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APPLICATIONS OF THE LORENTZ GROUP TO REGGE POLE ANALYSIS OF PION PHOTOPRODUCTION

Jan William Dash
(Ph. D. Thesis)

July 20, 1968

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UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

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ABSTRACT

We consider applications of Lorentz invariance and Regge pole theory to the phenomenological study of photoproduction of pions from nucleons and nucleon-nucleon scattering. We first work out the kinematics for the more general cases having all unequal and nonzero external masses, and then take the limit of equal baryon masses and zero photon mass. We then consider the applications of Lorentz symmetry at $t = 0$ (i.e. conspiracy), and explicitly obtain the photoproduction daughter sequence through analyticity arguments. The factorization of the photoproduction Regge residues is shown to be consistent with the conspiracy relations in the nucleon-nucleon and pion Compton scattering processes. Finally, we analyze high energy pion photoproduction data along with nucleon-nucleon data assuming the existence of an $M = 1$ pion parity doublet

conspiracy. We investigate the consequences of assuming a square-root type zero or a full zero in the $\pi N\bar{N}$ vertex function. The possibility of the existence of an M = 1 B-ρ' parity doublet conspiracy is investigated qualitatively through a photoproduction sum rule for the photon isoscalar amplitude, and by requiring consistency between phenomenological fits. Additional experimental tests for an M = 1 B trajectory are proposed

INTRODUCTION

It has been known for some time that the differential cross sections for positive pion photoproduction[1] show a marked forward peak very close to t=0, similar to the peak found in np charge exchange, with a width close to μ^2. The successful application of the assumption of an M=1 pion Lorentz pole with a zero in the $\pi N\bar{N}$ vertex function to np charge exchange[2] makes a similar conjecture attractive in pion photoproduction.[3] Although the unequal external π and γ masses make the direct application of the usual group theoretical arguments at t=0[4] more difficult, analyticity arguments and the asymptotic behavior of amplitudes can be used to give meaning to an M=1 pion daughter sequence, which possesses the same members as in NN scattering. We study the general kinematics of photoproduction and NN scattering by taking the limit of equal baryon masses and zero photon mass in the reactions involving all unequal and nonzero external masses. We next show that the constraints in photoproduction, pion Compton scattering, and NN scattering can be satisfied by the asymptotic Regge pole contributions to the scattering amplitudes assuming factorized Regge residues.

The existing positive pion photoproduction data range from laboratory momenta of 2.6 to 16 GeV/c. We find that the M=1 parity doublet (the pion π and its parity doublet partner π') provides a satisfactory explanation of the data up to $t=-0.5\,\mathrm{GeV}^2$ if the ρ, A_2, and B trajectories are also included (as they were in Ref. 2). These latter trajectories are all assumed to be M=0 trajectories with the $BN\bar{N}$ residue vanishing at t=0. (The B parent trajectory is completely

neglected here, i.e., we assume it decouples completely from the $N\bar{N}$ and $\gamma\pi$ channels). The only other known meson trajectory that could be exchanged here is the A_1 trajectory. Although the A_1 trajectory (with an M=0 assignment) seems necessary to fit certain resonance production data,[5] we do not include it here. Thus the fits here are consistent with the assumption of zero (or small) $A_1 N\bar{N}$ and $A_1\gamma\pi$ couplings.

The question of the order of the zero in the $\pi N\bar{N}$ residue function is also investigated (this dynamical zero is denoted here by t_0). We find that the assumption of a full zero in the vertex function is preferred over that of a square-root type zero in the above model. Constraints involving factorization have been imposed from previous fits,[6] and a fit assuming the existence of a double zero in the pion $N\bar{N} \to N\bar{N}$ residue function has been carried out for the reactions $np \to pn$ and $p\bar{p} \to n\bar{n}$. This double zero occurs around $t_0 \approx -2.5\ \mu^2$ rather than $t_0 \approx -\mu^2$ as is the case if a single zero is assumed.

We have also qualitatively investigated the possibility that the B trajectory is an $M = 1$ rather than an $M = 0$ object. For some time people have speculated about the possibility of the B trajectory conspiring with an as yet unknown trajectory, usually denoted by ρ', from certain high energy data.[7] Here we find evidence from two sources that this may be the case. The first is a photoproduction sum rule for the B trajectory, similar to the Bietti-Roy-Chu sum rule for the pion trajectory.[8] It was found that there was evidence from this sum rule for a conspiring pion with a zero in the pion residue

at $t_0 \approx -1.5\ \mu^2$, qualitatively (but not exactly) consistent with phenomenological fits of the data. We perform a similar calculation using the small photoproduction isoscalar amplitudes for the B trajectory and find similar results; the B residue is small and nonvanishing at $t = 0$ with a zero displaced by about $+5\ \mu^2$. The second source comes from the pion photoproduction and NN data, relying on the Regge fits. The small $M = 1$ B amplitude suggested by the sum rule seems inconsistent with the large $M = 0$ B amplitude found in the fit. Further, if one demands consistency of the position of the zero in the pion residue function found in these fits with the Bietti-Roy-Chu sum rule, an $M = 1$ B trajectory is preferred over $M = 0$. Experimental tests involving the $p\bar{p}$-$n\bar{n}$ and $\pi^- p$-γn reactions at small t are proposed to make a quantitative determination possible.

In Section I we give an account of the pion photoproduction and NN scattering formalism. In Section II we describe the data and the fits. Section III describes the photoproduction B sum rule. Section IV is concerned with qualitative remarks designed to support an M=1 assignment for the B trajectory.

I. KINEMATICS OF PION PHOTOPRODUCTION AND NN SCATTERING

A. Pion Photoproduction

We first consider the reaction with all the external masses being nonzero and unequal. We define the s and t channels as

$$s: \quad \gamma N_1 \to \pi N_2$$

$$t: \quad \gamma\pi \to \bar{N}_1 N_2$$

where m_γ, μ, m_1, and m_2 denote the masses of γ, π, N_1, and N_2 respectively. We denote the t channel scattering angle by θ_t ($z_t = \cos\theta_t$) and the t channel helicity amplitudes by $f^t_{\lambda_1\lambda_2,\lambda_\gamma}(s,t)$. Parity conservation reduces the number of independent amplitudes to six; hence we consider only those amplitudes with $\lambda_\gamma = 0$ or 1. We define $\lambda_i = \pm$ to mean $\lambda_i = \pm\frac{1}{2}$ (i=1,2). Extracting the half angle factors

$$(1+z_t)^{\frac{1}{2}|\lambda_1-\lambda_2+\lambda_\gamma|} \qquad (1-z_t)^{\frac{1}{2}|\lambda_1-\lambda_2-\lambda_\gamma|}$$

from the helicity amplitudes in the usual way, we define the six parity conserving amplitudes[9]

$$f^{\pm}_{++,1}(s,t) = \frac{1}{\sin\theta_t}\left(f^t_{++,1} \pm f^t_{--,1}\right)$$

$$f^{\pm}_{+-,1}(s,t) = \frac{f^t_{+-,1}}{1+z_t} \pm \frac{f^t_{-+,1}}{1-z_t}$$

$$f^{-}_{++,0}(s,t) = f^t_{++,0}$$

$$f^{-}_{+-,0}(s,t) = \frac{1}{\sin\theta_t} f^t_{+-,0} \qquad (1)$$

Notice that the amplitudes $f^{+}_{++,0}$ are zero via parity conservation. Now in the unequal mass case, the helicity amplitudes are presumed analytic at t=0, since no psuedothreshold or physical region boundary coincides with this point. Since $z_t \to 1 + O(t)$ as $t \to 0$, we see that $f^{\pm}_{++,1}$ and $f^{-}_{+-,0}$ have $t^{-\frac{1}{2}}$ singularities, while $f^{\pm}_{+-,1}$ have t^{-1} singularities. Further, since both $tf^{\pm}_{+-,1}$ only depend on $f^{t}_{-+,1}$ at t=0,

$$t f^{+}_{+-,1} = -t f^{-}_{+-,1} \qquad \text{as } t \to 0 \tag{2}$$

This relation is the unequal mass conspiracy relation. Conspiracy relations will be discussed in more detail later.

The parity conserving partial-wave amplitudes $f^{J\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(t)$ are defined through the usual partial wave expansions of the parity conserving amplitudes

$$f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(s,t) = \sum_J (2J+1)\left[e^{J+}_{\lambda\mu}(z_t) f^{J\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(t) + e^{J-}_{\lambda\mu}(z_t) f^{J\mp}_{\lambda_1\lambda_2,\lambda_\gamma}(t)\right] \tag{3}$$

where $\lambda=\lambda_\gamma$ and $\mu= \lambda_1-\lambda_2$.

The functions $e^{J\pm}_{\lambda\mu}(z_t)$ are defined and listed in Ref. 9. We remark here that $e^{J-}_{00}(z_t)=e^{J-}_{10}(z_t) =0$, accounting for the absence of the second term in the expansions of $f^{\pm}_{++,1}$ and $f^{-}_{+\pm,0}$.

Next, we use the L-S coupling method of Jackson and Hite[10] to extract the threshold and psuedothreshold kinematic singularities from the parity conserving amplitudes and to find their threshold and psuedothreshold relations. This method involves expanding the partial wave helicity amplitudes in terms of L-S coupling amplitudes and then using the known behavior of these amplitudes around thresholds

and psuedothresholds (psuedothresholds may be treated in exactly the same manner as thresholds if "psuedo" L-S amplitudes are defined with the parity of the lighter particle at each vertex being changed if it is a fermion; all parities otherwise remaining unchanged.)

The intrinsic spin and orbital angular momentum associated with the initial ($\gamma\pi$) and final ($\bar{N}_1 N_2$) states will be denoted by S,L and S′,L′ respectively. We define the L-S t-channel partial wave amplitudes $f^{J\pm}_{L'S'LS}$ with parities $(-1)^J$ and $(-1)^{J+1}$ respectively through the expansion

$$f^{J\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(t) = \sum_{SS'LL'} \delta_{S'} C^{\lambda_1,-\lambda_2}_{\frac{1}{2}\frac{1}{2}S'} C^{0\lambda_\gamma}_{L1J} C^{0,\lambda_1-\lambda_2}_{L'S'J} f^{J\pm}_{L'S'LS}(t) \qquad (4)$$

where $C^{m_1 m_2}_{j_1 j_2 J}$ are the usual Clebsch-Gordan coefficients. Near initial or final thresholds or psuedothresholds we expand the L-S amplitudes respectively about $T_i^2=0$, where $i=N,N',P,P'$ denotes $T_N^2 = t-(\mu+m_\gamma)^2$, $T_{N'}^2 = t-(m_1+m_2)^2$, $T_P^2 = t-(m_\gamma-\mu)^2$, and $T_{P'}^2 = t-(m_1-m_2)^2$. Near $T_N^2=0$ for example, we obtain

$$f^{J\pm}_{L'S'LS}(t) = T_N^L \sum_{n=0}^{\infty} T_N^{2n} \tilde{f}^{J\pm}_{L'S',L+2n,S} \qquad (5)$$

Next, we expand the $e^{J\pm}_{\lambda\mu}(z_t)$ functions in powers of z_t^{-2}, obtaining

$$e^{J\pm}_{\lambda\mu}(z_t) = z_t^{J-\lambda_M-\eta_\pm} \sum_{k=0}^{\infty} r_k^{J\lambda\mu\pm} z_t^{-2k} \qquad (6)$$

where $\lambda_M = \max(|\lambda|,|\mu|)$ and $\eta_\pm = \begin{Bmatrix}0\\1\end{Bmatrix}$. This expansion is valid near $T_i^2=0$ when all external masses are unequal, since near these points $z_t \to \infty$.

Multiplying the two series for $f^{J\pm}_{L'S'LS}$ and $e^{J\pm}_{\lambda\mu}$ together in the expansion of the full helicity amplitudes yields

$$f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(s,t) = \sum_{JSS'LL'} \Big\{ T_N^{L-J+\lambda_M} (2J+1)\, \delta_{S1}\, C^{\lambda_1,-\lambda_2}_{\frac{1}{2}\frac{1}{2}S'}\, C^{0\lambda_\gamma}_{L1J}\, C^{0,\lambda_1-\lambda_2}_{L'S'J} \, (T_N z_t)^{J-\lambda_M}$$

$$\cdot \sum_{n,k=0}^{\infty} (T_N z_t)^{-2k}\, T_N^{2n+2k} \Big[r_K^{J\lambda\mu+}\, \tilde{f}^{J\pm}_{L'S',L+2n,S} + \frac{T_N}{T_N z_t}\, r_K^{J\lambda\mu-}\, \tilde{f}^{J\mp}_{L'S',L+2n,S} \Big] \Big\} \quad (7a)$$

We define $L_i^{\pm}$ to be the minimum value of the orbital angular momentum L in the $\pm$ parity conserving state at $T_i^2=0$ $(i=N,N',P,P')$. We obtain $L_N^{\pm} = L_P^{\pm} = L_{P'}^{\pm} = \left\{ \begin{matrix} J \\ J-1 \end{matrix} \right\}$ and $L_{N'}^{\pm} = \left\{ \begin{matrix} J-1 \\ J \end{matrix} \right\}$ for $f^{J\pm}_{L'S'LS}$ respectively. Further, $f^{J+}_{L'S'LS}$ $(f^{J-}_{L'S'LS})$ do not couple to $S'=0$ at $T_{N'}^2=0$ $(T_{P'}^2=0)$ respectively due to the $\bar{N}N$ quantum numbers.

The kinematic singularities of the full helicity amplitudes $f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(s,t)$ at $T_N^2=0$ are just the factors $T_N^{L_N^{\pm}-J+\lambda_M}$ appearing in the $L=L_N^{\pm}$ term of the expansion (note that $L_N^{\pm}-J$ is independent of J). The threshold relations at $T_N^2=0$ are found by comparing expansions for the amplitudes $f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}$ in each order of T_N^{2n+2k}. For n=k=0 we get the usual threshold relations, and for $n,k>0$ we get derivative threshold relations when they exist.

Following this procedure we split off the $(L,n,k)=(L_N^{\pm},0,0)$ term from the expansion about $T_N^2=0$, obtaining

$$f^{\pm}_{\lambda_1\lambda_2,\lambda_3}(s,t) = T_N^{L_N^{\pm}-J+\lambda_M} \sum_{JSS'} (T_N z_t)^{J-\lambda_M} \Big\{ r_0^{J\lambda\mu+} \tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'}$$

$$+ \frac{T_N^{\alpha_\pm}}{T_N z_t} r_0^{J\lambda\mu-} \tilde{a}^{J\lambda\mu\mp}_{L_N^{\mp}SS'} + O(T_N^2)\Big\} \qquad (7b)$$

where $\alpha_\pm = L_N^{\mp} - L_N^{\pm} + 1$ and

$$\tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'} = \sum_{L'} (2J+1)\, C^{0\lambda}_{L_N^{\pm}1J}\, C^{0\mu}_{L'S'J}\, \delta_{S1}\, C^{\lambda_1,-\lambda_2}_{\frac{1}{2}\frac{1}{2}S'} \begin{Bmatrix} \delta_{S'0} \text{ for } f^{-}_{++,\lambda} \\ \delta_{S'1} \text{ otherwise} \end{Bmatrix} \tilde{f}^{J\pm}_{L'S'L_N^{\pm}S}$$

The terms contributing to $O(T_N^2)$ are $(L,n,k)=(L_N^{\pm},1,0),(L_N^{\pm},0,1)$ and $(L_N^{\pm}+2,0,0)$. There are too many unknown coefficients introduced to allow any derivative relations. Notice that the sum over S and S' collapses. Also notice that $|L_N^{\mp} - L_N^{\pm}| = 1$; hence the term $T_N^{\alpha_\pm}$ in the expansion may be of the same order as the first term.

As mentioned, the expansions about $T_{N'}^2$, T_P^2, and $T_{P'}^2=0$ can be similarly obtained. We do not write these out in detail, as they have been discussed by several authors.[11,12] We will, however, treat the more difficult case of $N\bar{N}$ scattering explicitly in Section ID.

These expansions lead us to define the parity conserving amplitudes free of all kinematic singularities

$$\tilde{F}_1 = \frac{1}{\sin\theta_t}\left(f^t_{++,1} + f^t_{--,1}\right)\frac{t^{\frac{1}{2}}}{\mathcal{J}(t-\Delta^2)^{\frac{1}{2}}}$$

$$\tilde{F}_2 = \frac{1}{\sin\theta_t}\left(f^t_{++,1} - f^t_{--,1}\right)\frac{t^{\frac{1}{2}}}{(t-4M^2)^{\frac{1}{2}}}$$

$$\tilde{F}_3 = \left(\frac{f^t_{+-,1}}{1+z_t} + \frac{f^t_{-+,1}}{1-z_t}\right)\frac{t}{\mathcal{J}(t-\Delta^2)^{\frac{1}{2}}}$$

$$\tilde{F}_4 = \left(\frac{f^t_{+-,1}}{1+z_t} - \frac{f^t_{-+,1}}{1-z_t}\right)\frac{t}{(t-4M^2)^{\frac{1}{2}}}$$

$$\tilde{F}_5 = f^t_{++,0}\quad \mathcal{J}(t-\Delta^2)^{\frac{1}{2}}$$

$$\tilde{F}_6 = \frac{1}{\sin\theta_t}\; f^t_{+-,0}\;\frac{t^{\frac{1}{2}}}{(t-4M^2)^{\frac{1}{2}}} \tag{8}$$

where $\mathcal{J} = \left\{\left[t-(m_\gamma+\mu)^2\right]\left[t-(m_\gamma-\mu)^2\right]\right\}^{\frac{1}{2}}$, $M = \frac{m_1+m_2}{2}$,

$\Delta = m_1 - m_2$. $\tilde{F}_5$ and $\tilde{F}_6$ refer to amplitudes with zero helicity for the massive photon.

By comparing the expansions for the helicity amplitudes it can be shown that the $\tilde{F}_i$ satisfy the relations

$$2M\tilde{F}_1 + \tilde{F}_3 = O(t - 4M^2) \qquad \text{(a)}$$

$$\Delta\tilde{F}_2 - \tilde{F}_4 = O(t - \Delta^2) \qquad \text{(b)}$$

$$\tilde{F}_5 + \Delta\,(4pk\cos\theta_t)\,\tilde{F}_6 = O(t - \Delta^2) \qquad \text{(c)}$$

$$\sqrt{2}\,(m_\gamma \pm \mu)\,\tilde{F}_6 - \tilde{F}_4 = O[t - (m_\gamma \pm \mu)^2] \qquad \text{(d)}$$

$$\sqrt{2}\,\tilde{F}_5 + (m_\gamma \pm \mu)(4pk\cos\theta_t)\,\tilde{F}_2 = O[t - (m_\gamma \pm \mu)^2]\,, \qquad \text{(e)}$$

(9)

where $4tk^2 = [t - (m_\gamma - \mu)^2][t - (m_\gamma + \mu)^2]$, $4tp^2 = (t - \Delta^2)(t - 4M^2)$.

The unequal mass conspiracy relation becomes

$$\Delta\,(\mu^2 - m_\gamma^2)\,\tilde{F}_3(s,0) = 2M\,\tilde{F}_4(s,0) \qquad (\mu \geq m_\gamma) \qquad (10a)$$

The unequal mass formalism given above is perfectly general, and would apply, for example to the reaction $\pi N \to K^* \Sigma$. We next take the limit $m_1 \to m_2$, thereby obtaining a formalism appropriate, for example, to $\pi N \to \rho N$. To do this, we must expand the kinematic singularity free amplitudes $\tilde{F}_j$ in powers of t and Δ around $\Delta = 0$.[12] While this procedure strictly speaking lies outside the framework of S-Matrix theory, it is most unlikely that a dynamical singularity in

Δ exists at $\Delta = 0$, so that the expansion should converge in some neighborhood of $\Delta = 0$.

Following this procedure, we write

$$\tilde{F}_j = \sum_{m,n=0}^{\infty} c_j^{mn} t^m \Delta^n$$

and insert this into the various relations, evaluating t at the appropriate values to make the right hand sides vanish. Since we have a power series in Δ equalling zero, the coefficient of each power of Δ must vanish separately. We then take the limit $\Delta \to 0$. It is easily seen that (9b) and (9c) imply that $c_4^{00} = c_5^{00} = 0$; hence $\tilde{F}_4$ and $\tilde{F}_5 \to O(t)$ in the equal mass limit. Next we consider the equal mass conspiracy relation. The unequal mass conspiracy relation yields

$$2M\, c_4^{01} = (\mu^2 - m_\gamma^2)\, c_3^{00}$$

while the coefficient of Δ in relation (9b) yields $c_4^{01} = c_2^{00}$. Associating the equal mass amplitudes $\tilde{F}_2^E$ and $\tilde{F}_3^E$ with c_2^{00} and c_3^{00} at t=0 yields (as $\Delta \to 0$)

$$2m\, \tilde{F}_2^E(s,0) = (\mu^2 - m_\gamma^2)\, \tilde{F}_3^E(s,0) \qquad (10b)$$

Finally we consider the zero mass limit of the photon. Writing $\tilde{F}_j^E = F_j^t + O(m_\gamma)$ and evaluating relations (9d) at $t=(m_\gamma+\mu)^2$ and at $t=(m_\gamma-\mu)^2$ respectively yields

$$\sqrt{2}\,(m_\gamma \pm \mu)\, F_6^t - F_4^t = O(m_\gamma)$$

Adding these equations and letting $m_\gamma \to 0$ yields $F_4^{\,t}(s,\mu^2) = 0$, provided that $\tilde{F}_6^{\,E}$ does not blow up as $m_\gamma \to 0$; hence $F_4^{\,t} \propto (t - \mu^2)$.

Next we consider relations (9e). Since $\tilde{F}_5$ is an amplitude for a zero helicity massive photon plus π having a transition to the singlet $\bar{N}N$ state, one may expect a pion pole in this amplitude. Except for the case of π^0 photoproduction where charge conjugation does not permit the pion pole, one cannot argue that as $m_\gamma \to 0$, $\tilde{F}_2$ becomes proportional to $t-\mu^2$. Rather, one obtains the normalization condition for the pion pole contributing to $F_2^{\,t}$. This condition and the $t-\mu^2$ factor in $F_4^{\,t}$ is the full content of gauge invariance for the t-channel helicity amplitudes of photoproduction.

For small enough fixed m_γ, both relations (9e) are valid arbitrarily close to $t=\mu^2$. Hence we may split off the pion pole term in $F_5^{\,t}$, obtaining

$$F_5^{\,t} = \frac{c_5^{(1)}}{t-\mu^2} + c_5^{(2)} \qquad \text{as } t \to \mu^2$$

Perturbation theory yields the coupling $c_5^{(1)}$ (in essence this defines the charge e)

$$c_5^{(1)} = \frac{\sqrt{2}}{4}\, e g m_\gamma (4\mu^2 - m_\gamma^2)\, \mu^2$$

We also write $F_2^{\,t} = c_2^{(1)} + (t - \mu^2)c_2^{(2)}$ for $t \approx \mu^2$.

Evaluating relations (9e) at $t=(m_\gamma \pm \mu)^2$ respectively yields

$$\sqrt{2}\left[\frac{c_5^{(1)}}{(m_\gamma \pm \mu)^2 - \mu^2} + c_5^{(2)}\right] + (m_\gamma \pm \mu) 4pk z_t \left[c_2^{(1)} + \{(m_\gamma \pm \mu)^2 - \mu^2\}\, c_2^{(2)}\right]\Big|_{t=(m_\gamma \pm \mu)^2} = 0$$

Subtracting these, we obtain to order m_γ

$$\frac{\sqrt{2}\, C_5^{(1)}}{m_\gamma \mu} + 8\mu p k z_t C_2^{(1)}\Big|_{m_\gamma=0} = O(m_\gamma)$$

Since $2pkz_t = s-m^2$ at $t=\mu^2$ we get the desired result as $m_\gamma \to 0$, namely

$$F_2^t(s,\mu^2) = C_2^{(1)} \to -\frac{eg\mu^2}{2(s-m^2)} \qquad (11)$$

Finally, then, we are led to define the amplitudes for massless photon photoproduction, denoted by F_i^t (redefining $F_4^t = \lim_{\substack{m_\gamma \to 0 \\ \Delta \to 0}} \frac{\tilde{F}_4}{t(t-\mu^2)}$)

$$F_1^t = \frac{1}{\sin\theta_t}\,(f^t_{++,1} + f^t_{--,1})\,\frac{1}{t-\mu^2}\,,$$

$$F_2^t = \frac{1}{\sin\theta_t}\,(f^t_{++,1} - f^t_{--,1})[t/(t-4m^2)]^{\frac{1}{2}}$$

$$F_3^t = \left(\frac{f^t_{+-,1}}{1+z_t} + \frac{f^t_{-+,1}}{1-z_t}\right)\frac{(t)^{\frac{1}{2}}}{t-\mu^2}\,,$$

$$F_4^t = \left(\frac{f^t_{+-,1}}{1+z_t} - \frac{f^t_{-+,1}}{1-z_t}\right)\frac{1}{(t-\mu^2)(t-4m^2)^{\frac{1}{2}}}\,, \qquad (12)$$

where $z_t = (s + \frac{t}{2} - m^2 - \frac{\mu^2}{2})/2kp = \nu/2kp$, $k = \frac{t-\mu^2}{2(t)^{\frac{1}{2}}}$, $p = \frac{1}{2}(t-4m^2)^{\frac{1}{2}}$.

and $f^t_{\lambda_1\lambda_2,\lambda_\gamma}$ are the $m_\gamma = 0$ photoproduction helicity amplitudes. The pion contributes to F_2^t only while sense-nonsense coupled triplet states contribute only to F_1^t. F_3^t and F_4^t in leading order are composed of nonsense-nonsense coupled triplet amplitudes and uncoupled triplet amplitudes respectively.

B. Reggeization of the Pion Photoproduction Amplitudes

Following the usual Reggeization procedure, we represent the amplitudes F_i^t for large s and small t by the Regge asymptotic series, neglecting cuts and the background integral. We obtain

$$F_1^t = \sum_i \frac{(1+\alpha_i)(1 \pm e^{-i\pi\alpha_i})}{2 \sin \pi\alpha_i} \bar{\gamma}_{SR}^{\,i}(t)\, G_{SR}^{\,i}(t) \left(\frac{\nu}{\nu_0}\right)^{\alpha_i - 1},$$

$$F_2^t = \sum_i \frac{(1+\alpha_i)(1 \pm e^{-i\pi\alpha_i})}{2 \sin \pi\alpha_i} \bar{\gamma}_{OI}^{\,i}(t)\, G_{OI}^{\,i}(t) \left(\frac{\nu}{\nu_0}\right)^{\alpha_i - 1},$$

$$F_3^t = \sum_i \frac{(1+\alpha_i)(1 \pm e^{-i\pi\alpha_i})}{2 \sin \pi\alpha_i} \Big[\alpha_i\, \bar{\gamma}_{NR}^{\,i}(t)\, G_{NR}^{\,i}(t)\,,$$

$$- \frac{1}{\nu}(\alpha_i - 1)(t - \mu^2)(t - 4m^2)\, \bar{\gamma}_{1I}^{\,i}(t)\, G_{1I}^{\,i}(t)\Big] \left(\frac{\nu}{\nu_0}\right)^{\alpha_i - 1},$$

$$F_4^t = \sum_i \frac{(1+\alpha_i)(1 \pm e^{-i\pi\alpha_i})}{2 \sin \pi\alpha_i} \Big[\alpha_i\, \bar{\gamma}_{1I}^{\,i}(t)\, G_{1I}^{\,i}(t)\,,$$

$$- \frac{1}{\nu}(\alpha_i - 1)\, \frac{t - \mu^2}{t}\, \bar{\gamma}_{NR}^{\,i}(t)\, G_{NR}^{\,i}(t)\Big] \left(\frac{\nu}{\nu_0}\right)^{\alpha_i - 1},$$

where $\nu_0 \equiv 1\ \mathrm{GeV}^2$. (13)

The residue functions $\bar{\gamma}_{ij}(t)$ have been given labels descriptive of the vertices. We label the singlet, uncoupled triplet, sense coupled triplet, and nonsense coupled triplet $N\bar{N}X$ vertices by 0, 1, S, N, and the regular $[P = (-1)^J]$ and irregular $[P = (-1)^{J+1}]$ $\gamma\pi X$ vertices by R and I. The residues may contain powers of α or t depending on the ghost-killing mechanisms and $t = 0$ coupling

schemes,[2,6] and are denoted by $G_{ij}(t)$ (Table I). The connection of the $\bar{\gamma}_{ij}$ with factorizable residues β_{ij} is given in Table I.

The cross section in the s channel in terms of helicity amplitudes is given by

$$\frac{d\sigma}{dt}(\mu b\ \mathrm{GeV}^{-2}) = \frac{389.5}{2\pi(s - m^2)^2}\left(|f^t_{++,1}|^2 + |f^t_{--,1}|^2 + |f^t_{+-,1}|^2 + |f^t_{-+,1}|^2\right), \tag{14a}$$

or in terms of the parity-conserving amplitudes,

$$\frac{d\sigma}{dt}(\mu b\ \mathrm{GeV}^{-2}) = \frac{389.5}{4\pi(s - m^2)^2}\left\{(z_t^2 - 1)\left[(\mu^2 - t)^2|F_1^t|^2 + \frac{4m^2 - t}{-t}|F_2^t|^2\right]\right.$$
$$+ (z_t^2 + 1)\left[\frac{(\mu^2 - t)^2}{-t}|F_3^t|^2 + (4m^2 - t)(\mu^2 - t)^2|F_4^t|^2\right]$$
$$\left. + 4z_t\left((4m^2 - t)/-t\right)^{\frac{1}{2}}(\mu^2 - t)^2\mathrm{Re}(F_3^{t*}F_4^t)\right\}. \tag{14b}$$

The photoproduction conspiracy relation is

$$2m\, F_2^t(s, 0) = \mu^2 F_3^t(s, 0)\ .$$

Notice that this constraint removes the apparent singularity in $\frac{d\sigma}{dt}$ at $t = 0$. In terms of the $M = 1$ parity doublet conspiracy between the π and π' we obtain the following relation between the residue functions $\bar{\gamma}_{OI}^{\pi}$ and $\bar{\gamma}_{NR}^{\pi'}$:

$$\bar{\gamma}_{NR}^{\pi'}(0) = -\frac{2m}{\mu^2}\,\bar{\gamma}_{0I}^{\pi}(0) \tag{15}$$

The gauge invariance relation giving the pion-nucleon coupling constant for π^+ photoproduction is

$$\lim_{t\to\mu^2} (t-\mu^2)[f^t_{++,1}(s,t) - f^t_{--,1}(s,t)] = -\mu^2\, eg \tag{16a}$$

Hence we obtain the connection between $g^2/4\pi$ and $\bar{\gamma}^{\pi}_{0I}$ for π^+ photoproduction:

$$g^2/4\pi = \frac{1}{4\pi}\left[\frac{2\bar{\gamma}^{\pi}_{0I}(\mu^2)(1-\mu^2/t_0)}{e\pi\mu^2}\right]^2, \tag{16b}$$

where $\bar{\gamma}^{\pi}_{0I}(\mu^2) = a_\pi e^{b_\pi \mu^2}$ (see Table II). The relation 16a also requires a factor of $(t-\mu^2)$ in the B residue; otherwise the B would contribute to the pion pole.

The constraint arising from factorization on the π' residue function from nucleon-nucleon fits is given by

$$\left(\bar{\gamma}_{SR}^{\pi'}/\bar{\gamma}_{NR}^{\pi'}\right)_{\text{photoproduction}} = \left(\gamma_{12}^{\pi'}/(\alpha_{\pi'})^{\frac{1}{2}}\ \gamma_{22}^{\pi'}\right)_{NN} , \tag{17}$$

where $\gamma_{12}^{\pi'}$ and $\gamma_{22}^{\pi'}$ are the same functions listed in Table II of Ref. 2.

Finally we remark on an amusing connection between the cross section calculated from the gauge-invariant Born terms and that calculated by using the $M = 1$ π-π' conspiracy, assuming $t_0 = \mu^2$. Namely, for small t and large s the Regge contribution is equivalent to the Born approximation. Satisfying the normalization condition and the conspiracy condition with the residues $\beta_\pi(t) \approx \frac{eg}{2}(1 + t/\mu^2)$ and $\beta_{\pi'}(t) \approx \frac{eg}{2}$, one obtains

$$\left(\frac{d\sigma}{dt}\right)_{\text{Regge}} \approx \frac{389.5}{4\pi(s - m^2)^2}\left\{\left|\frac{\beta_\pi(t)}{1 - t/\mu^2}\right|^2 + \left|\beta_{\pi'}(t)\right|^2\right\}$$

$$= \frac{389.5}{4\pi(s - m^2)^2}\ \frac{e^2g^2}{4}\ \frac{(1 + t/\mu^2)^2}{(1 - t/\mu^2)^2} = \left(\frac{d\sigma}{dt}\right)_{\text{Born}} . \tag{18}$$

This coincidence accounts for the abnormal success of gauge invariant perturbation theory in fitting the π^+ photoproduction cross sections.

C. Satisfaction of the Conspiracy Relation for Pion Photoproduction

We now show in detail how the assumption of an M=1 parity doublet conspiracy satisfies the constraints due to analyticity at the unequal mass ($\gamma\pi$) vertex and the conspiracy equation

$$2m\, F_2^t(s,0) = \mu^2\, F_3^t(s,0)$$

The Reggeized form of the amplitudes can be written formally as

$$2m\, F_2^t(s,t) = \nu^{\alpha-1} c_\pi(t)\, \bar{\gamma}_\pi(t) + \sum_{n=1}^{\infty} \nu^{\alpha-2n-1}\, t^{-n}\, c_2^{(n)}(t)$$

$$\mu^2\, F_3^t(s,t) = \nu^{\alpha-1} c_{\pi'}(t)\, \bar{\gamma}_{\pi'}(t) + \sum_{n=1}^{\infty} \nu^{\alpha-2n-1}\, t^{-n}\, c_3^{(n)}(t)$$

$$F_4^t(s,t) = t^{-1} \nu^{\alpha-2} \left[c_d(t)\bar{\gamma}_d(t) + c'_{\pi'}(t)\bar{\gamma}_{\pi'}(t) \right] + \sum_{n=1}^{\infty} \nu^{\alpha-2n-2}\, t^{-n-1}\, c_4^{(n)}(t) \qquad (19)$$

where for simplicity we have assumed all trajectories are parallel, and where we have split off the leading order contributions corresponding to the M=1 doublet π-π' and the first daughter d. The t^{-n} singularities coming from the $d^{\alpha}_{\lambda\mu}$ functions and the daughter residues are exhibited explicitly. The coefficients $c_i^{(n)}(t)$ are thus presumed regular in t about t=0 . Hence, expanding,

$$c_i^{(n)}(t) = \sum_{k=0}^{\infty} c_i^{(n,k)}\, t^k$$

We see that the nth derivative of $c_i^{(n)}(0)$ contributes a finite amount to the conspiracy equation. Thus the conspiracy equation yields

$c_\pi(0)\bar{\gamma}_\pi(0) = c_{\pi'}(0)\bar{\gamma}_{\pi'}(0)$ for the π and π' residues, and $\frac{d^n}{dt^n} c_2^{(n)}(0) = \frac{d^n}{dt^n} c_3^{(n)}(0)$ $(n=1,2,\ldots)$ where all the pion-like trajectories α_i with $\alpha_i(0) \geq \alpha_\pi(0) - 2n$ will contribute to $c_2^{(n)}$ and all π' and d-like trajectories with $\alpha_i(0) \geq \alpha_\pi(0) - 2n$ will contribute to $c_3^{(n)}$.

Analyticity of F_2^t and F_3^t implies that $c_i^{(n,k)} = 0$ for $k < n$ $(i=2,3)$. The existence of π and π'-like trajectories is actually sufficient to guarantee analyticity of F_2^t and F_3^t since a new π (π') type trajectory enters in each order of $t^{-n}c_2^{(n)}$ $(t^{-n}c_3^{(n)})$. However, analyticity of F_4^t forces the existence of the first daughter d to cancel off the t^{-1} singularity in F_4^t due to the π', and the other d-type trajectories are required for analyticity in higher orders in t^{-n} of F_4^t. Thus analyticity requirements due to the unequal $\gamma\pi$ mass kinematics coupled with the conspiracy equation, assuming only the existence of the first π-π' doublet, forces the existence of the whole M=1 Lorentz pole family. The constraints of analyticity and conspiracy do not suffice to determine all derivatives of the residue functions. For example, we have two constraints on the zeroth derivatives of the π, π', and d residues coming from F_4^t analyticity to order t^{-1} and the conspiracy equation. However, unknown functions are introduced in each t^{-n} term, so that the constraints tell us nothing about the nonzero derivatives of these residues. In general, the first n-2 derivatives will be completely determined for the nth $\pi\pi' d$ group, one derivative of order n-1 will be lacking, and no information will be available for the derivatives of order $\geq n$.

D. Nucleon-Nucleon Scattering

The Regge formalism for the equal mass $\bar{N}N$ scattering process has been summarized in Ref. 2. Here we give an account of the kinematic formalism appropriate for all unequal masses and show how the usual formalism results in the limit of vanishing mass differences.

We procede in a manner similar to that followed in Section IA. We define our reaction as $\bar{N}_1 N_2 - \bar{N}_3 N_4$ (t channel). Parity conservation and time reversal invariance limits the number of independent amplitudes for the unequal mass process to six (upon taking the limit of equal masses, G parity in the t channel or the Pauli principle in the s channel reduces this number to five). Defining the parity conserving helicity amplitudes $f^{\pm}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)$ in the usual way from the t-channel helicity amplitudes $f^{t}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)$, we make the partial wave expansion, obtain-amplitudes $f^{J\pm}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(t)$. We remark that the amplitudes f_i of GGMW[13] are related to ours up to a factor by f_i $(i=1,2,3,4,5) \propto (f^{-}_{++,++}$, $f^{+}_{++,++}$, $f^{-}_{+-,+-}$, $f^{+}_{+-,+-}$, $f^{+}_{++,+-})$. in the equal mass limit.

We next make the L-S expansion of the partial wave amplitudes, obtaining

$$f^{J\pm}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(t) = \sum_{SS'LL'} C^{\lambda_1,-\lambda_2}_{\frac{1}{2}\frac{1}{2}S} C^{\lambda_3,-\lambda_4}_{\frac{1}{2}\frac{1}{2}S'} C^{0\lambda}_{LSJ} C^{0\mu}_{L'S'J} f^{J\pm}_{L'S'LS}(t) \qquad (20)$$

where $\lambda=\lambda_1-\lambda_2$ and $\mu=\lambda_3-\lambda_4$. We define $T_N^2 = t-4M_{12}^2$, $T_{N'}^2 = t-4M_{34}^2$, $T_P^2 = t-\Delta_{12}^2$, and $T_{P'}^2 = t-\Delta_{34}^2$ where $M_{ij} = (m_i + m_j)/2$ and $\Delta_{ij} = m_i - m_j$. Further, we define $4tp_1p_2 = T_N T_{N'} T_P T_{P'}$. We expand $f^{J\pm}_{L'S'LS}$ about

threshold $T_N^2=0$

$$f^{J\pm}_{L'S'LS}(t) = T_N^{L} \sum_{n=0}^{\infty} T_N^{2n}\, \tilde{f}^{J\pm}_{L'S',\,L+2n,\,S} \tag{21}$$

Hence we obtain (splitting off the leading term)

$$f^{\pm}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t) = T_N^{L_N^{\pm}-J+\lambda_M} \sum_{JSS'} (T_N z_t)^{J-\lambda_M} \left\{ r_0^{J\lambda\mu+}\, \tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'} + \frac{T_N^{\alpha_\pm}}{T_N z_t}\, r_0^{J\lambda\mu-}\, \tilde{a}^{J\lambda\mu\mp}_{L_N^{\mp}SS'} \right] \tag{22}$$

where

$$\tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'} = \sum_{L'=J-1}^{J+1} (2J+1)\, C^{0\lambda}_{L_N^{\pm}SJ}\, C^{0\mu}_{L'S'J}\, C^{\lambda_1,-\lambda_2}_{\frac{1}{2}\frac{1}{2}S}\, C^{\lambda_3,-\lambda_4}_{\frac{1}{2}\frac{1}{2}S'} \begin{Bmatrix} \delta_{S0}\delta_{S'0} & \text{for } f^{-}_{++,++} \\ \delta_{S1}\delta_{S'0} & \text{for } f^{-}_{++,+-} \\ \delta_{S1}\delta_{S'1} & \text{otherwise} \end{Bmatrix} \tilde{f}^{J\pm}_{L'S'L_N^{\pm}S}$$

Here the absence of S=0 coupling for $f^{J+}_{L'S'LS}$ has been included along with some information about the Clebsch-Gordan coefficients in the Kronecker deltas. We have, further, $L_N^{\pm} = L_{N'}^{\pm} = \begin{Bmatrix} J-1 \\ J \end{Bmatrix}$ and $L_P^{\pm} = L_{P'}^{\pm} = \begin{Bmatrix} J \\ J-1 \end{Bmatrix}$ respectively. To leading order we obtain at $T_N^2 \approx 0$

$$f^{+}_{++,++} = \frac{1}{2T_N} \sum_J (T_N z_t)^J r_J \left[\frac{J}{2J-1}\, \tilde{f}^{J+}_{J-1,1,J-1,1} - \sqrt{\frac{J(J+1)}{(2J-1)(2J+3)}}\, \tilde{f}^{J+}_{J+1,1,J-1,1} + O(T_N^2) \right]$$

$$f^{+}_{+-,+-} = \frac{1}{2} \sum_J (T_N z_t)^{J-1} r_J \left[\frac{J}{2J-1}\, \tilde{f}^{J+}_{J-1,1,J-1,1} + \sqrt{\frac{J^3}{(J+1)(2J-1)(2J+3)}}\, \tilde{f}^{J+}_{J+1,1,J-1,1} + O(T_N^2) \right]$$

$$f^{+}_{++,+-} = -\frac{1}{2} \sum_J (T_N z_t)^{J-1} r_J \left[\frac{J}{2J-1}\, \tilde{f}^{J+}_{J-1,1,J-1,1} - \sqrt{\frac{J(J+1)}{(2J-1)(2J+3)}}\, \tilde{f}^{J+}_{J+1,1,J-1,1} + O(T_N^2) \right]$$

$$f^{-}_{++,++} = \frac{1}{2}\sum_J (T_N z_t)^J r_J \left[\tilde{f}^{J-}_{J0J0} + O(T_N^2)\right]$$

$$f^{-}_{+-,+-} = \frac{T_N}{2}\sum_J (T_N z_t)^{J-1} r_J \Big[\frac{J}{J+1}\tilde{f}^{J-}_{J1J1} + \frac{1}{T_N z_t}\Big(\frac{1-J}{2J-1}\tilde{f}^{J+}_{J-1,1,J-1,1}$$

$$+ \sqrt{\frac{J(1-J)^2}{(J+1)(2J-1)(2J+3)}}\,\tilde{f}^{J+}_{J+1,1,J+1,1} + O(T_N^2)\Big)\Big]$$

$$f^{-}_{++,+-} = \frac{T_N}{2}\sum_J (T_N z_t)^{J-1} r_J \left[\sqrt{\frac{J}{J+1}}\,\tilde{f}^{J-}_{J0J1} + O(T_N^2)\right] \qquad (23)$$

where $r_J = \dfrac{(2J+1)(2J)!}{2^J\,(J!)^2}$. The expansions at $T_{N'}^2=0$ are similar and will be discussed below.

The expansions at $T_P^2=0$ or $T_{P'}^2=0$ can be obtained by substituting $T_N \to T_P$ ($T_{N'} \to T_{P'}$), $f^{+}_{\lambda_3\lambda_4,\lambda_1\lambda_2} \leftrightarrow f^{-}_{\lambda_3\lambda_4,\lambda_1\lambda_2}$, and $f^{J+}_{\lambda_3\lambda_4,\lambda_1\lambda_2} \leftrightarrow f^{J-}_{\lambda_3\lambda_4,\lambda_1\lambda_2}$ (i.e. $\tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'} \to \tilde{a}^{J\lambda\mu\mp}_{L_N^{\pm}SS'}$). We also note that the amplitudes $f^{\pm\,\prime}_{\{\lambda_i\}}$ corresponding to the process $\bar{N}_1 N_2' \to \bar{N}_3 N_4'$ where N_2' and N_4' have quantum numbers $J^P=(\frac{1}{2})^-$ may be obtained either by substituting $T_N \leftrightarrow T_P$, $T_{N'} \leftrightarrow T_{P'}$, $f^{\pm}_{\{\lambda_i\}} \to f^{\pm\,\prime}_{\{\lambda_i\}}$, and $f^{J\pm}_{\{\lambda_i\}} \to f^{J\pm\,\prime}_{\{\lambda_i\}}$ or by substituting $f^{\pm}_{\{\lambda_i\}} \to f^{\mp\,\prime}_{\{\lambda_i\}}$, $f^{J\pm}_{\{\lambda_i\}} \to f^{J\mp}_{\{\lambda_i\}}$, and $T_i^2 \to T_i^2$. This same procedure yields the amplitudes for NN scattering. The $\bar{N}_1 N_2' \to \bar{N}_3 N_4'$ process is useful because it factorizes at the intermediate state pion pole into a product of $\pi N \to \pi N$ amplitudes.

Examining the threshold expansions, we find the relation

$$T_N T_{N'}\left(f^{+}_{++,++} + z_t f^{+}_{++,+-}\right) = O(T_N^2) \qquad (24a)$$

The expansions about $T_{N'}^2=0$ are the same for all amplitudes

except $f^{+}_{++,+-}$ if the replacements $T_N \to T_{N'}$ and $\tilde{f}^{J\pm}_{L'S'LS} \to \tilde{f}^{J\pm}_{LSL'S'}$ are made (note that symmetry is maintained in $f^{-}_{++,+-}$ which has only J=L couplings). The expansion for $f^{+}_{++,+-}$ is

$$f^{+}_{++,+-} = -\frac{1}{2}\sum_J{}' (T_{N'} z_t)^{J-1} r_J \Big[\frac{J}{2J-1} \tilde{f}^{J+}_{J-1,1,J-1,1} + \sqrt{\frac{J^2}{(J+1)(2J-1)(2J+3)}}\, \tilde{f}^{J+}_{J-1,1,J+1,1} + O(T_{N'}^2)\Big] \quad at \quad t \approx T_{N'}^2$$

Hence we obtain the threshold relation at $T^2_{N'} \approx 0$,

$$f^{+}_{+-,+-} + f^{+}_{++,+-} = O(T_{N'}^2) \tag{24b}$$

The unequal mass threshold relations become the GGMW threshold relations Eq. 7.15b, 7.15d in the equal mass limit. Eq. 7.15c is merely the statement that $f^{-}_{+-,+-}$ has a kinematic singularity as $T_N T_{N'} \to 4p^2$.

Next, we consider the psuedothreshold relations. Making the replacements indicated above, we see that the following relations are satisfied

$$f^{-}_{+-,+-} + f^{-}_{++,+-} = O(T_{p'}^2)$$

$$T_p T_{p'} f^{-}_{++,++} + T_p T_{p'} z_t f^{-}_{++,+-} = O(T_p^2) \tag{25}$$

Finally we have the unequal mass conspiracy relation, obtained as in Section IA

$$t f^{+}_{+-,+-} = -t f^{-}_{+-,+-} \quad as \quad t \to 0 \tag{26}$$

We next discuss the equal mass limit of the psuedothreshold and conspiracy equations. First, using the results of the L-S expansions, we define kinematic singularity free amplitudes $\tilde{\mathcal{H}}_i$ (with the usual $t^{\frac{1}{2}(|\lambda|+|\mu|)}$ factor also removed)

$$f^{+}_{++,++} = (T_N T_{N'})^{-1}\,\tilde{\mathcal{H}}_2 \quad , \qquad f^{-}_{++,++} = (T_p T_{p'})^{-1}\,\tilde{\mathcal{H}}_1$$

$$f^{+}_{+-,+-} = t^{-1} T_p T_{p'}\,\tilde{\mathcal{H}}_4 \quad , \qquad f^{-}_{+-,+-} = t^{-1} T_N T_{N'}\,\tilde{\mathcal{H}}_3$$

$$f^{+}_{++,+-} = t^{-\frac{1}{2}} T_p T_{p'}\,\tilde{\mathcal{H}}_5 \quad , \qquad f^{-}_{++,+-} = t^{-\frac{1}{2}} T_N T_{N'}\,\tilde{\mathcal{H}}_6 \qquad (27)$$

The $\tilde{\mathcal{H}}_i$ amplitudes then satisfy the relations

$$t^{-1} T_N T_{N'}\,\tilde{\mathcal{H}}_3 + t^{-\frac{1}{2}} T_N T_{N'}\,\tilde{\mathcal{H}}_6 = O(T_p^2) \qquad (a)$$

$$\tilde{\mathcal{H}}_1 + t^{-\frac{1}{2}} T_N T_{N'} T_p T_{p'} z_t\,\tilde{\mathcal{H}}_6 = O(T_p^2) \qquad (b)$$

$$T_p T_{p'}\,\tilde{\mathcal{H}}_4 + T_N T_{N'}\,\tilde{\mathcal{H}}_3 = O(t) \qquad (c) \qquad (28)$$

We next expand each amplitude in a triple power series in t, Δ_{12}, and Δ_{34}, writing

$$\tilde{\mathcal{H}}_i = \sum_{jkl} c_i^{jkl}\, t^j\, \Delta_{12}^{l}\, \Delta_{34}^{k}$$

We set $t = \Delta_{12}^2$ (or $t = \Delta_{34}^2$) in the two psuedothreshold relations, set t=0 in the conspiracy equation, and then equate coefficients of $\Delta_{12}^{m}\,\Delta_{34}^{n}$ separately (it is not sufficient to set $\Delta_{12} = \Delta_{34} = \Delta$ and equate coefficients of powers of Δ). Further, we must expand

z_t as

$$2P_1P_2z_t = s + \frac{\Delta_{12}^2}{2} - \frac{1}{2}\sum_i m_i^2 + 2\frac{\Delta_{34}}{\Delta_{12}}M_{12}M_{34} \quad at \quad t=\Delta_{12}^2$$

We obtain

From (28a): $c_3^{000} = c_3^{002} = c_6^{010} + c_3^{100} + c_3^{020} = c_3^{011} + c_6^{001} = 0$

From (28b): $c_1^{000} = c_1^{100} + c_1^{002} + 2(s-\frac{1}{2}\sum_i m_i^2)c_6^{001} = c_1^{020} + 4M_{12}M_{34}c_6^{010} = 0$

From (28c): $c_4^{000} + 4M_{12}M_{34}c_3^{011} = 0$

By symmetry we have $c_3^{020} = c_3^{002}$ and $c_1^{020} = c_1^{002}$. Hence we obtain

$$c_1^{100} + 4M_{12}M_{34}c_3^{100} + \frac{2(s-\frac{1}{2}\sum_i m_i^2)}{4M_{12}M_{34}}c_4^{000} = 0$$

We set $M_{12} = M_{34} = m$ and define $\tilde{f}_1(s,0) = c_1^{100}$, $\tilde{f}_3(s,0) = c_3^{100}$, and $\tilde{f}_4(s,0) = c_4^{000}$, where $\tilde{f}_i$ are the kinematic singularity free parity-conserving amplitudes for $\bar{N}N$ scattering (note $c_1^{000} = c_3^{000} = 0$, i.e. $\tilde{\mathcal{F}}_1$ and $\tilde{\mathcal{F}}_3$ pick up a factor t in the equal mass limit, as they must to cancel the unequal mass kinematic singularities $T_P^{-1}T_{P'}^{-1}$ or t^{-1} which are not present in the equal mass case). We have $f_1 = \tilde{f}_1$, $f_3 = T_N^2\tilde{f}_3$, and $f_4 = \tilde{f}_4$. Hence the equation becomes

$$f_1 - f_3 - z_t f_4 = 0 \quad at \quad t=0 \tag{29a}$$

or in terms of the GGMW ϕ_i^t amplitudes,

$$\phi_1^t - \phi_2^t = \phi_3^t - \phi_4^t \quad at \quad t=0 \tag{29b}$$

This is the equal mass conspiracy relation.

Thus we have shown that the usual $\bar{N}N$ scattering formalism results in the limit of equal masses in the unequal mass formalism.

E. Consistency Between Conspiracy Relations

We show here that the conspiracy relations for the three processes $\bar{N}N \to \bar{N}N$, $\gamma\pi \to \bar{N}N$, and $\gamma\pi \to \gamma\pi$ are consistent with each other and with factorization of the residues for an M=1 type parity doublet conspiracy. Denoting the full factorized residues by $\beta_{ij}=\gamma_i\gamma_j$ where i,j = 0,1,S,N,R,I are the vertex labels of Section IB, we obtain the following forms for the Reggeized amplitudes.

1. $\bar{N}N \to \bar{N}N$

We have , formally (the exact expressions will not be needed)

$$\phi_1^t - \phi_2^t = 2K_\pi(t) D^\alpha_{00}(z_t)\left[\gamma_0^\pi(t)\right]^2$$

$$\phi_3^t - \phi_4^t = K_{\pi'}(t)\left[D^\alpha_{11}(z_t) - D^\alpha_{1,-1}(z_t)\right]\left[\gamma_N^{\pi'}(t)\right]^2$$

$$+ K_d(t)\left[D^{\alpha_1}_{11}(z_t) + D^{\alpha_1}_{1,-1}(z_t)\right]\left[\gamma_1^d(t)\right]^2 \qquad (30a)$$

where $D^\alpha_{\lambda\mu}$ are the $d^\alpha_{\lambda\mu}$ with the Legendre polynomials P_α replaced by the usual $\mathcal{P}_\alpha$. Since at t=0, $z_t \to s$ we may write expansions for $D^\alpha_{\lambda\mu}$ in powers of z_t^{-2} . We have $\mathcal{P}_\alpha \approx A_\alpha z_t^\alpha$, $D^\alpha_{11} - D^\alpha_{1,-1} \approx \frac{2\alpha}{\alpha+1} A_\alpha z_t^\alpha$, and $D^\alpha_{11} + D^\alpha_{1,-1} \approx O(z_t^{\alpha-1})$. To leading order in z_t (where $\alpha_1(0)=\alpha(0)-1$, consistent with M=1), the conspiracy equation for $\bar{N}N$ scattering becomes [note $K_\pi(0) = K_{\pi'}(0)$]

$$\left[\gamma_0^\pi(0)\right]^2 = \frac{\alpha(0)}{\alpha(0)+1}\left[\gamma_N^{\pi'}(0)\right]^2 \qquad (30b)$$

2. $\gamma\pi \to \bar{N}N$

The connection of the photoproduction amplitudes F_i^t with the γ_i

is given by

$$2mF_2^t = -2K_\pi(t)\sqrt{\frac{\alpha}{\alpha+1}}\, A_\alpha z_t^{\alpha-1}\, \gamma_I^\pi(t)\gamma_0^\pi(t)$$

$$\mu^2 F_3^t = K_{\pi'}(t)\frac{\alpha}{\alpha+1} A_\alpha z_t^{\alpha-1}\gamma_R^{\pi'}(t)\gamma_N^{\pi'}(t) + K_1(t)\gamma_I^d(t)\gamma_1^d(t)\cdot O(z_t^{\alpha_1-2}) \qquad (31a)$$

We have expanded the d-functions in powers of z_t^{-2} even though as $t \to 0$ we have $z_t \to 0$ since these singularities are to be cancelled by the daughters as described in Section IC. Hence to leading order in z_t the conspiracy equation becomes

$$\gamma_I^\pi(0)\,\gamma_0^\pi(0) = -\sqrt{\frac{\alpha(0)}{\alpha(0)+1}}\;\gamma_R^{\pi'}(0)\,\gamma_N^{\pi'}(0) \qquad (31b)$$

3. $\gamma\pi \to \gamma\pi$

The pion Compton scattering amplitudes F_{RR} and F_{II} are analytic, where

$$F_{RR} = \left(\frac{f^t_{10,10}}{1+z_t} + \frac{f^t_{10,-10}}{1-z_t}\right)\frac{t}{(t-\mu^2)^2}$$

$$F_{II} = \left(\frac{f^t_{10,10}}{1+z_t} - \frac{f^t_{10,-10}}{1-z_t}\right)t \quad . \qquad (32a)$$

The unequal mass conspiracy relation

$$\frac{1}{\mu^4}F_{RR}(s,0) = -F_{II}(s,0) \quad , \qquad (32b)$$

is satisfied by the (full) factorized residues $\left(\gamma_I^\pi(0)\right)^2 = \left(\gamma_R^{\pi'}(0)\right)^2$ for the parity doublet solution. This relation eliminates the apparent pole in $f^t_{10,10}$ at $t = 0$.

II. THE DATA AND FITS FOR $\gamma p \to \pi^+ p$, $np \to pn$, AND $p\bar{p} \to n\bar{n}$ SCATTERING

A. The Data

The photoproduction data used[1] were positive pion photoproduction data at 2.6, 2.7, 3.4, 3.7, 5, 8, 11, and 16 GeV/c lab momentum. Reliable high energy negative pion photoproduction data are scarce; we used only one point at 3.4 GeV/c, $t = -0.37$ GeV^2 as a constraint.[14] We have included data up to $t = -0.5$ GeV^2, consistent with the NN fits. We have included the possibility of systematic errors quoted by the experimentalists on the order of $\pm 5\%$. In all, 62 photoproduction data points were used.

The $np \to pn$ and $p\bar{p} \to n\bar{n}$ data were described in Ref. (2). In all, 74 data points were used.

B. Parameterization of the Pion Photoproduction Fit

The most important part of the parameterization of the pion residue function is the zero at t_0. If one makes the assumption that the zero in the $N\bar{N} \to N\bar{N}$ pion residue is a single zero (i.e. a square root type zero in the $N\bar{N}_\pi$ vertex), then the square root zero must propagate throughout all vertex functions of the form $X\bar{Y}_\pi$. If, however, we assume that there is a full zero in the $N\bar{N}_\pi$ vertex and thus a double zero in the $N\bar{N} \to N\bar{N}$ pion residue, only reactions involving $N\bar{N}$ need have the zero. (Of course there is nothing to prevent any other

$X\bar{Y}\pi$ vertex function from having such a zero, but it is then not required to be in any specific place.) While the origin and full content of the zero is not well understood, it seems to be connected with the apparent incompatibility of M=1 with the zero spin of the physical pion[2,15] and recent work of Toller indicates that the hypothesis of a full zero in the $N\bar{N}\pi$ vertex function is preferred over that of a square-root type zero from group theoretic grounds. Since earlier fits to NN scattering assumed the square root type vertex zero, we have fit the NN data with the full $N\bar{N}\pi$ vertex function zero hypothesis and find that the zero is then required to be at around $t_0 = -2.5\ \mu^2$ rather than at $-\mu^2$. The photoproduction pion residue function has a single zero in any case (we assume nothing about the $\gamma\pi\pi$ vertex in the full vertex zero case). We find that consistent fits to all data can be obtained with the full vertex zero hypothesis but that some discrepancy exists between the values of $g^2/4\pi$ obtained in the NN and photoproduction fits if the square root vertex zero is assumed.

We next consider the zero in somewhat more theoretical detail. Mandelstam[15] has argued that since the M=1 pion must choose nonsense at $\alpha_{\pi 0} = 0$ for the massless pion ($\pi 0$) case, all soft pion amplitudes must be small. However, M can also be defined through the asymptotic behavior of unequal mass amplitudes, and one finds that the kinematic factors of $t^{\frac{1}{2}}$ yield small hard pion amplitudes as well. Evidently, one must either give up the hypothesis that the pion is M=1, or else hope that the soft pion amplitudes will have additional zeros relative to the hard pion ampli-

tudes . This can easily be accomplished in Bethe-Salpeter models[16] by regarding the pion trajectory as being composed as a superposition of crossing trajectories, which are regarded as the leading members of an M=1 and an M=0 family. A full zero is thereby introduced in the $\pi N\bar{N}$ vertex function, and the physical pion (which must choose sense at $\alpha_\pi=0$) is produced through the M=0 trajectory. Finally, by considering ghost killing mechanisms for equal mass scattering processes by looking at singularities in the group theoretic expansions at integer values of trajectories, Toller[17] concludes that the zero in the $\pi N\bar{N}$ vertex function exists, and moreover that it is most likely a full zero. If such an analysis could be carried out for unequal masses, it might happen that such zeros could depend on the external masses, the zeros moving in close to t=0 only when the equal mass limit is reached. Unfortunately, there are no unequal mass expansions that do not also make the unphysical assumption of parallel daughter trajectories.[18] An inverse Regge-Lorentz expansion would not possess this difficulty, since a separate inverse expansion could be written for each daughter; the sum of the daughter expansions would then collapse appropriately at t=0 for equal masses. Although such inverse unequal mass expansions can easily be perform i at asymptotic s (nonuniquely), it appears difficult to write expansions which have the correct analyticity properties as $t \to 0$.

We conclude this section by describing the rest of the parameterization. The parameterization of all trajectories and residue functions was made consistent with meson-nucleon and nucleon-nucleon fits.[2,6] The π, π', ρ, and A_2 trajectories were considered fixed and the B trajectory slope was assumed unknown. Factorization from meson-nucleon fits constrained the ρ and A_2 residues, which were taken to have the

Chew and Gell-Mann ghost-killing mechanisms at $\alpha = 0$, respectively. Thus this fit violates $\rho - A_2$ exchange degeneracy in this respect. The π' was made to choose nonsense at $\alpha = 0$, and its residues were constrained through factorization with the $N\bar{N}$ fit and the conspiracy equation. Altogether three couplings (ρ, A_2, B), five exponentials, one trajectory (B) slope, and the zero t_0 were used as variables. In addition, the 2.6, 5, 8, 11, and 16 GeV/c data were allowed to have systematic errors of less than ±7%.

C. The Fits

Photoproduction fits for both cases of a full vertex zero and a square root type vertex zero were obtained. The parameters obtained in the former case are listed in Table II. The amplitudes are pictured in Figs. 1 and 2, and the fit itself is pictured in Figs. 3 and 4. Although little effort has been made to test the nonuniqueness of the fits, it is probably true that they are not unique, so that these parameters should not be regarded quantitatively too seriously. We note in passing that the small ρ amplitudes found here seem consistent with the result of small $\gamma\pi\rho$ coupling found in photoproduction dispersion relation calculations.

A fit to the np-pn and $p\bar{p}$-$n\bar{n}$ data was obtained with the assumption of a full $N\bar{N}\pi$ vertex zero, and these parameters are also presented in Table II. The notation used is that of Ref. (2). Fits with the square root zero at various locations were also obtained, and will be discussed below.

The best photoproduction fit for the square root vertex zero case was obtained with $\chi^2 = 73$ for 62 points and a value of

$g^2/4\pi = 16.8$, in some disagreement with the value of $g^2/4\pi = 13$ obtained in the NN fit for this value of t_0 ($t_0 = -0.027$). Fits with larger values of t_0 tend to decrease $g^2/4\pi$ for photoproduction faster than $g^2/4\pi$ for NN scattering, so that the farther out we move t_0 the closer we come to consistency. However, for $t_0 = -0.034$ we obtain $g^2/4\pi = 15$ and 11.7, respectively, for γp and NN scattering; we cannot move t_0 farther out and retain an acceptable value of $g^2/4\pi$ for NN scattering. On the other hand, moving the zero in to $t_0 = -0.018$ only raises $g^2/4\pi$ to 15.7 in NN scattering. Thus some inconsistency seems to exist. This discrepancy may, however, not be serious, since we cannot be sure that there are no other $M = 1$ conspiring parity doublets (e.g., $B - \rho'$). If there were, one could put a zero at some $t_B < 0$ into the B residue function, and the data could then be fit with a wide range of values for t_0 since the coupling of the pion would then no longer be constrained at $t = 0$. We discuss this point more fully in Section IV.

The best photoproduction fit for the full vertex zero case was obtained with $\chi^2 = 66$ for 62 points (not significantly different from the previous case). With a value of $t_0 = -0.05$, nearly equal values of $g^2/4\pi = 15.4$ and 14.7 were obtained for photoproduction and NN scattering respectively. Thus problems of consistency do not seem to arise if the zero is assumed to be a full vertex zero.

The value of the π^-/π^+ cross section ratio at 3.41 GeV/c, $t = -0.37$ GeV^2 is measured to be $0.73/2.1 = 0.35$. We obtain

$\sigma(\pi^-)/\sigma(\pi^+) = 0.87/1.5 = 0.57$ for both types of zeros, giving a total χ^2 of about 3 for the π^- and π^+ cross sections in each case.

The photoproduction data can be fit well only out to about $t = -0.5$ GeV^2 with the models assumed here. Past this point, the data show a break which we do not quantitatively reproduce. This break may be related to the structure in the $p\bar{p} \to n\bar{n}$ cross sections past $t = -0.5$ which the NN fits could not quantitatively describe. It is possible that the inclusion of other trajectories (e.g., an $M = 1$ ρ' or some amount of A_1) could be used to affect quantitative reproduction of the data.

III. THE B TRAJECTORY PHOTOPRODUCTION SUM RULE

We begin by writing the sum rule, a positive moment sum rule for the even ν part of the t-channel photoproduction amplitude which contains the B (but not the π) trajectory. This t-channel amplitude is proportional to the photon isoscalar amplitude which in CGLN [19] notation is (we use CGLN's ν in this section)

$$\tilde{A}(\nu, t) \equiv A_1^{(0)}(\nu, t) + t\, A_2^{(0)}(\nu, t) \quad . \tag{33}$$

The sum rule is then

$$0 = \frac{1}{\pi} \int_{\nu_t}^{N} \nu \, \mathrm{Im}\, \tilde{A}(\nu,t) d\nu + \nu_B \frac{eg}{4M} \frac{t + \mu^2}{t - \mu^2} - \frac{1}{\pi} \bar{r}_{0I}^{\,B}(t)\, R(t) \frac{N^{\alpha_B + 1}}{\alpha_B + 1} \quad , \tag{34}$$

where $\bar{r}_{0I}^{\,B}(t)$ is the same residue used in the photoproduction fit,

$$4M\nu_B = -\mu^2 + t, \quad 4M\nu = s - u,$$

and

$$R(t) = 2\alpha_B(1 + \alpha_B)(2 + \alpha_B)(t/\mu^2)(2m)^{\alpha_B - 1}/(.389)^{\frac{1}{2}} \quad .$$

We evaluate $A(\nu, t)$ by writing its multipole expansion formally as

$$\tilde{A}(\nu, t) = \sum_i \mathcal{M}_i(\nu, t) + \frac{eg}{4M} \frac{t + \mu^2}{t - \mu^2} \left(\frac{1}{-\nu + \nu_B} + \frac{1}{\nu + \nu_B} \right), \tag{35}$$

where the multipole sum has the (real) Born term explicitly removed. The sum $\sum_i \mathcal{M}_i(\nu, t)$ is given in CGLN through the multipole expansion of $\mathcal{F}_i^{(0)}$ where

$$\tilde{A}(\nu, t) = \frac{4\pi}{k} [(M + E_1)/(M + E_2)]^{\frac{1}{2}} \mathcal{F}_1^{(0)}(\nu, t)$$

$$- \frac{4\pi}{q} [(M + E_2)/(M + E_1)]^{\frac{1}{2}} \mathcal{F}_2^{(0)}(\nu, t)$$

$$+ \frac{4\pi}{q} \frac{(Wt - M\mu^2) \mathcal{F}_3^{(0)}(\nu, t)}{(W - M)^2 [(M + E_2)(M + E_1)]^{\frac{1}{2}}}$$

$$- \frac{4\pi}{2k\, Wq^2} (Wt + M\mu^2) [(M + E_2)/(M + E_1)]^{\frac{1}{2}} \mathcal{F}_4^{(0)}(\nu, t) . \tag{36}$$

The final form of the sum rule is thus

$$+ \frac{eg}{(4M)^2} (t + \mu^2) + \frac{1}{\pi} \int_{\nu_t}^{N} \nu \, \mathrm{Im} \sum_i \mathcal{M}_i(\nu, t) d\nu = \frac{1}{\pi} \bar{\gamma}_{0I}^{B}(t) R(t) \frac{N^{\alpha_B + 1}}{\alpha_B + 1} . \tag{37}$$

We use the parameterization of the multipoles given by Walker[20] to evaluate the sum $\sum_i \mathcal{M}_i(\nu, t)$. This parameterization utilizes six resonances and a number of nonresonant parts, which are generally small.

The results of the calculation are presented in Table III, and the integrands of both the Bietti-Roy-Chu and the B - meson sum rules at $t = 0$ are plotted in Fig. 5.

It is seen that the B residue is finite at $t = 0$ and has a zero at $t_B \approx +5\mu^2$. The implication is that the B trajectory is an $M = 1$ trajectory, conspiring with an as yet unknown trajectory usually denoted as ρ'. Before turning to the relevance of this to scattering data, we remark that the form found for the B residue suggests an analogy with the pion residue function and perhaps suggests some correlation between the two trajectories in the sense of exchange degeneracy. If the B trajectory were to pass through the B meson and through zero at $t = 0$ the slope α_B' would be 0.7, which is not unreasonable.

We comment next on the reliability of the positive moment sum rule. First, we note a deficiency of the B sum rule that the corresponding pion sum rule does not possess. First the Born term here is depressed by a factor $(t - \mu^2)$ relative to the pion sum rule so that the inherent stability of the pion sum rule due to a large Born term is lost. Secondly, the small isoscalar amplitude is presumably not too reliably determined, as it involves cancellation of large and nearly equal resonant amplitudes for π^+ and π^- photoproduction. Thus, if there were important isoscalar resonant contributions at $k > 1.2$ GeV/c the sum rule would be inaccurate. We remark, however, that the integrand is positive over the whole region

$k = 0.2$ to 1.2 GeV/c; hence to reverse the sign of the integral (thus making the B an $M = 0$ trajectory), one would need to undo the total effect of the first six resonances. Since we are working with a positive moment sum rule, this is not inconceivable. However, the convergence of the integral over the first six resonances is good even with the positive moment, so the sum rule as it presently stands converges well. Notice that the "duality concept" as advanced by Schmid and Chew,[21] whereby dominant Regge trajectories provide a semi-local average to the energy dependence of the imaginary part of the amplitude at low energies in the resonant region, does not appear to hold in this energy region, as the contribution of the first six resonances to the B sum rule integrand produces only a wide positive bump over the whole region of integration. In fact, the Bietti-Roy-Chu sum rule integrand is even worse, being purely positive at momenta $0.2 < k < 0.7$ GeV/c and negative for $0.7 < k < 1.2$ GeV/c (see Fig. 5). Thus photoproduction amplitudes at these energies seem to violate the Schmid "duality concept," though there is no reason why it should not be valid over a larger energy region. Finally we remark on the zero in the B residue indicated by the sum rule. The zero is caused by cancellation of the Born term that rapidly increases in t with the nearly constant integral. If we double the integral, the zero moves outward to $t_B = +0.24$; if we cut the integral in half the zero moves in to $t_B = +0.03$. Since we cannot reliably estimate the errors on the integral, we cannot really be sure that the zero is not in fact at $t_B = 0$ (thus indicating an $M = 0$ B trajectory).

We have also investigated the possibility of evaluating the π and B residues using ordinary cutoff dispersion relations. The results are only roughly in agreement with the unsubtracted sum rules, yielding $M = 1$ π and B residues without any zeros and with magnitudes at $t = 0$ larger than those of the FESR by an order of magnitude. However, the cutoff dispersion relation is satisfied very nearly by the Born term and roughly by the resonances, so that the calculation of the Regge term is inherently inaccurate.

IV. IMPLICATIONS FOR SCATTERING DATA AND THE PION SUM RULE

The actual existence of an $M = 1$ B trajectory cannot conclusively be established from experimental evidence. As we have shown, an $M = 0$ B trajectory is certainly compatible with the existing data. We argue, however, that an $M = 1$ B trajectory is also compatible and perhaps preferred by existing data, but that exhaustive fits using it would be inappropriate until measurements at small t are made of the high energy cross sections for the processes $p\bar{p} \to n\bar{n}$ and $\gamma n \to \pi^- p$. These measurements should serve to determine the existence of an $M = 1$ B trajectory in a model where only the π and B trajectories have $M = 1$, since the π-B and π'-ρ' interference terms change sign between the processes $pn \to np$, $p\bar{p} \to n\bar{n}$ and between $\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$. If the B has the quantum number $M = 0$ these interference terms at $t = 0$ are zero in all cases. However, for an $M = 1$ assignment these interference terms would be nonzero at $t = 0$. Further, the small t behavior of the $p\bar{p} \to n\bar{n}$ reaction also provides a clear way to distinguish the type of zero in the πNN vertex function.

Another reaction which would be critical in determining the M quantum number of the B would be $\pi N \to \omega N^*$ near $t = 0$. Notice that this reaction is the analog of the reaction $\pi N \to \rho N^*$ involving π exchange. Finally, $pn \to np$ polarization measurements near $t = 0$ should affect this determination; these measurements are currently in progress.

We now consider the implications of consistency of high-energy data combined with the pion sum rule for an $M = 1$ assignment for the B trajectory.

A. Photoproduction

If we take the result of the B sum rule at least as an indication of the magnitude of the B residue, there appears to be a contradiction with the fit. For $|t| > \mu^2$ the fit with $t_0 = -0.03$ seems to require at least a factor of 30 times the B contribution given by the sum rule. The $M = 0$ B assumed in the $\gamma p \to \pi^+ n$ fit may therefore be interpreted as simulating the effect of a small $M = 1$ B amplitude together with the ρ' amplitudes. If we assume small $\pi\gamma B$ and $\pi\gamma\rho'$ couplings, a medium $N\bar{N}B$ and medium $N\bar{N}\rho'$ nonsense coupling, and a large $N\bar{N}\rho'$ sense coupling, the $M = 1$ B and ρ' will very nearly simulate the $M = 0$ B amplitude assumed in the photoproduction fit, being predominately equal to the sense-nonsense ρ' amplitude which vanishes at $t = 0$ (see Fig. 6).

It is possible that with different ρ or A_2 ghost killing mechanisms (or the inclusion of some amount of $M = 0$ A_1), less $M = 0$ B would be required to fit the data. In any case, the π^+ photoproduction fit can surely be made consistent with an $M = 1$ B-ρ' conspiracy.

Next, we consider implications of an $M = 1$ B trajectory for π^- photoproduction. Assuming the existence of an $M = 1$ B trajectory and the zeros indicated by the sum rules in the π and B residues,

it is phenomenologically clear that more constructive π-B and $(\rho + \rho') - (\pi' + A_2)$ interferences would give better results for the fit to the π^-/π^+ ratio at moderate t. This, unfortunately does not predict that the π^-/π^+ ratio near $t = 0$ would continue to be small since the ρ and A_2 terms vanish at $t = 0$, and these terms are significant at moderate t. Notice that local fluctuations (i.e. maxima or minima) should occur in the π^-/π^+ ratio in the $M = 1$ B model when the π or B residues vanish. Notice also that as $t \to 0$ an $M = 1$ B trajectory predicts that the π^-/π^+ ratio would be different from 1, whereas the model utilized in the fit with the $M = 0$ B residue vanishing at $t = 0$ yields the prediction of a ratio of 1 at $t = 0$. Even if the ρ' nonsense and B residues were small as indicated by the sum rule, interference with the large π and π' amplitudes would produce a noticeable effect. Hence, a measurement of the π^- photoproduction cross section near $t = 0$ would provide a critical test of the ρ'-B conspiracy.[14]

Next we consider π^0 photoproduction. Ader and Capeville and Braunschweig et al[22] have fit low energy π^0 photoproduction data utilizing an $M = 0$ B amplitude very similar in magnitude to what our $M = 0$ B would yield for π^0 photoproduction at small t (e.g., $t \approx -0.1$). For higher values of t, the ρ amplitudes in our fit would simulate the B amplitudes in these fits (which did not include the ρ). Hence the conjectured simulation of the $M = 0$ B by an $M = 1$ B + ρ' should fit the π^0 photoproduction data.

Finally we remark that the presence or absence of polarization in $\pi^- p \to \pi^0 n$ is not critical to any of these arguments, since we may always fit the polarization with a sufficiently small $\rho'\pi\pi$ residue.

B. NN Scattering

Next we consider implications of an $M = 1$ B-ρ' conspiracy for the $pn \to np$ and $p\bar{p} \to n\bar{n}$ reactions. First, suppose that the zero in the pion vertex function $(\pi N\bar{N})$ is of the square-root type. The value of the zero $t_0 = -1.5\ \mu^2$ is consistent in the sum rule and the photoproduction fits, but leads to some inconsistency in the NN fits, since $g^2/4\pi$ in the NN fit turned out to be rather low. However, an $M = 1$ B trajectory could easily remove this discrepancy by releasing the constraint on the pion residue at $t = 0$, thus allowing a higher value of $g^2/4\pi$ to be obtained in the NN fit via destructive interference of the π with the B at $t = 0$ [the ρ' and π' would also interfere destructively (see Fig. 7)]. Notice that since more parameters are introduced in an $M = 1$ B fit, the amount of freedom in fitting the NN data actually increases, so there is no doubt that a successful NN fit can be performed. The zero in the B residue would help to provide the necessary sharp peaks in the cross sections and the medium sized $N\bar{N}B$ and $N\bar{N}\rho'$ nonsense couplings would no doubt be nonviolent enough to achieve consistent fits. Thus, the pion could still be held accountable for a large role in making the sharp peaks. Notice that in this case destructive interference in $pn \to np$ implies

constructive interference in $p\bar{p} \to n\bar{n}$ so that the $p\bar{p} \to n\bar{n}$ cross sections should remain larger than the $pn \to np$ cross sections at $t = 0$ if this square-root type vertex zero model is correct.

Suppose now that the $\pi N\bar{N}$ vertex zero is a full zero. The value of this zero required to fit both photoproduction and NN data is $t_0 \approx -2.5\mu^2$. This value is not consistent with the pion sum rule [23] but could be made consistent if the $M = 1$ B trajectory were present. Moving the pion zero to $t_0 = -1.5\mu^2$ would lower the $t = 0$ contribution of the pion in the NN fits significantly (assuming fixed $g^2/4\pi$). The extra contribution needed in the $pn \to np$ cross section could then easily be provided by constructive interference of the B with the π, and the ρ' with the π' (see Fig. 7). Thus, in this case of a full vertex zero, the interference in $p\bar{p} \to n\bar{n}$ would be destructive so that there should actually be a dip in the $p\bar{p} \to n\bar{n}$ cross section for $|t| < 0.02$ GeV^2 (i.e., the $pn \to np$ and $p\bar{p} \to n\bar{n}$ cross sections should cross over).

To summarize, if the pion photoproduction sum rule is correct, the existing NN data seem to favor the existence of an $M = 1$ B trajectory regardless of the type of zero in the $\pi N\bar{N}$ vertex function. If the pion sum rule is yielding misleading results, there is no preference from NN scattering for an $M = 1$ B trajectory since it could be that a full $N\bar{N}\pi$ vertex zero at $t = -0.05$ would be consistent with the sum rule. The existence of higher resonances with large γN couplings could well change these sum rule results. In particular, measurements of the total cross section up to 2.6 GeV/c (where our Regge fits begin to work) would provide information on the γN partial widths of these resonances.

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23. That the consistency problem is not trivial can be seen in the following way. First, the pion sum rule is highly stable due to the large Born term; doubling the effect of the resonances only moves the zero to $t_0 = -0.04$. Secondly, a double zero NN fit for this value of t_0 yields $g^2/4\pi \approx 17.5$, which is already too

high. It is of course possible that other trajectories could change the position of the zero in the pion residue function, but the only candidate is the A_1 daughter, which is expected via kinematics to be small near $t = 0$.

Table I. Definition of $G_{ij}(t)$ and the full residues $\beta_{ij}(t)$

1. The factors $G_{ij}(t)$ in Eq. (2) are

$$G_{SR}(t) = \begin{Bmatrix} \alpha \text{ Chew } \rho \text{ with } M = 0 \\ \alpha \text{ Gell-Mann } A_2 \text{ with } M = 0 \\ \alpha \text{ Gell-Mann } \pi' \text{ with } M = 1 \end{Bmatrix} \qquad G_{NR}(t) = \begin{Bmatrix} \alpha t & \rho \\ t & A_2 \\ 1 & \pi' \end{Bmatrix}$$

$$G_{OI}(t) = \begin{Bmatrix} \alpha t(1 - t/\mu^2) & B \text{ with } M = 0 \\ \alpha(1 - t/t_0) & \pi \text{ with } M = 1 \end{Bmatrix} \qquad G_{1I}(t) = \begin{Bmatrix} \alpha \text{ Chew } A_1 \\ 1 \text{ Gell-Mann } A_1 \end{Bmatrix} \begin{Bmatrix} t & \text{uncoupled, } M = 0 \\ 1 & \text{coupled, } M = 0 \end{Bmatrix}$$

2. Define $X(\alpha) = \dfrac{(1 + \alpha)\Gamma(\alpha + 1)}{\sqrt{\pi}\,(2\alpha+1)\Gamma(\alpha+\frac{1}{2})}$. The connection of the full residue functions $\beta_{ij}(t)$ with the functions $\bar{\gamma}_{ij}(t)$ in Eq. (2) are

Table I (Continued).

Trajectories	Full residue	$N\bar{N}$ vertex	$\gamma\pi$ vertex	
$\begin{Bmatrix} \rho \\ A_2 \\ \pi' \end{Bmatrix}$	$\beta_{SR} =$	$\left[\sqrt{X}\, p^{\alpha-1} \begin{Bmatrix} 1 \\ \sqrt{\alpha} \\ \sqrt{\alpha}\ \sqrt{t} \end{Bmatrix}\right]$	$\left[\sqrt{X}\, k^{\alpha}\, (\alpha+1)^{\frac{1}{2}} \begin{Bmatrix} \sqrt{\alpha}\ \sqrt{t} \\ \sqrt{t} \\ 1 \end{Bmatrix}\right]$	$\bar{\gamma}_{SR}$
$\begin{Bmatrix} \rho \\ A_2 \\ \pi' \end{Bmatrix}$	$\beta_{NR} =$	$\left[\sqrt{X}\, p^{\alpha-1}\, (\alpha+1)^{\frac{1}{2}} \begin{Bmatrix} \sqrt{\alpha}\ \sqrt{t} \\ \sqrt{t} \\ 1 \end{Bmatrix}\right]$	$\left[\sqrt{X}\, k^{\alpha}\, (\alpha+1)^{\frac{1}{2}} \begin{Bmatrix} \sqrt{\alpha}\ \sqrt{t} \\ \sqrt{t} \\ 1 \end{Bmatrix}\right]$	$\bar{\gamma}_{NR}$
$\begin{Bmatrix} B \\ \pi \end{Bmatrix}$	$\beta_{OI} =$	$\left[\sqrt{X}\, p^{\alpha} \begin{Bmatrix} \sqrt{t} \\ (1 - t/t_0)^{\frac{1}{2}} \text{ or } 1 \end{Bmatrix}\right]$	$\left[\sqrt{X}\, k^{\alpha-1}\, [\alpha(\alpha+1)]^{\frac{1}{2}} \begin{Bmatrix} (1 - t/\mu^2) \\ t^{-\frac{1}{2}}(1 - t/t_0)^{\frac{1}{2}} \text{ or } 0 \end{Bmatrix}\right]$	$\bar{\gamma}_{OI}$
$\{A_1\}$	$\beta_{1I} =$	$\left[\sqrt{X}\, p^{\alpha}\, (\alpha+1)^{\frac{1}{2}} \begin{Bmatrix} \sqrt{t} \\ 1 \end{Bmatrix} \begin{Bmatrix} \sqrt{\alpha} \\ 1 \end{Bmatrix}\right]$	$\left[\sqrt{X}\, k^{\alpha}\, (\alpha+1)^{\frac{1}{2}} \begin{Bmatrix} t \\ \sqrt{t} \end{Bmatrix} \begin{Bmatrix} \sqrt{\alpha} \\ 1 \end{Bmatrix}\right]$	$\bar{\gamma}_{1I}$

Table II. Parameters for fits with full $N\bar{N}\pi$ vertex zero at $t_0 = -0.05$

Parameters fixed from meson-nucleon scattering[5]

$$\alpha_\rho = 0.58 + 1.11t$$

$$\alpha_{A2} = 0.5 + 0.86t$$

$$b_{12}^{\rho}/b_{11}^{\rho} = \bar{\gamma}_{NR}^{\rho}/\bar{\gamma}_{SR}^{\rho} = -8.8\ e^{0.4t}$$

$$b_{12}^{A2}/b_{11}^{A2} = \bar{\gamma}_{NR}^{A2}/\bar{\gamma}_{SR}^{A2} = 3.5\ e^{-0.11t}$$

Parameters obtained in nucleon-nucleon fit [notation and data normalizations correspond to Table II in Ref. (2)]. Residue units are proportional to $(mb)^{\frac{1}{2}}$. (equalling $(mb)^{\frac{1}{2}}$ $(GeV)^k$ for appropriate values of k).

$$\chi^2 = 89 \text{ for } 74 \text{ points}$$

$$\alpha_\pi = -0.025 + 1.25t \qquad \gamma_0^B = -800t\ (\alpha_B + 2)e^{10t}$$

$$\alpha_{\pi'} = -0.025 + t \qquad \gamma_0^\pi = 0.919(1 + t/0.05)^2\ e^{11t}$$

$$\alpha_B = -0.4 + 0.9t \qquad \gamma_{22}^{\pi'} = [b_0^\pi(0)/\alpha_\pi(0)]e^{4.8t}$$

$$\gamma_{11}^\rho = 0.35\ e^{-4.4t} \qquad \gamma_{12}^{\pi'} = -68\ (\alpha_{\pi'})^{\frac{1}{2}}\ e^{2.2t}$$

$$\gamma_{11}^{A2} = 1.8\ e^{11t} \qquad g^2/4\pi = 14.7$$

Table II (Continued).

Parameters obtained in π^+ photoproduction fit. Residue units not proportional to $(\mu b)^{\frac{1}{2}}$ (equalling $(GeV)^k$ for appropriate values of k).

χ^2 = 66 for 62 points

α_B	$= -0.4 + 0.95t$	$\bar{\gamma}_{OI}^{\pi}$	$= -0.078\ e^{9.1t}$
$\bar{\gamma}_{SR}^{\rho}$	$= 0.166\ e^{-9.0t}$	$\bar{\gamma}_{NR}^{\pi'}$	$= -(2m/\mu^2)\ \bar{\gamma}_{OI}^{\pi}(0)\ e^{3.7t}$
$\bar{\gamma}_{SR}^{A2}$	$= 0.77\ e^{-1.1t}$	$\bar{\gamma}_{SR}^{\pi'}/\bar{\gamma}_{NR}^{\pi'}$	$= 1.86\ e^{-2.6t}$
$\bar{\gamma}_{OI}^{B}$	$= -2.95\ e^{11t}$	$g^2/4\pi$	$= 15.4$

Table III. Results of the B meson sum rule

Eq. (37) reads $\frac{1}{\pi}\bar{\gamma}_{0I}{}^{B}(t)\, R(t)\, \frac{N^{\alpha_B+1}}{\alpha_B+1} = B(t) + I(t)$, where $B(t)$ is the Born term. The power series expansions of $I(t)$ and $B(t)$ around $t = 0$ are given by

$$I(t) = -0.027 - 0.06t + 0.06t^2$$

$$B(t) = +0.0058 + 0.294t.$$

The residue is zero at $t_B \approx +0.09$.

The contributions $[X(-10^3\ \mu^2)]$ to $I(t)|_{t=0}$ are given by

$P_{33}(1238)$	0.01	$S_{11}(1560)$	0.13	Nonresonant	0.01
$P_{11}(1470)$	0.21	$D_{15}(1652)$	0.00		
$D_{13}(1520)$	0.21	$F_{15}(1672)$	-0.01		

Table IV. TABLE OF NOTATION

Section IA

$f^{t}_{\lambda_1\lambda_2,\lambda_\gamma}$	t channel helicity amplitude for unequal mass process $\gamma\pi\to\bar{N}_1N_2$
$f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}$	t channel parity conserving helicity amplitude (same process)
$f^{J\pm}_{\lambda_1\lambda_2,\lambda_\gamma}$	t channel partial wave helicity amplitude (same process)
$f^{J\pm}_{L'S'LS}(t)$	t channel partial wave L-S amplitude (same process)
$\tilde{f}^{J\pm}_{L'S'LS}$	Coefficients of L-S amplitude expansion around threshold
$r_k^{J\lambda\mu\pm}$	Coefficients in expansion of $e^{J\pm}_{\lambda\mu}(z_t)$ functions
$\tilde{a}^{J\lambda\mu\pm}_{L_N^{\pm}SS'}$	Coefficients describing leading order behavior of helicity amplitudes around threshold
$\tilde{F}_i(s,t)$	Kinematic singularity free form of $f^{\pm}_{\lambda_1\lambda_2,\lambda_\gamma}(s,t)$
$\tilde{F}_i^{E}(s,t)$	$\lim\limits_{m_1\to m_2} \tilde{F}_i(s,t)$
$F_i^{t}(s,t)$	$\lim\limits_{m_\gamma\to 0} \tilde{F}_i^{E}(s,t)$ (in Eqn. 12, $F_4^{t} = \lim\limits_{m_\gamma\to 0;\,\Delta\to 0} \dfrac{\tilde{F}_4}{t(t-\mu^2)}$).

Section ID

$\tilde{\mathcal{F}}_i(s,t)$	Kinematic singularity free amplitudes for unequal mass process $\bar{N}_1N_2\to\bar{N}_3N_4$
$\tilde{f}_i(s,t)$	$\lim\limits_{m_1\to m_2;\,m_3\to m_4} \tilde{\mathcal{F}}_i(s,t)$
$c_i^{jkl}(s)$	Coefficient of $t^j\,\Delta_{12}^{\ell}\,\Delta_{34}^{k}$ in expansion of $\tilde{\mathcal{F}}_i(s,t)$

FIGURE CAPTIONS

Fig. 1. Real parts of the π^+ photoproduction amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0 = -0.05$. To leading order,

$$\left.\frac{d\sigma}{dt}\right|_{\pi^\pm\text{photoproduction}} = 2|\pi \pm B|^2 + 2|\pm\rho_{SR} + (A_2)_{SR} + \pi'_{SR}|^2$$

$$+ 2|\pm\rho_{NR} + (A_2)_{NR} + \pi'_{NR}|^2 .$$

Fig. 2. Imaginary parts of the π^+ photoproduction amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0 = -0.05$.

Fig. 3. π^+ photoproduction fit. Curves have been multiplied by 0.99, 1.03, 1.03, 0.97, 0.93, respectively for 2.6, 5, 8, 11, and 16 GeV/c.

Fig. 4. Small t region for π^+ photoproduction fit with the same normalization factors.

Fig. 5. Integrands at $t = 0$ for the $\bar{B}$ and pion photoproduction sum rules. ($\frac{-\mu^2\nu}{\pi}$ Im $A_1^{(0)}$ and $\frac{-\mu^2}{\pi}$ Im $A_1^{(-)}$, respectively).

Fig. 6. Conjectured simulation of $M = 0$ B amplitude found in photoproduction fit with $M = 1$ B, ρ' amplitudes.

Fig. 7. Conjectured $M = 1$ B amplitude and resulting π amplitude for $pn \to np$ scattering near $t = 0$.

——— π amplitude with $M = 0$ B assumed.

------ π amplitude with $M = 1$ B assumed.

—. —. $M = 1$ B amplitude.

The mark on the vertical axis yields $\frac{d\sigma}{dt}(pn \to np) = 1$ mb at 8 GeV.

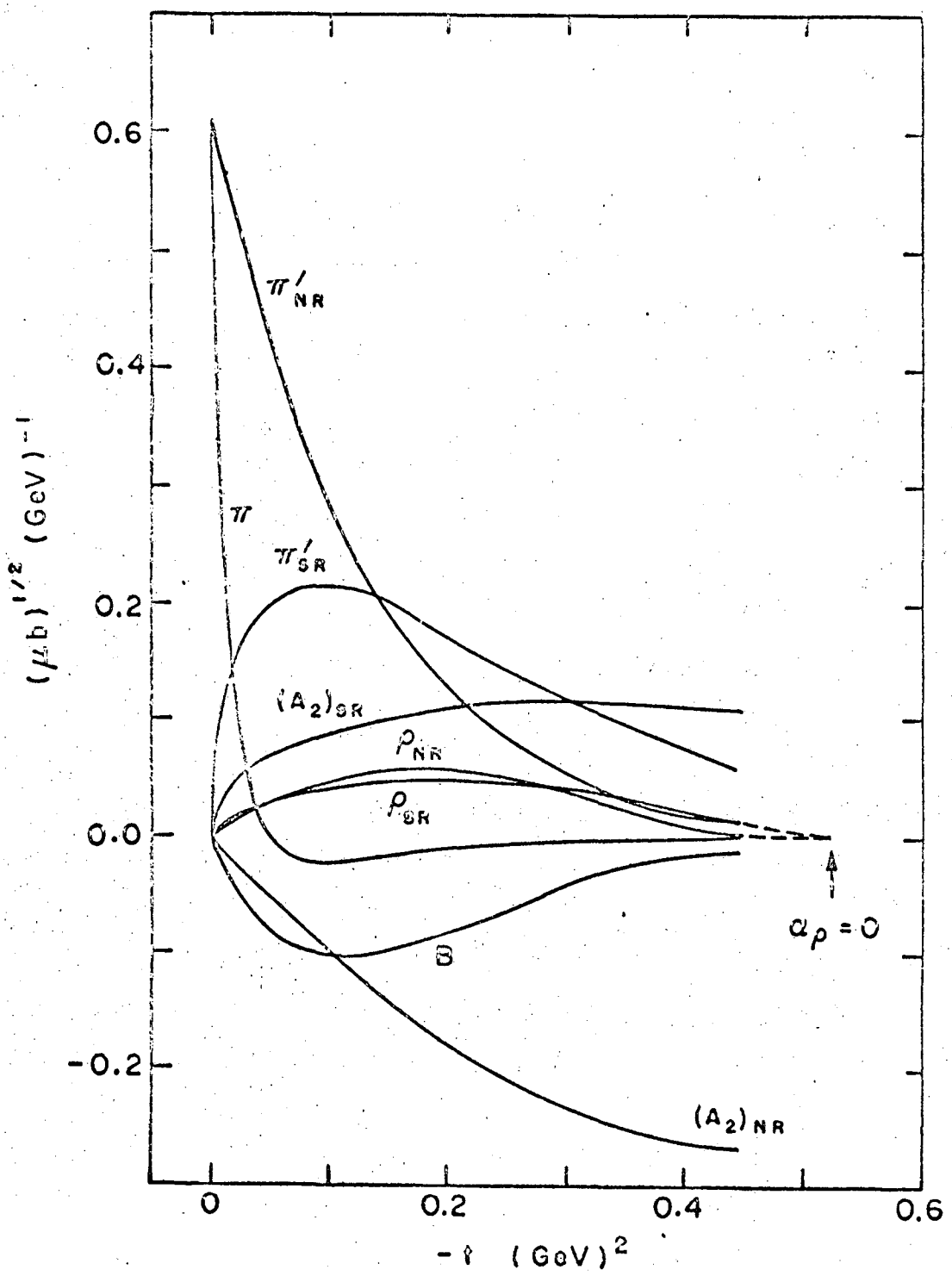
Fig. 8. Real parts of the nucleon-nucleon amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0 = -0.05$. To leading order,

$$\left.\frac{d\sigma}{dt}\right|_{\substack{pn \to np \\ p\bar{p} \to n\bar{n}}} = 2|\pi \pm B|^2 + 2|\pm\rho_{11} + (A_2)_{11} + \pi'_{11}|^2$$

$$+ 2|\pm\rho_{22} + (A_2)_{22} + \pi'_{22}|^2 + 4|\pm\rho_{12} + (A_2)_{12} + \pi'_{12}|^2 \quad .$$

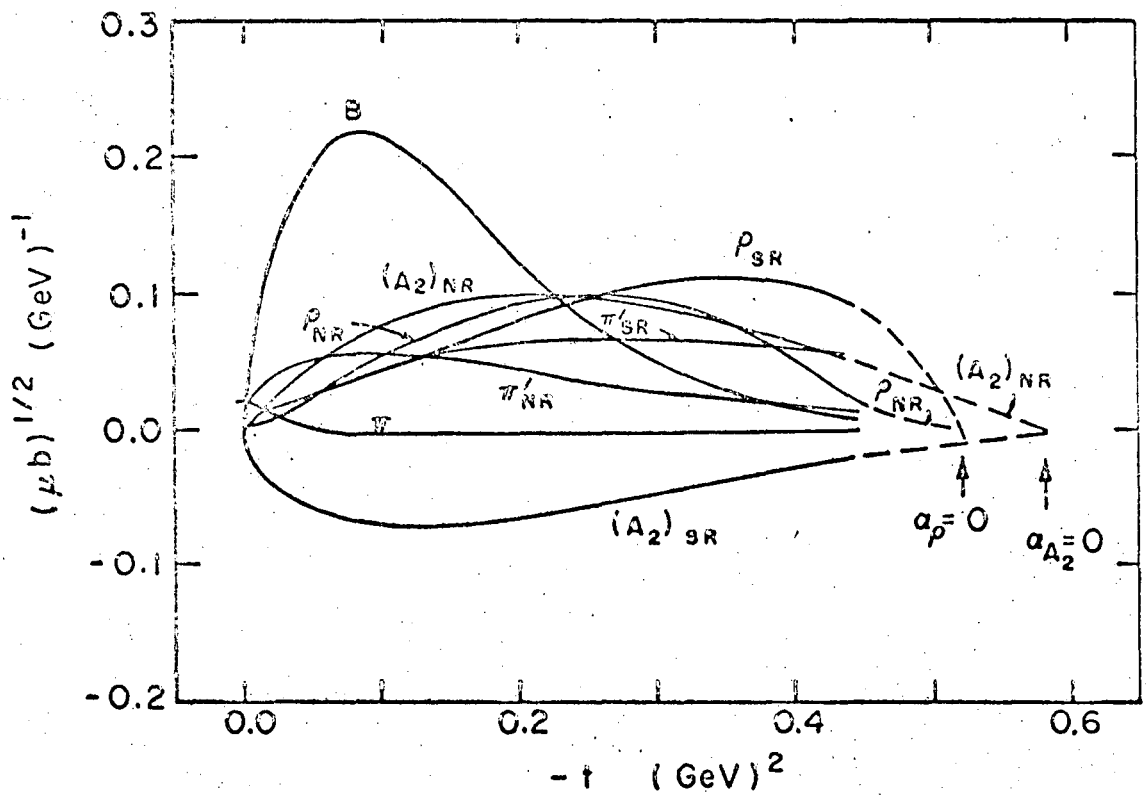
To leading order the coupled triplet amplitudes factorize; e.g., $\pi'_{11}\,\pi'_{22} = -(\pi'_{12})^2$. Note that the (12) amplitudes have additional weight in the cross sections.

Fig. 9. Imaginary parts of the nucleon-nucleon amplitudes at 8 GeV/c for full $N\bar{N}\pi$ vertex zero $t_0 = -0.05$.



XBL 685-2803

Fig. 1.



XBL685-2678

Fig. 2.

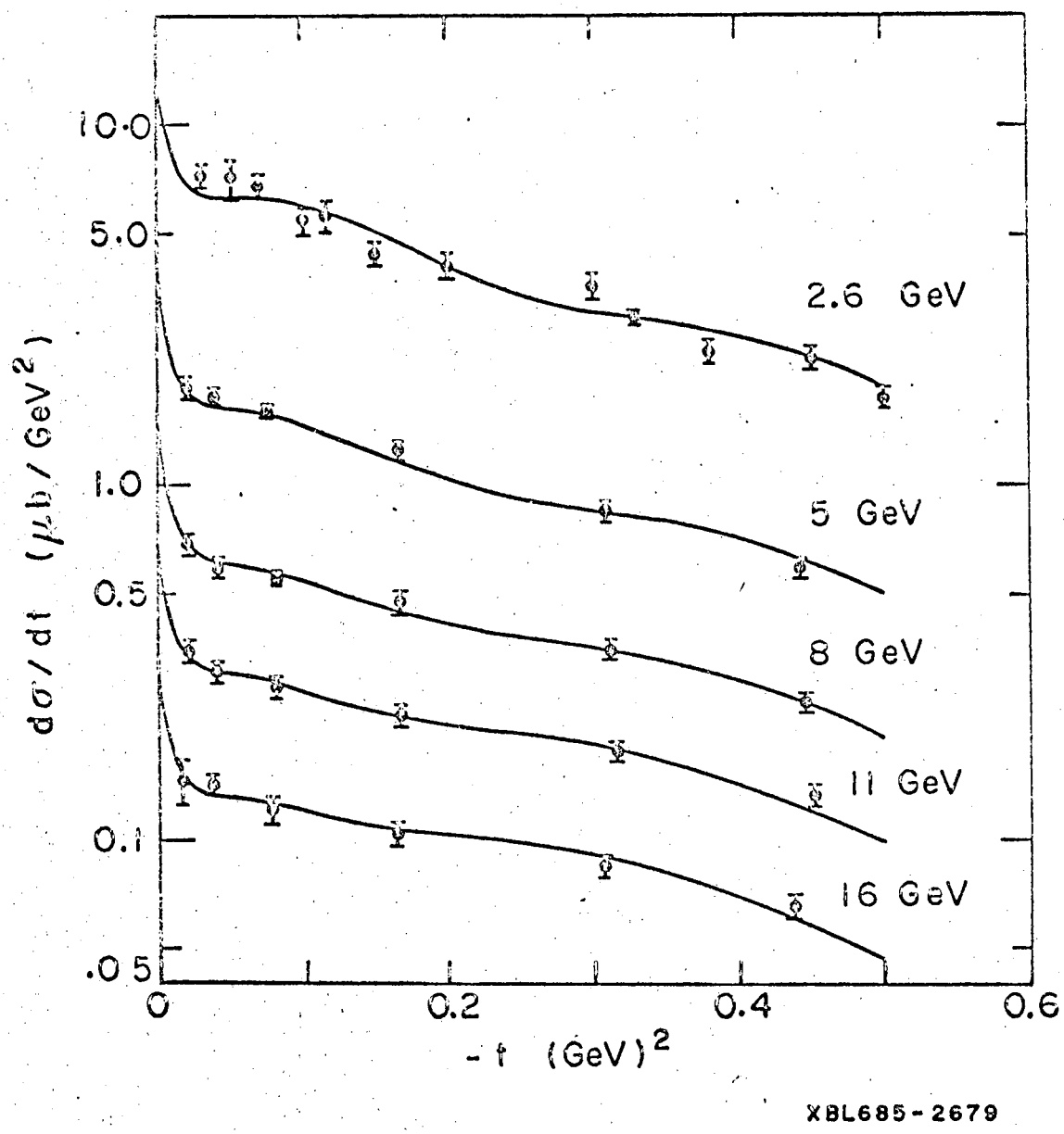
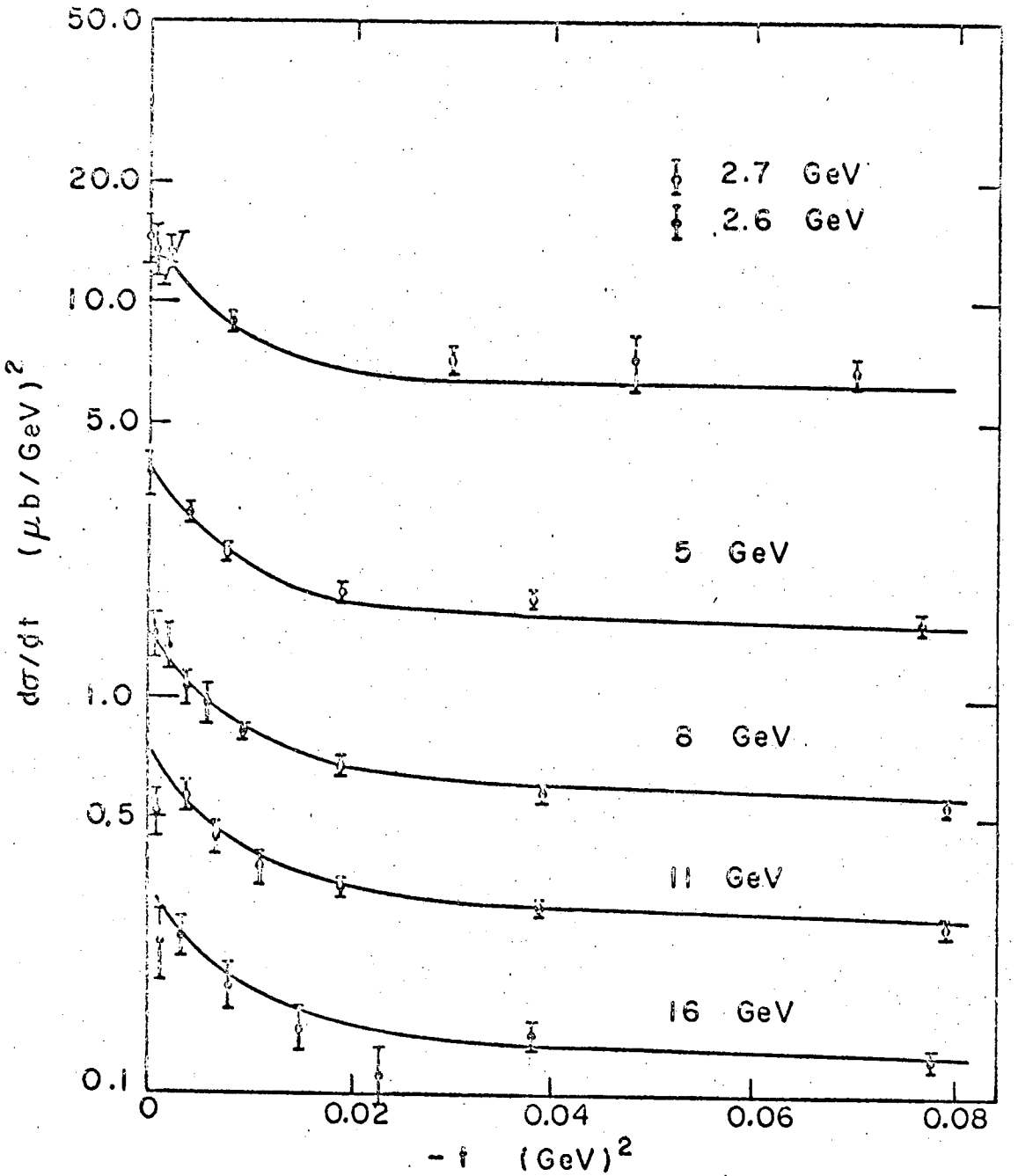

10.0
5.0
1.0
0.5
0.1
.05
0
0.2
0.4
0.6
dσ/dt (μb/GeV²)
- t (GeV)²
2.6 GeV
5 GeV
8 GeV
11 GeV
16 GeV
XBL685-2679

Fig. 3.



XBL605-2680

Fig. 4.

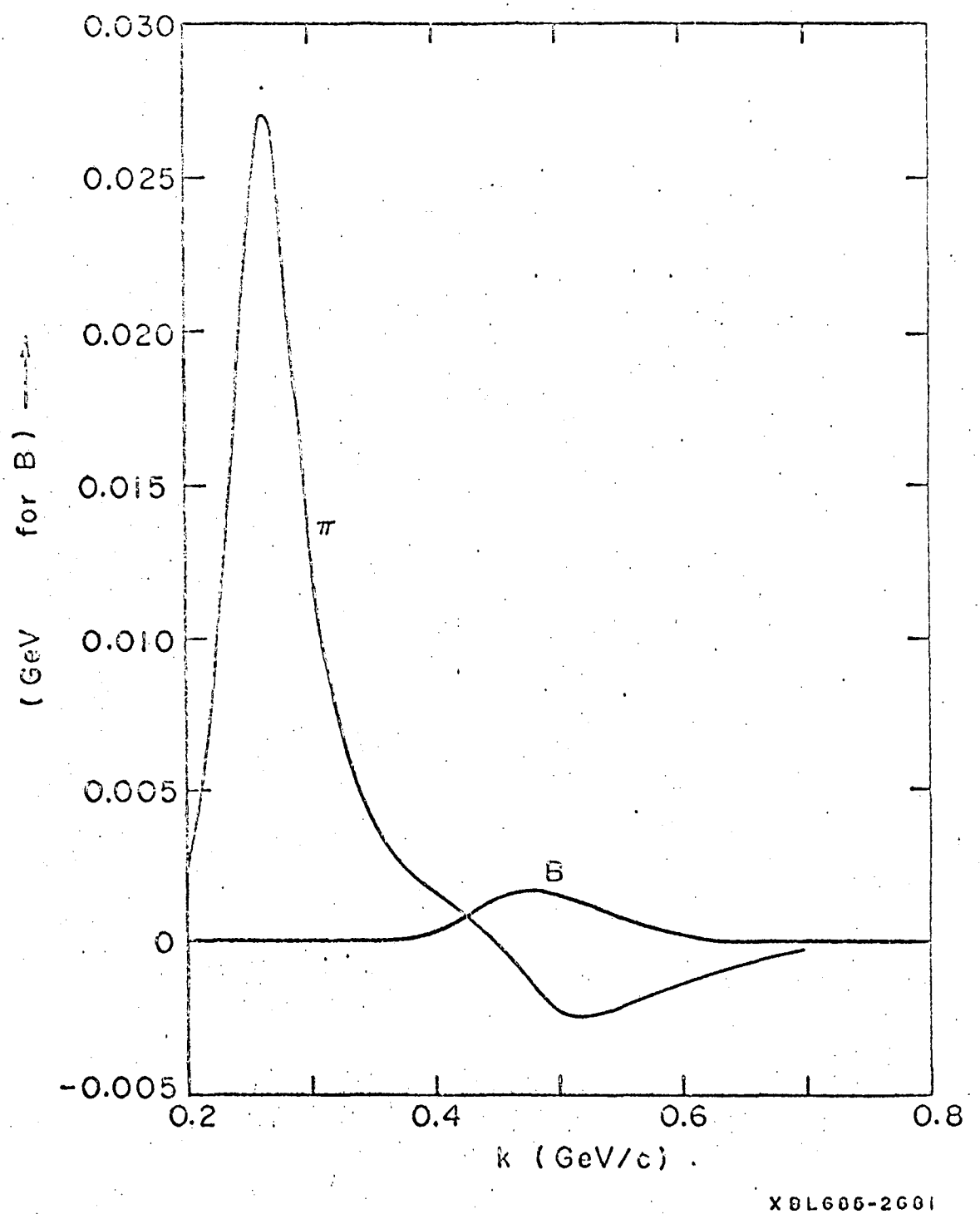
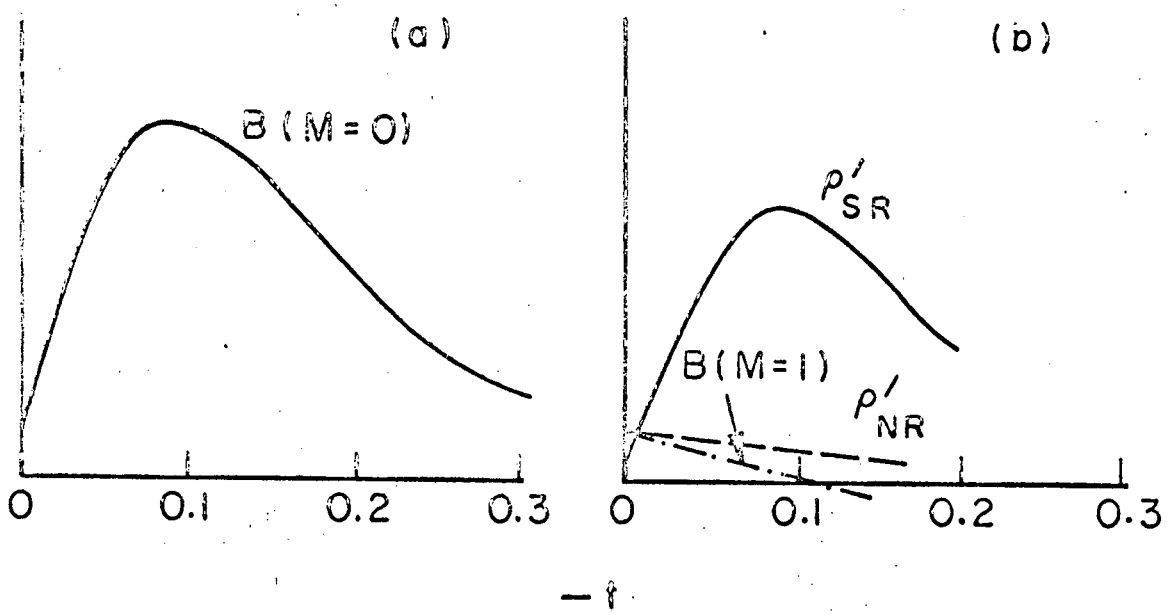
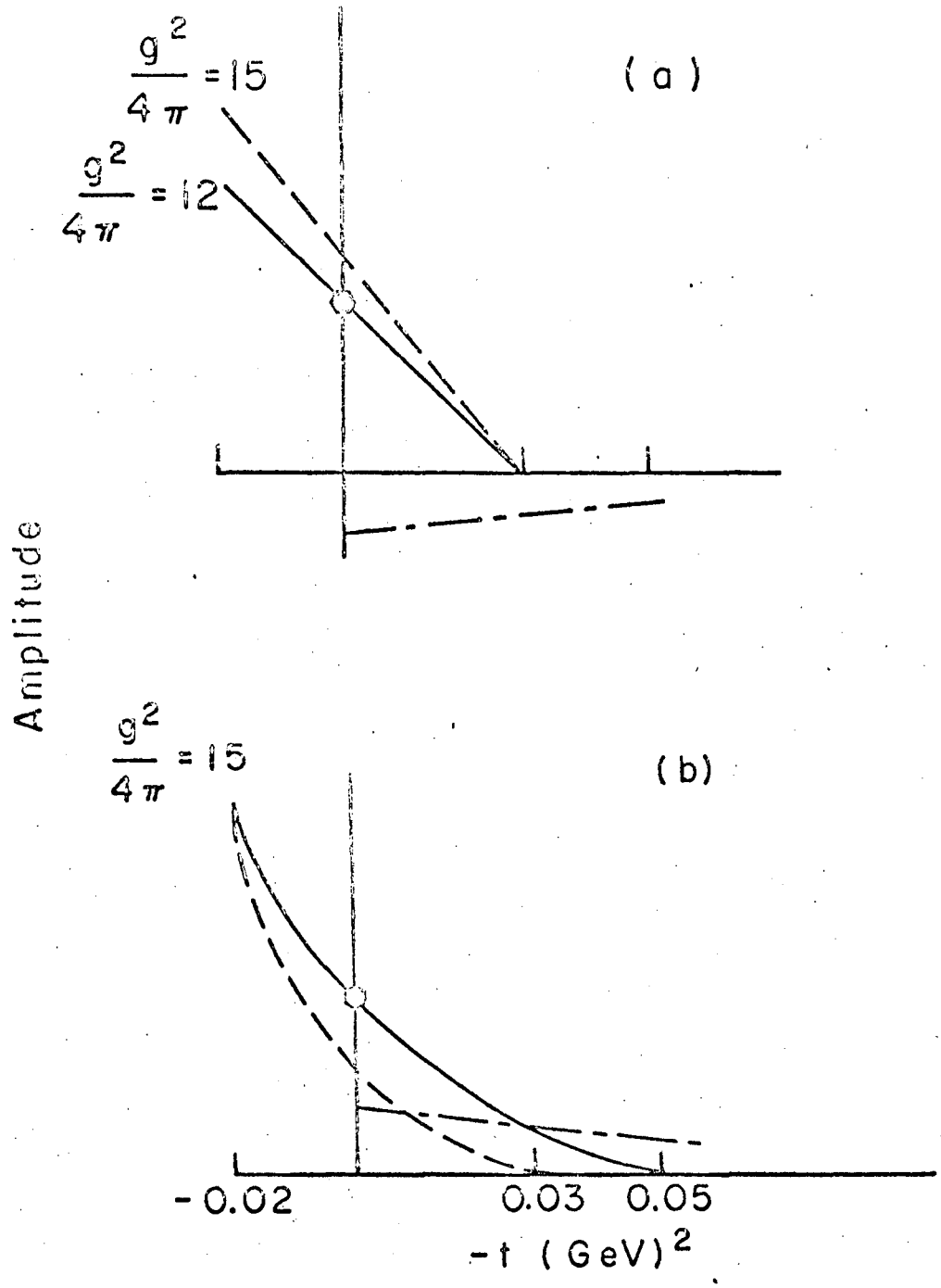

0.030
0.025
0.020
0.015
0.010
0.005
0
-0.005
0.2
0.4
0.6
0.8
k (GeV/c)
(GeV for B)
π
B
XBL606-2601

Fig. 5.



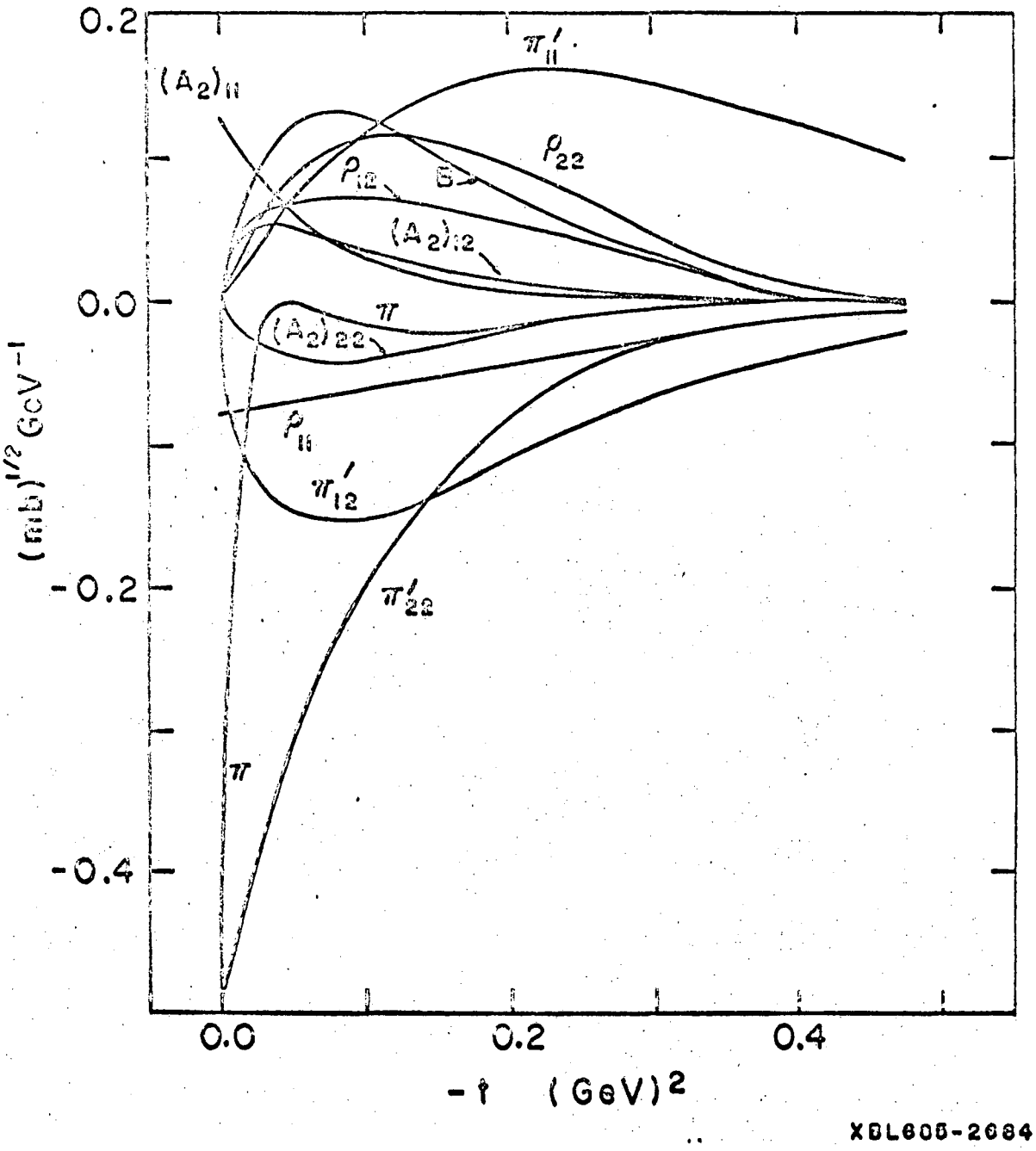
XBL685-2682

Fig. 6.



XBL685-2683

Fig. 7.



XBL605-2684

Fig. 8

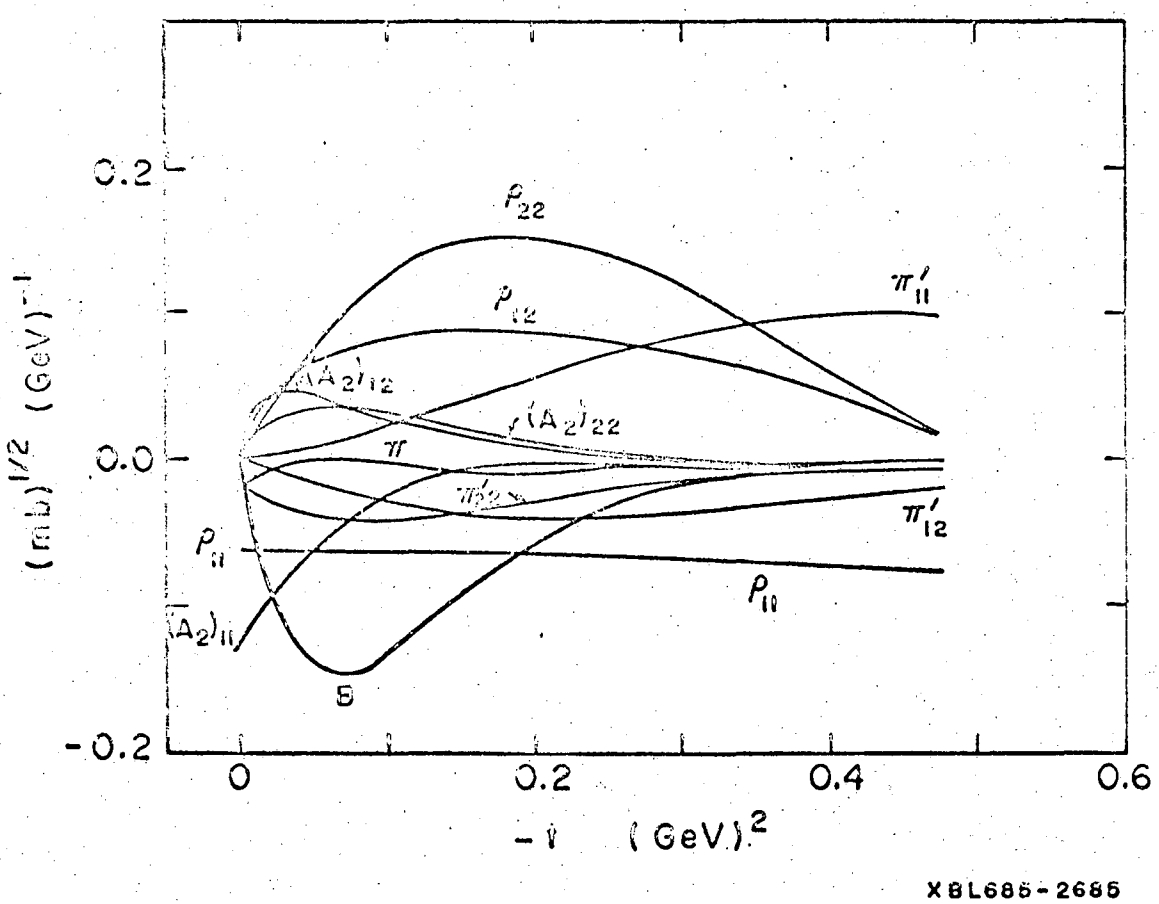

0.2
0.0
-0.2
0
0.2
0.4
0.6
$(mb)^{1/2}$ $(GeV)^{-1}$
$-t$ $(GeV)^2$
ρ_{22}
ρ_{12}
π'_{11}
$(A_2)_{12}$
$(A_2)_{22}$
π
π'_{22}
π'_{12}
ρ_{11}
ρ_{11}
$(A_2)_{11}$
B
XBL685-2685

Fig. 9.

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