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A MODEL FOR $\pi\pi$ SCATTERING

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September 1968

A MODEL FOR $\pi\pi$ SCATTERING*

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ABSTRACT

We propose a model for $\pi\pi$ scattering, using functions of the type recently suggested by Veneziano. We discuss the resulting threshold phenomena, and also the predicted widths and positions of the $\pi\pi$ resonances.

We discuss here a model for $\pi\pi$ elastic scattering, constructing our amplitude by using functions of the type recently proposed by Veneziano.¹ If we insist the amplitude be a linear combination of symmetric functions $F(x,y)$, then the most general isospin amplitudes satisfying crossing symmetry and Bose statistics, are

$$A_s^0 = c_2[3F(t,u) + F(s,u) + F(s,t)] + \frac{c_1}{2}[3F(s,u) + 3F(s,t) - F(t,u)] ; \quad (1a)$$

$$A_s^1 = (c_2 - c_1)[F(t,s) - F(u,s)] ; \quad (1b)$$

$$A_s^2 = c_1 F(t,u) + c_2[F(s,u) + F(s,t)] , \quad (1c)$$

where the c_i are constants and where $A_X^I(s,t,u)$ is the amplitude for isospin I in the x channel. We choose

$$F(s,t) = g \frac{\Gamma[1 - \alpha(s)] \Gamma[1 - \alpha(t)]}{\Gamma[1 - \alpha(s) - \alpha(t)]} \quad (2)$$

and

$$\alpha(s) = a + bs \quad (3)$$

We assume there exist no $I = 2$ $\pi\pi$ resonances. This can be assured by setting $c_2 = 0$ in (1).

The properties of $F(s,t)$ guarantee the correct Regge asymptotic behavior¹ and the saturation of the finite energy sum rules (FESR).² The three parameters a, b , and g are determined as follows. (There is no constraint on external mass here, so we take $\mu^2 = m_\pi^2$ from experiment.) We set $\Gamma_{\rho\pi\pi} = 112$ MeV and $m_\rho = 764$ MeV, leaving one free parameter which we take to be a . Further, in order to agree with the $I = 0$, $L = 0$ $\pi\pi$ phase shift,³ $\delta_0^0(m_{\pi\pi}^2)$, for $m_{\pi\pi} \leq 400$ MeV, we choose $a = 0.48$. This then gives a slope of⁴ $b = 0.90$ GeV⁻².

Our amplitude is now completely determined. It suffers from the following diseases:

- (1) The narrow resonance approximation violates unitarity.⁵
- (2) The Pomeron is ignored.
- (3) The amplitude is not unique.
- (4) Causality, in the form of positivity of the resonance widths, is not necessarily satisfied.

The narrow resonance approximation has been discussed by Mandelstam.⁵ An admittedly rough argument giving the order of magnitude of the Pomeron contribution goes as follows: The asymptotic $\pi\pi$ elastic scattering cross-section of ≈ 1 mb, estimated using factorization, can be compared with the $\pi\pi$ cross-section in the resonance region, assuming

the Pomeron contribution to the elastic scattering is roughly energy independent. This gives an effect of the order of a few percent.⁶ As pointed out by Veneziano,¹ (2) is not unique.⁷ We have chosen what seems to us the simplest possibility consistent with the absence of $I = 2$ resonances. We discuss the positivity condition below.

Our amplitude gives: (A) The masses and widths of the $\pi\pi$ resonances. (B) The low energy $\pi\pi$ phase shifts $\delta_L^I(s)$. (C) A prediction of the angular and energy dependence of high energy $\pi\pi$ charge-exchange scattering.

A. $\pi\pi$ Resonances

The model predicts the existence of families of ($J^P = \text{even}^+$, $I = 0$) and ($J^P = \text{odd}^-$, $I = 1$) $\pi\pi$ resonances, with fixed relative widths. The resonances lie on exchange degenerate parallel trajectories. For example, the ρ has a degenerate 0^+ partner, which can be identified with the broad enhancement sometimes called³ ϵ . The $f(1260)$ ($J^P = 2^+$) has degenerate $1^-(\rho')$ and $0^+(\epsilon')$ partners. Also there should be a $J^P = 3^-$ state at ≈ 1670 MeV with degenerate 2^+ , 1^- , and 0^+ companions.⁸

Our predicted masses and widths are shown in Table 1. The widths of the resonances lying on the first daughter trajectory are rather large. The most difficult case to deal with on comparison with experiment is the ρ' .⁹ The experimental data of Crennel et al.,¹⁰ can be combined with estimates of the ρ and ρ' production cross sections to give an upper limit to the ρ' width of

$$\Gamma_{\rho' \rightarrow \pi\pi} / \Gamma_{\rho \rightarrow \pi\pi} < 0.13, \quad (4)$$

assuming the ρ' is totally elastic.¹¹ Our simple picture can be preserved only if the ρ' is highly inelastic, so that the resonance is extremely broad.¹²

The requirement that all resonance widths be positive gives a bound on a . For the first daughter trajectory the condition is $3a + 4\mu^2 b \geq 1$.¹³ The condition for positivity of all widths seems to be that the 0^+ partner of the f , the ϵ' , have a positive width. We have reached this conclusion by numerical computation and are as yet unable to construct a proof. As can be seen in Table 1, our choice of a and b leads to a small negative width for ϵ' . Since $\Gamma_{\epsilon'}$ changes sign at $a \approx 0.496$ we do not consider this serious, and ascribe the negative width to uncertainties in $\delta_0^0(s)$ plus the inaccuracy of the model.

B. Threshold Properties

We confine ourselves here to a discussion of the s wave phase shifts $\delta_0^0(s)$, $\delta_0^2(s)$. Since our amplitude is pure real we can only compare our results with other analyses for δ small, say $\lesssim 35^\circ$ at best. There have been several semiphenomenological attempts to compute³ $\delta_L^I(s)$. Using double subtracted dispersion relations, and an input of Breit-Wigner forms for the f , ρ , and f' , Tryon³ has calculated $\delta_0^0(s)$ and $\delta_0^2(s)$ for $s \leq 950$ MeV. He also assumed the s -wave scattering lengths a_I satisfy $-\frac{1}{2} \leq a_0/a_2 \leq -\frac{7}{2}$ and $L = \frac{1}{6}(2a_0 - 5a_2) \approx 0.10/\mu$. Tryon's results seem relatively insensitive to the model dependent ratio a_0/a_2 . We are able to get a reasonable fit to Tryon's $\delta_0^0(s)$ for $s \sim 400$ MeV. His $\delta_0^2(s)$ is then also in fair agreement with ours. Near $a = \frac{1}{2}$, the a_I change sign because we are near a pole in one of our Γ functions, and we can therefore say nothing about a_0/a_2 . Our $L \approx 0.11/\mu$, and is relatively insensitive to a .¹⁴

C. Charge Exchange Diffraction

The analog of $\pi^- p \rightarrow \pi^0 n$ here, is $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$. Near $t = 0$ we have the usual form

$$\ln \left(\frac{d\sigma}{dt} \right) = c + dt,$$

where we compute $d = 1.80 \ln(0.90 s) + 6.3$ in units of GeV^{-2} . For $s \approx 30 \text{ GeV}^2$, $d \approx 12.3$, which is roughly the same as in πN and NN charge exchange.¹⁶

We have presented here a model which incorporates Regge behavior, pole structure, crossing, and analyticity. It seems to have some predictive success. On the other hand, besides suffering from the theoretical difficulties outlined above, it predicts two seemingly nonexistent experimental effects: the ρ' at the f mass, and $\beta_\rho(t = -0.54 \text{ GeV}^2) = 0$.¹⁵ Whether these features are inescapable remains to be seen.

The authors wish to express their thanks to Professor Geoffrey Chew for many helpful suggestions during the course of this work.

TABLE 1

Partial widths for $\pi\pi$ decay predicted by the present model. The widths are normalized to $\Gamma_{\rho\pi\pi} = 112$ MeV, and our ρ trajectory is $\alpha_{\rho}(t) = 0.48 + 0.90 t$.

8^+								5
7^-							6	4
6^+					13	10	12	
5^-				14	11	10	10	
4^+			34	27	20	21	14	
3^-		38	30	16	19	10	13	
2^+		96	82	27	39	16	25	12
1^-	<u>112</u>	112	14	36	10	20	8	14
0^+	565	-13	77	12	39	12	25	11
	764	1300	1670	1980	2240	2480	2690	2890

FOOTNOTES AND REFERENCES

- * This work was supported in part by the U.S. Atomic Energy Commission.
1. G. Veneziano, Nuovo Cimento 57, 190 (1968).
 2. The $\pi\pi$ problem in the context of finite energy sum rules has been studied by C. Schmid, Phys. Rev. Letters 20, 628 (1968). For discussions of the general problem of infinite families of Regge trajectories see N. N. Khuri, Phys. Rev. Letters 18, 1094 (1967); M. Kugler ibid, 21, 570 (1968).
 3. E. P. Tryon, Phys. Rev. Letters 20, 769 (1968); E. Malamud and P. Schlein, Phys. Rev. Letters 19, 1056 (1967); W. D. Walker et al., Phys. Rev. Letters 18, 630 (1967). Tryon's phase shifts seem to agree with Walker's. The energy dependence of his $\delta_0^2(s)$ seems to be rather different from that of Schlein and Malamud. Walker, and Schlein and Malamud find solutions for $\delta_0^0(s)$ which resonate near 750 MeV. These would be consistent with our broad $\epsilon(0^+)$ resonance. We wish to thank Dr. Tryon for a helpful discussion.
 4. This slope is on the low end of the commonly accepted range of values 0.9 - 1.1 GeV^{-2} . For example, F. Arbab and C. Chiu, Phys. Rev. 147, 1045 (1966) give $\alpha_\rho(t) = 0.56(\pm 0.01) + 1.08(\pm 0.03)t$ from an analysis of πN charge exchange scattering. B. Svensson, Proc. CERN School, Rattvik, (1967), p.153, gives $b = 0.91 \text{ GeV}^{-2}$.
 5. S. Mandelstam, Phys. Rev. 166, 1539 (1968). See also H. Cheng and D. Sharp, Ann. Phys.(N.Y.) 22, 481 (1963); Phys. Rev. 132, 1854 (1963).
 6. This procedure would be in accord with the general philosophy of H. Harari, Phys. Rev. Letters 20, 1395 (1968). We thank Geoffrey Chew for emphasizing this point to us.
 7. We have chosen what seems to us to be the simplest possibility consistent with our requirements, considering the $\pi\pi$ system in isolation.

On considering the internal resonances as external states and studying, e.g., $\pi\rho$ scattering, one may very well discover new terms must be added. For an alternate choice of the form of the amplitude see M. Virasoro, University of Wisconsin preprint (1968), unpublished. One of us (J.Y.), wishes to thank Dr. Virasoro for an interesting discussion.

8. The $\pi^+\pi^-$ $I = 0$ object at 1.05 BeV recently reported by D. H. Miller et al., Purdue preprint (1968) unpublished, would not fit into this scheme. Though the ρ' at ≈ 1300 MeV is the most difficult of these objects experimentally, the 1670 MeV 3^- object also presents difficulties, as the $g(1650)$ is somewhat lower in mass and is supposed to decay predominantly into 2π . The general situation in this mass region is deeply confused. (See N. Barash-Schmidt et al., UCRL-8030 Rev., August, 1968, unpublished.) More investigation into the dipion spectrum in the region above $m_{\pi\pi} = 1$ GeV would be highly desirable. The harmonic oscillator quark model of G. Zweig (Report to Philadelphia Meson Conference, April, 1968, unpublished) does not give the odd daughters.
9. Any fit to high energy data employing a ρ' trajectory runs into the problem of the existence of a resonant 1^- $I = 1$ state between ≈ 1000 and 1300 MeV, unless one assumes the ρ' represents an effective cut contribution, or unless one puts an arbitrary zero into $\beta_{\rho}(t)$. See, for example, V. Barger and R. Phillips, Phys. Rev Letters 21, 865 (1968).
10. D. Crennel et al., Phys. Rev Letters 18, 323 (1967); R. Eisner et al., Phys. Rev. 164, 1699 (1967).

11. J. D. Jackson and C. Quigg (private communication). Fig. 2(b) of Ref. 10 allows a generous upper limit for (ρ'/ρ) of $1/10$. The production cross section for ρ and ρ' is estimated using the absorptive OPE model which is known to work well in the energy range for ρ production. At $6 \text{ GeV}/c$ the result is $\sigma(\rho')/\sigma(\rho) \approx 3/4$. We thank Professor J. D. Jackson for numerous helpful comments and for the estimate above.
12. If we take the point of view of Schmid, Phys. Rev. Letters 20, 689 (1968), a given partial wave will be represented by an inward spiral on the Argand plot with the resonance becoming more and more inelastic as we go up in energy. We are now investigating this behavior in our model, and the result will be contained in a more detailed report.
13. This is related to the Veneziano supplementary condition. (See Ref. 1.) At the point $\alpha(s) + \alpha(t) + \alpha(u) = 4\mu^2 b + 3a = 1$, the odd daughter trajectories vanish. The condition $\Gamma_{\epsilon} > 0$ can be written as $a > \frac{1}{2} - \frac{32}{3} \mu^4 b^2 + \mathcal{O}(\mu^6 b^3) \approx 0.496$. In this note "second daughter trajectory" means second trajectory below leading trajectory.
14. The sum rule for the $I = 1$ amplitudes is saturated only if we keep the $\epsilon(760)$ contribution, since L is close to the current algebra result of $\approx 0.1/\mu$. This is in accord with Adler's original remarks in this connection. See S. L. Adler, Phys. Rev. 140, B736 (1965), Section IV.
15. Factorization implies that the zero we have in $\beta_{\rho}(t)$ at $\alpha_{\rho}(t) = 0$ also causes a dip in the πN charge exchange differential cross section. This does not seem to appear experimentally. Virasoro (see Ref. 7) attempts to circumvent this problem. We thank Haim Harari for pointing out the importance of this zero to us in a private communication.

16. R. K. Logan et al., Phys. Rev. Letters 18, 259 (1967); G. Manning et al., Nuovo Cimento 41, A167 (1966).

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