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THE ROLE OF DISLOCATION FLEXIBILITY IN THE STRENGTHENING OF METALS

Tibor Stefansky and John E. Dorn

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Tibor Stefansky and John E. Dorn

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I. INTRODUCTION

Mott and Nabarro[1-5] were first to illustrate the importance of the flexibility of dislocations in accounting for the strengthening that metals undergo due to the presence of localized internal-strain centers. Such strain centers might arise from a number of causes such as radiation damage, the presence of semi-coherent and coherent precipitates and clusters as well as individually dispersed interstitial and misfit substitutional solute atoms. Each strain center induces equal positive and negative shear stress fields on the slip planes of the surrounding alloy matrix. In order to effect plastic deformation therefore, the alloy must be subjected to sufficiently high applied shear stresses to push dislocations past all such resisting internal stress fields. If dislocations were ideally flexible, each blocked segment could be pushed separately over the local resisting internal stress field. At this extreme very high strengthenings would be obtained. On the other hand, if dislocations were inflexible and moved as rigid lines, they would be equally pushed and pulled by internal stress fields. At this extreme the strengthening would be vanishingly small. Dislocations, however, are neither ideally flexible nor infinitely rigid. Since the line tension of a dislocation attempts to keep the dislocation as straight as possible, it is one measure of its rigidity or flexibility. The observed strengthening, therefore, must lie between the maximum value given by an ideally flexible dislocation and the value of zero given by an infinitely rigid dislocation.

Cottrell[6] and Friedel[7,8] have independently presented lucid reviews of the Mott-Nabarro dislocation flexibility concept. More recently, however, some confusion inadvertently has been introduced[9,10] as a result of using the term "dislocation flexibility" to account for the effect of dislocation bowing on the link length between adjacent obstacle centers and related statistical topics. This "dislocation flexibility" concept differs appreciably from that originally intended by Mott and Nabarro despite the somewhat incidental fact that it too depends partly on the line tension of a dislocation. Friedel[7,8] has presented a steady state deformation analysis for the problem of cutting weak obstacles. An interesting statistical theory has been developed by Kochs[11,12] and further elaborated by Dorn, Guyot and Stefansky[13]. Recognizing the extreme complexity of this difficult statistical problem, Foreman and Makin[14] made what appears to be definitive computerized experiments which serve to elucidate the issues that are involved. The confusion that has been generated by the unfortunate use of the term "dislocation flexibility" for these statistical issues is now being extended to analyses of experimental data[10]. It is suggested here that the term "randomness correction" be henceforth employed for these essentially geometric statistical effects and the term "dislocation flexibility" be reserved to refer to the issues originally intended by Mott and Nabarro[1,5].

Although Mott, Nabarro, Cottrell, and Friedel have clearly expounded the significance of dislocation flexibility in the strain-center strengthening of alloys, their analyses were only semi-quantitative. This

report will be concerned with a more detailed analysis of the problem. In what transpires a correction will also be made for randomness effects but only the effects of dislocation flexibility will be specifically explored and emphasized.

It will be shown that for extremely low concentrations of atomic strain centers the strengthening increases with the square root of their concentration. For higher concentrations the strengthening rate becomes less rapid. At a critical concentration a maximum strength is obtained and a decrease in the strengthening takes place as the concentration of solute atoms is further increased. The later trends arise principally as a result of dislocation flexibility.

II. STRESS FIELDS ON SLIP PLANES

Internal stresses are introduced in a lattice whenever a host atom of radius r_o is replaced by another that has a different atomic radius. According to the classical theory of linear elasticity,[15] the stress field about a solute atom at the origin of a spherical coordinate system is given by

$$\sigma_{rr} = -4G\varepsilon \left(\frac{r_o}{r}\right)^3$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = 2G\varepsilon\left(\frac{r_o}{r}\right)^3 \quad \text{for } r \geq r_o\,(1+\varepsilon) \tag{1}$$

where G is the shear modulus of elasticity, r_o is the radius of a host atom, and $r = r_o\,(1+\varepsilon)$ is the radius in situ, of the substituted atom. As shown by Eshelby,[16] this approximation assumes that the bulk modulus of elasticity of the substituted atom is identical with that of the host species. Nabarro[17] suggested that the strain ε can be deduced experimentally from

$$\varepsilon = a^{-1}\,(da/dc) \tag{2}$$

where a is the lattice parameter and c is the atomic fraction of the solute species.

We will consider here only the interaction of the stresses on a slip plane in fcc metals with a dislocation that is substantially in edge orientation. We let X-Y be a slip plane at a height z above the solute atom center, where Y coincides with the Burger's vector of the dislocation

that lies parallel to the X axis, In this event the dislocation motion will depend only on the shear stresses σ_{zy} . Upon simple tensor transformation of Eq. 1.

$$\sigma_{zy} = \frac{6G\left(\frac{b}{2}\right)^3 \varepsilon zy}{\left(x^2+y^2+z^2\right)^{5/2}} , \qquad x^2+y^2+z^2 \geq \left(\frac{b}{2}\right)^2 \tag{3}$$

where for convenience r_o has been replaced by its substantial equivalent of b/2. Unfortunately Eq. 3 becomes somewhat inaccurate in the region of greatest interest here, namely when $x^2+y^2+z^2$ is small. The fact that the deduced shear stress σ_{zy} applies only for $x^2+y^2+z^2$ greater than an atomic radius squared is perhaps not as serious as the fact that when the core of the dislocation overlaps the volume of the substituted atom, the linear theory of elasticity becomes seriously in error. From a physical viewpoint it is easily judged that, where overlap takes place, the stress fields will be somewhat less than those suggested by Eq. 3. This issue, however, is not critical for demonstrating the effects of dislocation flexibility on solid-solution strengthening and consequently we might suggest that the right hand side of Eq. 3 be multiplied by the qualitatively justifiable compensating factor

$$1-e^{-\beta\frac{\left(x^2+y^2+z^2\right)}{b^2}}$$

Whereas in principle the constant β might be estimated from experimental data on binding energies, it will prove satisfactory for the present to arbitrarily select $\beta = 3$. This adjustment results in a decrease in the magnitude of the stress fields when overlap takes place and yet reproduces faithfully the results of Eq. 3, where it is yet reasonably valid. Accordingly we suggest that

$$\sigma_{zy} = \frac{3}{4} \frac{Gb^3 \varepsilon zy \left[1 - e^{-\frac{3}{b^2}\left(x^2+y^2+z^2\right)} \right]}{\left(x^2+y^2+z^2\right)^{5/2}} \tag{4}$$

where Eq. 4 refers to the stresses on a slip plane a distance z above or below the solute atom.

For a solute atom at $(x,y) = (o,o)$ on the first atomic plane $(z = b/\sqrt{6})$ above the slip plane, the shear stress distribution on the slip plane is that shown in Fig. 1. A positive edge dislocation lying parallel to the x axis, would be repelled by the solute atom except for the somewhat trivial condition when the dislocation is at $y = 0$. It is significant that the important region, where the absolute value of the stresses are great, is highly localized in the near vicinity say over about an area of $10b^2$ around the atom center. For a solute atom lying on the first atomic plane below the slip plane $(z = -b/\sqrt{6})$ the signs given in Fig. 1 are reversed. In this case a straight edge dislocation would be so attracted as to lie along the X-axis at $y = 0$. The interaction energy between an infinitely long straight edge dislocation and the stress field of a solute atom is

given by

$$U\left\{y\right\} = \int_{-\infty}^{y} \int_{-\infty}^{\infty} \sigma_{zy}\, b dx dy \tag{5}$$

for the case where the datum of energy equals zero is taken at $y = -\infty$. Introducing the expression given in Eq. 4 for σ_{zy} and integrating gives the results shown in Fig. 2. As shown the highest interaction energies arise from solute atoms on planes nearest the slip plane, i.e. $z = \pm b/\sqrt{6}$. It was noted that the interaction integral of Eq. 5 gave almost the same results when it was integrated over x from -5b to +5b in lieu of from -10b to +10b. This emphasizes the highly localized nature of dislocation solute atom interactions.

A force, F, acting on a long straight edge dislocation will hold it in equilibrium with a solute atom when F + Fi = 0 where Fi is the force arising from the internal interaction with a solute atom stress field. Consequently

$$F = -\int_{-\infty}^{\infty} \sigma_{zy} b dx \tag{6}$$

This force-displacement diagram is shown in Fig. 3 for the case where z is positive. For solute atoms lying below the slip plane the curve of Fig. 3 is merely reflected across the y = 0 axis. According to this somewhat naive approach then, the maximum force, Fm, to move a dislocation past the stress field of a solute atom is the same regardless of whether the stress field is attractive or repulsive. The maximum force, F_{2m}, to move a dislocation past a solute atom on the second nearest atomic plane

to the slip plane is given by $F_{2m} = \frac{0.20}{1.17} F_m$ and is therefore a small fraction of the force needed to move dislocations past atoms on the nearest planes.

III. YIELD STRENGTHS UNDER CONDITIONS LEADING TO HIGH DISLOCATION FLEXIBILITY

In this section we will estimate the yield strengths at the absolute zero for substitutional binary alloys assuming that the strengthening arises exclusively from the stress fields induced by substitutional solid-solution alloying, further assuming that the dislocations can behave as if they were ideally flexible. The concepts to be invoked here are substantially those previously presented by Fleischer[9]. The details, however, have been adjusted, as given in the preceeding section, to permit direct comparisons of deductions made here with those to be made later for the more realistic model that considers dislocation flexibility.

Solute atoms will first be viewed as forming a square array of strain centers. Neglecting all planes more distant than the two atomic planes nearest the slip plane, demands that the edge of the array, λ, is given in terms of the atomic fraction of solute that is present, c by

$$2c = b^2/\lambda^2 \tag{7}$$

The results for the random case will be deduced by applying the Foreman and Makin[14] correction to the square array.

Since the dislocations are assumed to be highly flexible each dislocation segment λ can be considered to be separately pushed past the resisting stress field of the solute atoms. Thus, the yield stress, τ^*_s at the absolute zero is given by

$$\tau^*_s \lambda b = \tau^*_s \frac{b^2}{\sqrt{2c}} = F_m = 1.17\ Gb^2\varepsilon \tag{8}$$

where the subscript s refers to the fact that a square array of strain centers was assumed. This analysis, however, must be limited to values of λ greater than about 5b where the strain fields of adjacent solute atoms do not overlap. The deduced variations of τ_s^*/G as a function of $\sqrt{c}$ for several values of ε are shown by curves marked s in Fig. 4.

To obtain the yield stress for cases where the solute atoms are distributed more or less at random, we apply the data of Foreman and Makin[14] as given in Fig. 5. The strength of the stress field is defined by in terms of the edge dislocation line energy of about $3Gb^2/4$ according to

$$\tau_s^*\lambda b = 3\alpha\ Gb^2/4 \tag{9}$$

Comparison with Eq. 8 reveals that for substitutional solid solution alloying $\alpha \simeq 1.56\varepsilon$. Applying the randomness correction to the data gives the broken curves of Fig. 4 marked R.

These calculations suggest that the yield stress at the absolute zero increases linearly with the $\sqrt{c}$.

Up to the present the effects of atoms on second nearest atomic planes to the slip plane have been neglected. We therefore demonstrate, here, that in large measure, so long as the atoms do not cluster, this neglect is permissible. Considering a square array of atoms on the four nearest planes, suggest that $4c = b^2/\lambda_4^2$. The force needed to cause the dislocations to surmount the weaker stress fields of solute atoms on the second nearest atomic planes to the slip plane is given by

$$\tau^*_{s4}\lambda_4\, b = \tau^*_{s4}\,\frac{b^2}{2\sqrt{c}} = 0.20\, Gb^2\varepsilon$$

whence $$\tau_{s4}/\tau_s = \frac{2 \times (.20)}{1.17\,\sqrt{2}} \simeq 1/4 \qquad (10)$$

This ratio is further decreased when the randomness correction is applied. Thus, as originally suggested by Fleischer,[9] the stress fields of misfit solute atoms residing on the second nearest planes to the slip plane are so small that the dislocations will be pushed past them at less than 1/4 of the stress needed to cause them to pass the stress fields of atoms on the planes immediately adjacent to the slip planes. It is necessary, however, to point out that although the behavior of solid solutions at the absolute zero of temperature is not sensitive to stress fields of more distant atoms than those in the immediate vicinity of the slip plane, the yield stress at higher temperatures, in the thermally activated range will depend on the stress fields of such more distant solute atoms.

IV. EFFECTS OF LIMITED DISLOCATION FLEXIBILITY

The model for solid-solution strengthening that was presented in Section III was based on the assumption that dislocations behave in an ideally flexible manner insofar as each segment of a dislocation was assumed to be able to pass the stress field arising from the presence of each nearby solute atom independently of adjacent segments. Due to their line tension, however, dislocations are not ideally flexible in this sense. Consequently all deductions, e.g. such as those presented in Fig. 4, based on the assumption of ideal dislocation flexibility must be greeted with some suspicion as to their validity.

In this section we present a more realistic model for solid-solution strengthening in which the questionable assumption of ideally flexible dislocation behavior will not be invoked. It will be demonstrated that the assumption of an ideally flexible dislocation is not too unreasonable for extremely dilute solutions. As the concentration increases however the assumption of ideal dislocation flexibility leads to serious errors. Whereas the assumption of ideal dislocation flexibility suggests that the strength of alloys continues to increase with $\sqrt{c}$ as the concentration, c, exceeds some critical value, in sharp contrast, the more realistic model, as might be expected from physical considerations, suggests a completely different trend of a decrease in the strengthening with additional increases in solute-atom concentration.

In order to permit an unbiased evaluation of the role of dislocation flexibility on solid-solution strengthening, we attempt to hold all remaining assumptions the same as those adopted for the ideally flexible dislocation model that was discussed in Section III. Therefore the stress centers arising from the solute atoms adjacent to the slip plane are placed on a square array as shown in Fig. 6. The value of λ, given by Eq. 7, also remains the same as that employed in the ideally flexible model. Since A and R of Fig. 6 refer to equal attractive and repulsive strain centers it is immediately apparent that a completely rigid edge dislocation could be moved through the stress fields of the solute atoms by an infinitely low applied stress. The more realistic strengthening behavior that we now seek must lie between the values of zero for an infinitely rigid dislocation and the results given in Fig. 4 for an ideally flexible dislocation.

As shown in Fig. 6 the origin of the X - Y slip plane was selected to be at a repulsive stress center. Consequently the significant stress at point (x,y) on the slip plane due to all solute atoms on the next nearest atomic planes is simply

$$\sigma'_{zy} = \frac{3}{4} Gb^3\varepsilon \sum_i \frac{(y-y_i)\, z_i \left\{ 1-e^{-\frac{3}{b^2}\left[\left(x-x_i\right)^2 + \left(y-y_i\right)^2 + z_i^2\right]} \right\}}{\left\{ \left(x-x_i\right)^2 + \left(y-y_i\right)^2 + z_i^2 \right\}^{5/2}} \tag{11}$$

where i refers to the ith stress center, and $z_i = \pm b/\sqrt{6}$. The stress fields about solute atoms are so localized that only a few atoms in the range $-6\lambda \leq x_i \leq 6\lambda$ and $-6\lambda \leq y_i \leq 6\lambda$ need be considered in the summation of Eq. 11.

It might be argued with considerable validity that the stress fields arising from near atoms on the second nearest atomic planes produce a much greater modification of the shear stress than the solute atoms on the nearest atomic planes that are more distant from the origin of the selected coordinate system. Such stresses, however, were not included here since we wished to preserve the basis of comparison with the results of Section III where the effect of solute atoms on second nearest planes was neglected. The small differences in the local stresses near solute atoms due to neighboring solute atoms is shown in Fig. 7 where $\sigma'_{zy}/G\varepsilon$ is plotted as a function of y/b for the cut at x=0. Since, as shown in Fig. 7, only small changes occur in the local stress field as a function of concentration of solute atoms, any difference between the deductions on solute atom strengthening obtained here and that presented in Section III must be ascribed almost exclusively to the effect of dislocation flexibility.

It is now our objective to ascertain the equilibrium shape of an edge dislocation having a Burgers vector b where the alloy is subjected to a resolved shear stress τ^*_s. Obviously the symmetry of internal stresses σ'_{zy} demands that for stable configurations, the dislocation line y = y{x} will be periodic in x with a period of 2λ. Thus in general, the energy of a dislocation line will (vide Dorn and Rajnak[18]) be

$$U = U_o + \int_{-\lambda}^{\lambda} \left\{ \Gamma \left[\sqrt{1+\left(\frac{dy}{dx}\right)^2} - 1 \right] - \int_0^y \sigma'_{zy} b\,dy - \tau^*_s by \right\} dx \qquad (12)$$

where U_o is the energy of a straight dislocation segment of length 2λ lying parallel to the x-axis at $y = 0$ and $\Gamma = 3/4Gb^2$ is the line energy per unit length. The first term in the integrand gives the increase in the line energy of the displaced dislocation and the second and third terms provide the work done by the internal and applied stresses.

Although several simplifying assumptions were made in arriving Eq. 12, they are known not to be too critical. The changes in line energy with dislocation slope as it veers from pure edge to partly screw orientation can be shown to be small in this example. The interaction between segments of the now curved dislocation line was neglected. Recent computer analyses by Foreman[19], however, have shown that such second order effects are small, particularly in the case of weak obstacles such as are being considered here.

Whereas Eq. 12 applies to any shape of dislocation $y = y\{x\}$, the equilibrium shape, of interest here, can be obtained by applying the calculus of variations to minimize U. Euler's equation, thus obtained, gives

$$\frac{d}{dx}\left(\frac{\partial U}{\partial p}\right) - \frac{\partial U}{\partial y} = 0 \tag{13}$$

where $p = dy/dx$. Consequently the differential equation for the equilibrium line is

$$\frac{d^2y/dx^2}{\left\{1 + (dy/dx)^2\right\}^{3/2}} = -\frac{b}{3/4Gb^2}\left(\tau_s^* + \sigma'_{zy}\right) \tag{14}$$

For convenience of computation Eq. 14 was placed in dimensionless form by letting $x' = x/\lambda$ and $y' = y/\lambda$. Upon introducing Eq. (7), the required relationship is given by

$$\frac{d^2y'/dx'^2}{\left\{1 + \left(dy'/dx'\right)^2\right\}^{3/2}} = -\frac{2\sqrt{2}}{3\sqrt{c}}\,\frac{\tau_s}{G} \tag{15}$$

$$-2\varepsilon c \sum_i \frac{(y' - y'_i)\, z'_i \left\{1 - e^{-\frac{3}{2c}\left[(x' - x'_i)^2 + (y' - y'_i)^2 + z_i'^2\right]}\right\}}{\left\{(x' - x'_i)^2 + (y' - y'_i)^2 + z'^2_i\right\}^{5/2}}$$

where $z'_i = \pm\sqrt{2c}/\sqrt{6}$

The integration of Eq. (15) will now be discussed in general terms. It contains three parameters $\tau_s^*/\sqrt{c}\,G$, εc, and c which in any case are fixed constants. A point $(0, y_o')$ was arbitrarily selected to start the calculation. Due to symmetry $(dy'/dx') = 0$ at $(0, y_o')$. The value of d^2y'/dx'^2 at this point was deduced from Eq. (15). An adjacent point on the dislocation line at $x_o' + \Delta x'$ was then determined from

$$(y')_{x_o' + \Delta x'} = y_o' + \left(\frac{dy'}{dx'}\right)_{x_o'} \Delta x' + \frac{1}{2}\left(\frac{d^2y'}{dx'^2}\right)_{x_o'} (\Delta x')^2 \tag{16}$$

and

$$\left(\frac{dy'}{dx'}\right)_{x_o' + \Delta x'} = \left(\frac{dy'}{dx'}\right)_{x_o'} + \left(\frac{d^2y'}{dx'^2}\right)_{x_o'} \Delta x' \tag{17}$$

where $\Delta x'$ was selected to be the small interval 0.001. This procedure was continued to the point $x' = 0.003$ following which the calculation was continued to $x' = \lambda'$ by the Adams[20] technique. To satisfy the symmetry conditions, an acceptable solution must result in $(dy'/dx') = 0$ at $x' = 1$ as well as $x' = 0$. Consequently a series of values of y_o' at $x' = 1$ were employed until the desired curve $y' = y'(x')$ having zero slopes at $x' = 1$ and nowhere else was obtained. A check using Eq. 12 illustrated that this was the minimum energy curve for the selected conditions.

In order to obtain the yield stress at the absolute zero, the parameter $\tau_s^*/\sqrt{c}\,G$ was increased to a series of new values, retaining εc and c constant. As the yield stress was approached the line shape changed sensitively to the values of $\tau_s^*/\sqrt{c}\,G$. If the selected value of $\tau_s^*/\sqrt{c}\,G$, however, exceeded the yield strength, dy'/dx' differed from zero at $x' = 1$ for all possible values of y_o'. In this way the yield strength could be approximated as closely as desired. An attempt was made to carry out the calculations on yield strength to an accuracy of about ±1%.

An example of the shape of the dislocation line at zero stress is given in Fig. 8. For all cases examined, only two equilibrium configurations were obtained. The equilibrium dislocation line never zig-zagged away from attractive stress centers along $y'=0$ and toward attractive centers near $y'=1$. Two somewhat different conditions for breakaway of the dislocation were obtained as illustrated schematically by the example presented in Fig. 9. The stress necessary to break the dislocation away from configuration (a) was less than that required for breakaway from (b). This is understandable since the internal stress field at the repulsive stress center in (a) cooperates with the applied stress in pulling the dislocation away from the attractive

stress centers whereas in (b) the applied and internal stresses from the attractive stress centers oppose each other. The variation of τ_s^*/G as a function of concentration of solute atoms when uncorrected for randomness is given in Fig. 10. The yield stress given by the solid lines refers to breakaway from the stronger configuration (b). The significantly lower breakaway stress for configuration (a) is also shown for the case of solute atomic strains of $\varepsilon = 0.06$. On the same graph are recorded the corresponding results, also without correction, for the randomness effect, that were obtained when the assumption of ideal flexibility was made.

We have already described how the randomness correction can be applied to the ideally flexible dislocation. It is not yet clear, however, as to what this correction should be for the more realistic model. This arises because the dislocation can no longer be visualized as being held up at highly localized points, but rather that it bends more gradually about somewhat more closely spaced internal stress fields. This makes the obstacles appear to be less efficient in resisting the motion of the dislocation. On the other hand, some randomness correction need be applied to the more realistic model now being considered. Since it is expected that the desired correction cannot differ greatly from that given by Forman and Makin[14], we suggest that their correction be tentatively adopted here also. Consequently the value of α was calculated for each point of the curves of Fig. 10 as suggested by Eq. 9. The $\tau^*/G - \sqrt{c}$ curves for the two dissimilar models following the randomness corrections are shown in Fig. 11.

V. DISCUSSION

The objective of this report was much less pertentious than the development of a complete theory for solid-solution strengthening. It centered principally on how the effect of limited dislocation flexibility might qualitatively influence the dependence of the effective yield stress at the absolute zero on the composition. Consequently it was permissible, without lack of generality, to neglect the modulus, electronic and all other interactions and to consider only the volumetric strain-energy interactions between solute atoms and dislocations that deviate at most only slightly from edge orientation. The fact that the volumetric interactions that were assumed at best approximate the interaction energy when the core of the dislocation overlaps the solute atom merely modifies the numerical results, but in no way changes that general trends that were deduced.

The results shown in Fig. 11 illustrate clearly that the more realistic dislocation model that considers limited dislocation flexibility gives trends that deviate seriously from those expected on the basis of an ideally flexible dislocation model.

Although the two models agree well for extremely dilute solutions where both models suggest that τ^*/G increases linearly $\sqrt{c}$, the more realistic model gives a less rapid increase in strength as the concentration increases to slightly higher values and finally, for yet higher concentrations, gives a reduction in the strengthening suggesting that an optimum

composition exists for maximum strengthening. This indeed is the thesis that was proposed by Mott and Nabarro on internal strain-center strengthening.

It would prove interesting if the qualitative trends deduced for the realistically flexible dislocation model could be compared with the experimental data. Despite the now extensive literature on solid solution strengthening, however, very few definitive results are available. In fact only those few data where attempts have been made to identify dislocation mechanisms are of some help in this respect. Even when such data as these are available they may not be suitable for the intended objective because many auxiliary indirect strengthening factors can and do effect the low-temperature effective stress, which is under scrutiny here. For example a number of reasonably definitive experiments[21,24] on strengthening by substitutional solid solution additions to fcc metals have shown that the dislocation intersection mechanism is largely responsible for their low-temperature mechanical behavior. The existing data suggest that alloying increases the density of dislocations and often modifies as well the stacking-fault energy. Such indirect effects, of course, in no way deny the nominal validity of the analyses given above on dislocation flexibility concepts; they merely reflect that the experiments were not conducted in a way that is appropriate to reveal the effect of strain centers on strength.

Several investigations [25, 28] on impurity interstitial solute atom strengthening do suggest that the effective stress at the absolute zero increases almost linearly with $\sqrt{c}$. In these cases, however, the concentrations of solute atoms were so low that the results could be interpreted

to agree with either the ideally flexible and realistically flexible dislocation model. None of the available experimental evidence in any way contradicts the general deductions based on the flexible dislocation model.

Recently, T. Suzuki[29] and T. Suzuki and T. Ishii[30] have approached the experimental problem in a way that can hopefully give definitive results on the yield stress needed to move dislocations past strain centers arising from solid-solution alloying. Single crystals of Cu containing various concentrations of Ni were so produced as to have the extremely low concentrations of dislocations of 10^3 to $10^5/cm^2$. Such single crystals were subjected to a stress pulses and the displacement of the dislocations were measured by etch-pitting before and after pulsing. In the pure copper crystals most dislocations exhibited no further displacement upon application of a second stress pulse of the same low magnitude as the first even when the pulse duration was increased. Whereas at 300°K the alloy crystals exhibited a similar behavior, at 77°K the number of moving dislocations increased with the pulse time suggesting a thermally activated process of overcoming barriers. Below a critical stress no multiplication of dislocations was observed; but when the crystals were pulsed near the yield stress all dislocations moved and extensive multiplication was obtained. As the initial dislocation density was decreased the measured yield strength at first decreased and finally remained constant independent of a further reduction in dislocation density. Only such stresses which seem to refer to those needed to push dislocations past solute atom strain centers were reported.

The yield stress for each alloy was determined over the temperature range from 4.2 to 300°K; the general trend of τ^*-T curves were observed to be consistent with those expected for dislocations passing localized strain centers. Pure Cu exhibited only athermal behavior over the full range of temperatures suggesting that the thermally activated dislocation-intersection mechanism, which controls the deformation of pure Cu at low temperatures when the dislocation density is 10^8 or more per cm^2, is no longer controlling at the dislocation densities investigated by T. Suzuki. For the alloys, the yield stress, τ, as well as the athermal component, τ_G, was found to depend on the concentration of solute atoms. The effective stress $\tau^*=\tau-\tau_G$, however, is a measure of the component necessary to push dislocations past the localized strain centers of solute atoms. To provide comparison with the theoretical deductions, τ^*/G, as extrapolated to the absolute zero, was plotted as a function of c in Fig. 12. These data can be compared with the theoretical curve for ε=.02 which is near the estimated value of ε for Ni in Cu.

Exact quantitative agreement between the theory presented here and the experimental results on solid-solution strengthening arising from additions of Ni to Cu could scarecely be expected. In an effort to maintain an easily tractable theory several assumptions regarding the dislocation orientation, i.e. principally edge, and the nature of the localized stresses about solute atoms were made. Numerous additional and often important factors such as modulus effects, chemical effects etc. were neglected. It is significant, however, that both theory and experiment are in nominal qualitative agreement regarding the fact that above some limiting concentration of solute atoms serious deviations from the commonly applied $\sqrt{c}$ law for strengthening are obtained. Models for solid solution strengthening must therefore be based on more realistic accounting of the effects of dislocation flexibility.

CONCLUSIONS

The strengthening of metals due to atomic strain centers cannot be properly described in terms of the cutting of simple point obstacles by ideally flexible dislocations. It is more realistic and indeed necessary to consider in greater detail the strain-energy interactions between solute atoms and dislocations having limited flexibility. The opposing internal stress along the dislocation can be considerably less when the latter is allowed to relax to its equilibrium shape.

As the spacing of solute atoms decreases the dislocation line straightens out rapidly and therefore a critical concentration exists for maximum strengthening. This result, which is in conflict with the predictions of ideally flexible dislocation models, is qualitatively confirmed by experimental data in which the effects of direct solute atom-dislocation interactions appear to have been successfully isolated.

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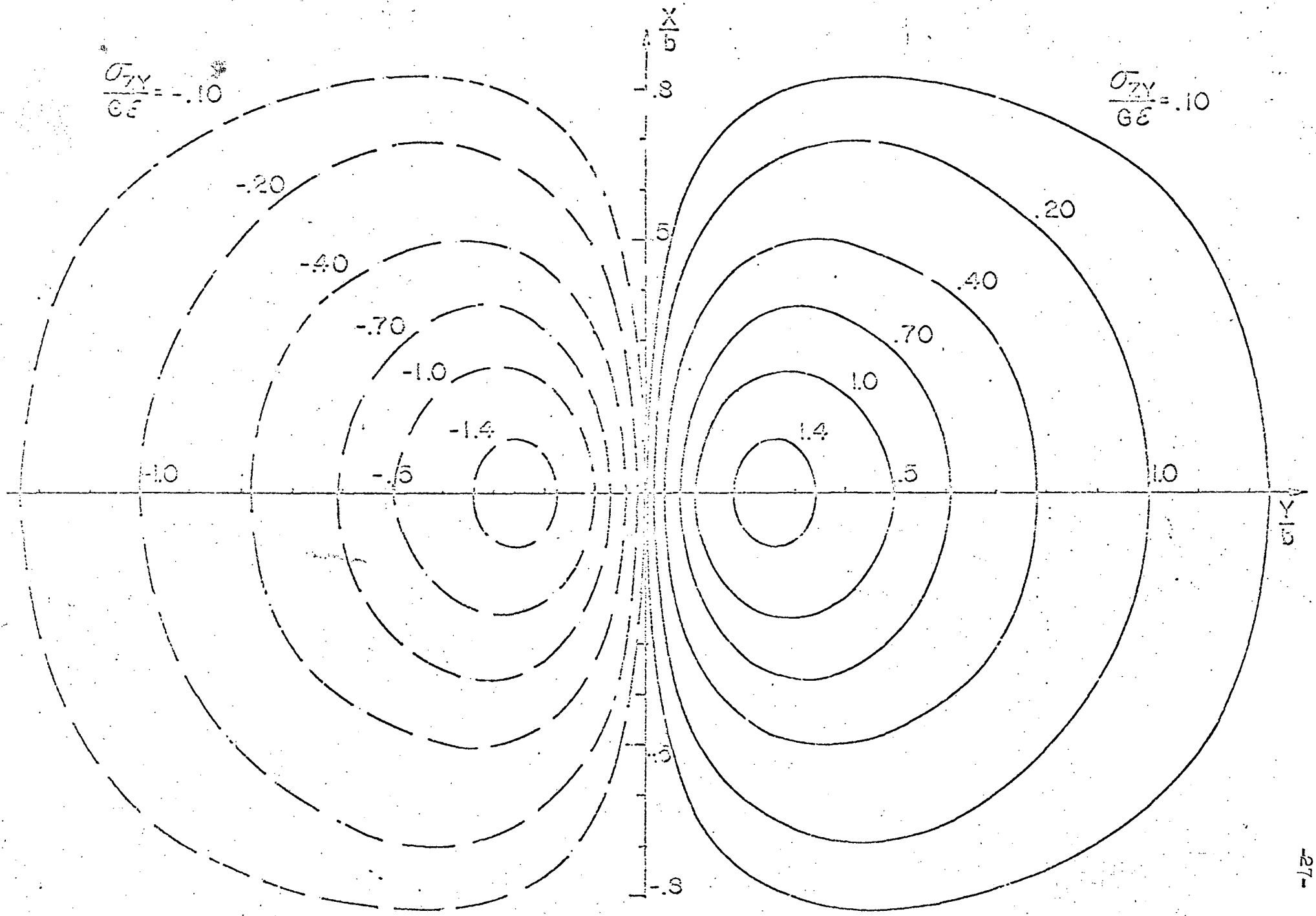

Fig. 1 Shear stresses on the slip plane due to a single solute atom.

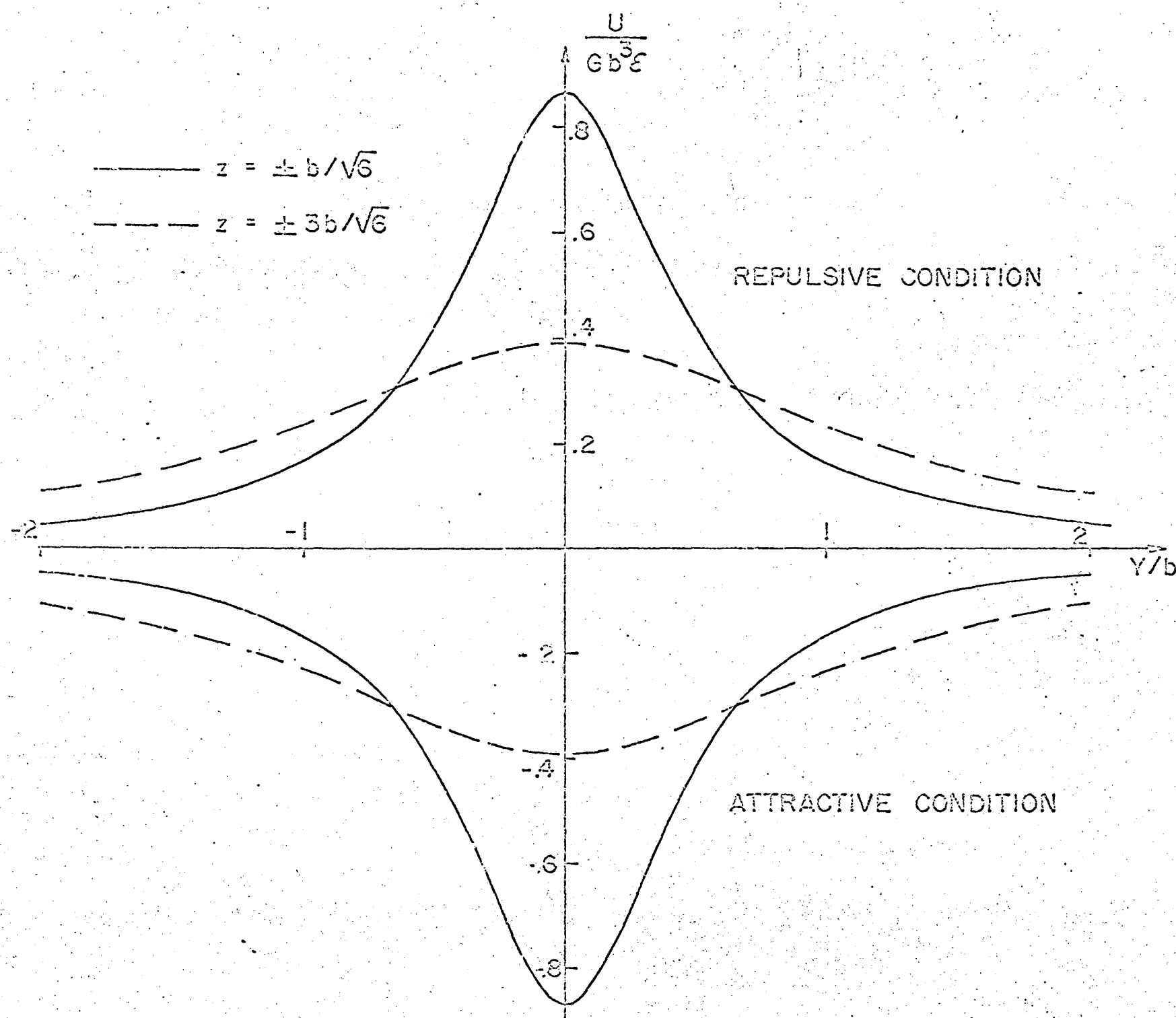

Fig. 2 Interaction energy between a long segment of an edge dislocation and a volumetric strain center.

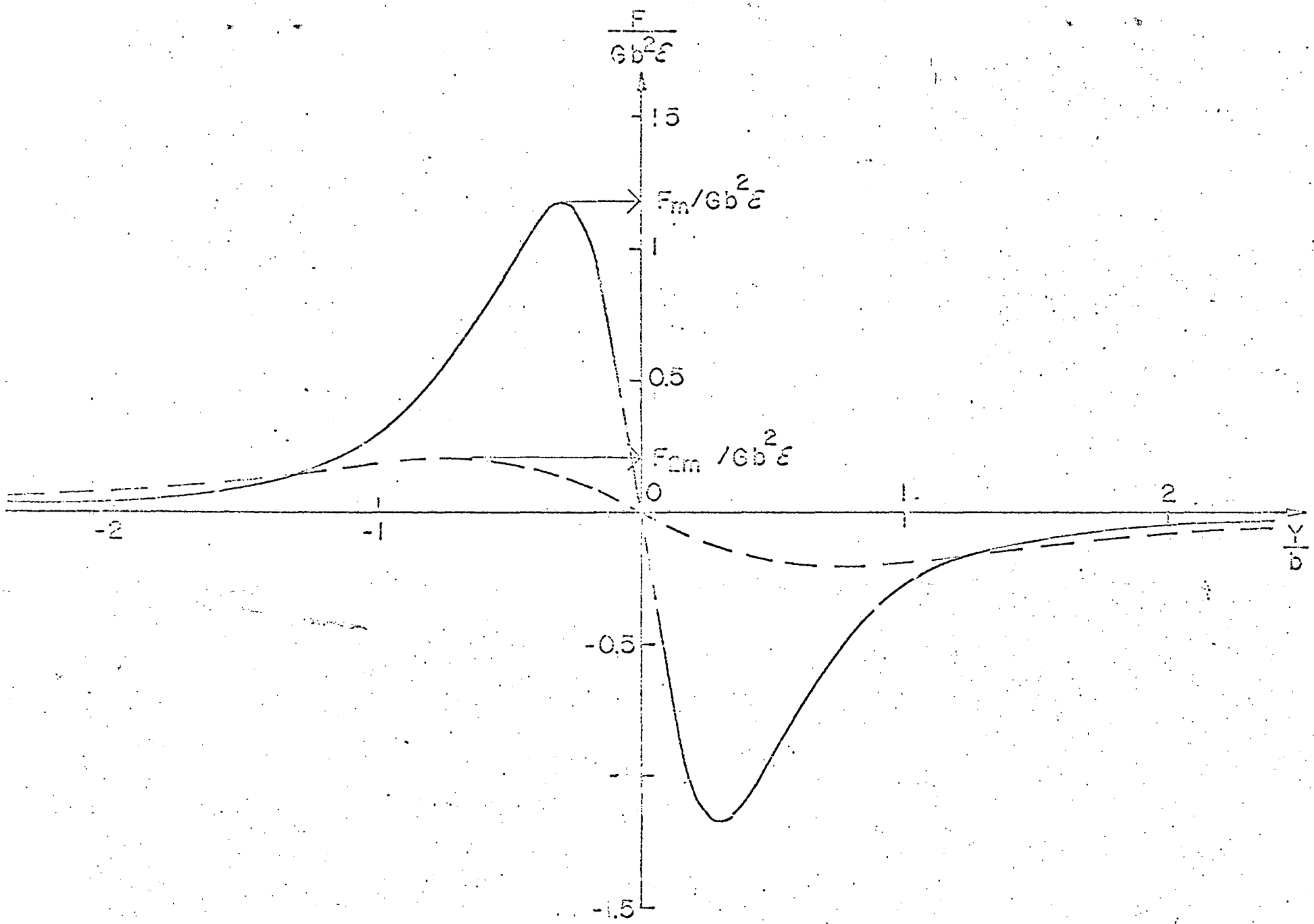

Fig. 3 Force-displacement diagram due to the elastic interaction between a long straight edge dislocation and the stress field of a substitutional solute atom.

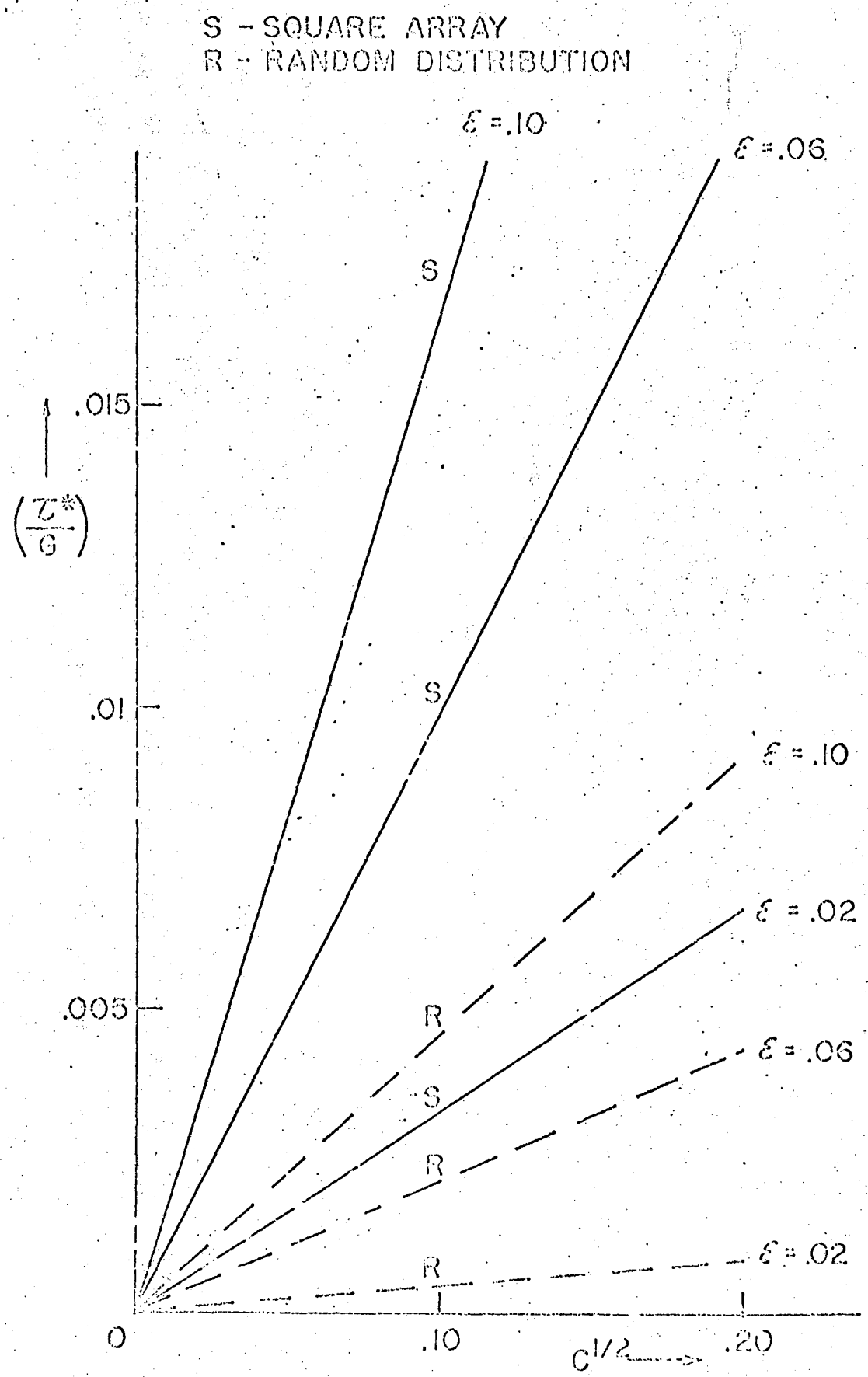

Fig. 4 Yield stress at 0°K for solute atom strengthening.

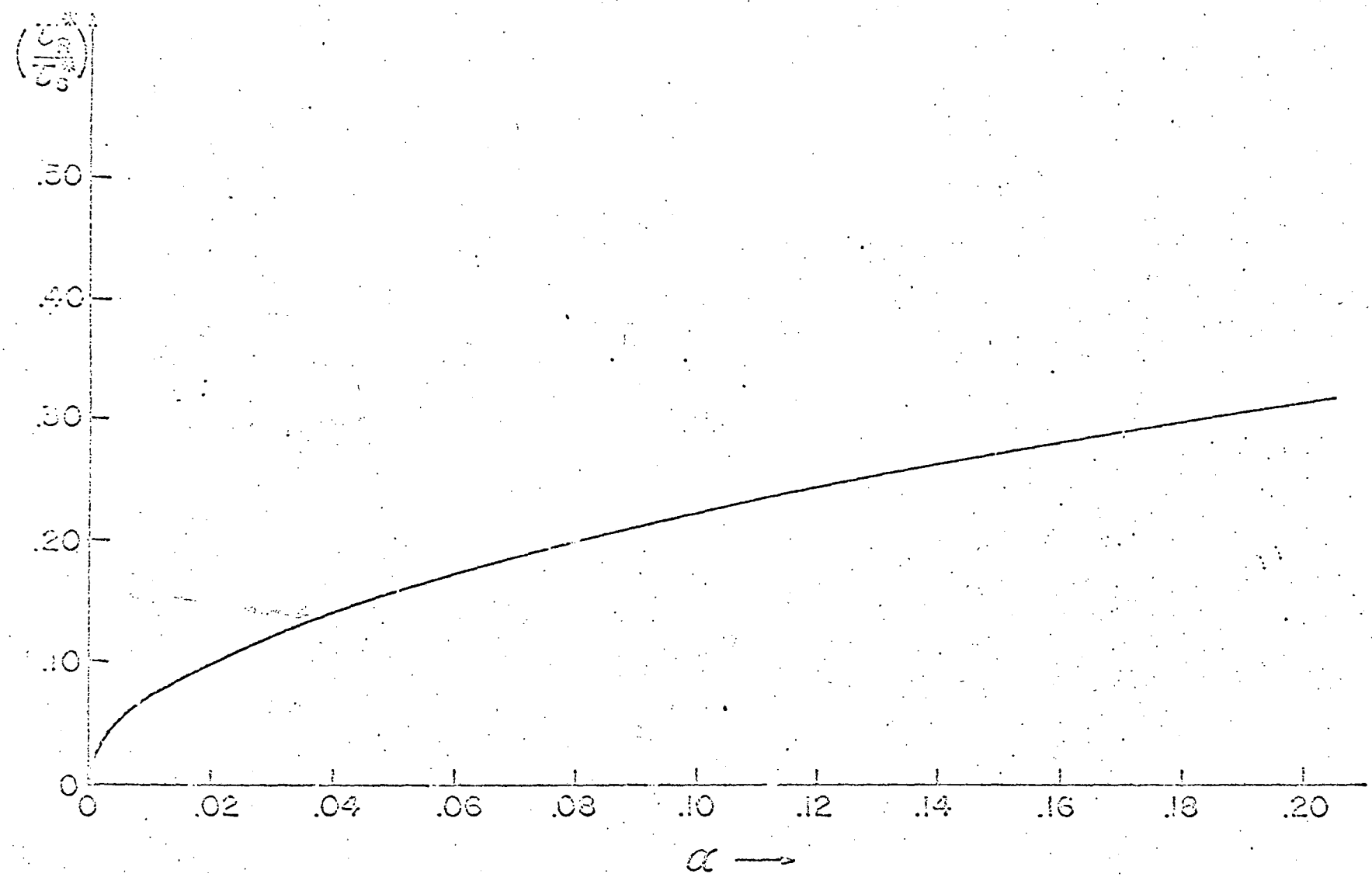

Fig. 5. The ratio of τ_r^* for a random distribution of solute atoms to τ_s^* for a square array as a function of obstacle strength α.

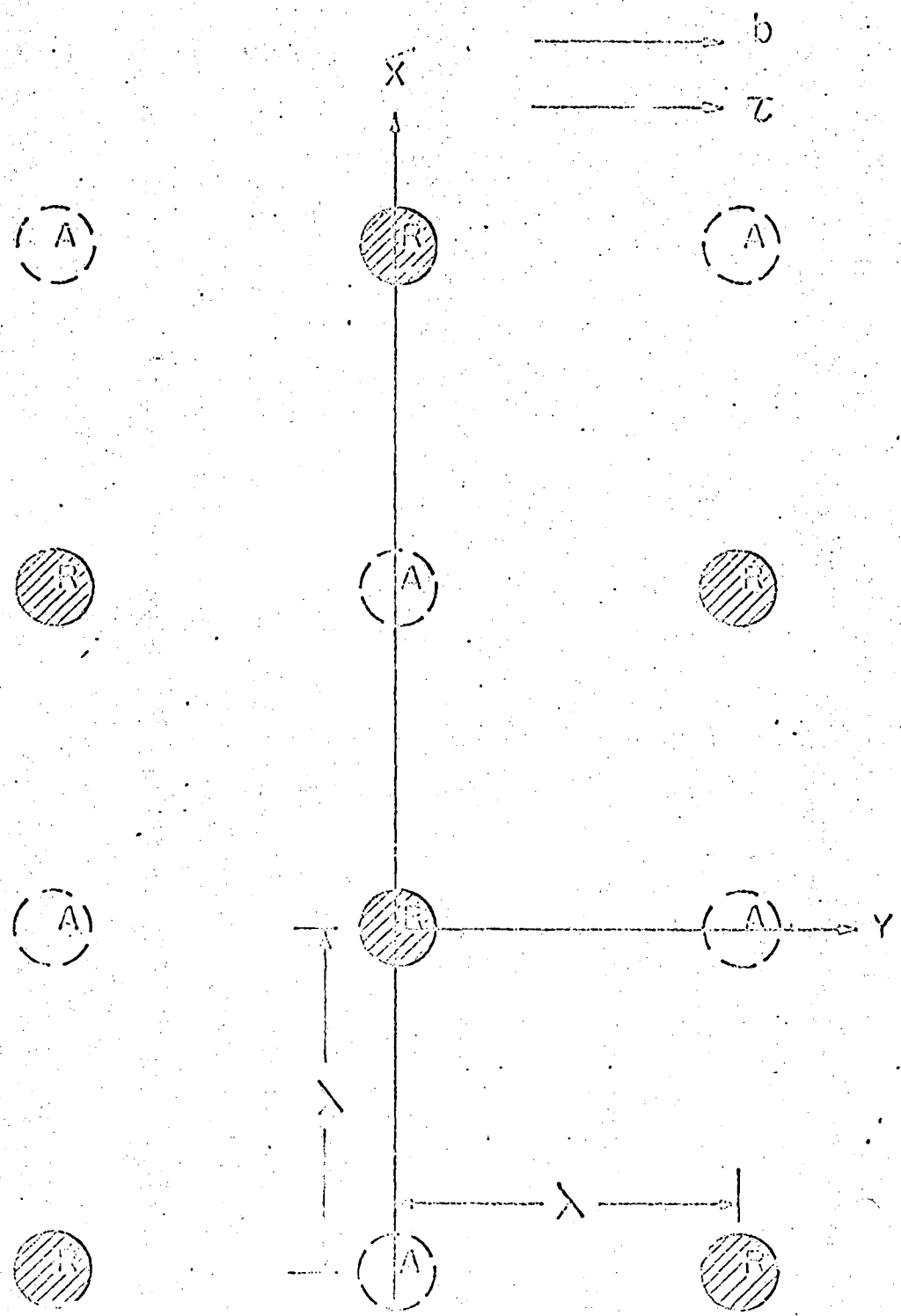

Fig. 6 A square array of attractive (A) and repulsive (R) stress centers.

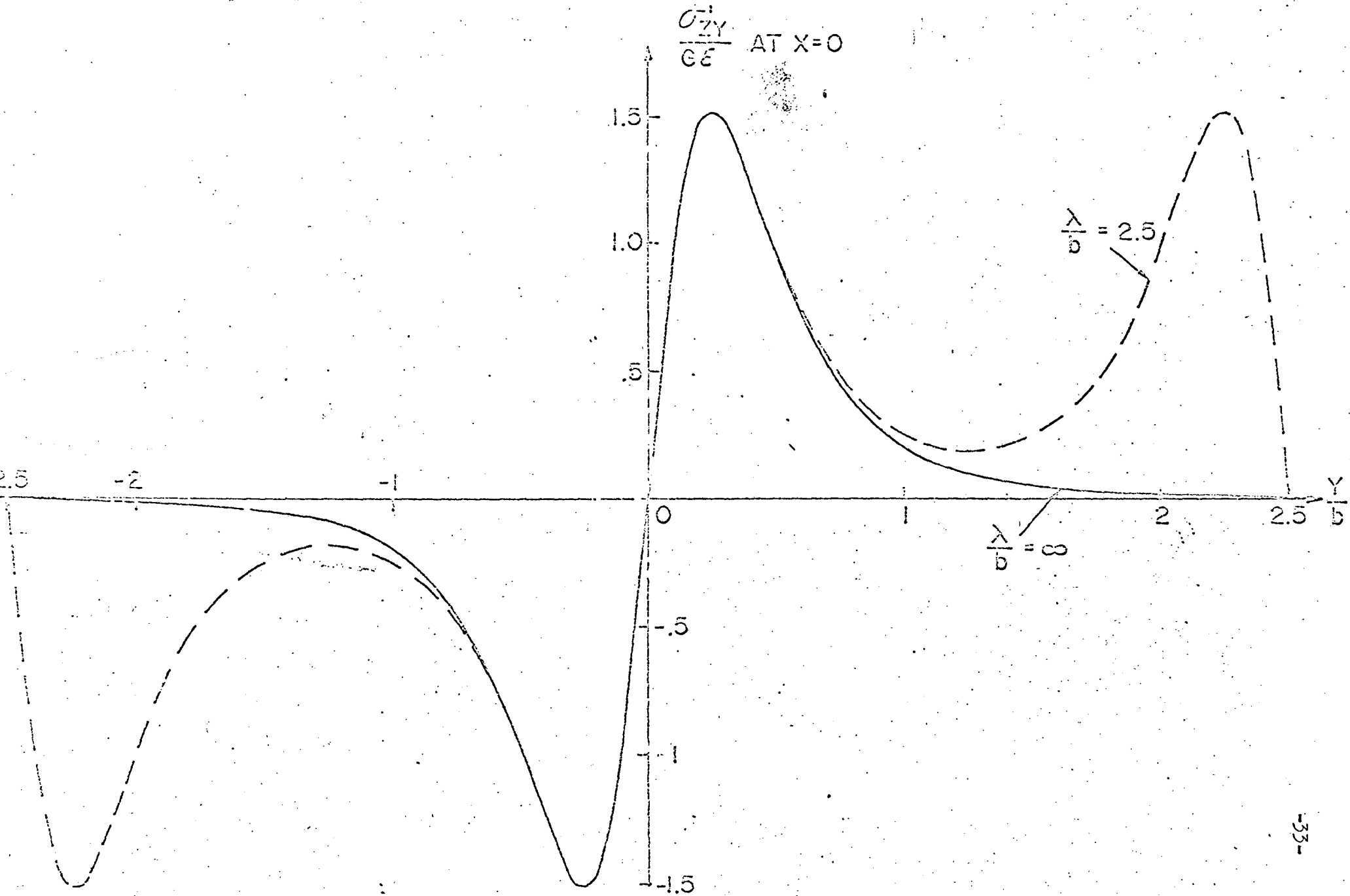

Fig. 7 Effect of neighboring atoms on the stress σ'_{zy} at x = 0 as a function of λ/b.

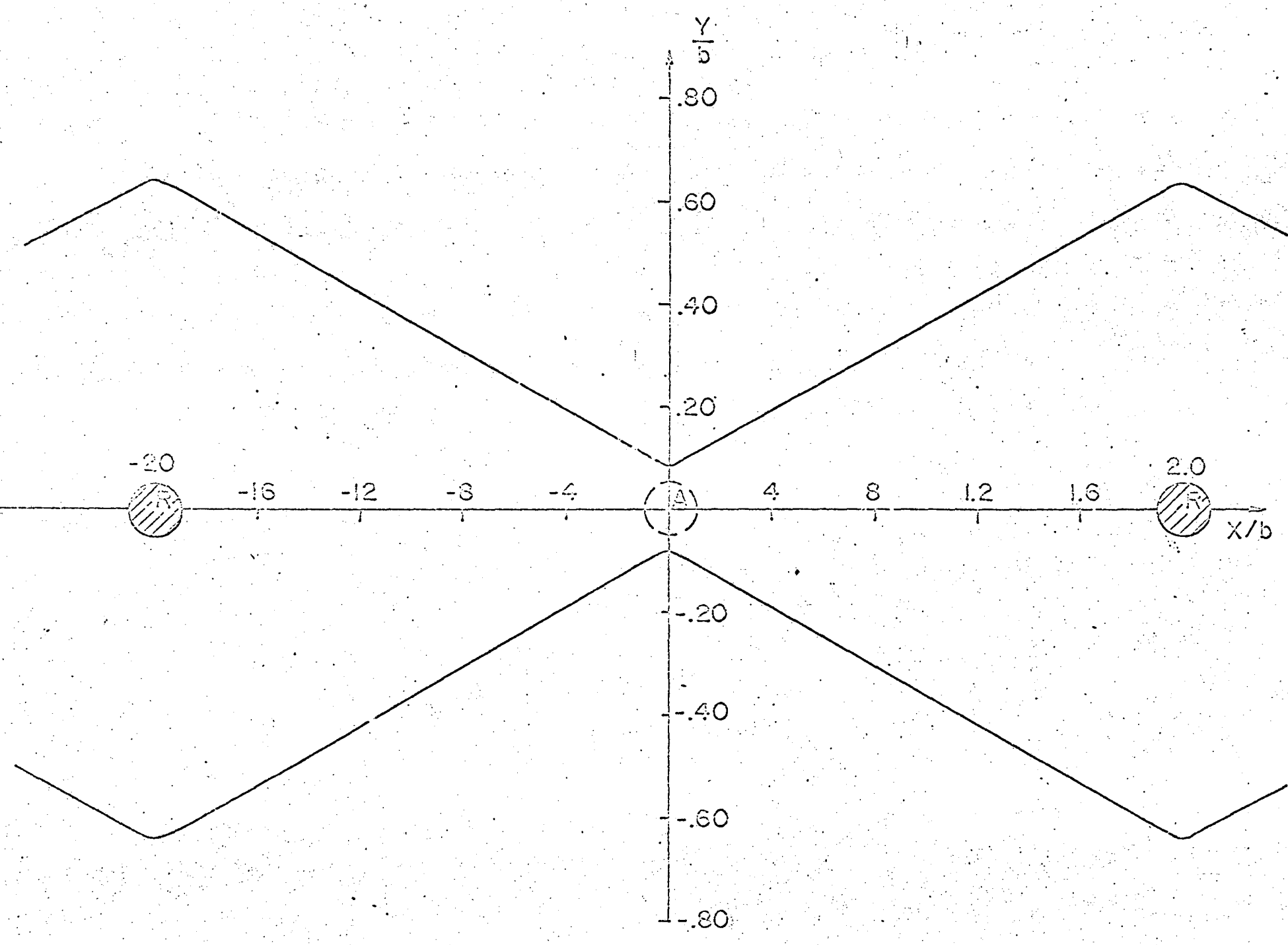

Fig. 8 Equilibrium line shape at zero applied stress (ε=.06, c=.00125).

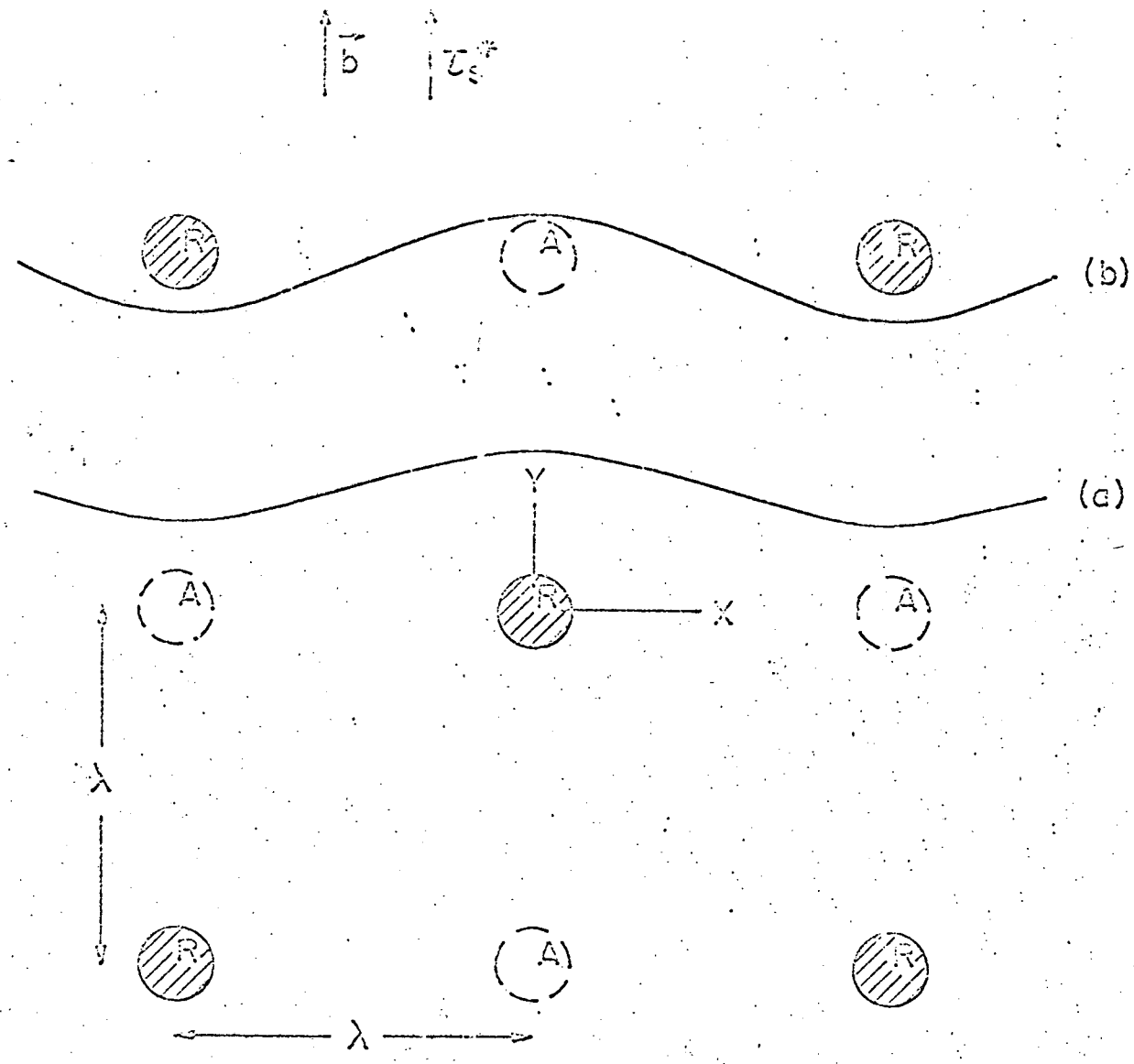

Fig. 9 Periodic dislocation line shapes.

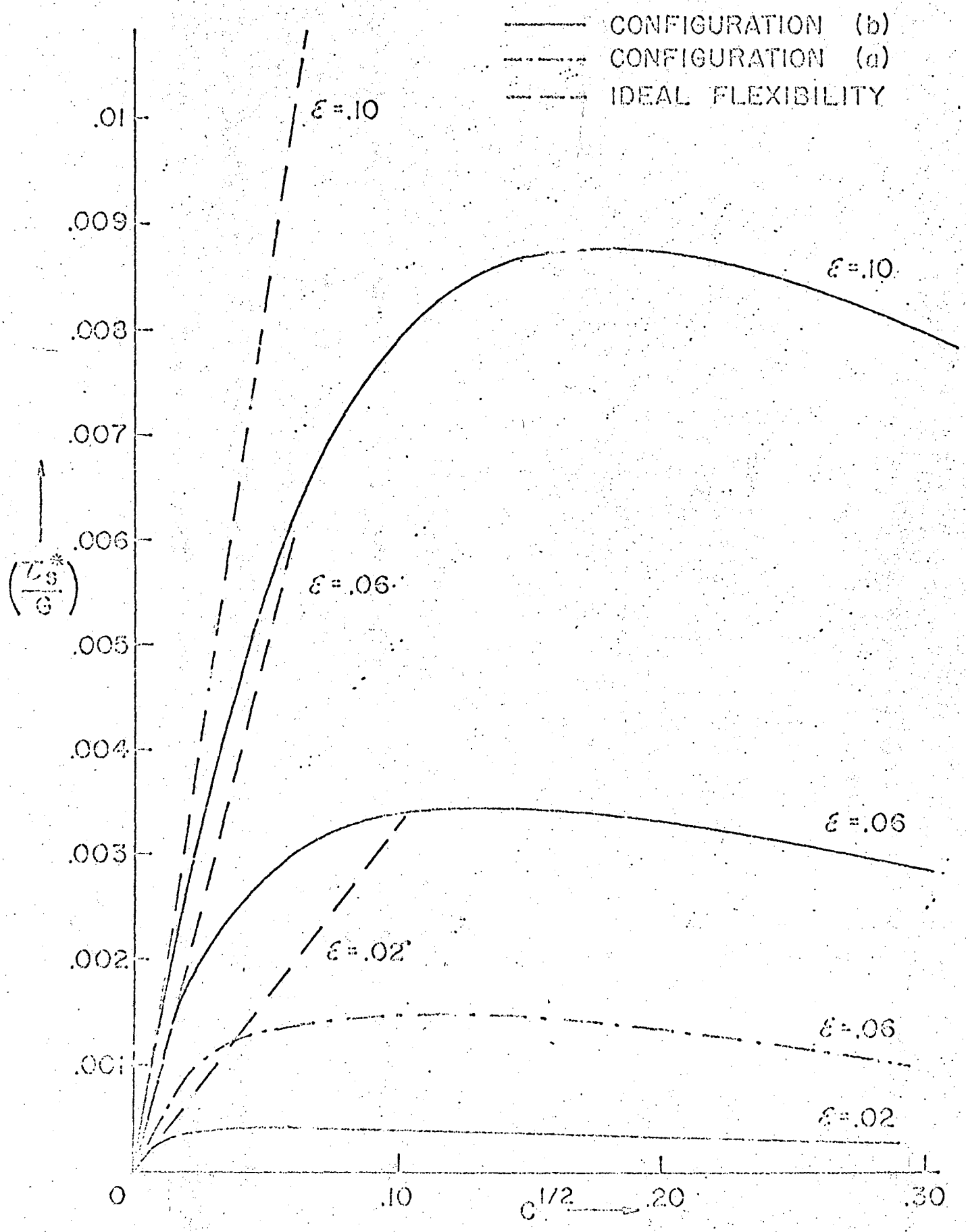

Fig. 10 Effect of concentration of solute atoms on the yield stress.

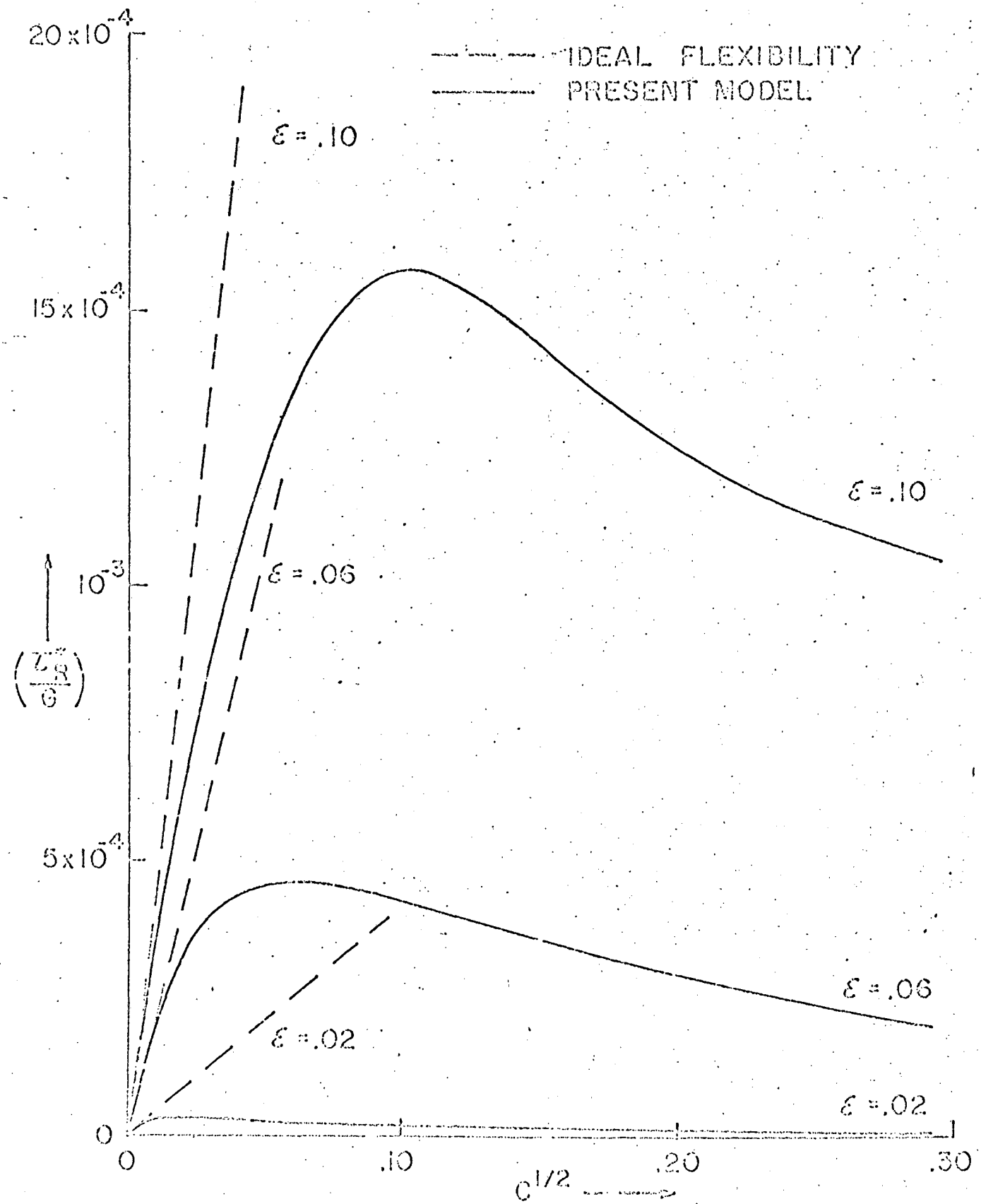

Fig. 11 Yield stress at 0°K for solute atom strengthening with randomness correction.

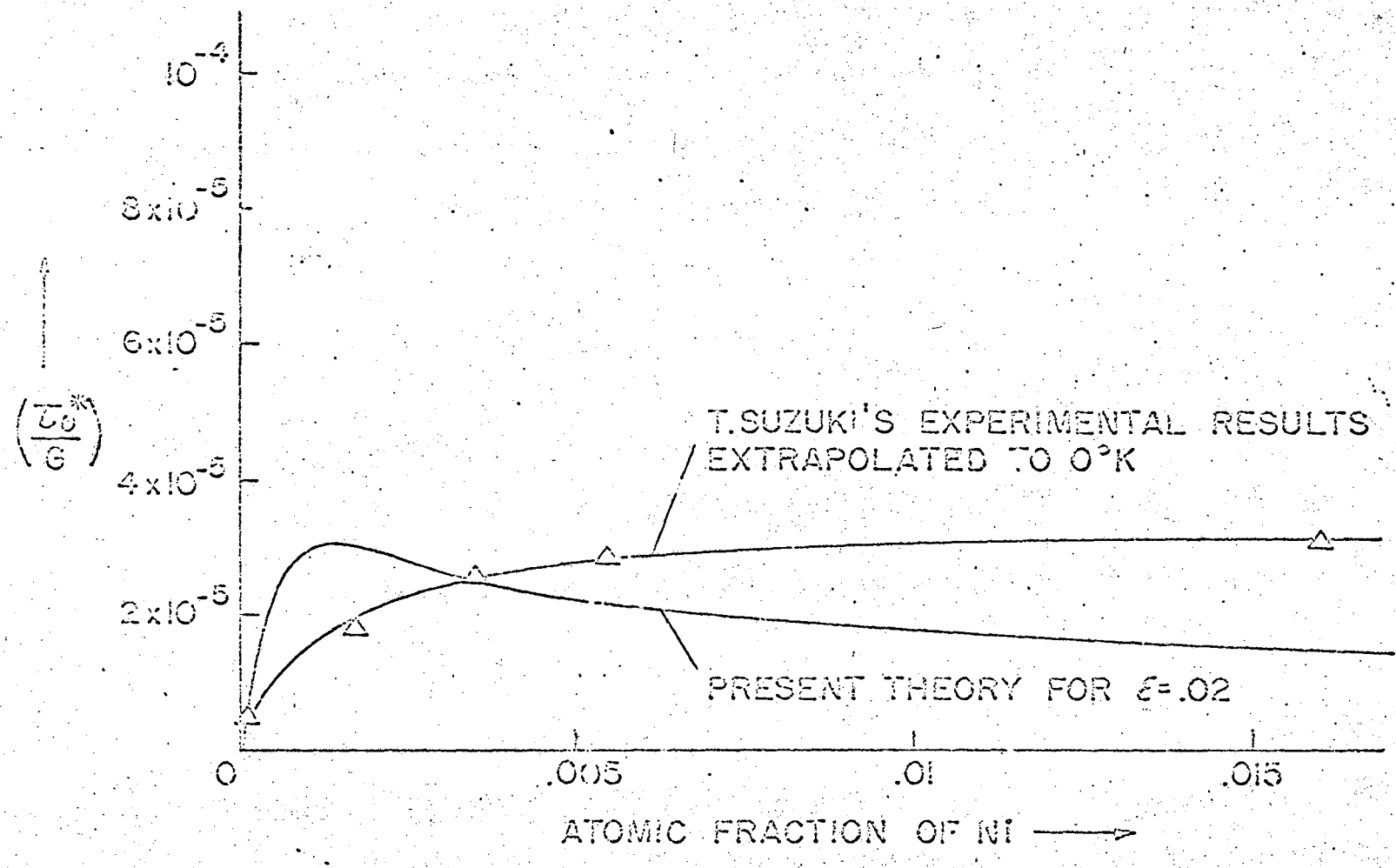

Fig. 12 Comparison with experimental results.

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