the axial vector current A^α. In accordance with our sidewise dispersion approach, we shall let the four momentum squared, $p^2=s$, of the initial baryon B_i take on any value above the threshold. The spectral function is then obtained by inserting a complete set of intermediate states between the initial and final states, and we simplify the calculation by neglecting all three and higher number body intermediate states. (There can be no single baryon intermediate state.) This is shown diagrammatically in fig. 1 .

Goldberger-Treimen relation is used to replace the axial vector current A^{α} by the meson state M_{α}. The spectral function is thus proportional to the product of a BBM vertex and a baryon meson to baryon-meson scattering amplitude.

The method of current algebra will be used together with exact SU(3), in the sense discussed later, approximation. The final expression obtained is of the form:

$$G_{A}^{fi\alpha} = (G_{A0}^{fi\alpha} - I^{-}_{fi\alpha})/(1 - I^{+}_{fi\alpha})$$

where the index 0 denotes the corresponding unrenormalized quantity,

obtainable from an exact SU(3) calculation, and the I's are dispersion integrals.

Section I and II show that the usual assumption $G_{A0} = G_V$ for beta decay is not compatible with the present theory, and that the classic SU(6) and current algebra results $G_A = 5/3$, $F/D = 2/3$ (assuming no representation mixing [5]) are the unrenormalized values. Section III and IV give a very crude method of evaluating the dispersion integrals I_+ and I_-, assuming that all the baryon masses and all the meson masses are degenerate, and that the dispersion integrals are dominated by low energy region. The result is: $G_A = 1.0$ to 1.16 , $F/D = 0.575$ to 0.60, dependent on the cut off value chosen. Section V takes into account the effect of mass splitting and gives $G_A = 1.22$, which is improved to 1.20 if we use the experimental values for the pion-nucleon s wave scattering length and p wave effective range in stead of the corresponding values calculated by Weinberg's [3] method. Furthermore the result obtained is almost completely cut off independent and is not very sensitive to small errors in the dispersion integrals. Finally the appendices give some details of calculating the scattering lenths and effective ranges.

Application to other processes are also considered.

We list all our basic assumptions below:

(a). The usual PCAC assumption,

$$\partial^{\lambda} A_{\lambda}^{\alpha}(x) = C_{\alpha}^{fi} \phi^{\alpha}(x) \tag{1}$$

with

$$C_{\alpha}^{fi} = -(M_i + M_f)\mu_{\alpha}^{2} G_A^{fi\alpha} / g_{fi\alpha} \tag{2}$$

where f, i, j etc always refer to baryon states and α, δ etc to meson states and M, μ are the baryon and meson masses respectively.

(b). The BBM vertex, defined in terms of the meson field of the above equations, goes to zero when the external mass of one of the baryons goes to infinity. This assumption includes Nambu's assumption and partially contains equation (1), and was used by Bardakci [1] in his calculations.

(c). The usual current algebra assumptions: If

$$V_{\mu}^{\alpha}(x) = \bar{\psi}(x) \frac{\lambda^{\alpha}}{2} \gamma_{\mu} \psi(x) \tag{3}$$

$$A_{\mu}^{\alpha}(x) = \bar{\psi}(x) \frac{\lambda^{\alpha}}{2} \gamma_5 \gamma_{\mu} \psi(x) \tag{4}$$

are the vector and axial vector currents and

$$Q^{\alpha} = \int V_0^{\alpha}(x)\, d^3x \tag{5}$$

$$Q_5^\alpha = \int A_o^\alpha(x)\ d^3x \tag{6}$$

their corresponding charges (called charges and chiral charges from now on) then the following commutation rules are true:

$$\begin{aligned} \left[Q^\alpha\ ,\ Q^\beta\right] &= if^{\alpha\beta\gamma}Q^\gamma \\ \left[Q^\alpha\ ,\ Q_5^\beta\right] &= if^{\alpha\beta\gamma}Q_5^\gamma \\ \left[Q_5^\alpha\ ,\ Q_5^\beta\right] &= if^{\alpha\beta\gamma}Q^\gamma \end{aligned} \tag{7}$$

The first commutation relation states that the charges Q^α generate the SU(3) algebra. This follows from the commutation rules for the λ matrics. The second commutation relation says that the chiral charges Q_5^α are vector operators in the octet representation of SU(3). The last relation is a basic assumption of current algebra; it clearly indicates that the chiral charges cannot form an algebra. Instead, the linear combinations $1/2.(Q^\alpha \pm Q_5^\alpha)$ together form the chiral SU(3) x SU(3) algebra.

(d). The dispersion integrals are dominated by its low energy region.

(e). Same F/D ratio at the strong (BBM) and weak vertics.

II. Ward Identity

Let $G^{fi\alpha}(q'^2, p^2, k'^2)$ be the BBM vertex function, with f, i the final and initial baryon states of momenta q' and p and meson state α with momentum k', then the Ward identity in our notation is, in terms of renormalized quantities,[2]

$$k'^{\mu}\Gamma^{fi\alpha}_{5\mu}(q'^2, p^2, k'^2) = iC^{fi}_{\alpha}G^{fi\alpha}(q'^2, p^2, k^2) + \tag{8}$$

$$+ \frac{1}{S'_F(q')}(\gamma_5\lambda^{\alpha}/2)_{fi} + (\gamma_5\lambda^{\alpha}/2)_{fi}\frac{1}{S'_F(p)}$$

Here $\Gamma^{fi\alpha}_{5\mu}$ is the axial vector current vertex function and can be expanded into positive and negative energy components as in Ref. 1:

$$\Gamma^{fi\alpha}_{5\mu} = (f^{fi\alpha}_{1+}\gamma_5\gamma_\mu + f^{fi\alpha}_{2+}\gamma_5 k'_\mu + f^{fi\alpha}_{3+}\gamma_5\sigma_{\mu\nu}k'^{\nu})\Lambda_+ \tag{9}$$

$$+ (f^{fi\alpha}_{1-}\gamma_5\gamma_\mu + f^{fi\alpha}_{2-}\gamma_5 k'_\mu + f^{fi\alpha}_{3-}\gamma_5\sigma_{\mu\nu}k'^{\nu})\Lambda_-$$

Similarly we write:

$$G^{fi\alpha} = iK^{fi\alpha}_{+}\gamma_5\Omega_+ + iK^{fi\alpha}_{-}\gamma_5\Omega_-$$

$$= iH^{fi\alpha}_{+}\gamma_5\Lambda_- + iH^{fi\alpha}_{-}\gamma_5\Lambda_- \tag{10}$$

where

$$\Omega_{\pm} = \frac{W \pm \not{p}}{2W} \quad ; \quad W^2 = s \tag{11}$$

and

$$\Lambda_{\pm} = \frac{M_i \pm \not{p}}{2M_i} \tag{12}$$

Substituting (9), (10) into the Ward identity we get two equations by letting $\not{p}$ go to zero and infinitity; eliminating f_{1-} between them and letting k'^2 go to zero leads to the following equations:

$$f_{1+}^{fi\alpha}(M_f^2, s, 0)\gamma_5 = (\gamma_5\lambda^{\alpha}/2)_{fi} - C_{\alpha}^{fi}/2M_i.(H_+^{fi\alpha}(M_f^2, s, 0) - H_-^{fi\alpha}(M_f^2, s, 0)) \tag{13}$$

In accordance with our basic assumption, H_+ and H_- go to zero as s approaches infinity. Therefore,

$$\lim_{s \to \infty} f_{1+}^{fi\alpha}(M_f^2, s, 0)\,\gamma_5 = (\gamma_5\lambda^{\alpha}/2)_{fi} \tag{14}$$

Note that we must consider $(\gamma_5\lambda^{\alpha}/2)$ as one operator. Now since $(\gamma_5\lambda^{\alpha}/2)$ do not form an algebra, we must evaluate the matrix element on the RHS by using the full chiral algebra SU(3) x SU(3). If we follow the usual assumptions, that the currents transform as the (1,8) and (8,1) representation while the baryons form the octet part of the (6,3) representation we obtain the well known result,

$$(\gamma_5\lambda^\alpha/2)_{fi} = \gamma_5(2/3.if_{f\alpha i} + d_{f\alpha i}) \tag{15}$$

Substituting into (14) and noting that

$$f_{1+}^{fi\alpha} = if_{f\alpha i}f_{1+}^{F} + d_{f\alpha i}f_{1+}^{D} \tag{16}$$

we get the following asymtotic values:

$$\begin{aligned} \lim_{s \to \infty} f_{1+}^{F}(M_f^2, s, 0) &= 2/3 \\ \lim_{s \to \infty} f_{1+}^{D}(M_f^2, s, 0) &= 1 \end{aligned} \tag{17}$$

If we follow Harrari's point of view [5] and say that the baryons are a linear combination of the octet part of (6,3) and (3,3*), with coefficients Cosω and Sinω respectively, then we get instead of (17), the following asymtotic values:

$$\begin{aligned} \lim_{s \to \infty} f_{1+}^{F}(M_f^2, s, 0) &= 2\cos^2\omega/3 \\ \lim_{s \to \infty} f_{1+}^{D}(M_f^2, s, 0) &= \cos^2\omega + \sin^2\omega/3 \end{aligned} \tag{18}$$

We shall restrict ourselves to the first point of view. Let us consider ordinary beta decay. By convention,

$$G_A^{pnT} = f_{1+}^{F}(M_N^2, M_N^2, 0) + f_{1+}^{D}(M_N^2, M_N^2, 0)$$

We can draw several conclusions:

(A). The classic results $G_A = 5/3$ and F/D are true as $s \to \infty$. That is, these are the unrenormalized values. (Assumption b.)

(B). It is not true that $G_{A0} = G_V$, where 0 denotes the corresponding unrenormalized value. In fact, the above expressions show that $G_{A0} = 5/3$ while $G_V = 1$

From now on we shall omit the indices pnT in G_A^{pnT} and πNN in $g_{\pi NN}$.

III. Dispersion relation for the axial vector current form factor: Degenerate mass case.

From the definition of the axial vector current vertex, we obtain on putting both baryons on mass shell, an expression for the axial vector current coupling constant $G_A^{fi\alpha}$ for leptonic decay from baryon state i to baryon state f through the axial vector current A^α, (From now on we shall omit the subscripts 1+ in f_{1+})

$$G_A^{fi\alpha} = -\sqrt{6}\begin{pmatrix} 8 & 8 & 8^a \\ i & \alpha & f \end{pmatrix} f^F - \sqrt{10/3}\begin{pmatrix} 8 & 8 & 8^s \\ i & \alpha & f \end{pmatrix} f^D \tag{19}$$

where DeSwart's notation has been used and again the convention is

$$G_A = f^F + f^D \qquad \text{for beta decay} \tag{20}$$

For specific physical processes, it is more transparent to use spherical basis as these have definite quantum numbers. Thus the evaluation of $G_A^{fi\alpha}$ in the degenerate mass approximation ends up with the evaluation of two form factors f^F and f^D. Since we know the asymtotic values of them we can write once subtracted dispersion relations,

$$f^F(M_f^2, M_i^2, 0) = f_0^F + 1/\pi . \int \frac{ds}{s - M_i^2} \; \mathrm{Im} f^F(M_j^2, s, 0) \tag{21}$$

$$f^D(M_f^2, M_i^2, 0) = f_0^D + 1/\pi . \int \frac{ds}{s - M_j^2} \, \mathrm{Im}\, f^D(M_j^2, s, 0) \qquad (22)$$

We can see from the above two expressions that the F/D ratio is no longer equal to 2/3, even without representation mixing, provided that the contribution from the dispersion integrals are not zero.

Using (10), (13) we can relate the spectral functions Im f^F, Im f^D to the absorptive part of the BBM vertex function. The final expressions are as follows:

$$\begin{aligned} \mathrm{Im}\, f^F(s) &= - C_\alpha^{fi}/2W . \, \mathrm{Im}\, (K_+^F(s) - K_-^F(s)) \\ \mathrm{Im}\, f^D(s) &= - C_\alpha^{fi}/2W . \, \mathrm{Im}\, (K_+^D(s) - K_-^D(s)) \end{aligned} \qquad (23)$$

where, whenever there is no confusion, we shall write

$$f(s) \qquad \text{in stead of} \qquad f(M^2, s, 0)$$

From the unitarity relation for the "meson form factor" $K^{f\alpha i}$ we get, using straightforeward SU(3) manipulations, unitarity conditions for the two reduced meson form factors K^F, K^D:

$$\begin{aligned} \mathrm{Im}\, K_{\pm}^F(s) &= \rho \left\{ M^{aa}_{\substack{1-\\0+}} K_{\pm}^F(s)^* + M^{as}_{\substack{1-\\0+}} K_{\pm}^D(s)^* \right\} \\ \mathrm{Im}\, K_{\pm}^D(s) &= \rho \left\{ M^{sa}_{\substack{1-\\0+}} K_{\pm}^F(s)^* + M^{ss}_{\substack{1-\\0+}} K_{\pm}^D(s)^* \right\} \end{aligned} \qquad (24)$$

where ρ is the phase space factor and M^{aa}, M^{as}, M^{sa}, M^{ss} are the four octet-octet scattering amplitudes and 1- , 0+ denote the particular partial wave. Note that we have only 8 to 8 amplitudes in the exact SU(3) approximation, because the initial baryon is in the octet representation.

We write,

$$K^{F}_{\pm}(M_j^2, s, k^2) = \frac{(1-\alpha) g F^{\pm}(M_j^2, s, \mu_\delta^2)}{\mu_\delta^2 - k^2}$$

$$K^{D}_{\pm}(M_j^2, s, k^2) = \frac{\alpha g F^{\pm}(M_j^2, s, \mu_\delta^2)}{\mu_\delta^2 - k^2} \tag{25}$$

Here α and $1 - \alpha$ are the usual symmetric and antisymmetric part of the pion-nucleon coupling constant, and by convention, F^+ is normalized to one when all the particles are on the mass shell,

$$F^+(M_j^2, M_i^2, \mu_\delta^2) = 1 \tag{26}$$

To find the corresponding value for F^-, we go back to the Ward identity and obtain after some algebra,

$$K^{j\delta i}_{-}(M_j^2, M_i^2, 0) = \frac{2M_i}{M_i + M_j} \frac{(\gamma_5 \lambda^\alpha/2)_{ji}}{G_A^{ji\delta}} \frac{g_{j\delta i} F^-(M_j^2, M_i^2, 0)}{\mu_\delta^2} \tag{27}$$

Note that $(\gamma_5 \lambda^\alpha/2)$ is essentially $G_{\Lambda 0}^{fi\alpha}$.

We can express (27) in terms of reduced matrix elements, and extrapolate equation (26) to zero mass by assuming

$$F^{+}(M^2,M^2,0) \approx 1 \tag{29}$$

we get,[4]

$$\begin{aligned} K_{+}^{F}(M^2,M^2,0) &\approx (1-\alpha)g/\mu^2 \\ K_{+}^{D}(M^2,M^2,0) &\approx \alpha g/\mu^2 \\ K_{-}^{F}(M^2,M^2,0) &\approx f_0^F g/G_A \mu^2 \\ K_{-}^{D}(M^2,M^2,0) &\approx f_0^D g/G_A \mu^2 \end{aligned} \tag{30}$$

We write out the spectral function again, in terms of the meson form factors:

$$\mathrm{Im}\ f^F(M^2,s,0) = \frac{M^2 G_A}{Wg}\ \mathrm{Im}\ (\ K_{+}^{F}(M^2,s,0) - K^F(M_{-}^2,s,0)\ ) \tag{31}$$

with an analogous expression for the D part. Note that the meson form factors must satify the coupled unitarity equation (24). The eventual problem to solve is thus to find a suitable expression for the K(s)'s and for the four octet-octet scattering amplitudes such that unitarity is satisfied both at the BBM vertex and for the scattering process.

IV. Solution of the coupled unitarity equation

We shall rewrite the coupled unitarity equation (24) in matrix form. Let

$$K_{\pm} = \begin{pmatrix} K^{F}_{\pm} \\ K^{D}_{\pm} \end{pmatrix}$$

$$M_{1-} = \begin{pmatrix} M^{aa}_{1-} & M^{as}_{1-} \\ M^{sa}_{1-} & M^{ss}_{1-} \end{pmatrix}$$

with similar expression for M_{0+} . In terms of matrices eqution (24) takes the form,

$$\begin{aligned} \mathrm{Im}\, K_{+}(s) &= \rho(s) M_{1-}(s) K_{+}(s)^{*} \\ \mathrm{Im}\, K_{-}(s) &= \rho(s) M_{0+}(s) K_{-}(s)^{*} \end{aligned} \tag{32}$$

Note the matrix M is symmetric, since $M_{as} = M_{sa}$. Let Q be the orthogonal matrix that diagonalizes it:

$$M' = Q^{T} M Q = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix} \tag{33}$$

On the new basis the K matrix becomes

$$K' = Q^{T} K \quad ; \quad K = Q K' \tag{34}$$

Let us call the matrix elements of K' K_1 and K_2 ,

$$K' = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

The unitarity equation becomes

$$\mathrm{Im}\, K'(s) = \rho(s) M'(s) K'(s)^* \tag{35}$$

or in component form,

$$\begin{aligned} \mathrm{Im}\, K_1(s) &= \rho(s) \Lambda_1(s) K_1(s)^* \\ \mathrm{Im}\, K_2(s) &= \rho(s) \Lambda_2(s) K_2(s)^* \end{aligned} \tag{36}$$

Since Λ_1, Λ_2 are scattering amplitudes, they must satisfy the unitarity equation

$$\mathrm{Im}\, \Lambda(s) = \rho(s) \Lambda(s) \Lambda(s)^* \tag{37}$$

We shall unitarize Λ by a scattering length type approximation. If we define

$$\begin{aligned} \Lambda^i_{0+}(s_0) &= L^i_{0+} \\ \Lambda^i_{1-}(s_0) &= \lim_{s \to s_0} L^i_{1-} k' k \end{aligned} \qquad i = 1, 2 \tag{38}$$

where s_0 is the threshold value of s and whenever there is no confusion, k and k' will stand for the absolute value of the center of mass meson three momenta $\underset{\sim}{k} = \underset{\sim}{q}_j$ and $\underset{\sim}{k}' = \underset{\sim}{p}'$ (see fig.1), then the amplitudes

$$\Lambda^{i}_{0+}(s) = \frac{1}{(L^{i}_{0+})^{-1} - i\rho(s)} \tag{39}$$

$$\Lambda^{i}_{1-}(s) = \frac{1}{(L^{i}_{1-}kk')^{-1} - i\rho(s)} \quad ; \quad i = 1,2$$

satisfy unitariy equation (37), and the unitarity equation for K determines

$$K^{i}_{+}(s) = K^{i}_{+}(M^2)\left[1-i\rho(M^2)\ L^{i}_{1-}k(M^2)k'(M^2)\right]/\left[1-i\rho(s)\ L^{i}_{1-}k(s)k'(s)\right]$$

$$K^{i}_{-}(s) = K^{i}_{-}(M^2)\left[1-i\rho(M^2)L_{0+}(M^2)\right]/\left[1-i\rho(s)L^{i}_{0+}\right] \tag{40}$$

Note that at $s = M^2$, where M is the SU(3) mean mass, the above equations are reduced to two trivial identities, as they should. Taking the imaginary parts both sides we have

$$\mathrm{Im}\ K^{i}_{+}(s) = K^{i}_{+}(M^2)\left[1-i\rho(M^2)\ L^{i}_{1-}k(M^2)k'(M^2)\right]\frac{s\rho^2(s)\rho'(s)/4L^{i}_{1-}}{(L^{i}_{1-})^{-2} + \frac{[s\rho^2(s)\rho'(s)]^2}{16}} \tag{41}$$

$$\mathrm{Im}\, K_-^i(s) = K_-^i(M^2)\left[1-i\rho(M^2)L_{0+}^i\right]\frac{\rho(s)/L_{0+}^i}{(L_{0+}^i)^{-2}+\rho^2(s)} \tag{42}$$

where $k(M^2)$ and $\rho(M^2)$ are the meson momentum and phase space factor evaluated at the unphysical point $s = M^2$, and are purely imaginary numbers. In the present case $k = k'$, but we keep the prime so that the expressions can be easily compared with those obtained in later sections, when mass splitting is taken into account.

Let

$$Q^p = \begin{pmatrix} e_{11}^p & e_{21}^p \\ e_{12}^p & e_{22}^p \end{pmatrix} \tag{43}$$

with a similar expression for Q^s, where the indices p and s represent the corresponding quantities for the p and s partial waves. From (34),

$$\begin{aligned} \mathrm{Im}\, K_+^F(s) &= e_{11}^p\ \mathrm{Im} K_+^1(s) + e_{21}^p\ \mathrm{Im}\, K_+^2(s) \\ \mathrm{Im}\, K_+^D(s) &= e_{12}^p\ \mathrm{Im}\, K_+^1(s) + e_{22}^p\ \mathrm{Im}\, K_+^2(s) \end{aligned} \tag{44}$$

with similar expressions for $\mathrm{Im}\, K_-^F(s)$ and $\mathrm{Im}\, K_-^D(s)$. Using (44), (22), (23) and noting that

$$f^F + f^D = G_A$$

we get

$$G_A = (G_{A0} - I_-^F - I_-^D)/(1 - I_+^F - I_+^D) \tag{45}$$

$$f^F = f_0^F + G_A I_+^F - I_-^F \tag{46}$$

$$f^D = f_0^D + G_A I_+^D - I_-^D \tag{47}$$

where

$$I_+^F,\ I_-^D,\ I_+^F,\ I_-^D = M/\pi g. \int_{s_0}^{s_{max}} \mathrm{Im}\ (\ K_+^F,\ K_-^D,\ K_+^F,\ K_-^D) \frac{ds}{\sqrt{s}(s-M^2)} \tag{48}$$

Equations (46), (47) are our final results for SU(3) symmetric calculation. The axial vector current coupling constant $G_A^{fi\alpha}$ for any leptonic decay of baryon B_i to baryon B_f are given by (46), (47) and (19).

Now since the dispersion integrals depend on the parameter α as well as G_A itself, equation (45) must be solved by numerical methods. We begin with any resonable values of G_A and α, calculate the s wave scattering length and p wave effective range and use (41),(42),(45),(46) and (47) to calculate G_A and the F/D ratio. By assumption (e),

$$\alpha = f^D/(f^F + f^D\ ) \tag{49}$$

so that the parameter α can be calculated as well. We repeat the calculation until consistent solution for both α and G_A are obtained. The results are as follows: If we choose $s_{max} = (2M)^2$ with M = 1.13 Bev we get,

$$G_A = 1.16 \qquad F/D = 0.600$$

If we choose $s_{max} = (3M)^2$ we get

$$G_A = 1.00 \qquad F/D = 0.575$$

The details of calculating the scattering amplitudes are shown in the Appendix. We see that the solutions are quite cut off dependent. In fact, if we extend s_{max} to infinity G_A would get well below 1.0 and it is difficult to get consistent solutions for both G_A and F/D . In view of our low energy approximation, a cut off value of much higher than $(2M)^2$ is not justified anyway. Therefore we can say that the results for an exact SU(3) calculation is G_A probably lies between 1.0 and 1.2 and F/D ration probably lies between 0.57 to 0.60. Noting that our unrenormalized values for G_A and for theF/D ratio are 5/3 and 2/3 respectively, the above results are not bad for such a crude calculation.

It will be seen that in the following broken symmetry calculation the results are far less cut off dependent.

V. Effect of mass splitting

Let us repeat the previous calculations, taking into account the effect of mass splitting, but otherwise the underlying symmetry group is still that of SU(3). This implies, along with others, that the intermmediate state B_j, M_δ and the final state B_f, M_α must combine to form octets, since we assume no symmetry breaking at the BBM vertex and the scattering bubble. However, the scattering process is now considered as a multichannel one. The channels involved are all those communicating with the initial baryon state B_i, If we again assume that two body channels dominate, then the number of channels is finite. The meson form factors $K_{\pm}(M^2,s,0)$ in general is a column matrix with matrix elements $K_{\pm}^{j\delta,i}$ where j,δ is the baryon-meson two body channel communicating with B_i. The scattering amplitude is an n x n symmetric matrix (see appendix II) $M_{1\pm}$. Unitarity at the BBM vertex requires

$$\mathrm{Im}\, K_{\pm} = M_{\substack{1-\\0+}} \rho K_{\pm}^{*} \tag{50}$$

while unitarity in the scattering requires

$$\mathrm{Im}\, M_{\substack{1-\\0+}} = M_{\substack{1-\\0+}}^{*} \rho\, M_{\substack{1-\\0+}} \tag{51}$$

Here ρ is the usual diagonal phase space factor. Again we construct M_{1-}, M_{0+} by a scattering length type approximation. Let

$$(B_{0+})^{f\alpha,j\delta} = L_{0+}^{f\alpha,j\delta} \tag{52}$$

$$(B_{1-})^{f\alpha,j\delta} = kk' L_{1-}^{f\alpha,j\delta} \tag{53}$$

and B_{0+}, B_{1-} the corresponding matrices, where L_{0+} and L_{1-} are the scattering length matrices given in appendix II. Let

$$Y_{0+} = 1/2.(\rho B_{0+} + B_{0+}\rho) \tag{54}$$

with a similar expresion for Y_{1-}, then

$$M_{0+} = \rho^{-1} Y_{0+}(1 - iY_{0+})^{-1} \tag{55}$$

with a similar expression for M_{1-}, satisfies (51) and is symmetric, while

$$K_{\pm} = \rho^{-1}(1-iY_{\substack{1-\\0+}})^{-1}\rho\, C_{\pm} \tag{56}$$

where C_+, C_- are constant matrices, satisfy (50). To find C_+ , C_-, we impose the condition that at $s = M_f^2 = M^2$, $K^{j\delta,i}(M^2,M^2,0)$ must give back the exact SU($\bar{3}$) coupling constants $g_{j\delta i}$. Noting that

$$K_{\pm}^{j\delta,i}(M_j^2,M_i^2,0) = g_{j\delta i}\; F_{\pm}(M_j^2,M_i^2,0)/\mu_\delta^2 \tag{57}$$

where

$$F_+(M_j^2,M_i^2,0) \approx 1 \tag{58}$$

$$F_-(M_i^2, M_j^2, 0) \approx \frac{2M_i G_{A0}^{ji\delta}}{(M_i + M_j) G_A^{ji\delta}} \tag{59}$$

we have

$$C_+ = \left[1 - i\,\rho^0(M^2) B_{1-}^0(M^2)\right] K_+^0 \tag{60}$$

$$C_- = \left[1 - i\,\rho^0(M^2) B_{0+}^0(M^2)\right] K_-^0 \tag{61}$$

where the argument M^2 means that the corresponding quantity is evaluated at the unphysical point $s = M^2$, and the superscript 0 means that all the masses involved are set equal to M, the SU(3) mean mass. Define

$$R_{1-}(s) = \rho^{-1}\, Y_{1-}(1 + Y_{1-}^2)^{-1} \rho \tag{62}$$

$$R_{0+}(s) = \rho^{-1}\, Y_{0+}(1 + Y_{0+}^2)^{-1} \rho \tag{63}$$

then the spectral function becomes

$$\mathrm{Im}\, f^{fi\alpha}(M_f^2, s, 0) = (M_i + M_f) G_A^{fi\alpha} \mu_\alpha^2 / 2g_{fi\alpha} \quad \times \tag{64}$$

$$\sum_{j\delta} \left\{ R_{1-}^{f\alpha, j\delta}(s) C_+^{j\delta, i} - R_{0+}^{f\alpha, j\delta}(s) C_-^{j\delta, i} \right\}$$

As we have pointed out before, the coupling constants $g_{fi\alpha}$, $g_{ji\delta}$ etc are charge independent ones (like $g_{NN\pi}$, $g_{\Sigma\Xi K}$) while the axial vector current coupling constants are basically dependent

on the charge states of the baryons, the ratios $G_A^{ji\delta}/g_{ji\delta}$ is no longer a constant, but is obtainable by an isospin analysis. Let $g'_{ji\delta}$ be the charge dependent coupling constant correponding to $g_{ji\delta}$, then,

$$g'_{ji\delta} = -\sqrt{3}\,c(j,\delta,i)\,g_{ji\delta} \tag{65}$$

$$\left(\text{example: } g_{pn\pi} = -\sqrt{3}\,(-\sqrt{2/3})\,g_{NN\pi}\right)$$

where $c(j,\delta,i)$ are Clebsch-Gordan coefficients of the isospin group. Thus,

$$G_A^{ji\delta}/g_{ji\delta} = c(j,\delta,i)\,G_A^{fi\alpha}/c(f,\alpha,i)\,g_{fi\alpha} \tag{66}$$

From which,

$$G_A^{fi\alpha} = (G_{A0}^{fi\alpha} - I_-^{fi\alpha})/(1 - I_+^{fi\alpha}) \tag{67}$$

where,

$$I_+^{fi\alpha} = (M_i + M_f)\mu_\alpha^2/2\pi \cdot \sum_{j\delta,n} \int_{s_n}^{s_{n+1}} ds\, \frac{{}_nR_{1-}^{f\alpha,j\delta}\,{}_nC_+^{j\delta i}\,c(j,\delta,i)}{\sqrt{s}\,(s-M_i^2)\,c(f,\alpha,i)\,g_{f\alpha i}} \tag{68}$$

$$I_-^{fi\alpha} = (M_i + M_f)\mu_\alpha^2/2\pi \cdot \sum_{j\delta,n} \int_{s_n}^{s_{n+1}} ds\, \frac{{}_nR_{0+}^{f\alpha,j\delta}\,{}_nC_-^{j\delta i}\,G_{A0}^{ji\delta}}{\sqrt{s}\,(s-M^2)\,g_{j\delta i}} \tag{69}$$

Some explaination must be given to the summation over n. Suppose m channels are involved. Let us denote the channels by $A_1, A_2, \ldots A_m$ in order of increasing threshold values s_1, $s_2, \ldots s_m$. When $s_1 < s < s_2$, only A_1 is open and R_{1-}, R_{0+} must be 1 x 1 matrices corresponding to the leading 1 x 1 submatrices of L_{1-} and L_{0+}; and C_+, C_- are one row column matrices corresponding to the first row of the column matrices K_+^0, K_-^0. When $s_2 < s < s_3$ both A_1 and A_2 are open and R_{1-}, R_{0+} are 2 x 2 matrices corresponding to the leading 2 x 2 submatrices of L_{1-}, L_{0+} and C_+, C_- are two-row column matrices corresponding to the first two rows of K_+^0, K_-^0, and so on. Finally when all the channels are open ($s > s_m$) the full matrices L_{1-}, L_{0+}, K_+, K_- are used. The summation over n is the summation over the above pieces. s_{n+1} is thus an arbitrary cut off value.

VI. Results and conclusion

In appendix II the matrices L_{1-}, L_{0+} are calculated using a method very similar to that of Weinberg's [3]. The input values of G_A and α (note that the dispersion integrals contain G_A and α) being 1.16 and 0.635 (F/D ratio = 0.575) respectively. These trial values are suggested by the results of section IV. The coupling constants are calculated with exact SU(3), as in Carruthers [6]. Three decay processes are considered: ordinary beta decay; $\Xi^- \to \Lambda$ and $\Lambda \to p$. The results are listed below, together with their respective unrenormalized values (according to the presesnt theory).

Process	F part	D part	G_{A0}	G_A
$n \to p$	1	1	1.667	1.22
$\Xi^- \to \Lambda$	$3/\sqrt{6}$	$-1/\sqrt{6}$	.408	.411
$\Lambda \to p$	$-3/\sqrt{6}$	$-1/\sqrt{6}$	-1.225	-1.01

A cut off value of $(5M_i)^2$ has been used. If we increase the cut off value to infinity, or decrease it to $(3M_i)^2$, the results will change by less than 3 per cent. So the calculation is reasonally cut off independent.

If we like we can repeat the entire calculation, using G_A = 1.22 as trial value. But this is hardly necessary since 1.22 is already quite near to the first trial value G_A=1.16 and the results are not too sensitive to small errors in the dispersion integrals.

In the first case (beta decay), we found that 80 per cent of the contribution to the dispersion integrals comes from the elastic region. Since the s wave scattering length and p wave

effective range for the I = 1/2, πN to πN scattering are known experimentally ($L_{0+} \approx 1.0$, $L_{1-} \approx -20$ BeV^{-3} in our units[8,9]), it is worthwhile to evaluate G_A again, using these experimental values, while keeping all other scattering lengths and effective ranges as those calculated in appendix II. In this way we found $G_A = 1.20$. The change 0.02 thus represents roughly the effect of SU(3) breaking at the BBM vertex and the scattering bubble. (The scattering lengths given in appendix II is calculated with "mass corrected" SU(3), see also beginning of section V.)

Thus we see, for beta decay at least, agreement with experiment is very good. We can check the results for the other two cases by calculating the leptonic branching ratio, using the values of $G_A^{fi\alpha}$ calculated by our method. (See, eg, Bell's lecture [7].) The results are listed below:

	Branching ratio		
process	present method	other method	experimental
$\Xi^- \to \Lambda$	$.75 \times 10^{-3}$	$.60 \times 10^{-3}$	$2.4 \pm 1.4 \times 10^{-3}$
$\Lambda \to p$	1.09×10	$.96 \times 10^{-3}$	$1.0 \pm 0.1 \times 10^{-3}$

The results under "other method" is obtained by using $G_A^{fi\alpha}$ calculated as follows: first take $f^F + f^D = 1.18$ and F/D ratio = 0.5 as known experimentally, then calculate $G_A^{fi\alpha}$, assuming exact SU(3)[7]. We see that our method does give better agreement with experiment than the simple minded theory, for the decay $\Xi^- \to \Lambda$; and for the decay $\Lambda \to p$ the results obtained by both methods are compatible with experiment.

Another merit of the present method is that the result is stable against small errors in the dispersion integrals, which contain the scattering amplitudes and meson form factors, both cannot be determined accurately. Thus, for example, an error of say 20 per cent in I_+ changes the value of G_A by about 0.03 .

The axial vector current coupling constant for all other processes can be calculated by the same method. In some cases it is more tedious as the number of channels involved increases to five.

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Appendix I. Evaluation of the octet-octet scattering amplitudes using Weinberg's low energy theorem.[3]

Consider the scattering of $B_j(q)$, $M_\delta(k)$ into $B_f(q')$, $M_\alpha(k')$. The S-matrix element is given by:

$$(S-1)_{fj} = \int d^4x\, d^4y\, (\Box_x^2+\mu_\alpha^2)(\Box_y^2+\mu_\delta^2) \times$$

$$\times \langle f(q')|\, T\{\phi_\alpha(x), \phi_\delta(y)\}\,|j(q)\rangle \frac{e^{ik'y-ikx}}{(2\pi)^3\sqrt{4\omega\omega'}}$$

$$= -i(2\pi)^4\delta^4(q'+k'-q-k)\,\frac{\bar{u}(q')\, T^{f\alpha, j\delta}\, u(q)}{(2\pi)^6\sqrt{\frac{4EE'\omega\omega'}{M_f M_j}}} \qquad \text{(A1)}$$

Current algebra gives

$$k^\mu k'^\nu \int d^4x\, d^4y\, (\mu_\delta^2-k^2)(\mu_\alpha^2-k'^2)\, e^{ik'y-ikx} \times$$

$$\times \langle f(q')|\, T\{A_\nu^\alpha(x), A_\mu^\delta(y)\}\,|j(q)\rangle$$

$$\approx C_\alpha^{fi} C_\delta^{ji} \Big\{ d^4x\, d^4y\, (\mu_\delta^2-k^2)(\mu_\alpha^2-k'^2)\, e^{ik'y-ikx} \times$$

$$\times \langle f(q')|\, T\Big\{\frac{\phi_\alpha(x)}{2}, \frac{\phi_\delta(y)}{2}\Big\}\,|j(q)\rangle\Big\} - \qquad \text{(A2)}$$

$$- i f_{\alpha\delta\gamma} \langle f(q')|\, V_\nu^\gamma(0)\,|j(q)\rangle \frac{(k+k')}{2}(\mu_\alpha^2-k'^2)(\mu_\delta^2-k^2)$$

If we take the gradient coupled Born approximation and let k^2, k'^2 go to zero (this implies both low energy and small meson mass, the latter assumption is unfortunately not quite justified except for pions) we have

$$\bar{u}(q')\, T^{f\alpha, j\delta}\, u(q) = \tag{A3}$$

$$= \bar{u}(q')\left\{ \frac{Z_d^{f\alpha, j\delta}\, g^2}{(M_j+M_i)(M_f+M_i)}\, \gamma_5 \not{k}' \, \frac{\not{q}+\not{k}+M_p}{s-M_p^2}\, \gamma_5 \not{k} \; + \right.$$

$$\left. \frac{Z_x^{f\alpha, j\delta}\, g^2}{(M_j+M_i)(M_f+M_i)}\, \gamma_5 \not{k} \, \frac{\not{q}'-\not{k}'-M_{p'}}{u-M_{p'}^2}\, \gamma_5 \not{k} \right\} u(q) \; +$$

$$+ \sum_{\gamma} \frac{1}{2\pi} \sqrt{\frac{E'E}{M_j M_f}}\, \frac{\mu_\alpha^2 \mu_\beta^2}{C_\alpha^{fi}\, C_\delta^{ji}}\, 4i f_{\alpha\delta\gamma} \left\langle f(q') \middle| V_\nu^\gamma(0) \middle| j(q) \right\rangle \frac{(k'+k)_\nu}{2}$$

Here $Z^{f\alpha, j\delta}$ are octet-octet scattering factors to be determined later.

The first term corresponds to the direct baryon octet pole; the second term corresponds to the exchanged baryon octet pole, and the final term corresponds to the equal time commutator. In our calculation, the initial baryon B_i is in the octet respresentation, therefore under our assumption that there is no symmetry breading at the BBM vertex and the scattering bubble, the intermediate baryon-meson states B_j, M_δ must couple to form an octet. Similarly the final baryon-meson states B_f, M_α must couple to form an octet. Hence the scattering amplitudes are of the octet-

octet type and is therefore a 2 x 2 symmetric matrix with matrix elements equal to the odd-odd, odd-even, even-odd and even-even amplitudes. Furthermore, the direct pole must be a baryon pole. The exchanged pole can be a baryon octet, a baryon decuplet, or anything. However, it can be shown that the contribution from the exchanged decuplet to the scattering amplitude is one order in k^2 smaller and therefore negligible in our approximation.

We write in the usual way,[10]

$$T^{f\alpha,j\delta} = A^{f\alpha,j\delta} + 1/2.(\not{k} + \not{k}')B^{f\alpha,j\delta} \tag{A4}$$

where A, B are scalar functions. We can find the contribution to A, B from the direct Born, crossed Born and commutator terms by the usual method. After some straightforeward but tedious algebra we find, (the subscripts d, x, c stand for contibutions from the direct Born, crossed Born and commutator term)

$$A_d^{f\alpha,j\delta} = \frac{Z_d^{f\alpha,j\delta}\, g^2}{(M_j + M_i)(M_f + M_i)} \left\{ \frac{(M_j+M_p)(s-M_f^2)}{2(s-M_p^2)} + \frac{(M_f+M_p)(s-M_j^2)}{2(s-M_p^2)} \right\} \tag{A5}$$

$$A_x^{f\alpha,j\delta} = \frac{Z_x^{f\alpha,j\delta}\, g^2}{(M_j + M_i)(M_f + M_i)} \left\{ \frac{(M_j+M_{p'})(u-M_f^2)}{2(u-M_{p'}^2)} + \frac{(M_f+M_{p'})(u-M_j^2)}{2(u-M_{p'}^2)} \right\} \tag{A6}$$

$$B_d^{f\alpha,j\delta} = -\frac{Z_d^{f\alpha,j\delta}\, g^2}{(M_j + M_i)(M_f + M_i)} \left\{ \frac{s+M_jM_f+M_fM_p+M_pM_j}{s-M_p^2} \right\} \tag{A7}$$

$$B_x^{f\alpha,j\delta} = \frac{Z_x^{f\alpha,j\delta}\, g^2}{(M_j+M_i)(M_f+M_i)} \left\{ \frac{u+M_jM_f+M_fM_{p'}+M_pM_j}{u-M_{p'}^2} \right\} \tag{A8}$$

where p and p' denote the direct and crossed poles.

To find the Z's, we assume exact SU(3) but for the time being keep the subscripts to the M's. Then $f\alpha=8_\gamma$, $j\delta=8_{\gamma'}$, where γ=a or s. There are all together six independent factors:

$$
\begin{aligned}
Z_d^{ss} &= 20/3.\alpha^2 \\
Z_d^{as} &= 4\sqrt{5}\alpha(1-\alpha) = Z_d^{sa} \\
Z_d^{aa} &= 12(1-\alpha)^2 \\
Z_x^{ss} &= -\ 2(4\alpha^2 - 6\alpha + 3) \\
Z_x^{aa} &= 2/3.(4\alpha^2 - 18\alpha + 9) \\
Z_x^{as} &= Z_x^{sa} = 0
\end{aligned}
\tag{A9}
$$

Note that Z depends on the parameter α. To find corresponding expressions for the commutator term we note

$$
1/2.(k'+k)_\nu\, i f_{\alpha\delta\gamma} \left\langle f(q') \middle| V_\gamma^\nu(0) \middle| j(q) \right\rangle
$$

$$
\sim\ \bar{u}(q')\left\{ f_e \gamma_\nu + \frac{i\sigma_{\mu\nu} f_m}{M_f + M_j}(q'-q)^\mu \right\} u(q)(-\underset{\sim}{F}^M.\underset{\sim}{F}^B)(k'+k)^\nu/2
$$

where f_e = 1.0 and f_m = 3.71 are electric and magnetic moment form factors, and $\underset{\sim}{F}^M$, $\underset{\sim}{F}^B$ stand for the F-spin operators for the meson

octet and the baryon octet. Using Gordan decomposition we can write the right hand side as

$$\bar{u}(q')\left\{(f_e+f_m)(\not{k}+\not{k}')/2 - f_m(q'+q)(k+k')/2(M_f+M_j)\right\} u(q) \times$$
$$\times\ (-\underset{\sim}{F}^M.\underset{\sim}{F}^B)$$

Comparing with (A4) we get,

$$B^{aa}_{c,t} = -(f_e+f_m)\frac{4g^2}{(M_i+M_j)(M_i+M_f)G_A^2}\ \frac{1}{2}\ (\underset{\sim}{F}^2 - \underset{\sim}{F}^{B^2} - \underset{\sim}{F}^{M^2}) \qquad \text{(A10)}$$

$$A^{aa}_{c,t} = +\ \frac{f_m}{8s(M_j+M_f)}\left\{4s^2 - \left[(M_j^2+M_f^2) - (\mu_\delta^2+\mu_\alpha^2)\right]^2\right\} \times$$

$$\times\ \frac{4g^2}{(M_j+M_i)(M_f+M_i)G_A^2}\ \frac{1}{2}\ (\underset{\sim}{F}^2 - \underset{\sim}{F}^{B^2} - \underset{\sim}{F}^{M^2}) \qquad \text{(A11)}$$

$$A^{ss}_{c,t} = A^{as}_{c,t} = B^{ss}_{c,t} = B^{as}_{c,t} = 0 \qquad \text{(A12)}$$

where $\underset{\sim}{F}$ is the F-spin operator for the combined baryon and meson octet and the subscript t reminds us of the fact that we are working in the t channel. Only the odd-odd amplitude is non-zero because of the presence of the factor $\underset{\sim}{F}^M.\underset{\sim}{F}^B$. In deriving (A11) we have neglected a term containing a factor $(\underset{\sim}{k}+\underset{\sim}{k}').(\underset{\sim}{k}+\underset{\sim}{k}')$ in consistency with our low energy assumption.

In accordance with the usual assumption in current algebra,

the commutator of two current octets is again an octet of currents. Therefore

$$\underset{\sim}{F}^2 = \underset{\sim}{F}^{M^2} = \underset{\sim}{F}^{B^2} = 3 \tag{A13}$$

Finally using the crossing matrix in the octet-octet space[6] we can find the contribution from the commutator to the scattering amplitudes in the s channel

$$B_c^{ss} = B_c^{aa} = \frac{3(f_e + f_m)g^2}{(M_j + M_i)(M_i + M_f)G_A^2} \tag{A14}$$

$$A_c^{ss} = A_c^{aa} = -\frac{3f_m g(4s^2 - [(M_f^2+M_j^2)-(\mu_\alpha^2 + \mu_\delta^2)]^2}{8s(M_i+M_f)(M_i+M_j)(M_f+M_j)G_A^2} \tag{A15}$$

$$B_c^{as} = A_c^{as} = 0 \tag{A16}$$

The partial wave amplitudes $f_\ell^{\gamma\gamma'}$ is related to A, B by[10]:

$$f_{\ell\pm} = 1/2. \int_{-1}^{1} dx \left\{ f_1(x)P_\ell(x) + f_2(x)P_{\ell\pm1}(x) \right\} \tag{A17}$$

$$f_1 = \frac{1}{8\pi W}(E+M_j)^{1/2}(E'+M_f)^{1/2}\left\{A + (W - \frac{M_j+M_f}{2})B\right\} \tag{A18}$$

$$f_2 = \frac{1}{8\pi W}(E-M_j)^{1/2}(E'-M_f)^{1/2}\left\{-A + (W + \frac{M_j+M_f}{2})B\right\} \tag{A19}$$

The partial wave amplitudes must satisfy the unitarity equation

$$\mathrm{Im}\, f_{\ell\pm}^{\gamma\gamma'} = \sum_n k_n f_{\ell\pm}^{\gamma\gamma_n *} f_{\pm}^{\gamma_n \gamma'} \tag{A20}$$

where $k_n = |\underset{\sim}{k}_n|$ is the momentum of the intermediate state in their center of mass frame.

It is more customary to define the partial wave amplitude $M_{\ell\pm}^{\gamma\gamma'}$ related to the above partial wave amplitudes by

$$M_{\ell\pm}^{\gamma\gamma'} = W/2.f_{\pm}^{\gamma\gamma'} \tag{A21}$$

and satisfies the unitarity equation

$$\mathrm{Im}\, M_{\ell\pm}^{\gamma\gamma'} = \sum_n \rho_n M_{\ell\pm}^{\gamma\gamma_n *} M_{\ell\pm}^{\gamma_n \gamma} \tag{A22}$$

with

$$\rho_n = 2k_n/W \tag{A23}$$

so that $\rho_n \to 1$ as $s \to \infty$.

We define the s wave scattering length and p wave effective range L_{0+}, L_{1-}, by the following equations

$$M_{0+}^{\gamma\gamma'}(s_0) = L_{0+}^{\gamma\gamma'} \tag{A24}$$

$$M_{1-}^{\gamma\gamma'}(s_0) = \lim_{k,k'\to 0} kk'\, L_{1-}^{\gamma\gamma'} \tag{A25}$$

where 0 denotes the threshold value.

Tables 1 and 2 give the values of L_{0+}, L_{1-}, as well as their eigenvalues, for different values of α. The mean masses are taken as M=1.13BeV, μ= .49BeV and G_A is 1.16 .

Appendix II Octet-octet scattering amplitudes with mass splitting.

We shall give in the following section expressions for the octet part of the baryon-meson scattering amplitudes, taking into account the effect of mass splitting, but with coupling constants and the contribution from the commutator as given by exact SU(3). The first approximation may be controversial. Frautschi and Dashen [11] have shown, in a theoretical calculation based on self-consistency, that the coupling shifts in broken SU(3) is typically 100 per cent of the values given by exact SU(3). Kim [12] in an analysis based on $\bar{K}N$ scattering data, showed that $N\Lambda K$ and $N\Sigma K$ coupling constants are consistent with those obtained from exact SU(3), contrary to Dashen and Frautschi's conclusion. In view of this uncertainty we shall carry out the calculation with coupling constants as given by exact SU(3)[6]. The scattering amplitudes will be written in the form of symmetric m x m matrics where m is the number of communicating two-body channels. The matrix element $M^{f\alpha,j\delta}$ represents the scattering from j,δ to f,α. We define, as in appendix I,

$$M_{0+}^{f\alpha,j\delta}(s_0) = L_{0+}^{f\alpha,j\delta} \tag{A26}$$

$$M_{1-}^{f\alpha,j\delta}(s_0) = \lim_{k,k'\to 0} kk' L_{1-}^{f\alpha,j\delta} \tag{A27}$$

All the results in appendix I can be carried over to this section, with the following modifications:

(i). The unitarity equation becomes

$$\mathrm{Im}\, M_{\ell\pm} = M^{*}_{\ell\pm}\, \rho\, M_{\ell\pm} \tag{A28}$$

where ρ is a diagonal matrix. If the channel is open, the corresponding matrix element ρ_n is as defined in (A23), otherwise it is zero. The * stands for complex conjugation.

(ii). The factors $Z_d^{f\alpha,j\delta}$, $Z_x^{f\alpha,j\delta}$, $Z_c^{f\alpha,j\delta}$ can be expressed in terms of the nine factors Z_d^{aa}, Z_x^{as}, Z_c^{ss} (note that from (A14)-(A16), $Z_c^{aa} = Z_c^{ss} = 1$, $Z_c^{as} = 0$) by noting that [6]

$$Z^{f\alpha,j\delta} = \sum_{\mu_\gamma \mu_{\gamma'}} \left(\begin{array}{cc|c} 8 & 8 & \mu_\gamma \\ j & \delta & \lambda \end{array}\right) \left(\begin{array}{cc|c} 8 & 8 & \mu_{\gamma'} \\ f & \alpha & \lambda \end{array}\right) Z^{\mu_\gamma \mu_{\gamma'}} \tag{A29}$$

where $\left(\begin{array}{cc|c} 8 & 8 & \mu_\gamma \\ j & \delta & \lambda \end{array}\right)$ are isoscalar factors as we are neglecting electromagnetic effects. In particular, if we keep only the octet part of the scattering amplitude the sum extends over Z^{aa}, Z^{ss} and Z^{as} only.

Using (A5) to (A8), (A14) to (A19) and (A29) we can calculate the s wave scattering length and p wave effective range. These are put into the form of symmetric matrices: (We use $G_\Lambda = 1.16$, $\alpha = .635$)

I = 1/2, Y = 1

$$L_{1-} = \begin{array}{c} \begin{array}{cccc} N\pi & N\eta & \Lambda K & \Sigma K \end{array} \\ \begin{pmatrix} -27.057 & -.772 & 1.779 & -6.218 \\ & 3.167 & -1.522 & 3.914 \\ & & -.882 & -2.614 \\ & & & 5.383 \end{pmatrix} \end{array} \quad \mathrm{BeV}^{-2}$$

$$L_{0+} = \begin{pmatrix} .393 & .257 & -.932 & -1.583 \\ & .670 & -.944 & .780 \\ & & .259 & -.824 \\ & & & 2.226 \end{pmatrix}$$

I = 1/2, Y = -1

$$L_{1-} = \begin{array}{c} \begin{array}{cccc} \Xi K & \Lambda \bar{K} & \Sigma \bar{K} & \Xi \eta \end{array} \\ \begin{pmatrix} 1.375 & 3.311 & 2.031 & -2.254 \\ & 9.302 & 2.457 & -.773 \\ & & -6.609 & 2.361 \\ & & & -1.509 \end{pmatrix} \end{array} \quad \mathrm{BeV}^{-2}$$

$$L_{0+} = \begin{pmatrix} .302 & .013 & -1.074 & -.206 \\ & 3.117 & 1.714 & -.653 \\ & & 1.233 & -1.329 \\ & & & .179 \end{pmatrix}$$

<u>I = 0, Y = 0</u>

$$L_{1-} = \begin{array}{c} \begin{array}{cccc} \Sigma\pi & N\bar{K} & \Lambda\eta & \Xi K \end{array} \\ \begin{pmatrix} 12.759 & 11.982 & -1.105 & -1.274 \\ & -4.618 & 1.955 & 1.497 \\ & & -.226 & 2.087 \\ & & & .223 \end{pmatrix} \end{array} \quad \mathrm{BeV}^{-2}$$

$$L_{0+} = \begin{pmatrix} 2.641 & 1.553 & .368 & -.779 \\ & .980 & -.500 & .842 \\ & & .149 & .616 \\ & & & .134 \end{pmatrix}$$

Table I : Octet-octet p-wave effective range L_{1-} and its eigenvalues for different values of α.

α	ss	as	ss	eigenvalues 1	2
.615	.035	-10.419	1.912	11.435	-9.488
.625	-.401	-10.314	2.482	11.455	-9.347
.635	-.841	-10.199	3.039	11.481	-9.283
.645	-1.285	-10.076	3.584	11.515	-9.216
.655	-1.734	-9.944	4.116	11.556	-9.174

Table II : Octet-octet s-wave scattering length L_{0+} and its eigenvalues for different values of α.

α	ss	as	ss	eigenvalues 1	2
.615	1.724	-.301	1.548	1.950	1.322
.625	1.708	-.298	1.580	1.949	1.339
.635	1.691	-.295	1.612	1.950	1.354
.645	1.675	-.291	1.643	1.950	1.368
.655	1.660	-.287	1.674	1.954	1.380

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Figure Caption

Fig. 1 : The axial vector current vertex and its absorptive part.

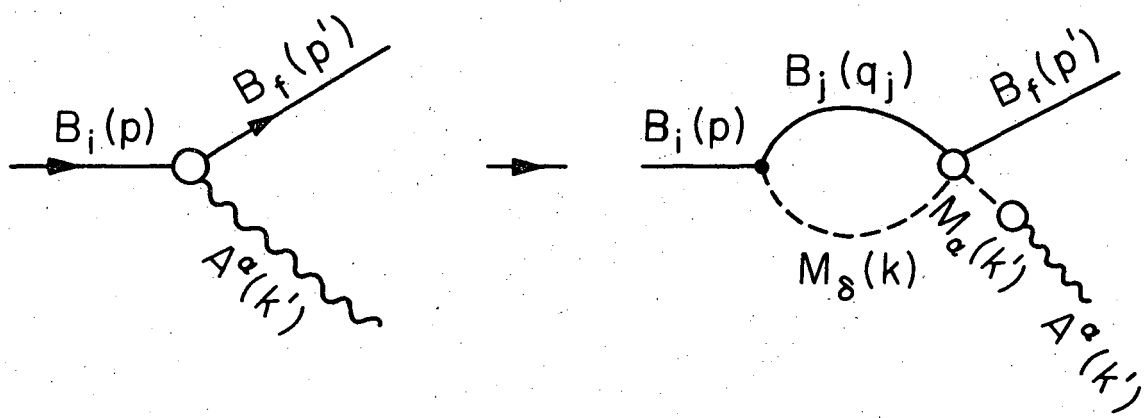

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Fig. 1

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