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ELLIPTICAL VACUUM CHAMBER STRESS AND DEFLECTIONS*

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Summary

Equations are derived for the bending moments and stresses at all points and for deflections at the major and minor axes in a uniform-wall tubular chamber of elliptical cross section subjected to uniform pressure loading. Graphs are included to facilitate computations.

Introduction

A cross section of the elliptical chamber is shown in Fig. 1. Internal pressure is taken positive to agree with pressure vessel literature. Thus, p is negative for internal vacuum (external pressure). In the analysis, the following simplifying assumptions were made: (1) axial effects are neglected; (2) deflections are small and linear theory is used; (3) uniform pressure; (4) uniform wall thickness; (5) stability effects are not considered.

Bending Moments

Because of symmetry, only one quadrant of the ellipse needs to be considered. A free-body diagram and associated reactions for static equilibrium are shown in Fig. 2. Since the ellipse is statically indeterminate, the necessary additional equation is derived by noting that the slopes at the major and minor axes remain unchanged during the application of the load.

By the method of virtual work,[1] the moments (per unit length) at minor and major axes are derived as[2]

$$M_1 = (-pa^2/6)\,[-1+2k^2+(1-k^2)F/E] = -K_1pa^2$$

$$M_2 = (pa^2/6)\,[1+k^2)-(1-k^2)F/E] = K_2pa^2,$$

where F and E are complete elliptic integrals of the first and second kind, and $k = [1-(b/a)^2]^{1/2}$. K_1 and K_2 are given as functions of b/a in Fig. 3.

From the free-body diagram (Fig. 2), the moment at any point along the ellipse is

$$M = M_1 + (pk^2x^2/2) = K_xpa^2,$$

where K_x is given as a function of x/a for selected values of b/a in Fig. 4.

*Work performed under auspices of U. S. Atomic Energy Commission.

Stresses

In a chamber with uniform wall thickness, the bending stress is $\sigma_m = \pm Mc/I = \pm 6M/t^2$, and the circumferential hoop stress is $\sigma_h = pb/\{t[1-k^2(x/a)^2]^{1/2}\}$. The total stress is the algebraic sum of these two. The maximum total stresses at the major and minor axes are:

$$|\sigma_2|_{max} = |6K_2pa^2/t^2| + |pa/t|$$

$$|\sigma_1|_{max} = |6K_1pa^2/t^2| + |pb/t|.$$

In many cases, the hoop stress is negligible compared to the bending stress. If the wall is thick, the stress distribution should be modified according to curved-beam theory.

Deflections

A virtual unit load, w=1, is applied at the minor axis. From the expression for the bending moment m due to this unit load and from the previously derived expression for the actual bending moment, the actual deflection is

$$\delta_y = \int_0^{S_0} \frac{Mm}{D}\,ds = \frac{pa^4}{D}\left[\frac{K_1}{2} + \frac{3k^2-1}{16} + \frac{b^2(8K_1+1+3k^3)}{32\,ka^2}\,\ln\left(\frac{1+k}{1-k}\right)\right] = \frac{C_1pa^4}{D},$$

where D is the flexural rigidity of the wall. A similar analysis for a virtual unit load placed at the major axis gives

$$\delta_x = \frac{pa^4}{D}\left[K_1\left(\frac{b^2}{2a^2} + \frac{b}{2ka}\arcsin k\right) - \frac{1-3k^2+2k^4}{16} + (b/16\,ka)\arcsin k\right] = C_2pa^4/D.$$

C_1 and C_2 are plotted as functions of b/a in Fig. 5. The foregoing virtual work methods can be extended for deflections at any point and for other chamber configurations, possibly using graphical or numerical integration.

References

1. A.S. Niles and J.S. Newell, Airplane Structures (John Wiley & Sons, Inc., N.Y., 1945), 3rd ed.
2. R.T. Avery and G.A. Tidrick, "Structural Analysis of an Elliptical Chamber with a Uniform Pressure Loading," UCID-3215, 1968.

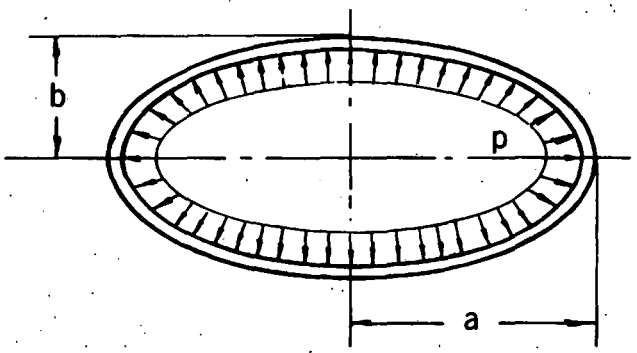

Fig. 1. Elliptical chamber cross section.

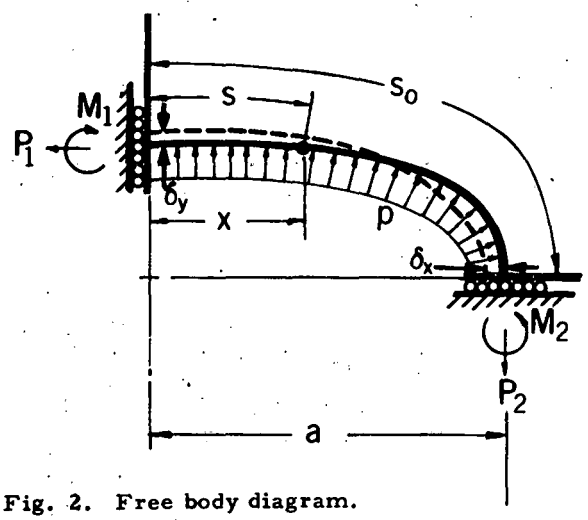

Fig. 2. Free body diagram.

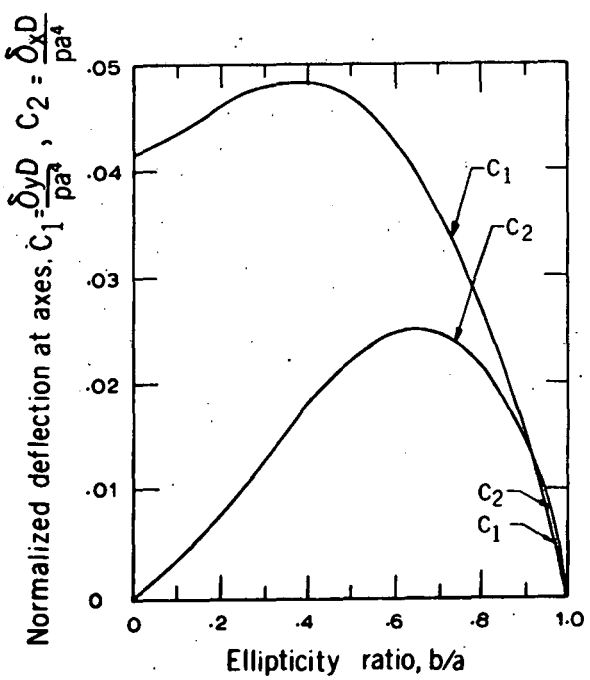

Fig. 3. Bending moments at principal axes.

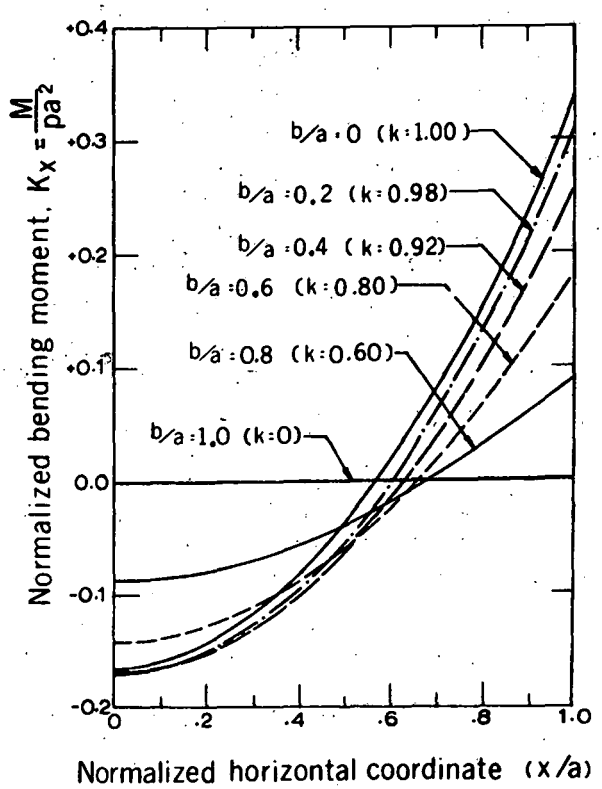

Fig. 4. Bending moments at intermediate points.

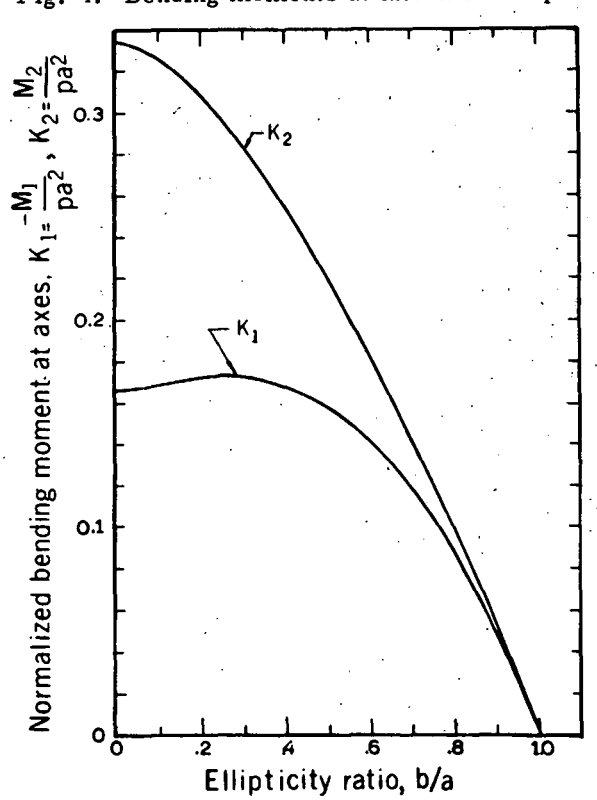

Fig. 5. Deflections at Principal Axes.

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