Submitted to Transactions of the
Metallurgical Society of the AIME

UCRL-18405 Rev.
Preprint

cy. 2

THE ROLE OF DISLOCATION FLEXIBILITY IN THE STRENGTHENING OF METALS

Tibor Stefansky and John E. Dorn

RECEIVED
LAWRENCE
RADIATION LABORATORY
APR 28 1969
LIBRARY AND
DOCUMENTS SECTION

April 1969

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

UCRL

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

UCRL-18405 Rev.

cy. 2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

THE ROLE OF DISLOCATION FLEXIBILITY IN THE STRENGTHENING OF METALS

T. Stefansky and J. E. Dorn

Inorganic Materials Research Division, Lawrence Radiation Laboratory,
and Department of Materials Science and Engineering,
College of Engineering,
University of California, Berkeley, California

ABSTRACT

The stress required to drive a single dislocation through a fixed array of isotropic strain centers due to solute atoms is calculated using linear elasticity and numerical methods. The Mott-Nabarro "Dislocation Flexibility" concept is extended by allowing the dislocation to relax to its equilibrium shape in the stress fields of such atoms. The increase in the critical resolved shear stress varies linearly with the square root of the strain-center concentration only in the very dilute range and above some critical concentration a reduction in strengthening is observed.

I. INTRODUCTION

Mott and Nabarro[1-5] were first to illustrate the importance of the flexibility of dislocations in accounting for the strengthening that metals undergo due to the presence of internal-strain centers. Such strain-centers might arise from a number of causes such as radiation damage, the presence of semi-coherent and coherent precipitates and clusters, as well as individually dispersed interstitial and misfit substitutional solute atoms. Each strain-center induces positive and negative shear stress fields on the slip planes of the surrounding alloy matrix. In order to effect plastic deformation, therefore, the alloy must be subjected to sufficiently high applied shear stresses to push dislocations past all such internal stress fields. If dislocations were inflexible and moved as rigid lines, they would be equally pushed and pulled by internal stress fields. At this extreme the strengthening would be vanishingly small. Dislocations, however are not rigid; they are flexible and extensible. They wiggle between positive and negative strain-centers, thus increasing their line energy at the expense of their interaction energy with strain-centers. Cottrell[6] and Friedel[7,8] have independently presented lucid reviews of the significance of the Mott-Nabarro dislocation-flexibility concept to the strain-center strengthening of alloys. Their analyses however were only semi-quantitative. There is evidence in the recent literature of renewed interest in a more rigorous analysis of flexibility. Thus, Gleiter[9] and Gerold[10] have shown that the calculated strain-energy interaction between a dislocation and the stress fields of coherent precipitates can be seriously in error if the flexibility of the dislocation is not taken into account.

This report will be concerned with a more detailed analysis of the problem based on the following approximations:

(1) Although many types of strain-centers exist, the following analysis will be limited to the case of solute-atom strain-centers in f.c.c. substitutional alloys.

(2) To facilitate the analysis by avoiding statistical considerations, the strain-centers are assumed to be in regular arrays on the slip plane. Although the quantitative results are somewhat dependent on the type of array that is assumed, the qualitative trends are similar for various arrays that were studied.

(3) An adjustment of the results from those for square arrays to those for the more realistic case of a random distribution of strain-centers is accomplished by adopting the interesting statistical theory first introduced by Kochs[11,12], elaborated on by Dorn, Guyot and Stefansky[13] and analyzed by Foreman and Makin[14] in their computerized experiments.

(4) For simplicity the dislocation lines are taken to wiggle about their edge orientation and no correction is applied to their line energy for deviations from edge configurations or for the elastic interaction between segments of the now curved dislocation.

It will be shown that for extremely low concentrations of atomic strain-centers the strengthening increases with the square root of their concentration. For slightly higher concentrations the strengthening rate becomes less rapid. At a critical concentration a maximum strength

is obtained and a decrease in the strengthening takes place as the concentration of solute atoms is further increased. These trends are a direct consequence of dislocation flexibility. They arise from changes in the amplitude of the waviness of the dislocation line at the yield stress with solute atom concentration. Despite the simplifying assumptions that were made, the results agree qualitatively with appropriate experimental data.

II. STRESS FIELDS ON SLIP PLANES

Internal stresses are introduced in a lattice whenever a host atom of radius r_o is replaced by another that has a different atomic radius. According to the classical theory of linear elasticity,[15] the stress field about a solute atom at the origin of a spherical coordinate system is given by

$$\sigma_{rr} = -4G\varepsilon \left(\frac{r_o}{r}\right)^3$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = 2G\varepsilon \left(\frac{r_o}{r}\right)^3 \text{ for } r \geq r_o\ (1 + \varepsilon) \qquad (1)$$

where G is the shear modulus of elasticity and $r_a = r_o\ (1 + \varepsilon)$ is the radius in situ, of the substituted atom. As shown by Eshelby,[16] this approximation assumes that the bulk modulus of elasticity of the substituted atom is identical with that of the host species. Nabarro[17] suggested that the strain ε can be deduced experimentally from

$$\varepsilon = a^{-1}\ (da/dc) \qquad (2)$$

where a is the lattice parameter and c is the atomic fraction of the solute species.

We will consider here only the interaction of the stresses on a slip plane in f.c.c. metals with a dislocation that is substantially in edge orientation. We let X-Y be a slip plane at a height z above the solute atom center, where Y coincides with the Burger's vector of the dislocation that lies parallel to the X axis. In this event the

dislocation motion will depend only on the shear stresses σ_{zy}. Upon simple tensor transformation of Eq. (1)

$$\sigma_{zy} = \frac{6G\left(\frac{b}{2}\right)^3 \varepsilon zy}{\left(x^2 + y^2 + z^2\right)^{5/2}}, \quad x^2 + y^2 + z^2 \geq \left(\frac{b}{2}\right)^2 \tag{3}$$

where for convenience r_o has been replaced by its substantial equivalent of b/2. Unfortunately Eq. (3) becomes somewhat inaccurate in the region of greatest interest here, namely when $x^2 + y^2 + z^2$ is small. The fact that the deduced shear stress σ_{zy} applies only for $x^2 + y^2 + z^2$ greater than an atomic radius squared is perhaps not as serious as the fact that when the core of the dislocation overlaps the volume of the substituted atom, the linear theory of elasticity becomes seriously in error. From a physical viewpoint it is easily judged that, where overlap takes place, the stress fields will be somewhat less than those suggested by Eq. (3). This issue, however, is not critical for demonstrating the effects of dislocation flexibility on solid-solution strengthening and consequently we suggest that the right-hand side of Eq. (3) be multiplied by the qualitatively justifiable compensating factor

$$1-e^{-\beta\frac{(x^2 + y^2 + z^2)}{b^2}}$$

Whereas the constant β might be estimated from experimental data on binding energies, in principle it will prove satisfactory for the present to arbitrarily select $\beta = 3$. This adjustment results in a

decrease in the magnitude of the stress fields when overlap takes place and yet reproduces faithfully the results of Eq. (3), where it is yet reasonably valid. Accordingly we suggest that

$$\sigma_{zy} = \frac{3}{4} \frac{Gb^3 \varepsilon zy \left[1 - e^{\frac{-3}{b^2}(x^2+y^2+z^2)}\right]}{\left(x^2+y^2+z^2\right)^{5/2}} \tag{4}$$

where Eq. (4) refers to the stresses on a slip plane a distance z above or below the solute atom.

For a solute atom at (x,y) = (o,o) on the first atomic plane ($z = b/\sqrt{6}$) above the slip plane, the shear stress distribution on the slip plane is that shown in Fig. 1. A positive edge dislocation lying parallel to the x axis, would be repelled by the solute atom except for the somewhat trivial condition when the dislocation is at y = 0. It is significant that the important region, where the absolute value of the stresses are great, is highly localized in the near vicinity say over about an area of $10b^2$ around the atom center. For a solute atom lying on the first atomic plane below the slip plane ($z = -b/\sqrt{6}$) the signs given in Fig. 1 are reversed. In this case a straight edge dislocation would be so attracted as to lie along the X-axis at y = 0. The interaction energy between an infinitely long straight edge dislocation and the stress field of a solute atom is given by

$$U\{y\} = \int_{-\infty}^{y} \int_{-\infty}^{\infty} \sigma_{zy}\, b\,dx\,dy \tag{5}$$

for the case where the datum of energy equals zero is taken at $y = -\infty$. Introducing the expression given in Eq. (4) for σ_{zy} and integrating gives the results shown in Fig. 2. As shown the highest interaction energies arise from solute atoms on planes nearest the slip plane, i.e. $z = \pm b/\sqrt{6}$. It was noted that the interaction integral of Eq. (5) gave almost the same results when it was integrated over x from -5b to +5b in lieu of from -10b to +10b. This emphasizes the highly localized nature of dislocation solute atom interactions.

A force, F, acting on a long straight edge dislocation will hold it in equilibrium with a solute atom when F + Fi = 0 where Fi is the force arising from the internal interaction with a solute atom stress field. Consequently

$$F = -\int_{-\infty}^{\infty} \sigma_{zy} b dx \qquad (6)$$

This force-displacement diagram is shown in Fig. 3 for the case where z is positive. For solute atoms lying below the slip plane the curve of Fig. 3 is merely reflected across the y = 0 axis. According to this somewhat naive approach then, the maximum force, Fm, to move a dislocation past the stress field of a solute atom is the same regardless of whether the stress field is attractive or repulsive. The maximum force, F_{2m}, to move a dislocation past a solute atom on the second nearest atomic plane to the slip plane is given by $F_{2m} = \frac{0.20}{1.17} F_m$ and is therefore a small fraction of the force needed to move dislocations past atoms on the nearest planes.

III. YIELD STRENGTHS UNDER CONDITIONS LEADING TO HIGH DISLOCATION FLEXIBILITY

In this section we estimate the strengthening at the absolute zero due to substitutional alloying assuming that the stress fields due to solute atoms can be replaced by an "equivalent" interaction force localized at the center of the stress field. The concepts to be invoked here are substantially those previously presented by Fleischer[18]. The details, however, have been adjusted, as given in the preceeding section, to permit direct comparisons of deductions made here with those to be made later for the more realistic model that considers dislocation flexibility in the stress fields due to the solute atoms.

Solute atoms will first be viewed as forming a square array of strain centers. Neglecting all planes more distant than the two atomic planes nearest the slip plane, demands that the edge of the array, λ, is given in terms of the atomic fraction of solute that is present, c, by

$$2c = b^2/\lambda^2 \tag{7}$$

The results for the random case will be deduced by applying the Foreman and Makin[14] correction to the square array.

Each dislocation segment λ, can be considered to be pushed against the localized resisting force arising from the stress fields about the solute atoms. Thus, the yield stress, τ_s^* at the absolute zero is given by

$$\tau_s^* \lambda b = \tau_s^* \frac{b^2}{\sqrt{2c}} = F_m = 1.17\ Gb^2\varepsilon \tag{8}$$

where the subscript s refers to the fact that a square array of strain centers was assumed. This analysis, however, must be restricted to values of λ much greater than about 5b where the stress fields of adjacent solute atoms do not overlap. The deduced variations of τ_s^*/G as a function of $\sqrt{c}$ for several values of ε are shown by curves marked s in Fig. 4.

To obtain the yield stress for cases where the solute atoms are distributed more or less at random, we apply the data of Foreman and Makin[14] as given in Fig. 5. The strength of the stress field is defined by in terms of the edge dislocation line energy of about $3Gb^2/4$ according to

$$\tau_s^* \lambda b = 3\alpha\, Gb^2/4 \tag{9}$$

Comparison with Eq. (8) reveals that for substitutional solid solution alloying $\alpha \simeq 1.56\varepsilon$. Applying the randomness correction to the data gives the broken curves of Fig. 4 marked R. These calculations suggest that the yield stress at the absolute zero increases linearly with the $\sqrt{c}$.

Up to the present the effects of atoms on second nearest atomic planes to the slip plane have been neglected. We now demonstrate that in large measure, so long as the atoms do not cluster, this neglect is permissible. Considering a square array of atoms on the four nearest planes, suggest that $4c = b^2/\lambda_4^2$. The force needed to cause the dislocations to surmount the weaker stress fields of solute atoms on the second nearest atomic planes to the slip plane is given by

$$\tau_{s4}^* \lambda_4\, b = \tau_{s4}^* \frac{b^2}{2\sqrt{c}} = 0.20\, Gb^2\varepsilon$$

whence

$$\tau_{s4}/\tau_s = \frac{2 \times (.20)}{1.17\sqrt{2}} \simeq 1/4 \tag{10}$$

This ratio is further decreased when the randomness correction is applied. Thus, as originally suggested by Fleischer[18], the stress fields of misfit solute atoms residing on the second nearest planes to the slip plane are so small that the dislocations will be pushed past them at less than 1/4 of the stress needed to cause them to pass the stress fields of atoms on the planes immediately adjacent to the slip planes. It is necessary, however, to point out that although the behavior of solid solutions at the absolute zero of temperature is not sensitive to stress fields of more distant atoms than those in the immediate vicinity of the slip plane, the yield stress at higher temperatures in the thermally activated range will depend on the stress fields of such more distant solute atoms.

IV. EFFECTS OF LIMITED DISLOCATION FLEXIBILITY

The model for solid-solution strengthening that was presented in Section III was based on the assumption that the stress fields due to solute atoms could be represented by "equivalent" localized forces. In this section we present a more realistic model for solid-solution strengthening in which the dislocations are permitted to bow flexibly around the stress fields due to the solute atoms. It will be demonstrated that the stress fields can be replaced by "equivalent" localized forces only for extremely dilute solutions. As the concentration increases, however, this approximation leads to serious errors. Whereas the assumption of an "equivalent" localized force suggests that the strength of alloys continues to increase with $\sqrt{c}$, the more realistic model, suggests a completely different trend giving a maximum strengthening at a critical solute-atom concentration followed by a decrease in the strengthening with yet additional increases in concentration.

In order to permit an unbiased evaluation of the role of dislocation flexibility in solute atom stress fields, we hold all remaining assumptions the same as those adopted for the localized force approximation that was discussed in Section III: The stress centers arising from the solute atoms adjacent to the slip plane are placed on a square array as shown in Fig. 6. The value of λ, given by Eq. (7), also remains the same as that previously employed. Since A and R of Fig. 6 refer to equal attractive and repulsive strain centers it is immediately apparent that a completely rigid edge dislocation could be moved through the stress fields of the solute atoms by an infinitely low applied stress.

As shown in Fig. 6 the origin of the X-Y slip plane was selected to be at a repulsive stress center. Consequently the significant stress at point (x,y) on the slip plane due to all solute atoms on the next nearest atomic planes is simply

$$\sigma'_{zy} = \frac{3}{4} Gb^3\varepsilon \sum_i \frac{(y-yi)\, z_i \left\{ 1-e^{\frac{-3}{b^2}\left[\left(x-x_i\right)^2 + \left(y-y_i\right)^2 + z_i^{\,2}\right]} \right\}}{\left\{\left(x-x_i\right)^2 + \left(y-y_i\right)^2 + z_i^{\,2}\right\}^{5/2}} \tag{11}$$

where i refers to the ith stress center, and $z_i = \pm b/\sqrt{6}$. The stress fields about solute atoms are so localized that only a few atoms in the range $-6\lambda \leq x_i \leq 6\lambda$ and $-6\lambda \leq y_i \leq 6\lambda$ need be considered in the summation of Eq. (11). It might be argued with considerable validity that the stress fields arising from near atoms on the second nearest atomic planes produce a much greater modification of the shear stress than the solute atoms on the nearest atomic planes that are more distant from the origin of the selected coordinate system. Such stresses, however, were not included here since we wished to preserve the basis of comparison with the results of Section III where the effect of solute atoms on second nearest planes was neglected. The small differences in the local stresses near solute atoms due to neighboring solute atoms is shown in Fig. 7 where $\sigma'_{zy}/G\varepsilon$ is plotted as a function of y/b for the cut at x = 0. Since, as shown in Fig. 7, only small changes occur in the local stress field as a function of concentration of solute atoms, any difference between deductions on solute atom strengthening obtained here and that presented in Section III must be ascribed almost exclusively to the effect

of dislocation flexibility in the stress fields of the solute atoms.

It is now our objective to ascertain the equilibrium shape of an edge dislocation having a Burger's vector b where the alloy is subjected to a resolved shear stress τ_s^*. Obviously the symmetry of internal stresses σ'_{zy} demands that, for stable configurations, the dislocation line $y = y\{x\}$ will be periodic in x with a period of 2λ. Thus in general, the energy of a dislocation line will (vide Dorn and Rajnak[19]) be

$$U = U_o + \int_{-\lambda}^{\lambda} \left\{ \Gamma \left[\sqrt{1 + \frac{dy}{dx}}^{\,2} - 1 \right] - \int_0^y \sigma'_{zy} b dy - \tau_s^* b y \right\} dx \tag{12}$$

where U_o is the energy of a straight dislocation segment of length 2λ lying parallel to the x-axis at $y = 0$ and $\Gamma = 3/4Gb^2$ is the line energy per unit length. The first term in the integrand gives the increase in the line energy of the displaced dislocation and the second and third terms provide the work done by the internal and applied stresses.

Although several simplifying assumptions were made in arriving at Eq. (12), they are known not to be too critical. The changes in line energy with dislocation slope as it veers from pure edge to partly screw orientation can be shown to be small in this example. The interaction between segments of the now curved dislocation line was neglected. Recent computer analyses by Foreman[20], however, have shown that such second order effects are small, particularly in the case of weak obstacles such as are being considered here.

Whereas Eq. (12) applies to any shape of dislocation $y = y\{x\}$, the equilibrium shape, of interest here, can be obtained by applying the

calculus of variations to minimize U. Euler's equation, thus obtained, gives

$$\frac{d}{dx}\left(\frac{\partial U}{\partial p}\right) - \frac{\partial U}{\partial y} = 0 \tag{13}$$

where p = dy/dx. Consequently the differential equation for the equilibrium line is

$$\frac{d^2y/dx^2}{\{1 + (dy/dx)^2\}^{3/2}} = -\frac{b}{3/4Gb^2}\left(\tau_s^* + \sigma'_{zy}\right) \tag{14}$$

For convenience of computation Eq. (14) was placed in dimensionless form by letting $x' = x/\lambda$ and $y' = y/\lambda$. Upon introducing Eq. (7), the required relationship is given by

$$\frac{d^2y'/dx'^2}{\left\{1 + (dy'/dx')^2\right\}^{3/2}} = -\frac{2\sqrt{2}}{3\sqrt{c}}\frac{\tau_s}{G} \tag{15}$$

$$-2\varepsilon c\sum_i \frac{(y'-y'_i)\, z'_i\left\{1 - e^{-\frac{3}{2c}\left[(x'-x'_i)^2 + (y'-y'_i)^2 + z'^{\,2}_i\right]}\right\}}{\left\{(x'-x'_i)^2 + (y'-y'_i)^2 + z'^{\,2}_i\right\}^{5/2}}$$

where $z'_i = \pm\sqrt{2c}/\sqrt{6}$

The integration of Eq. (15) will now be discussed in general terms. It contains three parameters $\tau_s^*/\sqrt{c}\,G$, εc, and c which in any case are fixed constants. A point $(0,y_o')$ was arbitrarily selected to start the

calculation. Due to symmetry $(dy'/dx') = 0$ at $(0, y_o')$. The value of d^2y'/dx'^2 at this point was deduced from Eq. (15). An adjacent point on the dislocation line at $x_o' + \Delta x'$ was then determined from

$$(y')_{x_o' + \Delta x'} = y_o' + \left(\frac{dy'}{dx'}\right)_{x_o'} \Delta x' + \frac{1}{2}\left(\frac{d^2y'}{dx'^2}\right)_{x_o'} (\Delta x')^2 \tag{16}$$

and

$$\left(\frac{dy'}{dx'}\right)_{x_o' + \Delta x'} = \left(\frac{dy'}{dx'}\right)_{x_o'} + \left(\frac{d^2y'}{dx'^2}\right)_{x_o'} \Delta x' \tag{17}$$

where $\Delta x'$ was selected to be the small interval 0.001. This procedure was continued to the point $x' = 0.003$ following which the calculation was continued to $x' = \lambda'$ by the Adams[21] technique. To satisfy the symmetry conditions, an acceptable solution must result in $(dy'/dx') = 0$ at $x' = 1$ as well as $x' = 0$. Consequently a series of values of y_o' at $x' = 1$ were employed until the desired curve $y' = y'(x')$ having zero slopes at $x' = 1$ and nowhere else was obtained. A check using Eq. (12) illustrated that this was the minimum energy curve for the selected conditions.

In order to obtain the yield stress at the absolute zero, the parameter $\tau_s^*/\sqrt{c}\,G$ was increased to a series of new values, retaining εc and c constant. As the yield stress was approached, the line shape changed sensitively to the values of $\tau_s^*/\sqrt{c}\,G$. If the selected value of $\tau_s^*/\sqrt{c}\,G$, however, exceeded the yield strength, dy'/dx' differed from zero at $x' = 1$ for all possible values of y_o'. In this way the yield strength could be approximated as closely as desired. An attempt was made to

carry out the calculations on yield strength to an accuracy of about ±1%.

An example of the shape of the dislocation line at zero stress is given in Fig. 8. For all cases examined, only two equilibrium configurations were obtained. The equilibrium dislocation line never zig-zagged away from attractive stress centers along $y' = 0$ and toward attractive centers near $y' = 1$. Two somewhat different conditions for breakaway of the dislocation were obtained as illustrated schematically by the example presented in Fig. 9. The stress necessary to break the dislocation away from configuration (a) was less than that required for breakaway from (b). This is understandable since the internal stress field at the repulsive stress center in (a) cooperates with the applied stress in pulling the dislocation away from the attractive stress centers whereas in (b) the applied and internal stresses from the attractive stress centers oppose each other. The variation of τ_s^*/G as a function of concentration of solute atoms when uncorrected for randomness is given in Fig. 10. The yield stress given by the solid lines refers to breakaway stress for configuration (a) is also shown for the case of solute atomic strains of $\varepsilon = 0.06$. On the same graph are recorded the corresponding results, also without correction for the randomness effect, that were obtained when the assumption "equivalent" localized forces was made.

We have already described how the randomness correction can be applied to the "equivalent" localized force model. It is not yet clear, however, as to what this correction should be for the more realistic model. This arises because the dislocation can no longer be visualized as being held up at highly localized points. It bends more gradually about the somewhat more closely spaced internal stress fields at higher

concentrations. This makes the obstacles appear to be less efficient at higher concentrations in resisting the motion of the dislocation. On the other hand, some randomness correction need be applied to the more realistic model now being considred. Since it is expected that the desired correction cannot differ greatly from that given by Foreman and Makin,[14] we suggest that their correction be tentatively adopted here also. Consequently the value of α was calculated for each point of the curves of Fig. 10 as suggested by Eq. (9). The $\tau^*/G - \sqrt{c}$ curves for the two dissimilar models following the randomness corrections are shown in Fig. 11.

V. DISCUSSION

The objective of this report was much less pretentious than the development of a complete theory for solid-solution strengthening. It centered principally on how the effect of limited dislocation flexibility might qualitatively influence the dependence of the effective yield stress at the absolute zero on the composition. Consequently it was permissible, without lack of generality, to neglect the modulus, electronic and all other interactions and to consider only the volumetric strain-energy interactions between solute atoms and dislocations that deviate at most only slightly from edge orientation. The fact that the volumetric interactions that were assumed at best approximate the interaction energy when the core of the dislocation overlaps the solute atom merely modifies the numerical results, but should in no way change the general trends that were deduced.

The results shown in Fig. 11 illustrate clearly that the more realistic dislocation model that considers dislocation flexibility about stress fields gives trends that deviate seriously from those expected on the basis of "equivalent" localized forces.

Although the two models agree well for extremely dilute solutions where both models suggest that τ^*/G increases linearly $\sqrt{c}$, the more realistic model gives a less rapid increase in strength as the concentration increases to slightly higher values and finally, for yet higher concentrations, gives a reduction in the strengthening suggesting that an optimum composition exists for maximum strengthening. This indeed is the thesis that was proposed by Mott and Nabarro on internal strain-center strengthening.

It would prove interesting if the qualitative trends deduced for the realistically flexible dislocation model could be compared with the experimental data. Despite the now extensive literature on solid solution strengthening, however, very few definitive results are available. In fact only those few data where attempts have been made to identify dislocation mechanisms are of some help in this respect. Even when such data as these are available they may not be suitable for the intended objective because many auxiliary indirect strengthening factors can and do effect the low-temperature effective stress, which is under scrutiny here. For example a number of reasonably definitive experiments[22-25] on strengthening by substitutional solid solution additions to f.c.c. metals have shown that the dislocation intersection mechanism is largely responsible for their low-temperature mechanical behavior. The existing data suggest that alloying increases the density of dislocations and often modifies as well the stacking-fault energy. Such indirect effects, of course, in no way deny the nominal validity of the analyses given above on dislocation flexibility concepts; they merely reflect that the experiments were not conducted in a way that is appropriate to reveal the effect of stress centers on strength.

Several investigations[26-29] on impurity interstitial solute atom strengthening do suggest that the effective stress at the absolute zero increases almost linearly with $\sqrt{c}$. In these cases, however, the concentrations of solute atoms were so low that the results could be interpreted to agree with either dislocation model. None of the available experimental evidence in any way contradicts the general deductions based on the flexible dislocation model.

Recently, T. Suzuki[30] and T. Suzuki and T. Ishii[31] have approached the experimental problem in a way that can hopefully give definitive results on the yield stress needed to move dislocations past strain centers arising from solid-solution alloying. Single crystals of Cu containing various concentrations of Ni were so produced as to have the extremely low concentrations of dislocations of 10^3 to $10^5/cm^2$. Such single crystals were subjected to a stress pulse and the displacement of the dislocations were measured by etch-pitting before and after pulsing. In the pure copper crystals most dislocations exhibited no further displacement upon application of a second stress pulse of the same low magnitude as the first even when the pulse duration was increased. Whereas at 300°K the alloy crystals exhibited a similar behavior, at 77°K the number of moving dislocations increased with the pulse time suggesting a thermally activated process of overcoming barriers. Below a critical stress no multiplication of dislocations was observed; but when the crystals were pulsed near the yield stress all dislocations moved and extensive multiplication was obtained. As the initial dislocation density was decreased the measured yield strength at first decreased and finally remained constant independent of a further reduction in dislocation density. Only such stresses which seem to refer to those needed to push dislocations past solute atom strain centers were reported.

The yield stress for each alloy was determined over the temperature range from 4.2 to 300°K; the general trend of τ^*-T curves were observed to be consistent with those expected for dislocations passing strain centers. Pure Cu exhibited only athermal behavior over the full range

of temperatures suggesting that the thermally activated dislocation-intersection mechanism, which controls the deformation of pure Cu at low temperatures when the dislocation density is 10^8 or more per cm^2, is no longer controlling at the dislocation densities investigated by T. Suzuki. For the alloys, the yield stress, τ, as well as the athermal component, τ_G, was found to depend on the concentration of solute atoms. The effective stress $\tau^* = \tau - \tau_G$, however, is a measure of the component necessary to push dislocations past the localized strain centers of solute atoms. To provide comparison with the theoretical deductions, τ^*/G, as extrapolated to the absolute zero, was plotted as a function of c in Fig. 12. These data can be compared with the theoretical curve for $\varepsilon = .02$ which is near the estimated value of ε for Ni in Cu. As yet unpublished experiments[32] indicate that similar trends appear in Mg-Zn solid solution single crystals orientated for prismatic slip.

Exact quantitative agreement between the theory presented here and the experimental results on solid-solution strengthening arising from additions of Ni to Cu could scarcely be expected. In an effort to maintain an easily tractable theory several assumptions regarding the dislocation orientation, i.e. principally edge, the nature of the stresses about solute atoms, and the distribution of solute atoms were made. Numerous additional and often important factors such as modulus effects, chemical effects etc. were neglected. It is significant, however, that both theory and experiment are in nominal qualitative agreement regarding the fact that above some limiting concentration of solute atoms serious deviations from the commonly applied $\sqrt{c}$ law for strengthening are obtained. Models for solid solution strengthening

must therefore be based on more realistic accounting of the effects of dislocation flexibility.

VI. CONCLUSIONS

The strengthening of metals due to atomic strain centers cannot be properly described in terms of the cutting of simple point obstacles by flexible dislocations. It is more realistic and indeed necessary to consider in greater detail the strain-energy interactions between the stress fields of solute atoms and dislocations. The opposing internal force along the dislocation can be considerably less when the latter is allowed to relax to its equilibrium shape in the stress fields of solute atoms. This effect becomes more pronounced as the concentration is increased. As the spacing of solute atoms decreases the dislocation line straightens out somewhat and a critical concentration is obtained for maximum strengthening. This result, which is in conflict with the predictions of "equivalent" localized force models, is qualitatively confirmed by experimental data in which the effects of direct solute atom-dislocation interactions appear to have been successfully isolated.

ACKNOWLEDGMENT

This research was conducted as part of the activities of the Inorganic Materials Research Division of the Lawrence Radiation Laboratory of the University of California, Berkeley, and was done under the auspices of the U. S. Atomic Energy Commission.

REFERENCES

1. N. F. Mott and F. R. N. Nabarro, Proc. Phys. Soc. 52, 86 (1940).
2. N. F. Mott, J. Inst. Metals, 72, 367 (1946).
3. F. R. N. Nabarro, Proc. Phys. Soc., 58, 669 (1949).
4. N. F. Mott and F. R. N. Nabarro, Rep. Conf. Strength Solids, p.1, Bristol (1948).
5. N. F. Mott, Imperfections in Nearly Perfect Crystals, p. 173, Wiley, New York (1952).
6. A. H. Cottrell, Dislocations and Plastic Flow in Crystals, Oxford (1958).
7. J. Friedel, Dislocations, Addison-Wesley (1964).
8. J. Friedel, Electron Microscopy and Strength of Crystals, p. 364, Interscience (1963).
9. H. Gleiter, Acta Met. 16, 829 (1968).
10. V. Gerold, Acta Met. 16, 823 (1968).
11. U. F. Kocks, Phil. Mag. 13, 541 (1966).
12. U. F. Kocks, Canadian J. Phys. 45, 737 (1967).
13. J. E. Dorn, P. Guyot and T. Stefansky, in Physics of Strength and Plasticity, (The Orowan 65th Anniversary Volume), ed. A. S. Argon (M.I.T. Press, Cambridge, Mass., in press).
14. A. J. E. Foreman and M. J. Makin, Phil. Mag. 14, 911 (1966).
15. A. E. H. Love, The Mathematical Theory of Elasticity, Dover, (1944).
16. J. D. Eshelby, S. S. P. 3, 79 (1956).
17. F. R. N. Nabarro, Proc. Phys. Soc. 58, 669 (1946).
18. R. L. Fleischer, Strengthening of Metals, p. 93, Reinhold (1964).
19. J. E. Dorn and S. Rajnak, Trans. AIME, 230, 10521 (1964).
20. A. J. R. Foreman, Phil. Mag. 15, 1011 (1967).

21. I. S. Sokolinikoff and R. M. Redheffer, Mathematics of Physics and Modern Engineering, p. 690 ff, McGraw-Hill (1966).

22. S. K. Mitra and J. E. Dorn, Trans. AIME 224, 1062 (1962).

23. A. A. Hendrickson and M. E. Fine, ibid. 221, 967 (1961).

24. T. J. Koppenaal and M. E. Fine, ibid. 224, 347 (1962).

25. K. R. Evans and W. F. Flanagan, Phil. Mag. 16, 535 (1968).

26. S. S. Lau and J. E. Dorn, Submitted to Scripta Met., April 1968.

27. C. Wert, Trans. AIME. 188, 1242 (1950).

28. D. F. Stein and J. R. Low, Acta Met. 14, 1183 (1966).

29. P. R. V. Evans, J. Less Common Metals 4, 78 (1962).

30. T. Suzuki, Tech. Rept. of ISSP, Ser. 4, No. 265, July (1967).

31. T. Suzuki and T. Ishii, Tech. Rept. of ISSP, Ser. 4, No. 283, November (1967).

32. A. Akhtar and E. Teghtsoonian, Presented at the 97th AIME Annual Meeting, 1968. To be published.

FIGURE CAPTIONS

Fig. 1 Shear stresses on the slip plane due to a single solute atom.

Fig. 2 Interaction energy between a long segment of an edge dislocation and a volumetric strain center.

Fig. 3 Force-displacement diagram due to the elastic interaction between a long straight edge dislocation and the stress field of a substitutional solute atom.

Fig. 4 Yield stress at 0°K for solute atom strengthening.

Fig. 5 The ratio of τ_r^* for a random distribution of solute atoms to τ_s^* for a square array as a function of obstacle strength α.

Fig. 6 A square array of attractive (A) and repulsive (R) stress centers.

Fig. 7 Effect of neighboring atoms on the stress σ'_{zy} at $x = 0$ as a function of λ/b.

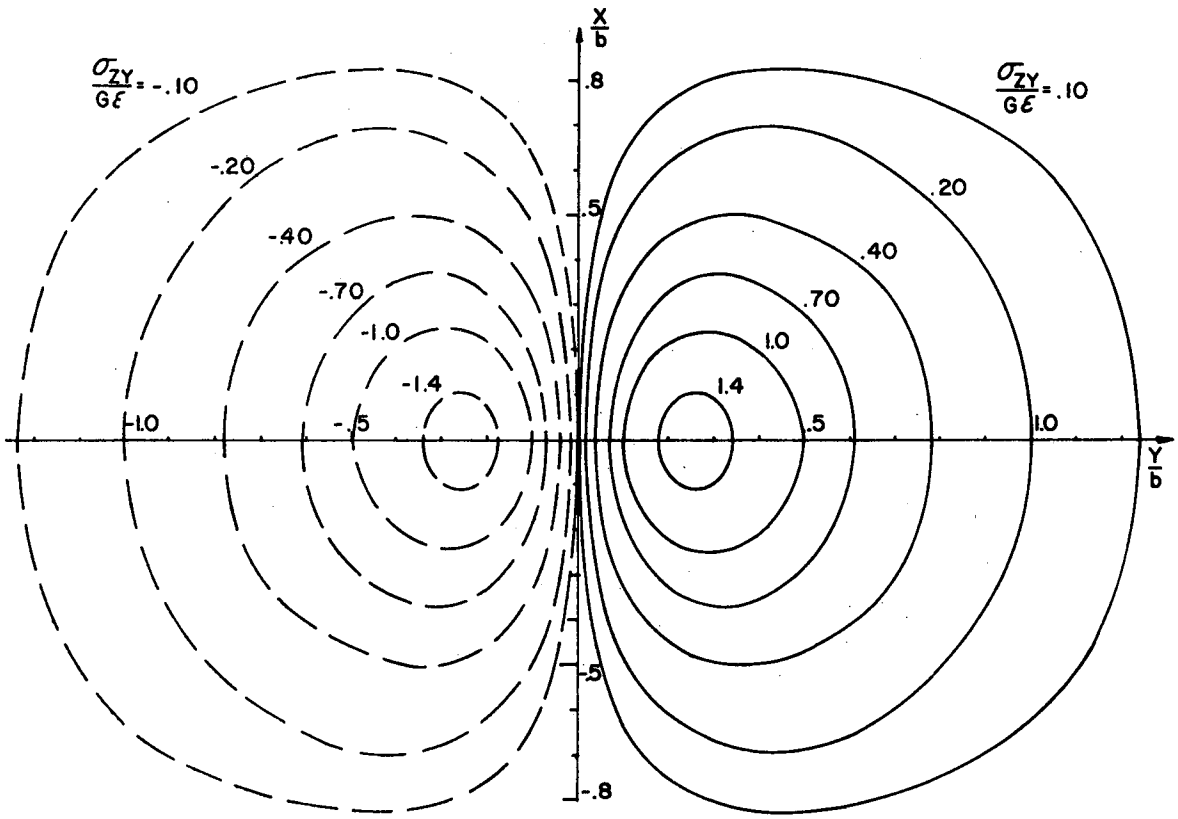
Fig. 8 Equilibrium line shape at zero applied stress ($\epsilon = .06$, $c = .00125$).

Fig. 9 Periodic dislocation line shapes.

Fig. 10 Effect of concentration of solute atoms on the yield stress.

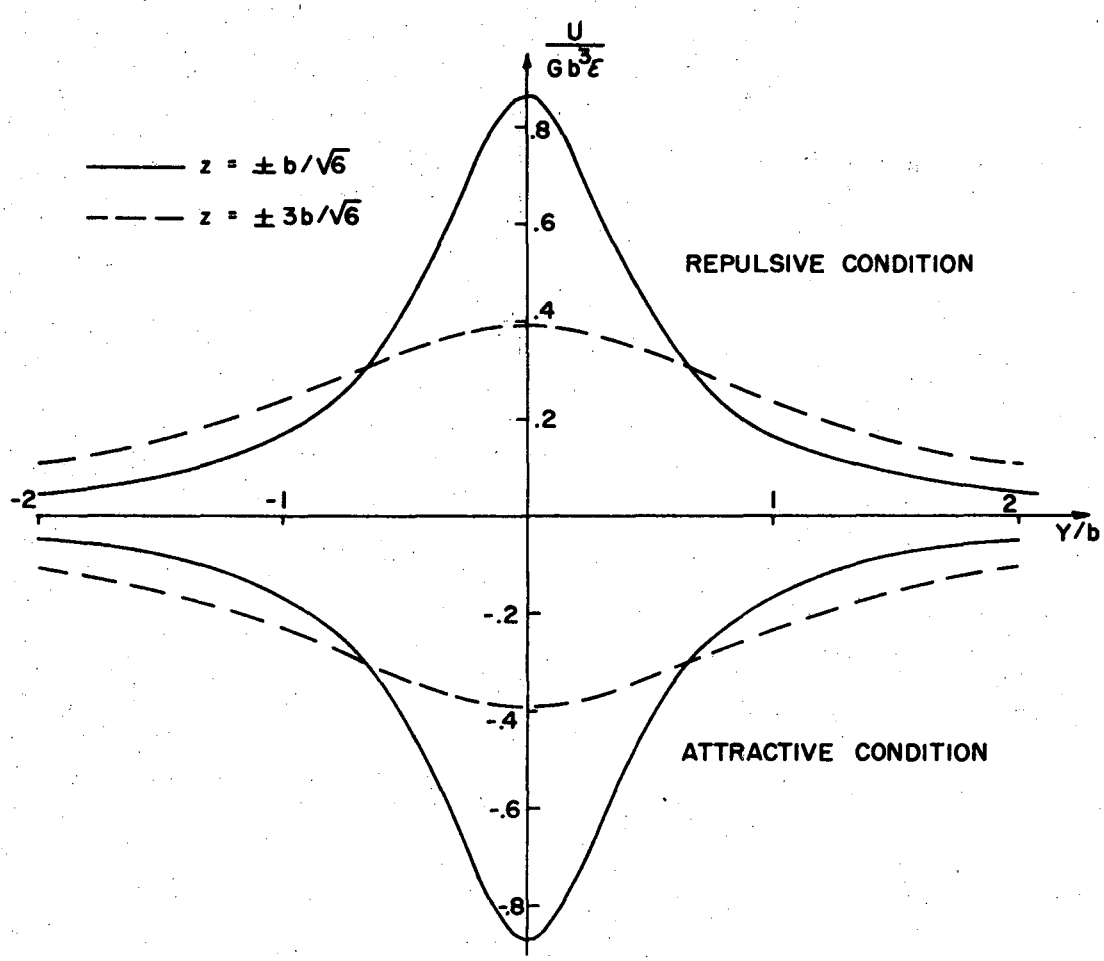
Fig. 11 Yield stress at 0°K for solute atom strengthening with randomness correction.

Fig. 12 Comparison with experimental results.



XBL 692-209

Fig. 1



XBL 692-207

Fig. 2

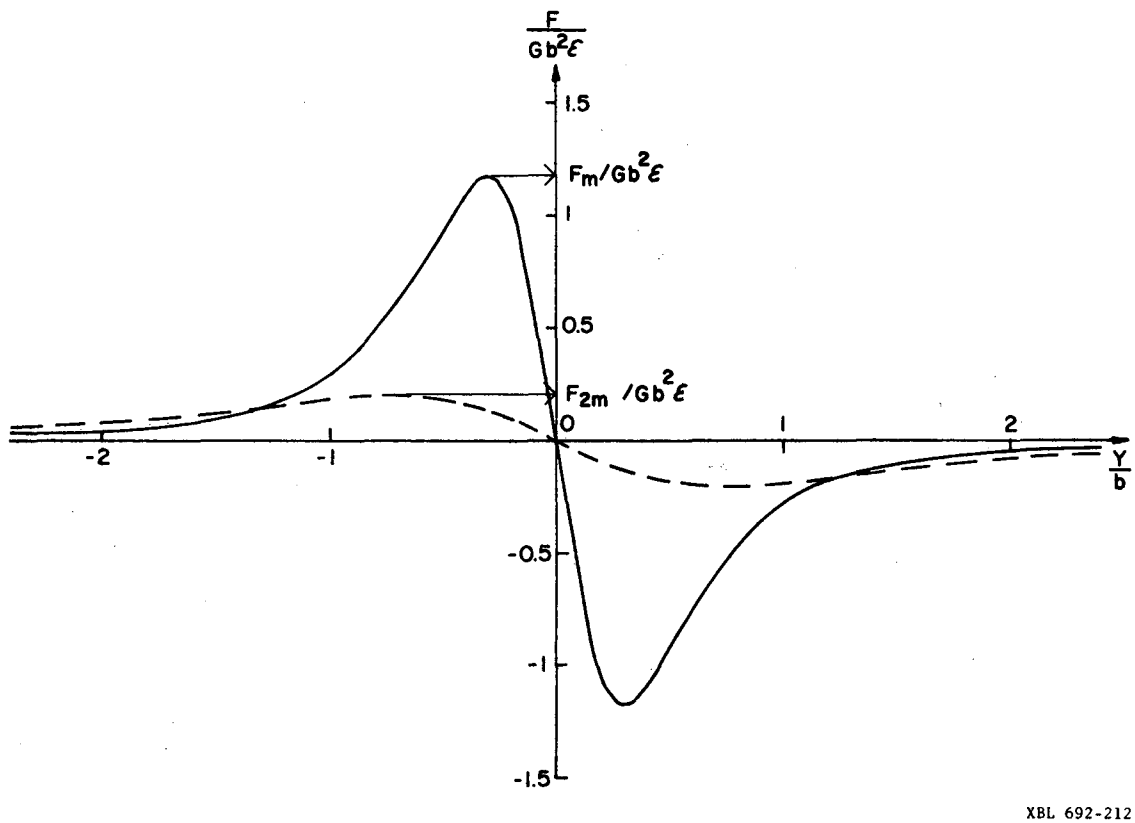


Fig. 3

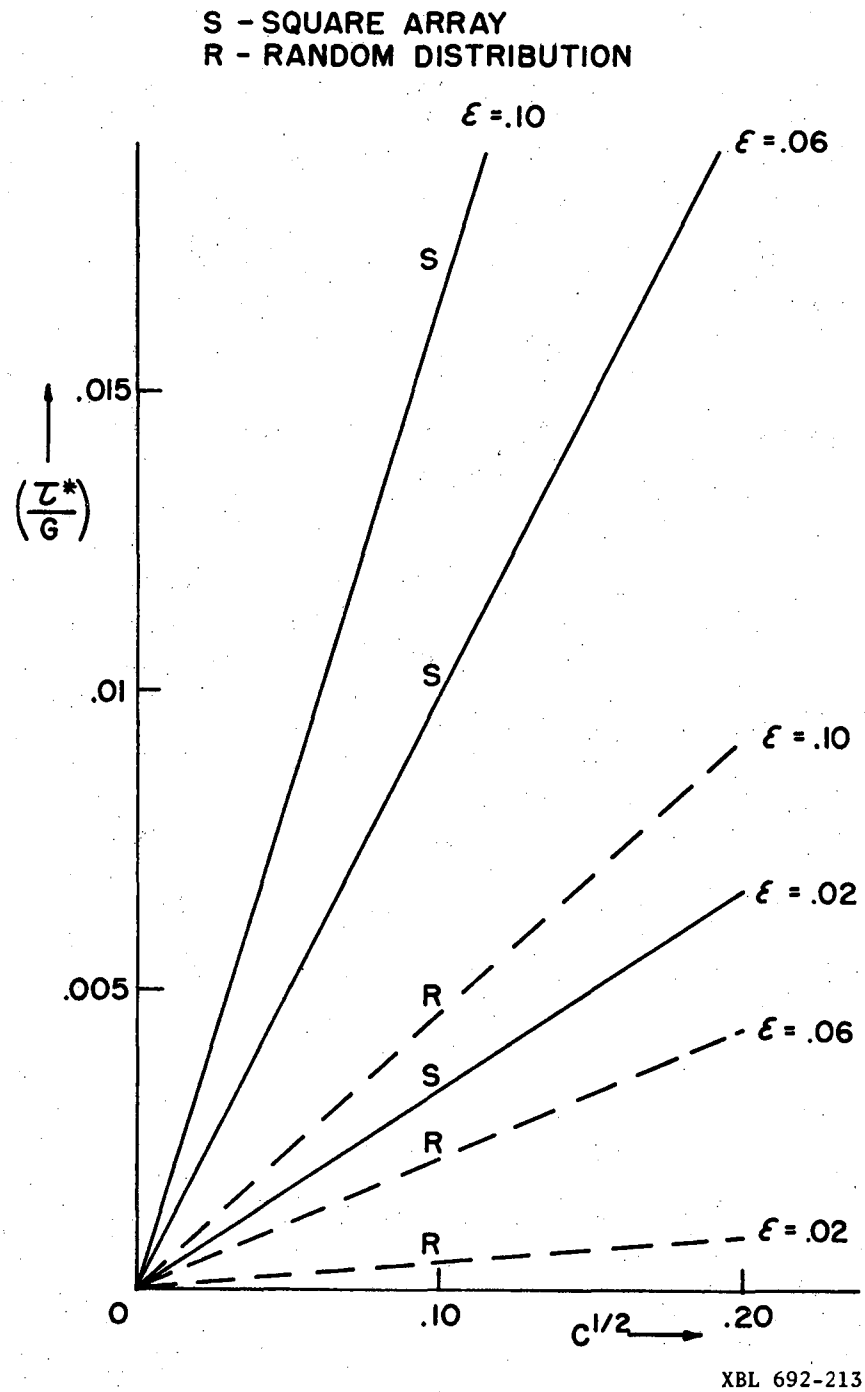
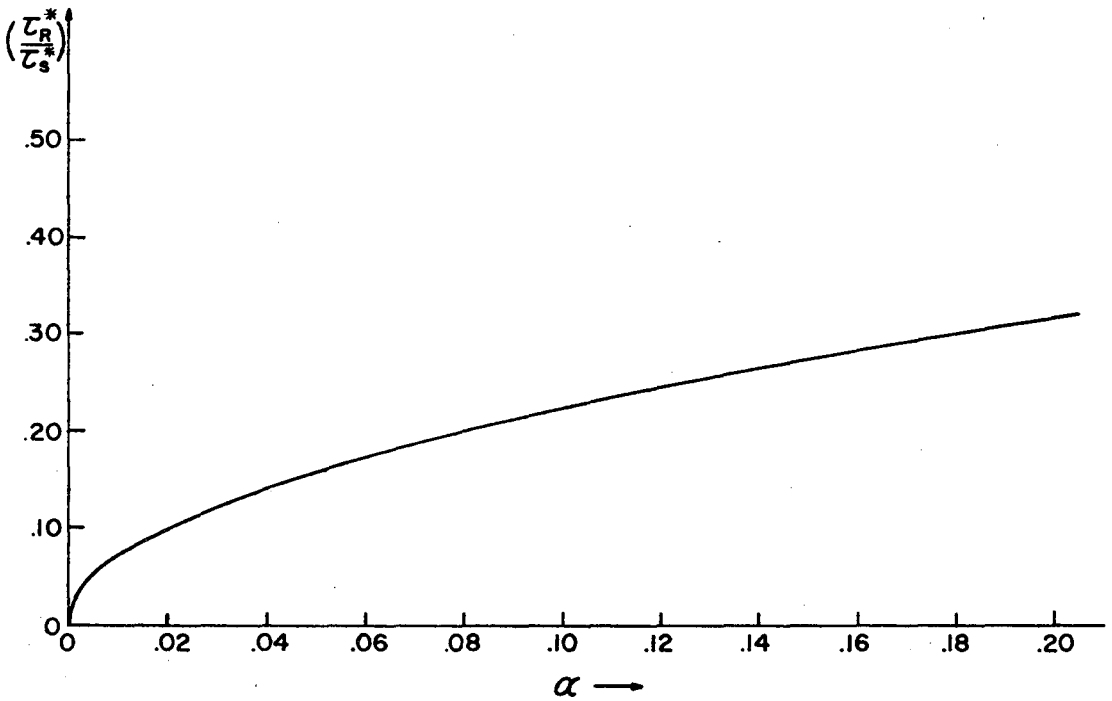
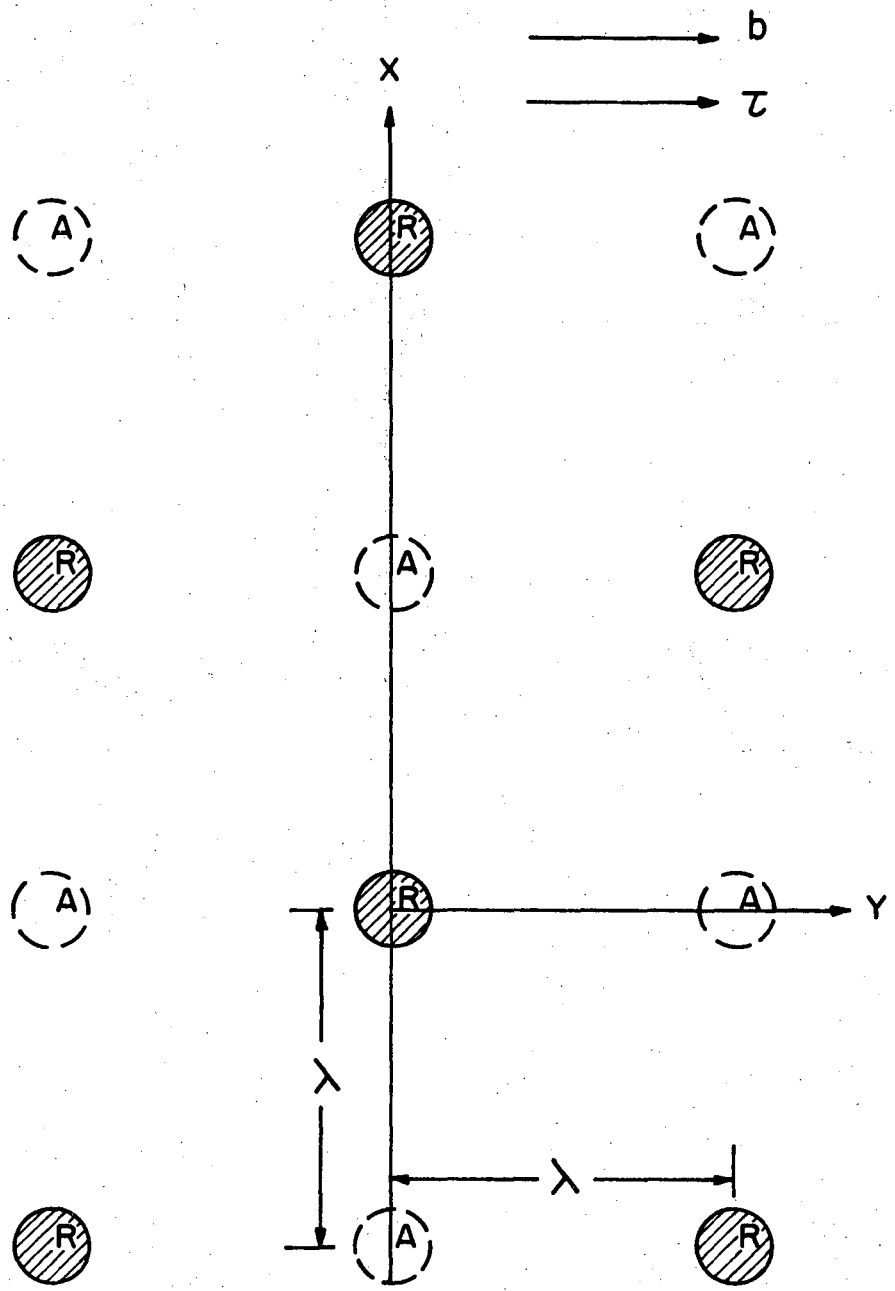

S - SQUARE ARRAY
R - RANDOM DISTRIBUTION
ε = .10
ε = .06
S
S
.015
.01
.005
0
(τ*/G)
ε = .10
ε = .02
ε = .06
ε = .02
R
S
R
R
.10
.20
C^1/2
XBL 692-213

Fig. 4



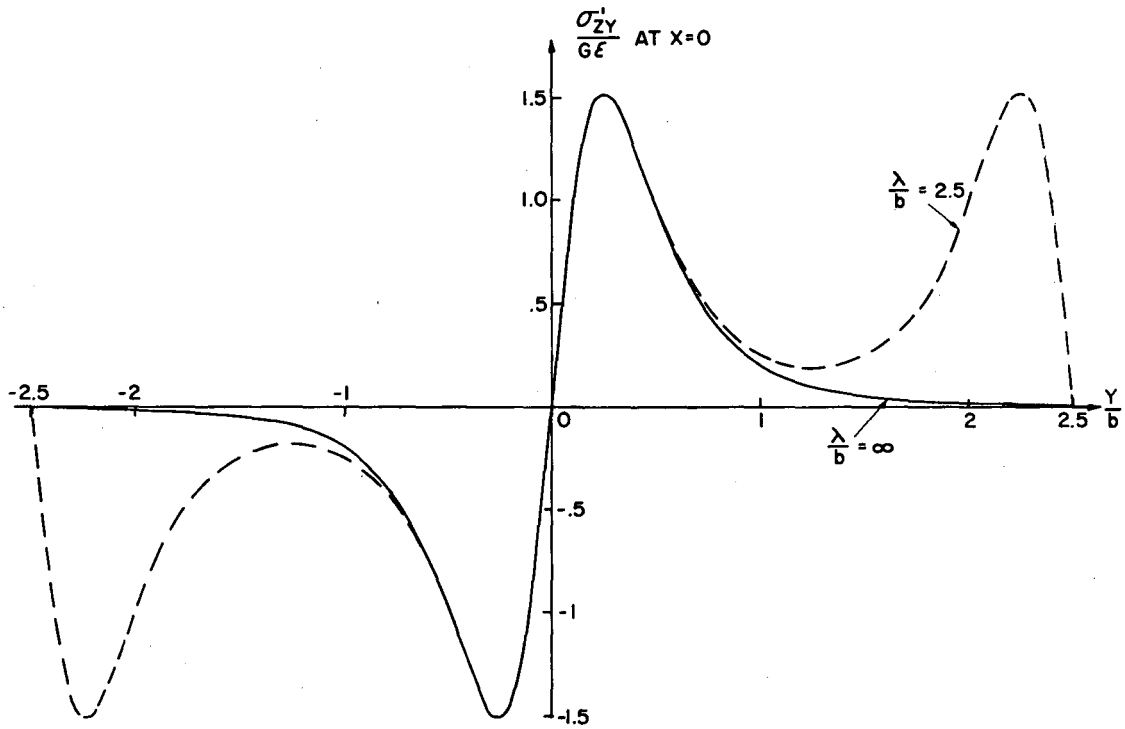
XBL 692-210

Fig. 5



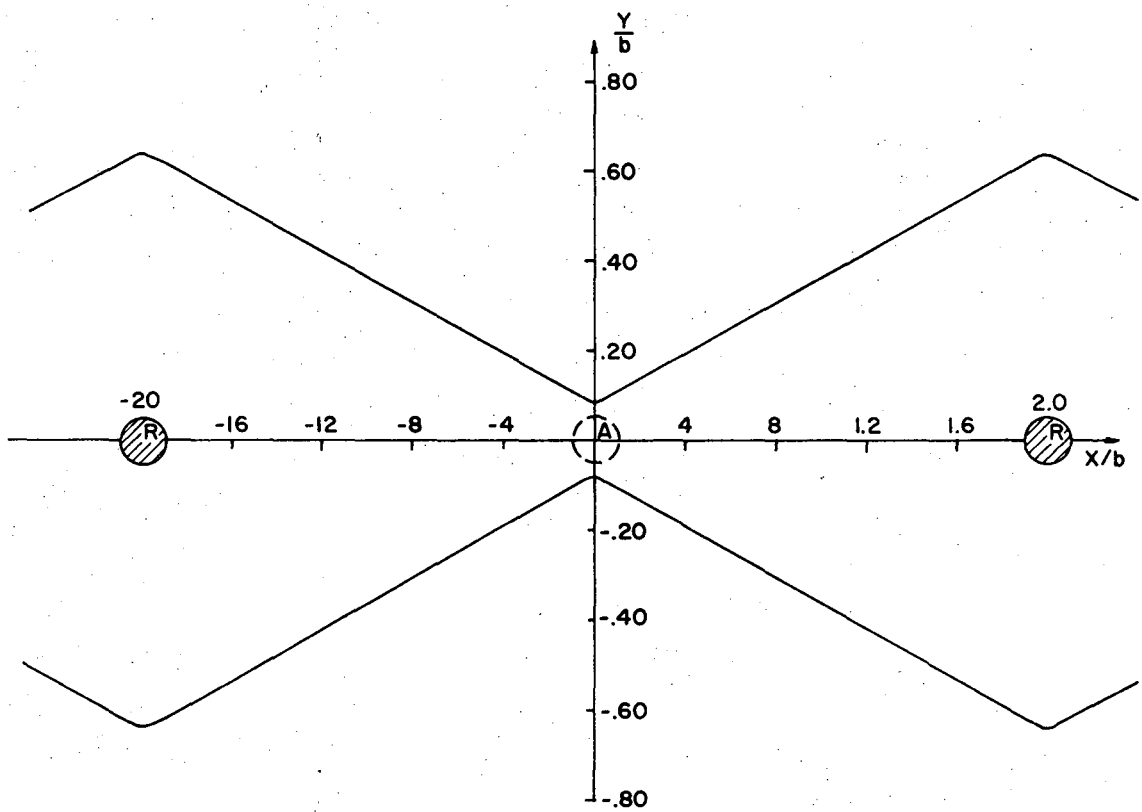
XBL 692-211

Fig. 6



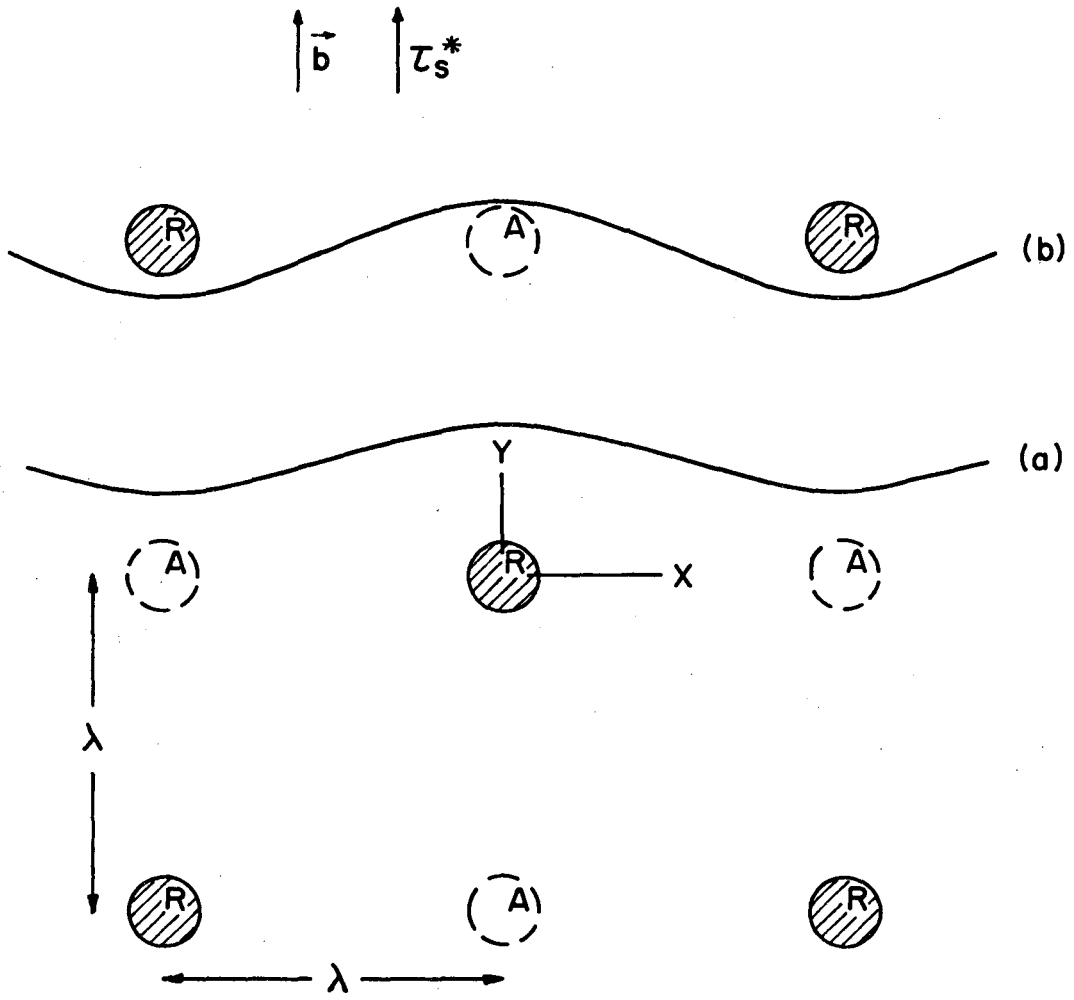
XBL 692-214

Fig. 7



XBL 692-215

Fig. 8



XBL 692-206

Fig. 9

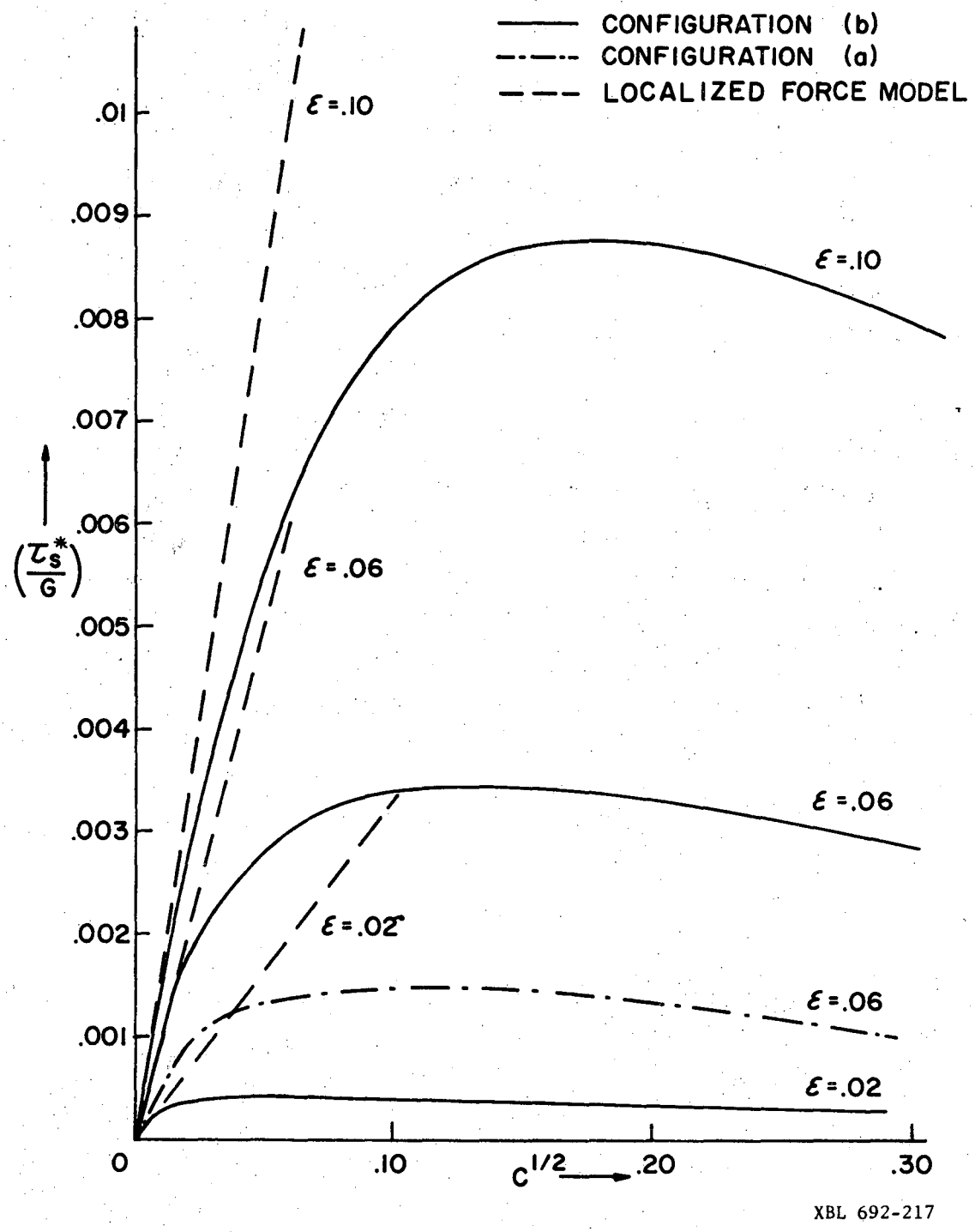

CONFIGURATION (b)
CONFIGURATION (a)
LOCALIZED FORCE MODEL
.01
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
.10
.20
.30
$\left(\frac{\tau_s^*}{G}\right)$
$c^{1/2}$
ε =.10
ε =.06
ε =.02
ε =.10
ε =.06
ε =.06
ε =.02
XBL 692-217

Fig. 10

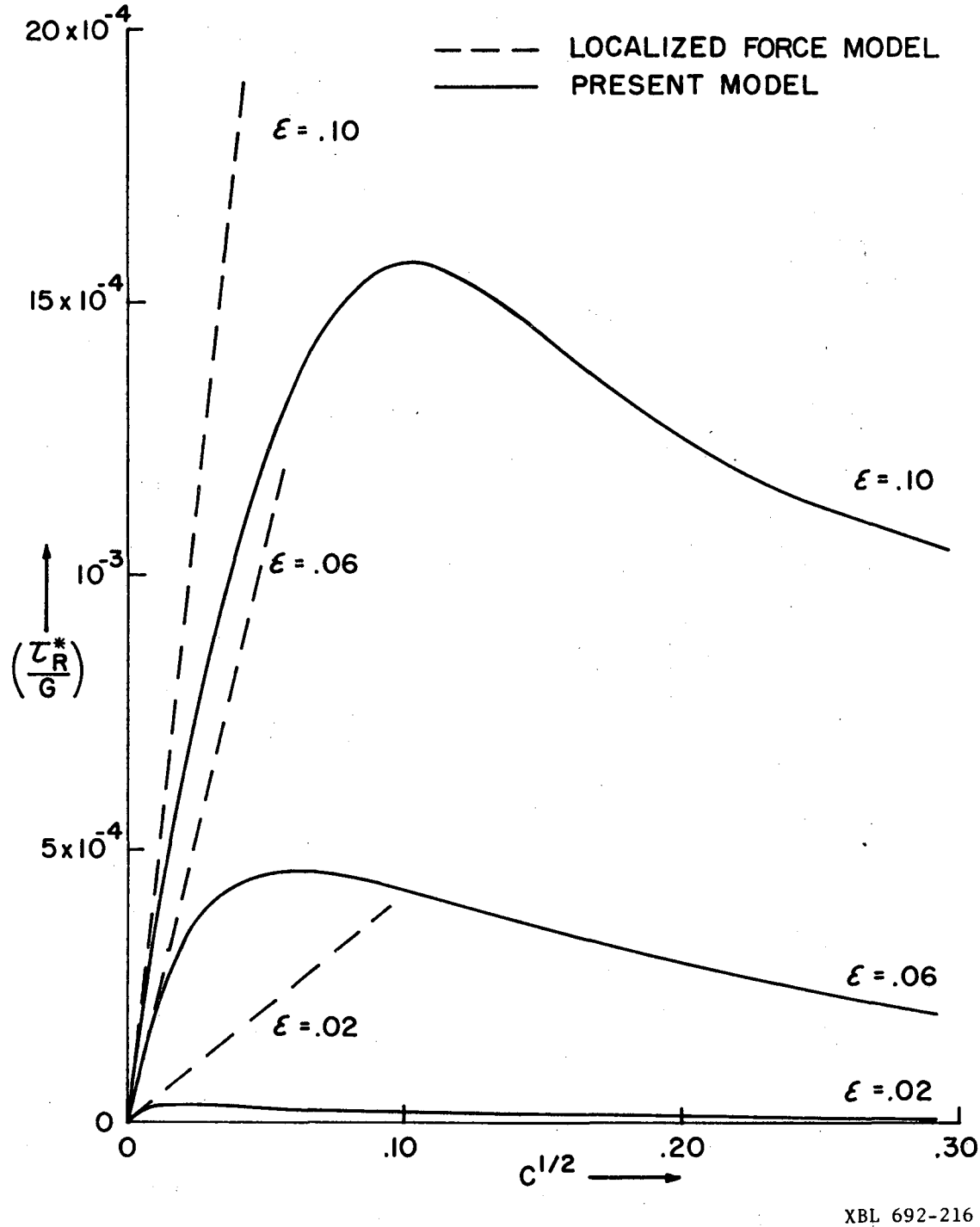
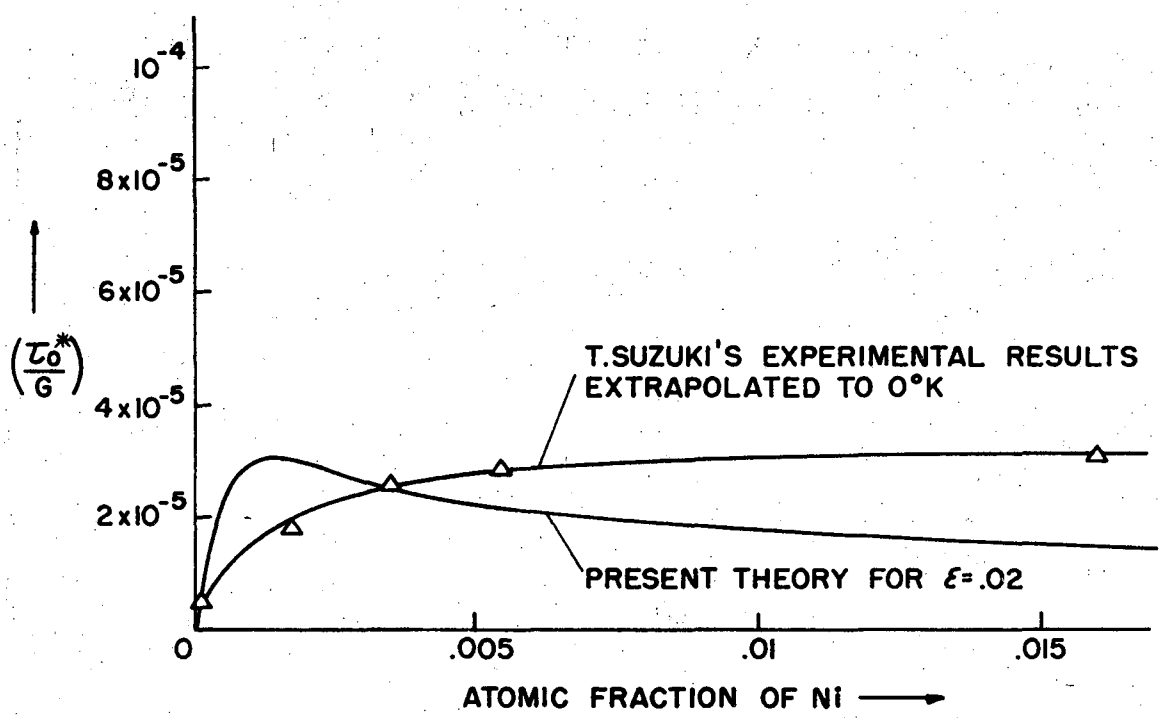

LOCALIZED FORCE MODEL
PRESENT MODEL
20 x 10^-4
15 x 10^-4
10^-3
5 x 10^-4
0
(τ*R / G)
ε = .10
ε = .06
ε = .02
ε = .10
ε = .06
ε = .02
0
.10
.20
.30
c^1/2
XBL 692-216

Fig. 11



XBL 692-208

Fig. 12

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720