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November 20, 1968

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ABSTRACT

In this work we demonstrate that the basic equations of the Landau hydrodynamics may be derived directly from the quantum mechanical microscopic N-body Hamiltonian, which is expressed as a functional of the local observables, namely particle and current densities. The Landau quantum hydrodynamic equations are then the Heisenberg equations of motion of these quantities.

I. INTRODUCTION

As the experimental[1] and theoretical[2] picture of liquid helium becomes less diffuse, it appears that the studies of Landau towards a quantum mechanical theory of hydrodynamics[3] are "...germane and fundamental".[2] Initially the Landau quantization of hydrodynamics stimulated a number of attempts at extension. These studies began with varying forms of classical hydrodynamics expressed in terms of velocity potentials.[4] The formalism of canonical quantization was then applied to the classical theory of fluids. These theories, due to their highly nonlinear character, give rise to cut-off problems.

It is the purpose of this work to demonstrate that the basic equations of the Landau hydrodynamics may be derived directly from the quantum mechanical microscopic N-body Hamiltonian, thus avoiding the difficult and physically uncertain quantization of the classical theory of fluids.

Let us review briefly the Landau construction of quantum hydrodynamics[5] for a Bose fluid. Since the natural variables for describing a liquid are densities and currents, Landau constructs these quantities as

$$\rho(\underset{\sim}{x}) = \sum_{i=1}^{N} m_i \, \delta(\underset{\sim}{x} - \underset{\sim}{x}_i) \tag{1.1}$$

and

$$\underset{\sim}{J}(\underset{\sim}{x}) = \frac{1}{2i} \sum_{i=1}^{N} \left[\underset{\sim}{\nabla}_i \, \delta(\underset{\sim}{x} - \underset{\sim}{x}_i) + \delta(\underset{\sim}{x} - \underset{\sim}{x}_i) \underset{\sim}{\nabla}_i \right] . \tag{1.2}$$

Now the classical hydrodynamic description of the liquid provides an expression for the energy of the macroscopic movement of the liquid. The energy density of the system is $\frac{1}{2}\rho v^2 + \rho\,\epsilon[\rho]$, where $\epsilon[\rho]$ is the internal energy of a unit mass of the liquid, which is supposed to depend only on the density ρ of the liquid. This classical expression provides Landau with the quasimacroscopic Hamiltonian of his theory,

$$H = \int d^3x \left\{ \frac{1}{2}\, \underset{\sim}{v}(\underset{\sim}{x})\, \rho(\underset{\sim}{x})\, \underset{\sim}{v}(\underset{\sim}{x}) + \rho(\underset{\sim}{x})\, \epsilon[\rho(\underset{\sim}{x})] \right\} . \tag{1.3}$$

However, the representation of this expression as an operator necessitates the definition of a velocity operator $\underset{\sim}{v}(\underset{\sim}{x})$.[6] The definition of the classical velocity, $\underset{\sim}{v}(\underset{\sim}{x}) = \underset{\sim}{J}(\underset{\sim}{x})/\rho(\underset{\sim}{x})$, suggests a quantum mechanical representation of the velocity operator as

$$\underset{\sim}{v}(\underset{\sim}{x}) = \frac{1}{2}\left[\frac{1}{\rho(\underset{\sim}{x})}\, \underset{\sim}{J}(\underset{\sim}{x}) + \underset{\sim}{J}(\underset{\sim}{x})\, \frac{1}{\rho(\underset{\sim}{x})} \right] . \tag{1.4}$$

From the known commutation relations of the fields $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$, one obtains the commutation relations for the components of this velocity operator,

$$\left[v_i(\underset{\sim}{x}),\ v_j(\underset{\sim}{y}) \right] = i\, \delta(\underset{\sim}{x} - \underset{\sim}{y})\, \frac{1}{\rho(\underset{\sim}{x})} \left(\frac{\partial}{\partial x_i}\, v_j(\underset{\sim}{x}) - \frac{\partial}{\partial x_j}\, v_i(\underset{\sim}{x}) \right) . \tag{1.5}$$

The Landau quantum hydrodynamics then consists of the Heisenberg equations of motion for $\rho(\underset{\sim}{x})$ and $\underset{\sim}{v}(\underset{\sim}{x})$ with the given Hamiltonian (1.3).

In our derivation of the quantum hydrodynamic equations we start directly from the Hamiltonian describing N identical interacting particles of the Bose type, which can be written in the language of

second quantization in terms of creation and annihilation operators satisfying canonical commutation relations. However, we prefer not to describe quantum liquids with these canonical fields as the set of quantum mechanical "coordinates." Rather, following Landau, we will give a complete description of quantum liquids in terms of observable quantities only. The natural variables for this description are the particle and current densities $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$, respectively. These variables are not canonical coordinates.

The idea of formulating theories in terms of observable current densities has recently become quite attractive in strong-interaction physics.[7] There the weak and electromagnetic hadron current densities are local observables and one supposes that these currents, which treat all strongly interacting particles (hadrons) on an equal footing, are useful coordinates in a dynamical theory of hadronic matter. The formulation of dynamical theories in terms of local observables[8] is not new and is implied by the formulation of field theory in terms of "rings of local observables."[9] The local observables are just the objects which we are working with, namely, current and density operators. These are the local fields in the sense of abstract field theory.

II. N-BODY HAMILTONIAN IN TERMS OF DENSITIES AND CURRENTS

We consider a system of N identical spinless bosons interacting through a local two-body potential which is described by the Hamiltonian

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i<j}^{N} V_{ij} . \tag{2.1}$$

The expression for the Hamiltonian in the language of second quantization is given by

$$H = \frac{1}{2m} \int d^3x \, \underset{\sim}{\nabla} \psi^+(\underset{\sim}{x}) \, \underset{\sim}{\nabla} \psi(\underset{\sim}{x}) + \frac{1}{2} \iint d^3x \, d^3y \, \psi^+(\underset{\sim}{x}) \, \psi^+(\underset{\sim}{y}) \, V(|\underset{\sim}{x}-\underset{\sim}{y}|) \, \psi(\underset{\sim}{x}) \, \psi(\underset{\sim}{y}) , \tag{2.2}$$

where the field operators $\psi^+(\underset{\sim}{x})$ and $\psi(\underset{\sim}{x})$ satisfy the following canonical commutation relations:

$$\begin{aligned} [\psi^+(\underset{\sim}{x}), \psi^+(\underset{\sim}{y})] &= [\psi(\underset{\sim}{x}), \psi(\underset{\sim}{y})] = 0 , \\ [\psi(\underset{\sim}{x}), \psi^+(\underset{\sim}{y})] &= \delta(\underset{\sim}{x} - \underset{\sim}{y}) . \end{aligned} \tag{2.3}$$

Since we do not wish to describe quantum liquids with these nonobservable canonical fields as a set of quantum mechanical "coordinates," we rewrite the Hamiltonian (2.2) in terms of observable quantities only. These are the density $\rho(\underset{\sim}{x})$ and the current $\underset{\sim}{J}(\underset{\sim}{x})$. In the second-quantized formalism, these operators are given by

$$\begin{aligned} \rho(\underset{\sim}{x}) &= m \, \psi^+(\underset{\sim}{x}) \, \psi(\underset{\sim}{x}) , \\ \underset{\sim}{J}(\underset{\sim}{x}) &= \frac{1}{2i} [\psi^+(\underset{\sim}{x}) \, \underset{\sim}{\nabla} \psi(\underset{\sim}{x}) - \underset{\sim}{\nabla} \psi^+(\underset{\sim}{x}) \, \psi(\underset{\sim}{x})] . \end{aligned} \tag{2.4}$$

The physical interpretation of $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$ is clear: they represent particle or mass density and flux. From the commutation relations (2.3) of the fields $\psi^{+}(\underset{\sim}{x})$ and $\psi(\underset{\sim}{x})$ one may determine the algebra satisfied by $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$:

$$[\rho(\underset{\sim}{x}), \rho(\underset{\sim}{y})] = 0 ,$$

$$[\rho(\underset{\sim}{x}), J_i(\underset{\sim}{y})] = -i \frac{\partial}{\partial x_i} [\delta(\underset{\sim}{x} - \underset{\sim}{y}) \rho(\underset{\sim}{x})] ,$$

$$[J_i(\underset{\sim}{x}), J_j(\underset{\sim}{y})] = -i \frac{\partial}{\partial x_j} [\delta(\underset{\sim}{x} - \underset{\sim}{y}) J_i(\underset{\sim}{x})] + i \frac{\partial}{\partial y_i} [\delta(\underset{\sim}{x} - \underset{\sim}{y}) J_j(\underset{\sim}{x})] . \tag{2.5}$$

One can easily show that, according to Schur's lemma, $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$ represent a complete set of "coordinates."[10] This result tells us that the states of our system of N identical particles span a single irreducible representation of the algebra.[11] We consider the states, which form a basis in the Hilbert space, as being the complete set of eigenvectors associated with a maximal commuting set of density and current operators, i.e., in our case $\rho(\underset{\sim}{x})$. Thus we choose the following set of eigenstates of ρ :

$$\rho(\underset{\sim}{x}) \, |\underset{\sim}{x}_1 \cdots \underset{\sim}{x}_N\rangle = m \sum_{i=1}^{N} \delta(\underset{\sim}{x} - \underset{\sim}{x}_i) \, |\underset{\sim}{x}_1 \cdots \underset{\sim}{x}_N\rangle . \tag{2.6}$$

Moreover the irreducibility of the "coordinates" $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$ implies that every operator is a function of $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$. For example, the total momentum $\underset{\sim}{P}$ and the total angular momentum $\underset{\sim}{L}$ are given by

$$\underset{\sim}{P} = \int d^3x \, \underset{\sim}{J}(\underset{\sim}{x}) \, ,$$
$$\underset{\sim}{L} = \int d^3x \, \underset{\sim}{x} \times \underset{\sim}{J}(\underset{\sim}{x}) \, . \tag{2.7}$$

With the help of the identities

$$\underset{\sim}{\nabla} \rho(\underset{\sim}{x}) + 2i \, \underset{\sim}{J}(\underset{\sim}{x}) = 2\psi^+(\underset{\sim}{x}) \underset{\sim}{\nabla} \psi(\underset{\sim}{x}) \, ,$$

$$\underset{\sim}{\nabla} \rho(\underset{\sim}{x}) - 2i \, \underset{\sim}{J}(\underset{\sim}{x}) = 2\underset{\sim}{\nabla} \psi^+(\underset{\sim}{x}) \, \psi(\underset{\sim}{x}) \, ,$$

one obtains the following form for the Hamiltonian (2.2) expressed in terms of the particle density and flux operators:

$$H = \frac{1}{8} \int d^3x \, [\underset{\sim}{\nabla} \rho(\underset{\sim}{x}) - 2i \, \underset{\sim}{J}(\underset{\sim}{x})] \, \frac{1}{\rho(\underset{\sim}{x})} \, [\underset{\sim}{\nabla} \rho(\underset{\sim}{x}) + 2i \, \underset{\sim}{J}(\underset{\sim}{x})]$$
$$+ \frac{1}{2} \iint d^3x \, d^3y \, \rho(\underset{\sim}{x}) \, V(|\underset{\sim}{x}-\underset{\sim}{y}|) \, \rho(\underset{\sim}{y}) \, . \tag{2.8}$$

We want to emphasize that although we have derived the form of the Hamiltonian and the commutation relations of the density and current operators from the underlying representation in terms of creation and annihilation operators, their form could also have been deduced from other considerations. The algebra (2.5) is the simplest possible, i.e., no extra derivatives of δ-functions, consistent with the interpretation of $\rho(\underset{\sim}{x})$ as particle density and $\underset{\sim}{P} = \int d^3x \, \underset{\sim}{J}(\underset{\sim}{x})$ as generator of spatial displacements.

If the operators in the Hamiltonian (2.8) are replaced by c-numbers, so that the ordering can be neglected, it reduces to the

quasimacroscopic hydrodynamic Hamiltonian of Landau (1.3), with

$$\rho(\underset{\sim}{x})\ \epsilon[\rho(\underset{\sim}{x})] = \frac{1}{8}\left(\underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right)^2 \frac{1}{\rho(\underset{\sim}{x})} + \frac{1}{2}\int d^3y\ \rho(\underset{\sim}{x})V(|\underset{\sim}{x}-\underset{\sim}{y}|)\ \rho(\underset{\sim}{y}). \tag{2.9}$$

We observe that if one adopts the Landau definition (1.4) for the fluid velocity operator $\underset{\sim}{v}(\underset{\sim}{x})$, one can calculate the commutation relations of its components with the help of the algebra (2.5) and the identity

$$\left[\frac{1}{\rho(\underset{\sim}{x})},\ J_k(\underset{\sim}{y})\right] = \frac{\delta}{\delta\rho}\left(\frac{1}{\rho(\underset{\sim}{x})}\right)[\rho(\underset{\sim}{x}),\ J_k(\underset{\sim}{y})]\ , \tag{2.10}$$

where $\frac{\delta}{\delta\rho}$ denotes the functional derivative. This calculation yields

$$[v_i(\underset{\sim}{x}),\ v_j(\underset{\sim}{y})] = i\ \delta(\underset{\sim}{x}-\underset{\sim}{y})\ \frac{1}{\rho(\underset{\sim}{x})}\left(\frac{\partial}{\partial x_i}v_j(\underset{\sim}{x}) - \frac{\partial}{\partial x_j}v_i(\underset{\sim}{x})\right), \tag{2.11}$$

which is Landau's result (1.5) as well.[12]

III. HEISENBERG EQUATIONS OF MOTION FOR DENSITIES AND CURRENTS

One can now calculate the Heisenberg equations of motion for the observable quantities $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$, which satisfy the algebra (2.5), using the Hamiltonian (2.8). For the particle density ρ we obtain the equation of continuity:

$$\dot{\rho}(\underset{\sim}{x}) = i\,[H,\ \rho(\underset{\sim}{x})] = -\underset{\sim}{\nabla}\,\underset{\sim}{J}(\underset{\sim}{x})\ . \tag{3.1}$$

The equation for $\dot{\underset{\sim}{J}}$ can be calculated in a similar manner by using the identity (2.10). We obtain

$$\begin{aligned}
\dot{\underset{\sim}{J}}(\underset{\sim}{x}) = i[H,\ \underset{\sim}{J}(\underset{\sim}{x})] &= \left(\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right)\int d^3y\ V(|\underset{\sim}{x}-\underset{\sim}{y}|)\ \rho(\underset{\sim}{y}) \\
&+ \sum_k \Bigg[\ \frac{1}{4}\,\frac{1}{\rho(\underset{\sim}{x})}\left(\nabla_k\rho(\underset{\sim}{x})\right)\left(\nabla_k\,\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right) \\
&\quad + \frac{1}{8}\left(\nabla_k\rho(\underset{\sim}{x})\right)^2\left(\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right)\frac{\delta}{\delta\rho}\left(\frac{1}{\rho(\underset{\sim}{x})}\right) + \frac{1}{2}J_k(\underset{\sim}{x})\frac{\delta}{\delta\rho}\left(\frac{1}{\rho(\underset{\sim}{x})}\right)\left(\underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right)J_k(\underset{\sim}{x}) \\
&+ \frac{1}{2}\left\{J_k(\underset{\sim}{x})\,\frac{1}{\rho(\underset{\sim}{x})}\left(\underset{\sim}{\nabla}\,J_k(\underset{\sim}{x}) - \nabla_k\,\underset{\sim}{J}(\underset{\sim}{x})\right) + \left(\underset{\sim}{\nabla}\,J_k(\underset{\sim}{x}) - \nabla_k\underset{\sim}{J}(\underset{\sim}{x})\right)\frac{1}{\rho(\underset{\sim}{x})}\,J_k(\underset{\sim}{x})\right\} \\
&+ \frac{i}{4}\Bigg\{\left(\nabla_k\rho(\underset{\sim}{x})\right)\frac{1}{\rho(\underset{\sim}{x})}\left(\underset{\sim}{\nabla}\,J_k(\underset{\sim}{x})-\nabla_k\underset{\sim}{J}(\underset{\sim}{x})\right) - \left(\underset{\sim}{\nabla}\,J_k(\underset{\sim}{x})-\nabla_k\underset{\sim}{J}(\underset{\sim}{x})\right)\frac{1}{\rho(\underset{\sim}{x})}\left(\nabla_k\rho(\underset{\sim}{x})\right) \\
&\quad + \left(\nabla_k\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right)\frac{1}{\rho(\underset{\sim}{x})}\,J_k(\underset{\sim}{x}) - J_k(\underset{\sim}{x})\,\frac{1}{\rho(\underset{\sim}{x})}\left(\nabla_k\,\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right) \\
&\quad + \left(\nabla_k\rho(\underset{\sim}{x})\right)\frac{\delta}{\delta\rho}\left(\frac{1}{\rho(\underset{\sim}{x})}\right)\left(\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right)J_k(\underset{\sim}{x}) \\
&\qquad - J_k(\underset{\sim}{x})\,\frac{\delta}{\delta\rho}\left(\frac{1}{\rho(\underset{\sim}{x})}\right)\left(\underset{\sim}{\nabla}\,\rho(\underset{\sim}{x})\right)\left(\nabla_k\,\rho(\underset{\sim}{x})\right)\Bigg\}\Bigg]\ .
\end{aligned} \tag{3.2}$$

Looking at the above equation of motion (3.2) for $\dot{\underset{\sim}{J}}$, we see that it involves inverse operators. However, in liquid helium $\rho(\underset{\sim}{x})$ is nearly a constant and one can replace $\rho^{-1}(\underset{\sim}{x})$ in the Hamiltonian (2.8) by the c-number $\rho_0^{-1}(\underset{\sim}{x})$, where $\rho_0(\underset{\sim}{x})$ is the ground-state expectation value of $\rho(\underset{\sim}{x})$. The resulting Hamiltonian is bilinear in $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$ and is therefore much easier to handle. In this approximation the Heisenberg equation of motion for $\rho(\underset{\sim}{x})$ remains unchanged and the Heisenberg equation of motion for $\underset{\sim}{J}(\underset{\sim}{x})$ simplifies to[13]

$$
\begin{aligned}
\dot{\underset{\sim}{J}}(\underset{\sim}{x}) = \left(\underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right) & \int d^3y\, V(|\underset{\sim}{x} - \underset{\sim}{y}|)\, \rho(\underset{\sim}{y}) \\
& + \sum_k \Bigg[\frac{1}{4}\frac{1}{\rho_0(\underset{\sim}{x})} \left(\nabla_k \rho(\underset{\sim}{x})\right)\left(\nabla_k \underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right) \\
& \quad + \frac{1}{2\rho_0(\underset{\sim}{x})} \Big\{ J_k(\underset{\sim}{x})\left(\underset{\sim}{\nabla} J_k(\underset{\sim}{x}) - \nabla_k \underset{\sim}{J}(\underset{\sim}{x})\right) \\
& \qquad + \left(\underset{\sim}{\nabla} J_k(\underset{\sim}{x}) - \nabla_k \underset{\sim}{J}(\underset{\sim}{x})\right) J_k(\underset{\sim}{x}) \Big\} \\
& + \frac{i}{4\rho_0(\underset{\sim}{x})} \Big\{ \left(\nabla_k \rho(\underset{\sim}{x})\right)\left(\underset{\sim}{\nabla} J_k(\underset{\sim}{x}) - \nabla_k \underset{\sim}{J}(\underset{\sim}{x})\right) - \left(\underset{\sim}{\nabla} J_k(\underset{\sim}{x}) - \nabla_k \underset{\sim}{J}(\underset{\sim}{x})\right)\left(\nabla_k \rho(\underset{\sim}{x})\right) \\
& \quad + \left(\nabla_k \underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right) J_k(\underset{\sim}{x}) - J_k(\underset{\sim}{x})\left(\nabla_k \underset{\sim}{\nabla}\rho(\underset{\sim}{x})\right) \Big\} \Bigg] .
\end{aligned}
\tag{3.3}
$$

Finally we would like to point out the equivalence of the Heisenberg equations of motion (3.1) and (3.2) and the algebra (2.5) from the point of view of physical content. Solving the latter means that one has to introduce a functional representation of the algebra (2.5). By this we mean that we specify a basis in Hilbert space consisting of the complete set of eigenvectors associated with a maximal commuting set of density and current operators [see Eq. (2.6) and the remarks above]. Then an arbitrary state $|\phi\rangle$ is represented by giving its components along each of the basis vectors, i.e., by a wave functional. With the states represented in this way the effect of applying any density or current operator to a state $|\phi\rangle$ can be represented either as multiplication of the associated wave functional by a c-number function, or by c-number functional differentiation applied to the wave functional. With the density and current operators realized in this way, the algebra (2.5) is automatically satisfied and the dynamics is contained in the solutions to a functional Schrödinger equation.

IV. DISCUSSION

We have seen that the local, observable density and current operators provide a natural description of the macroscopic behavior of a quantum mechanical many-body system. The Hamiltonian (2.8) together with the Heisenberg equations of motion (3.1) and (3.2) for $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$, or equivalently the functional representation of the algebra (2.5), show an alternative way in which one may view the Landau quantum hydrodynamics. Many attempts to derive the Landau theory, and indeed to formulate quantum mechanical theories of collective motion, have taken up the problem of quantization of classical hydrodynamics. No justification has been given for this procedure.[14] In fact, several of these studies carried out the quantization procedure with the assumption of the existence of a dynamical variable canonically conjugate to the local density operator. This has been shown to lead to contradiction.[15] The representation in terms of noncanonical variables circumvents these difficulties. The Landau theory thus appears as an exact consequence of the introduction of a mathematical velocity operator and the non-canonical representation in terms of density and current operators.

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FOOTNOTES AND REFERENCES

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10. This is due to the fact that the irreducibility of the canonical fields $\psi^+(\underset{\sim}{x})$ and $\psi(\underset{\sim}{x})$ implies irreducibility of $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$.

11. For a brief discussion on how the proper irreducible representation can be determined, see the remarks in Chapter III after Eq. (3.3).

12. In a recent paper by R. Farelli and R. E. Struzynski, Phys. Rev. 173, 248 (1968), it was claimed that the right-hand side of (2.11) vanishes in disagreement with Landau's result (1.5). However, these authors derived their commutation relations for $\rho(\underset{\sim}{x})$ and $\underset{\sim}{J}(\underset{\sim}{x})$ from the canonical commutation relations for $\psi^+(\underset{\sim}{x})$ and $\psi(\underset{\sim}{x})$ incorrectly.

13. We emphasize that the replacement of $\rho^{-1}(\underset{\sim}{x})$ by the ground-state expectation value $\rho_0^{-1}(\underset{\sim}{x})$ leaves the algebra (2.5) unchanged. The substitution occurs only in the Hamiltonian (2.8) and in the quantities expressed in terms of $\rho(\underset{\sim}{x})$, e.g., in the fluid velocity operator (1.4), which now becomes $\underset{\sim}{v}(\underset{\sim}{x}) = \left(1/\rho_0(\underset{\sim}{x})\right)\underset{\sim}{J}(\underset{\sim}{x})$. The commutation relations (2.11) remain unchanged, except with $\rho^{-1}(\underset{\sim}{x})$ replaced by $\rho_0^{-1}(\underset{\sim}{x})$.

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