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DEFINITIONS AND CONVENTIONS IN ELLIPSOMETRY

Rolf H. Muller

November 1968

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DEFINITIONS AND CONVENTIONS IN ELLIPSOmetry

Rolf H. Muller

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Definitions and Conventions in Ellipsometry*

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ABSTRACT

Many parameters which appear in the theory of ellipsometry crucially depend on the choice of arbitrary conventions and definitions. This dependence is discussed for nine two-fold choices. On the basis of literature usage, preferences are expressed for one of each of the alternatives and for the nomenclature of the variables involved.

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In the derivation of the basic equations of ellipsometry from the electromagnetic theory of light reflection, for monochromatic plane waves, arbitrary choices have to be made at many points. Some of these choices, although of profound consequence, are not recognizable from the results, and the comparison of data from different sources may be impossible. It should be realized that to some extent the separation between cause and effect is arbitrary. The alternatives chosen here as "cause" are those which present themselves in the step by step derivation of the classical theory of ellipsometry from the Maxwell equations.

In the following discussion, nine two-fold alternatives of definitions and conventions have been singled out, and are stated in their most concise and fundamental form. Their direct effects are discussed and some of the multiply connected interactions are indicated. However, no attempt is made to discuss the effects of all possible 512 combinations. The alternatives listed in the first and second columns of Tables I to IX will be referred to as alternatives one and two. The first column represents the author's tentative preference⁷ which is strong, however, only in the cases represented in Tables VI (coordinate system) and IX (angle independent optical constants).

In order to establish past and present usage, the literature has been searched for authors who make specific statements on the conventions and definitions they use. Only one publication has been selected from each author and is referred to by his name. If a convention had to be inferred from the results, the name appears in parenthesis.

Formal Definitions of Ellipsometer Parameters

Table I gives the two alternative formulations for the relative amplitude attenuation $\tan \psi$ in terms of the Fresnel coefficients r for

the electric field components normal (subscript s) and parallel (subscript p) to the plane of incidence¹. The resulting alternative angles ψ are complementary to each other. The ranges of ψ values for bare surfaces given in the table hold in combination with the conventions contained in the first columns of the other tables, and under the assumption that values between 0° and 90° are admitted for the description of elliptic polarization. Although most of the literature employs alternative 2, the first one is preferred here⁷ in order to be consistent with the choice of Δ discussed below.

Table II gives the two alternative formulations for the relative phase change Δ in terms of the absolute phase changes² of s and p components upon reflection. The first alternative is almost exclusively used in the literature and therefore preferred here. It would seem that the first alternative of ψ in Table I requires the second alternative of Δ in Table II. However, the preferred choices given in Tables III, IV and V, primarily the negative exponent of the time-factor, result in a reversal of the sign of Δ and require the first alternative in Table II. The ranges of Δ values for bare surfaces given in the table again hold in combination with the other preferred conventions and are basically due to the restriction in the angle of incidence to 0 to 90° .

Table III gives two alternative formulations of the complex relative amplitude attenuation ρ . The first alternative is almost exclusively used in the literature and therefore preferred here, although there is some logic in using the second alternative in combination with the negative exponent of the time dependence factor.

Time Dependence Factor

Table IV shows two alternatives for the complex formulation of sinusoidal oscillations. As a basis of discussion, the negative exponent is preferred⁷ here because of its use in most of modern physics, particularly quantum mechanics, although alternative two is firmly established in electrical engineering and can be found in the older physics texts.

A direct consequence of the choice in sign for the exponent in the time-factor is the sign of the imaginary part of the complex refractive index. A combination of signs other than those indicated in Table IV results in intrinsically negative values for the imaginary part regardless of which formulation of the refractive index (Table VIII) is chosen.

Absolute Phase

Little attention has been given in the literature to the effect which results from the formulation of the absolute phase change upon reflection. Although four alternatives are shown in Table V, the choice is again basically two-fold and formally lies in the relative sign of time and phase exponent. In the preferred⁷ first alternative of Table V, a positive absolute phase change δ is equivalent to an increase in time, or a phase advance. Correspondingly, a negative absolute phase represents a phase retardation. The third alternative, most frequently found in the literature, is physically equivalent to the first within the framework of a positive time exponent.

Coordinate System

Table VI illustrates two possible choices for the positive coordinate direction of the reflected electric field component in the plane of incidence. The resulting values of the relative phase Δ differ by an additive

quantity of π .

This effect can most easily be understood by comparing mirror images of ellipses in Fig. 2. The coordinate system used in literature references can be inferred from statements of the Fresnel coefficient for the p-component which differs in sign for the two alternatives and is given in its most compact form in Table VI. ϕ is the angle of incidence, ϕ' the angle of refraction.

The literature preference is divided between the two alternatives. Although the second alternative results in identical reflection coefficients for s and p components at normal incidence, which is physically reasonable, it is rejected here because (if used with the first alternative of Table VII) it involves a change from a right to a left hand coordinate system for the directions of \vec{E}_p , \vec{E}_s and \vec{k} between incident and reflected beam. Since azimuth circles on instruments usually show the same sense of rotation for incident and reflected beam, alternative 1 is strongly favored despite its lack of beauty at normal incidence.

Table VII shows the two possibilities for defining the positive coordinate directions of the reflected electric field component normal to the plane of incidence. It can be parallel or opposite to that of the incident wave. A dot stands for an arrow coming out of, a cross for one going into, the plane of the drawing which is also the plane of incidence. The literature is almost unanimous in the use of the first alternative except for Winterbottom whose equations do not seem to agree with drawings if the arrows are taken to indicate positive coordinate directions rather than electric field orientation. The preferred coordinate system with the nomenclature employed is also shown in Fig. 1. This system is also best suited for application of the Poincaré sphere

in tracing the state of polarization through the different elements of an ellipsometer.

Complex Refractive Index

Two alternative formulations of the complex refractive index, irrespective of the sign chosen for the imaginary part, are given in Table VIII. The imaginary unit $\sqrt{-1}$ is designated by the letter i . The factored form, alternative 1, seems to be prevalent in the literature,⁷ however the use of the symbols k and κ is not uniform. A comparison of nomenclature for the different parts of the complex refractive index as formulated in Table VIII is given in Table X.

The proposed preferred names are:

n - refractive index

κ - absorption index

k - extinction index

With either of the above formulations, the complex refractive index can be defined in two basically different ways, such that its value is either a constant or depends on the angle of incidence. This is the most far-reaching of the nine choices. Table IX shows the two alternatives in the form of the first alternative of Table VIII. n' and κ' stand for the values which are dependent on angle of incidence, n and κ for those which are independent. The relationships given between the two sets of values show that they are not simply connected to each other but become identical at normal incidence. The difference in the two definitions shows most clearly in the description of refraction at a dielectric-metal interface when put in the form of Snell's law.³ As shown in Table IX, the formulation with the angle-independent complex refractive index requires the introduction of a complex angle of refraction ϕ'_{complex} which has no recognizable

physical meaning. The angle-dependent complex refractive index, on the other hand, is chosen so that its real part and the real angle of refraction ϕ'_{real} , which describes the direction of the wave vector \vec{k} in the absorbing medium, and is shown in Fig. 1, appear in Snell's law. The phase velocity in the absorbing medium is simply connected to the angle-dependent optical constants; so is the attenuation of the electric field amplitude when formulated as a function of the real direction z normal to the surface of the absorbing medium (Table IX). This fact has been a reason for the widespread, often unrecognized, application of this definition in the literature. On the other hand, the square of the angle-independent refractive index is simply related to magnetic permeability μ , dielectric constant ϵ and conductivity σ , as shown in Table IX.

A strong preference is expressed here for the use of the angle-independent definition of the complex refractive index. Although this definition entails mathematical complications in some parts of the theoretical derivations, it retains the physical meaning of the refractive index as a material constant rather than being a device of mathematical expedience and is, for a given frequency, independent of the way in which it is determined.

As indicated in Table IX, the literature is divided in applying the two alternatives. Indirect evidence for the simultaneous use of both can be found in many references. Discussions of the two alternatives are presented by Ditchburn, Koenig, Vasicek and Valasek.

Numerical examples of the interrelationship between the two sets of optical constants are given in Figures 3 and 4. They show that the dependence of the refractive index n' on angle of incidence is strong only for small values of n and large values of κ , while the angle-dependence

of the absorption index κ' is small only for high values of n . It can be seen that for a material like silver n' changes by about a factor of ten, κ' by a factor of two between normal and grazing incidence.

A question of nomenclature arises in the formulation of the amplitude attenuation

$$E = E_0 e^{-\alpha z}$$

given in Table IX. The exponent contains the coordinate z normal to the surface of the absorbing medium and the quantity

$$\alpha = \frac{2\pi n' \kappa'}{\lambda_0}$$

which should be called the "amplitude absorption coefficient." Its reciprocal, the "amplitude penetration depth," is a physically well recognizable measure of light absorption. (λ_0 designates the wavelength in vacuum.) Most of the formulations of light absorption found in the literature are given in terms of intensity rather than amplitude

$$I = I_0 e^{-2\alpha z}$$

As indicated in Table XI, the quantity

$$2\alpha = \frac{4\pi n' \kappa'}{\lambda_0}$$

usually identified by " α ", is called absorption coefficient. To avoid any ambiguity, it should be named "intensity absorption coefficient." Correspondingly, its reciprocal is the "intensity penetration depth."

Summary⁸

The student of ellipsometry is confronted with a confusing multiplicity of conventions and definitions employed in the classical papers on the subject, and this problem persists in the present literature. In an effort to untangle this situation, nine conventions and definitions have been singled out, where arbitrary choices between two alternatives have to be made in the theory of ellipsometry. Since some of these choices have the same or opposite effects, not all possible combinations lead to different results. Nevertheless, according to this analysis, reflection from any given surface can be represented by 8 numerically different combinations (including sign) of ellipsometer parameters ψ and Δ . Conversely, from any measured set of ψ and Δ values, 16 different combinations of optical constants, which are all valid within the respective framework, can be derived.

For the electric field strength⁴ formulated as

$$\vec{E} = E_0 e^{-i(\omega t + \delta)} e^{i \vec{k} \cdot \vec{r}} e^{-\alpha z}$$

the complex relative amplitude attenuation

$$\rho \equiv \frac{r_s}{r_p}$$

can be formulated as⁵

$$\rho = \frac{\frac{|E_s''|}{|E_s|} e^{-i\delta_s}}{\frac{|E_p''|}{|E_p|} e^{-i\delta_p}}$$

This expression can be re-written in the form

$$\rho = \frac{\frac{|E_s''|}{|E_p''|}}{\frac{|E_s|}{|E_p|}} e^{+i(\delta_p - \delta_s)}$$

or⁶

$$\rho = \frac{\tan \psi_r}{\tan \psi_i} e^{i\Delta}$$

$$\rho = \tan \psi e^{i\Delta}$$

Thus, the preferred formulations of the two ellipsometer parameters are

$$\tan \psi = \frac{|r_s|}{|r_p|}$$

$$\Delta = \delta_p - \delta_s$$

The coordinate system is preferred as given in Fig. 1. The complex refractive index of an absorbing medium is formulated as

$$n_c \equiv n(1 + i\kappa)$$

and defined to be independent of angle of incidence.

The preferred nomenclature is

$\tan \psi$ - relative amplitude attenuation

Δ - relative phase change

n - refractive index

κ - absorption index

$k \equiv n\kappa$ - extinction index

Although the general use of a standardized set of definitions and conventions may not be feasible, it is hoped that this discussion leads to a greater appreciation of the problem, and results in the more frequent statement of the arbitrary choices made by authors. Thus, communication within a growing technical community would be greatly facilitated.

| Jonathan Z. Shohar has assisted in the literature search for this study.

Modification of Preferences

Contributions to the discussion of this paper, particularly by Dr. H. E. Bennett* and Professor F. Abeles**, have convinced the author to adopt the positive exponential for the time factor (alternative 2, Table IV) and the unfactored form for the complex refractive index (alternative 2, Table VIII). As a consequence of this change, several other choices have to be reversed, so that the modified set of preferred definitions and conventions is given by

Tables	I	II	III	IV	V	VI	VII	VIII
Alternatives	2	1	1	2	3	1	1	2

The summary, to take the place of the one given in the paper, can therefore be restated as follows: For the electric field strength formulated as:

$$\vec{E} = E_0 e^{i(\omega t + \delta)} e^{i\vec{k} \cdot \vec{r}} e^{-\alpha z}$$

The complex relative amplitude attenuation is:

$$\rho \equiv \frac{r_p}{r_s} = \frac{\frac{|E_p''|}{|E_s''|}}{\frac{|E_p|}{|E_s|}} e^{i(\delta_p - \delta_s)} = \tan \psi e^{i\Delta}$$

where

$$\tan \psi = \frac{|r_p|}{|r_s|}$$

$$\Delta = \delta_p - \delta_s$$

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The preferred coordinate system is shown in Fig. 1. The complex refractive index, defined to be independent of angle of incidence, is preferably formulated as

$$n_c \equiv n - ik$$

Table I. Formal Definition of Relative Amplitude Parameter ψ

Alternatives:	$\tan \psi = \frac{r_s}{r_p}$	$\tan \psi = \frac{r_p}{r_s}$
Effect on ψ	ψ	$90^\circ - \psi$
Bare surface values:	$45^\circ \leq \psi \leq 90^\circ$	$0^\circ \leq \psi \leq 45^\circ$
Authors:	Born and Wolf p. 617	Abelès p. 46
	Fry p. 22	Archer p. 10
	Mayer p. 90	Born p. 262
		Drude p. 286
		(Foersterling p. 115)
		Hall p. 911
		Heavens p. 198
		Holmes and Feucht p. 467
		Jenkins and White p. 523
		Koenig p. 226
		Laue p. 162
		Leberknight and Lustman p. 62
		McCrackin and Colson p. 62
		Pohl p. 154
		Rothen p. 1057
		Tronstad p. 19
		Valasek p. 251
		Vasicek p. 289
		Winterbottom p. 16
		Wood p. 552

Table II. Formal Definition of Relative Phase Parameter Δ

Alternatives:	$\Delta = \delta_p - \delta_s$	$\Delta = \delta_s - \delta_p$
Effect on Δ :	Δ	$-\Delta$
Bare surface values:	$0 \leq \Delta \leq +180^\circ$	$0 \geq \Delta \geq -180^\circ$
Authors:	Abeles p. 46	(Fry p. 22)
	Archer p. 10	Mayer p. 90
	Born and Wolf p. 617	
	Drude p. 292	
	(Foersterling p. 119)	
	Hall p. 911	
	(Heavens p. 198)	
	(Holmes and Feucht p. 467)	
	Jenkins and White p. 524	
	Koenig p. 222	
	Laue p. 162	
	Leberknight and Lustman p. 62	
	Rothen p. 1057	
	Tronstad p. 19	
	Valasek p. 251	
	Vasicek p. 289	
	Winterbottom p. 16	
	Wood p. 552	

Table III. Formal Definition of Complex Relative Amplitude Attenuation

Alternatives:	$\rho = \tan \psi e^{i\Delta}$	$\rho = \tan \psi e^{-i\Delta}$
Effect on Δ :	Δ	$-\Delta$
Authors:	Abelès p. 46	Born and Wolf p. 617
	Archer p. 10	
	Born p. 262	
	Drude p. 292	
	Foersterling p. 129	
	Fry p. 22	
	Hall p. 912	
	Heavens p. 198	
	Holmes and Feucht p. 467	
	Koenig p. 241	
	Laue p. 162	
	(Leberknight and Lustman p. 62)	
	Mayer p. 90	
	McCrackin and Colson p. 62	
	Pohl p. 154	
	Tronstad p. 19	
	Valasek p. 251	
	Vasicek p. 289	
	Winterbottom p. 16	
	Wood p. 552	

Table IV. Time Dependence Factor

Alternatives:	$e^{-i\omega t}$	$e^{+i\omega t}$
Effect on Complex Refractive Index:	$n(1 + i\kappa)$	$n(1 - i\kappa)$
Authors:	Born and Wolf p. 24 Fry p. 143 Stone p. 35 (Winterbottom p. 15)	Abelès p. 42 (Archer p. 11) Born p. 21 Condon p. 6-7 Ditchburn p. 40 Drude p. 299 Foersterling p. 75 Françon p. 387 (Hall p. 911) (Heavens p. 198) Koenig p. 153 Laue p. 161 (Leberknight and Lustman p. 66) Mayer p. 125 (McCrackin and Colson p. 64) Pohl p. 149 Rossi p. 368 Valasek p. 243 Vasicek p. 15 Wood p. 416

Table V. Absolute Phase

Alternatives:	$e^{-i(\omega t + \delta)}$	$e^{-i(\omega t - \delta)}$	$e^{+i(\omega t + \delta)}$	$e^{+i(\omega t - \delta)}$
Effect on Δ :	Δ	$-\Delta$	Δ	$-\Delta$
Authors:	Born and Wolf p. 24	Stone p.35	Born p.21 Ditchburn p.40 (Drude p.292) Foersterling p.75 Koenig p.153 (Mayer p.140) (Pohl p.126) (Rossi p.269) Valasek p.219 Vasicek p.15 Wood p.416	Francon p.393

Table VI. Coordinate System for p-Component

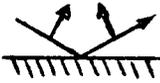
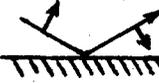
Alternatives:		
Effect on	Δ	$\Delta \pm 180^\circ$
Effect on Fresnel Reflection Coeff. r_p :	$+\frac{\tan(\phi-\phi')}{\tan(\phi+\phi')}$	$-\frac{\tan(\phi-\phi')}{\tan(\phi+\phi')}$
Authors:	Archer p. 2	(Condon p.6-8)
	(Bashara and Peterson p. 1329)	Francon p. 391
	Born p. 27	(Koenig p. 211)
	(Born and Wolf p. 40)	Rossi p. 373
	Ditchburn p. 533	(Stone p. 397)
	Drude p. 280	Valasek p. 199
	(Foersterling p. 114)	
	Heavens p. 196	
	Holmes and Feucht p. 467	
	Jenkins and White p. 529	
	(Laue p. 129)	
	Leberknight and Lustman p. 66	
	(Mayer p. 137)	
	(McCrackin and Colson p. 61)	
	Pohl p. 147	
	(Tronstad p. 17)	
	Vasicek p. 27	
	Winterbottom p. 15	
	Wood p. 411	

Table VII. Coordinate System for s-Component

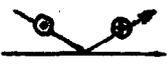
Alternatives:		
Effect on Δ :	Δ	$\Delta \pm 180^\circ$
Effect on Fresnel Reflection Coeff. r_s :	$-\frac{\sin(\phi - \phi')}{\sin(\phi + \phi')}$	$+\frac{\sin(\phi - \phi')}{\sin(\phi + \phi')}$
Authors:	(Archer p. 11)	Winterbottom p. 15
	(Born p. 29)	
	(Born and Wolf p. 40)	
	(Condon p. 6-8)	
	(Ditchburn p. 535)	
	(Drude p. 282)	
	(Foersterling p. 114)	
	Francon p. 393	
	Heavens p. 196	
	Holmes and Feucht p. 467	
	Jenkins and White p. 529	
	Koenig p. 211	
	(Laue p. 129)	
	(Mayer p. 137)	
	(McCrackin and Colson p. 61)	
	(Pohl p. 144)	
	Rossi p. 373	
	Stone p. 392	
	(Tronstad p. 17)	
	Valasek p. 199	
	Vasicek p. 27	
	(Wood p. 410)	

Table VIII. Formal Definition of Complex Refractive Index n_c

Alternatives:	$n(1 \pm ik)$	$n \pm ik$
Authors:	Born	p. 260
	Born and Wolf	p. 613
	Condon	p. 6-7
	Ditchburn	p. 551
	Drude	p. 363
	Foersterling	p. 128
	Hall	p. 911
	Jenkins and White	p. 533
	Koenig	p. 241
	Laue	p. 160
	Leberknight and Lustman	p. 64
	Mayer	p. 126
	Rossi	p. 408
	Tronstad	p. 20
	Valasek	p. 244
	(Vasicek	p. 287)
	Winterbottom	p. 15
	(Wood	p. 546)
	Abelès	p. 50
	Archer	p. 11
	Bashara and Peterson	p. 1329
	Françon	p. 387
	Fry	p. 23
	Heavens	p. 198
	McCrackin and Colson	p. 71
	Pohl	p. 150
	Stone	p. 372

Table IX. Angle of Incidence Dependence of Refractive Index

Alternatives:	independent	dependent
Designation:	$n(1 + i\kappa)$	$n'(1 + i\kappa')$
Relationships:	$n^2(1 - \kappa^2) = n'^2(1 - \kappa'^2)$ $n^2 \kappa = n'^2 \kappa' \cos \phi'$	real
Snell:	$n_o \sin \phi = n(1 + i\kappa) \sin \phi'$ complex	$n_o \sin \phi = n' \sin \phi'$ real
Phase Velocity:		$v = \frac{c}{n'}$
Amplitude Attenuation:		$E \sim E_o e^{-\frac{2\pi n' \kappa' z}{\lambda_o}}$
Dielectric Constant:	$n^2 + i2n^2\kappa - n^2\kappa^2 = \mu(\epsilon + i \frac{4\pi \sigma}{\omega})$	
Authors:	(Born p. 260) (Born and Wolf p. 613) (Condon p. 6-9) Ditchburn p. 590 (Drude p. 361) (McCrackin and Colson p. 61) (Wood p. 546)	Hall p. 911 Koenig p. 207 Valasek p. 245 Vasicek p. 290 (Winterbottom p. 15)

Table X. Nomenclature for Optical Constants

Authors:	Names For:		
	n	κ	$k \equiv n\kappa$
Archer	p. 11		ext. coeff.
Bashara and Peterson	p. 1329	{real part of refr. index	abs. coeff.
Born	p. 260	abs. index	
Born and Wolf	p. 613	att. index	
Condon	p. 6-7	abs. index	
Ditchburn	p. 551	ext. coeff.	
Drude	p. 360	coeff. of abs.	
Françon	p. 387	refr. index	ext. index
Fry	p. 23		ext. coeff.
Hall	p. 911	refr. index	abs. coeff.
Jenkins and White	p. 533	refr. index	abs. index
Koenig	p. 189	abs. index	ext. modul
Laue	p. 160	refr. index	ext. coeff.
Mayer	p. 127		abs. index
McCrackin and Colson	p. 79	{real part of refr. index	{imag. part of refr. index
Stone	p. 372	refr. index	ext. index
Tronstad	p. 18	refr. index	abs. index
Vasicek	p. 286		index of abs.
Winterbottom	p. 15	refr. index	ext. coeff.
Wood	p. 546	refr. index	abs. index

Table XI. Nomenclature for Light Attenuation

	$I \sim I_0 e^{-2\alpha z}$	Names For: $2\alpha = \frac{4\pi n'k'}{\lambda_0}$
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Author:	Archer	p. 11	abs. coeff.
	Born and Wolf	p. 614	abs. coeff
	Ditchburn	p. 551	abs. coeff.
	Koenig	p. 189	abs. coeff
	Mayer	p. 127	abs. coeff.
	Rossi	p. 409	abs. coeff.

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FOOTNOTES

1. Defined as the plane which contains incident and reflected beams or wave vectors.
2. Phase of the reflected with respect to the incident wave in the reflecting surface.
3. Real refractive index of dielectric - n_0 , complex refractive index of metal $n(1 + ik)$ or $n'(1 + ik')$. Angle of incidence ϕ (real), angle of refraction ϕ' (complex or real, depending on choice of complex refractive index).
4. E_0 - reference amplitude, i - imaginary unit, ω - angular frequency, δ - absolute phase (see footnote 2), \vec{k} - wave vector, \vec{r} - position vector, α - amplitude absorption coefficient, z - inward normal to reflecting surface.
5. $|E|$ and $|E''|$ - electric field amplitude in incident and reflected wave, respectively.
6. $\tan \psi_i$ and $\tan \psi_r$ - relative amplitudes of s and p components in incident and reflected wave, respectively.
7. As a result of the discussion, the author's preference has been modified to alternative 2 in Tables I, IV and VIII and to alternative 3 in Table V.
8. See end of discussion for a summary of the modified preferences.

FIGURE CAPTIONS

- Fig. 1 Reflection and refraction at a dielectric-metal interface, designation of symbols. E , E' and E'' -positive coordinate direction of electric field amplitude in incident, refracted and reflected waves for components parallel (subscript p) and normal (subscript s) to the plane of incidence. The circles represent arrows pointing out of the plane of the drawing. k , k' and k'' - propagation direction of the three waves. ϕ - angle of incidence, ϕ_r - real angle of refraction, n_o - refractive index of incident medium, n_c or n'_c - refractive index of reflecting medium, z - coordinate direction for amplitude attenuation.
- Fig. 2 Dependence of elliptic polarization on phase difference Δ between orthogonal linearly polarized components of amplitude A and B.
- Fig. 3 Effect of the angle of incidence ϕ on the angle-dependent refractive index n' for different combinations of the angle-independent optical constants n and κ .
- Fig. 4 Effect of the angle of incidence ϕ on the angle-dependent absorption index κ' for different combinations of the angle-independent optical constants n and κ .

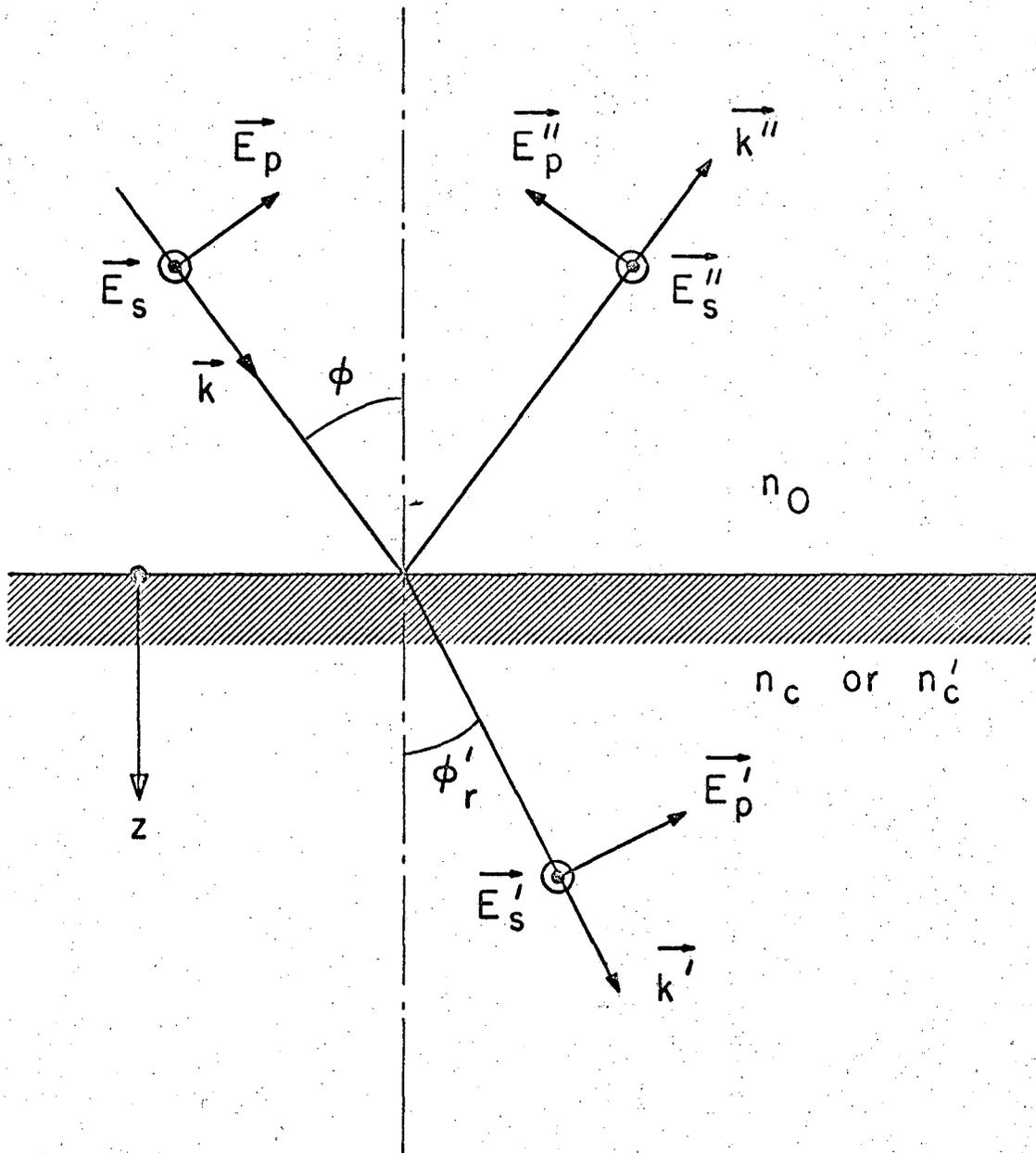
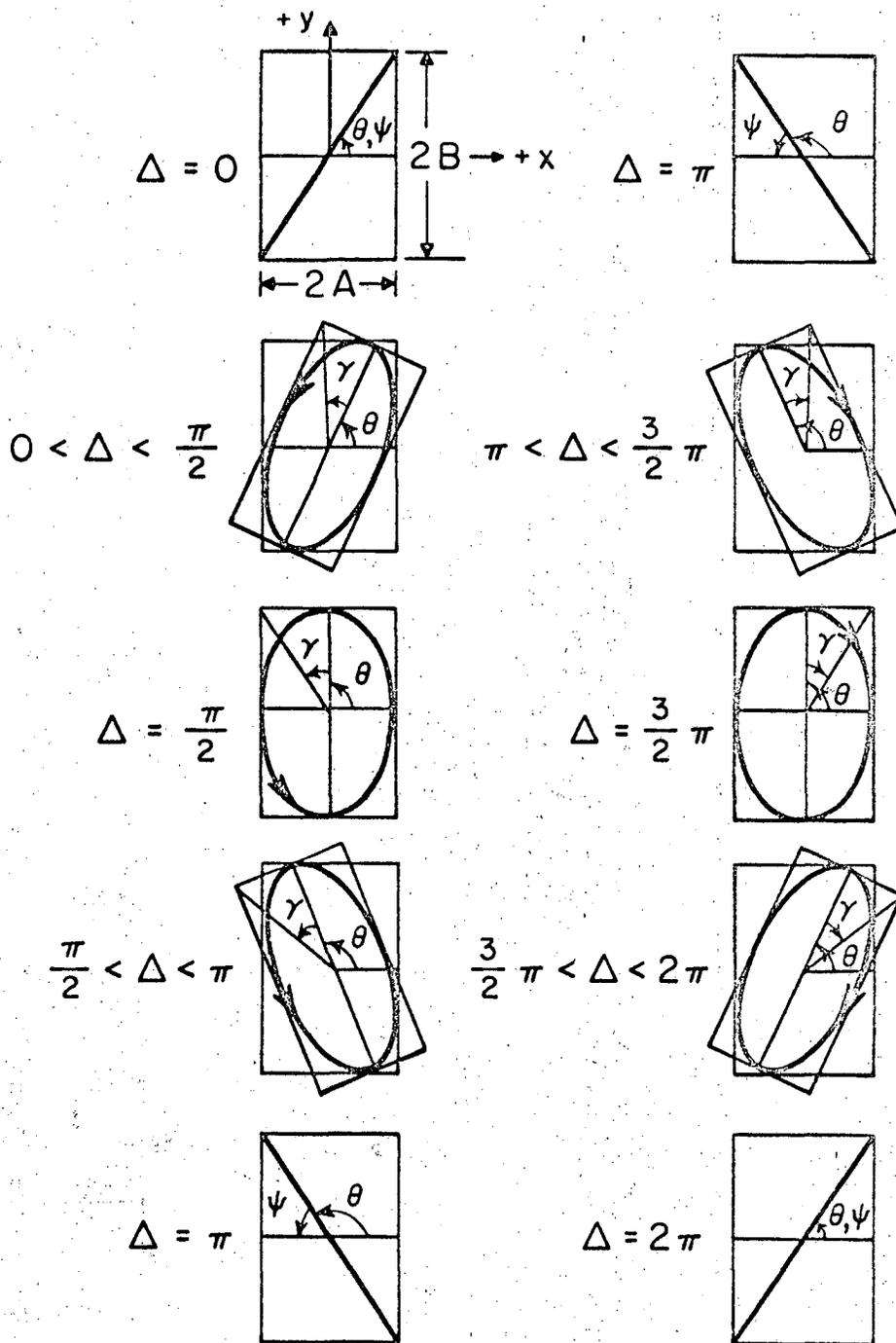


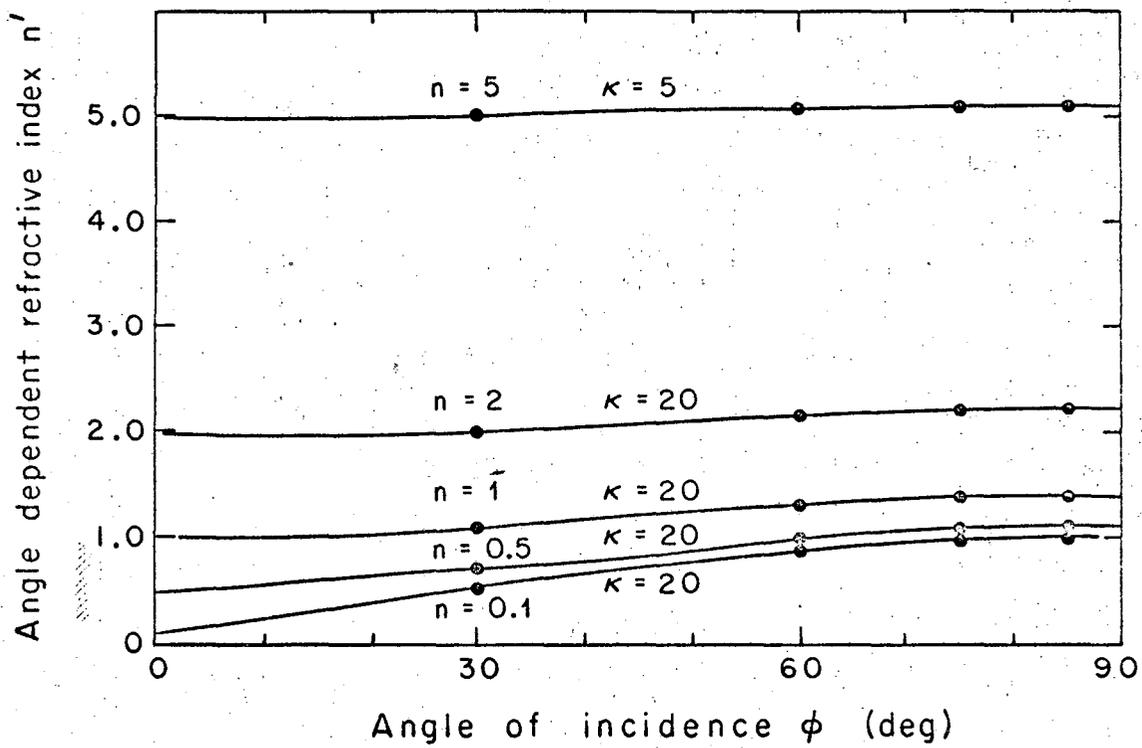
Fig. 1

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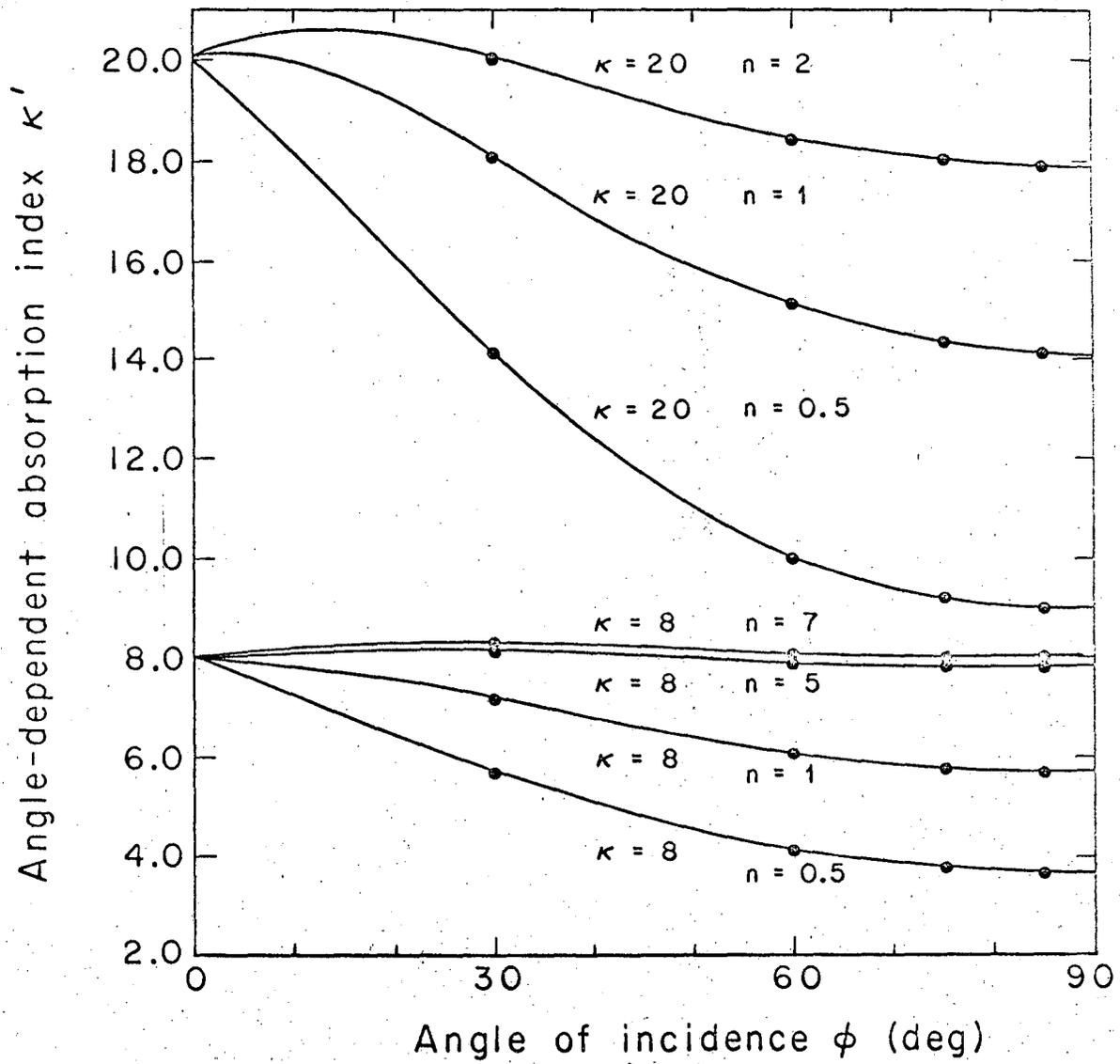
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Fig. 2



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Fig. 3



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Fig. 4

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