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December 1968

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ABSTRACT

Experimental evidence is presented for feedback narrowed far infrared response of a point contact Josephson junction which is strongly coupled to a resonant cavity. The observed response shows high sensitivity (NEP $\lesssim 10^{-14}$ W/$\sqrt{\text{Hz}}$) and frequency selectivity ($Q \gtrsim 10^3$). A model is described which relates this narrowed response to the non-linear coupling of the junction current to a resonant cavity.

Point contact junctions have been previously[1] operated as broad-band detectors of millimeter and submillimeter radiation by monitoring the radiation induced changes in the zero voltage current. When such a junction is strongly coupled to a resonant cavity, current steps[2] appear on the I-V characteristic at voltages given by the Josephson relation,[3] $2eV = \hbar\omega_c$, for the resonant frequencies of the cavity, ω_c. We have observed that the height of such current steps is extremely sensitive to externally applied radiation at the center of the cavity mode. We find spectral sensitivities corresponding to a noise equivalent power of less than 10^{-14} watts/$\sqrt{\text{Hz}}$ in a bandwidth of less than 0.01 cm^{-1} at 6 cm^{-1}.

Our experiments were performed using the techniques of Fourier Transform far infrared spectroscopy so that measurements could be made as a function of frequency in the millimeter and submillimeter range. The output of a lamellar grating interferometer with a Hg arc source was conveyed to the cryostat through a 1.1 cm i.d. light pipe. The light pipe terminated in a focusing cone with a 1.6 mm opening on the axis of a cylindrical cavity, typically 4 mm in length and diameter, which was immersed in liquid He at 4.2°K. The point contact junction was made from two Nb wires, positioned along the axis of the cavity, one of which continued through the cone into the light pipe, providing a coaxial coupling lead.

The coupling between the cavity and the Josephson junction produces a current step on the I-V characteristic whenever the ac Josephson frequency equals a cavity resonance frequency. Associated with this step is a high differential resistance region connecting the step

to the rest of the (resistive) I-V characteristic. As the height of the current step changes (due to the applied radiation, which is chopped at 33 cps) the high resistance region follows. The junction is biased with a constant current on the high differential resistance region next to the current step. A change in the step height thus causes a change in the dc voltage bias, which is detected with a lock-in amplifier and recorded as a function of the interferometer path difference.

Fig. 1 shows typical junction characteristics. The upper trace is the differential resistance of the junction. The trace goes to zero at $V_{dc} = 0$ (the familiar zero voltage current) and at ± 393 μv (a cavity mode step). The high differential resistance region shows clearly. The lower trace shows the junction response to broadband radiation (0 - 40 cm^{-1}). The major response occurs at the high resistance regions associated with the current steps. The large signal near $V_{dc} = 0$ is the response to broadband radiation, while just below the 393 μv step narrowband response is seen due to the cavity mode. The strength of this latter response is remarkable since it is sensitive to less than one percent of the rf power contributing to the response near $V_{dc} = 0$.

Fig. 2 shows the measured spectral response of the resonant junction biased near the cavity mode of Fig. 1. The peak response is at the Josephson frequency corresponding to the voltage of the current step and not the actual voltage bias. Further, the shape of the response is not dependent on the particular bias selected within a given high resistance region. Thus, neither the changes in bias voltage with external radiation nor noise in the bias circuit is expected to broaden the response peak.

The shape of the peak shown in Fig. 2 is very nearly the $[\sin 2\pi (v - v_0) \Delta_{max}]/2\pi (v - v_0) \Delta_{max}$ spectral window of the interferometer for the maximum path difference $\Delta_{max} = 5$ cm employed, indicating that the response width was small compared with the 0.2 cm^{-1} instrumental resolution.[4] In this case of a single narrow peak dominating the spectrum, the raw data from the interferometer appear as a damped cosine function of path difference. From a direct measurement of the damping factor we place an upper limit of 0.01 cm^{-1} = 300 MHz on the bandwidth. The complete absence of harmonics on the computed spectrum implies that the response voltage is proportional to the rf power (square law detector).

The spectral power density entering the cavity was estimated from measurements of the interferometer output and attenuation in the coaxial coupler using a calibrated bolometer. Assuming a response bandwidth of .01 cm^{-1} we computed an NEP $\simeq 10^{-14} W/\sqrt{Hz}$. A smaller bandwidth would imply a correspondingly smaller NEP. This is a conservative estimate since it is probable that only a few of the spatial modes entering the cavity actually couple to the point contacts.

The narrow bandwidth, high sensitivity and expected high speed[2] of the observed response indicate the utility of this effect as a practical detector of millimeter and submillimeter radiation. In this regard it should be mentioned that the response peak is stepwise tunable by adjusting the voltage bias to the various cavity modes, or continuously tunable if the cavity size is changed.

The observed behavior of the cavity mode response can be understood by extending the theory of Werthamer[5] and Werthamer and Shapiro[6] to

include the effect of the applied rf power. The resonant cavity is treated as a driven harmonic oscillator:

$$[\frac{d^2}{dt^2} + \gamma \frac{d}{dt} + \omega_c{}^2]\ \Omega(t) = F(t). \qquad (1)$$

Here $\Omega(t) = 2eV(t)/\hbar$ is the normalized voltage across the junction, γ the damping constant, and ω_c the resonant frequency. The cavity is driven by both the Josephson current in the junction and the applied rf power, so

$$F(t) = A \frac{dI}{dt} + B \frac{d}{dt} Z_{rf} \cos(\omega_{rf}t+\phi_{rf}), \qquad (2)$$

where $$I(t) = \cos[\Omega_{dc}t + \int^t \Omega(t')dt']. \qquad (3)$$

Here, A and B are constants measuring the strength of the Josephson current and its coupling to the cavity, and the coupling of the rf into the cavity respectively. $Z_{rf}\cos(\omega_{rf}t+\phi_{rf})$ is the (dimensionless) rf voltage applied.[7] A useful analytical solution can be obtained by assuming a narrow enough resonance that only one frequency component of the Josephson current couples to the cavity. The rf voltage in the cavity is calculated for $\omega \approx \omega_c$ by assuming

$$\int^t \Omega(t')dt' = Z \sin(\omega_{rf}t+\phi) \qquad (4)$$

where Z and ϕ are real. The dc bias is assumed equal to the rf frequency ($\Omega_{dc} = \omega_{rf}$), and Z and ϕ are found by solving (1) at the frequency ω. After some manipulation we obtain

$$[2\Gamma J_1(Z)J_1'(Z)]^2 = [ZJ_1'(Z)(Z + \Gamma_e Z_{rf}\cos(\phi_{rf} - \phi))]^2 + [J_1(Z)(Z\tan\theta + \Gamma_e Z_{rf}\sin(\phi_{rf} - \phi))]^2 \qquad (5)$$

and $\cos\phi = \frac{Z}{2\Gamma J_1(Z)}[Z + \Gamma_e Z_{rf}\cos(\phi_{rf} - \phi)]$ (6)

where $\Gamma = A/\gamma\omega_{rf}$ measures the coupling of the junction current to the resonant mode, $\Gamma_e Z_{rf}$ is the external rf voltage coupled into the cavity, and $\theta = \arctan[(\omega_{rf}^2 - \omega_c^2)/\gamma\omega]$ is the phase difference between the response $\int^t \Omega(t')dt'$ and the driving term (3). $J_1(Z)$ is the first order Bessel function of argument Z. From (3) and (6) we can show the experimentally measured quantity, I_{dc}, to be

$$I_{dc} = \frac{Z}{2\Gamma}(Z + \Gamma_e Z_{rf}\cos(\phi_{rf} - \phi)). \qquad (7)$$

From (5) and (6) we see that both Z and ϕ depend on Z_{rf} and ϕ_{rf}, the amplitude and phase of the applied rf, so I_{dc} depends on the applied rf both explicitly, on Z_{rf}, and implicitly through Z and ϕ.

In order to obtain the weak signal response, we expand I_{dc} in powers of Z_{rf}. Terms linear in Z_{rf} are also linear in $\cos\phi_{rf}$. Over the ~ 1 sec duration of our measurement such terms average to zero except for the negligible amount of radiation within ~ 1 Hz of the feedback frequency. Terms quadratic in Z_{rf} represent square-law response of the detector. The calculated square-law response is plotted in Fig. 3 as a function of tan θ with Γ as a parameter. For $\Gamma = 0$, the response Q is the same as that of the cavity resonance mode. As Γ increases, the response sharpens and is singular for $\Gamma \geq 2.91$ and $\omega_{rf} = \omega_c$ ($\tan\theta = 0$). The singular response arises from a quadratic term proportional to $[J_1'(Z)]^{-3}$ which diverges when the cavity rf voltage saturates at $J_1'(Z) = 0$.

We can understand this singular response qualitatively by considering the dependence of the ac current, I_ω, on Γ when $\omega_{rf} = \omega_c$ and $Z_{rf} = 0$.

From Fig. 4 of Ref. 5 we see that when $\Gamma = 0$, there is no ac voltage, Z, across the junction, and I_ω has its maximum value. As Γ increases, so does Z, and I_ω decreases. When Γ reaches 2.91, Z saturates, and further increase in Γ decreases I_ω without changing Z. That is, a change in I_ω does not change Z. But Z_{rf} adds directly to Z and so I_ω, and consequently I_{dc}, changes discontinuously.

We may consider the analogy to a regenerative receiver whose spectral response is governed by positive feedback through a resonant circuit. The dependence of spectral width on feedback amplitude and the singular response (oscillation) when the loop gain becomes unity are essentially the same as found for coupled Josephson junctions.

The model discussed here is the simplest one which explains the important features of our data. Werthamer and Shapiro have verified the method of solution of Eqs. 1-3 for the case of $Z_{rf} = 0$ by comparing the predicted currents with exact calculations done by analogue computer.

Our model assumes that the dc bias is set equal to the rf frequency, $\Omega_{dc} = \omega_{rf}$. We have investigated the somewhat more general case where $\Omega_{dc} \neq \omega_{rf}$ by adding a second term (with frequency Ω_{dc}) to (4) and solving (1) at ω_{rf} and Ω_{dc}. The effect of the feedback at ω_{rf} is, however, small, and the response of the junction to Z_{rf} is essentially that given in Ref. 6. Feedback is of course important at Ω_{dc} since it generates the dc current step. Only where $\Omega_{dc} = \omega_{rf}$ do we get the narrowed response observed experimentally. A correct calculation would include both the finite impedances of the junction and bias source[8] and the proper bias, which is actually set to the peak in the differential resistance of the junction and not to ω_{rf}.

Observation of the resonant response described above for junctions biased near ω_c requires strongly coupled cavity modes (large Γ) which produce pronounced steps on the I-V characteristic of the junction. In addition to these modes, less strongly coupled modes usually exist which produce no visible steps or weak steps with no observable sensitivity. The existance of even these weak modes, however, strongly modifies the broadband response of the zero-voltage current. This response is computed by setting $\Omega_{dc} = 0$, solving (1) at ω_{rf}, and calculating the zero voltage critical current dependence on Z_{rf}. Since $I_\omega = 0$, there is no junction-cavity feedback, and the voltage in the cavity $Z = \Gamma_e Z_{rf} \cos\theta$. Since the measured quantity for small signals

$$I_{dc} = J_o(Z) \simeq 1 - \frac{1}{4}(\Gamma_e Z_{rf})^2 \cos^2\theta \qquad (8)$$

contains the Lorentzian resonance factor $\cos^2\theta$, the zero voltage response (occuring at ± 100 μv in Fig. 1) measures the power spectrum in the cavity. This consequence of our model[7] is supported by the published data for the spectral response of the zero voltage current which we believe to be dominated by resonances in the surroundings of the junctions even for the case of conical cavities.[1]

In the experiment described here the broadband zero-voltage response showed peaks with $Q \leq 10$ at frequencies of strongly coupled cavity modes. These can be compared with the narrowed response peaks with Q from 50 to our resolution limit of 10^3 when the junction was biased near the corresponding current step. Such response narrowing corresponds to values of $\Gamma > 2.3$.

Evaporated film Josephson junctions are known to show pronounced current steps on their I-V characteristics due to strongly coupling

internal resonant modes.[9] They are expected to show resonant response in the microwave frequency region similar to that described here. The well-known difficulties of coupling radiation to thin film junctions may, in practice, limit their sensitivity.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig. 1(a) Differential resistance of junction plotted against dc voltage bias; arrows indicate the cavity modes.

(b) Far infrared response of junction vs. dc voltage bias.

Fig. 2 Spectral response of cavity mode current step. Response peak at 6.34 cm^{-1}.

Fig. 3 Response line shapes for various values of Γ plotted against $\tan\theta = (\omega_{rf}^2 - \omega_c^2)/\gamma\omega_{rf}$. The response is normalized to unity at $\tan\theta = 0$ and is singular at $\omega = \omega_c$ for $\Gamma \gtrsim 2.9$.

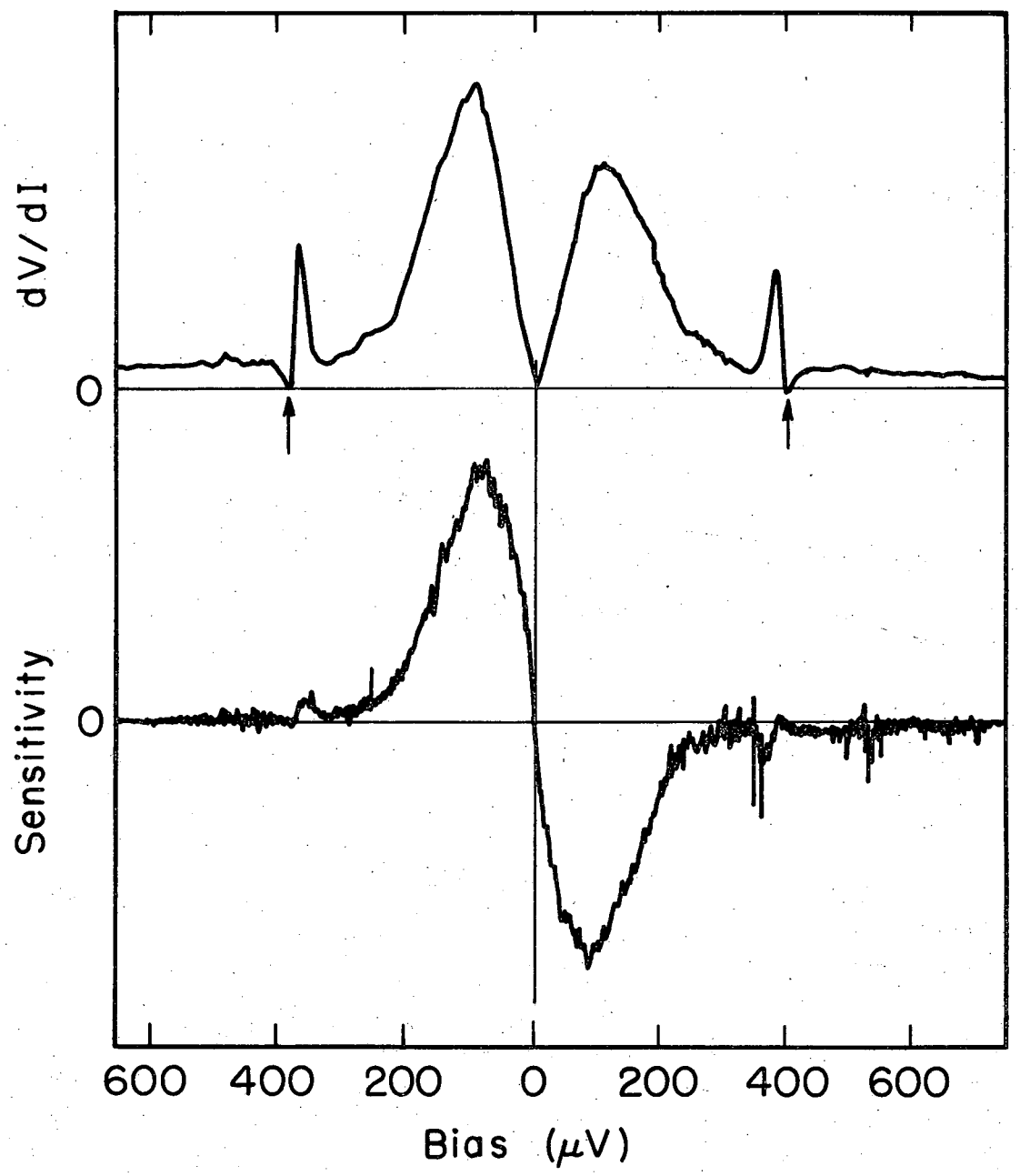

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Fig. 1

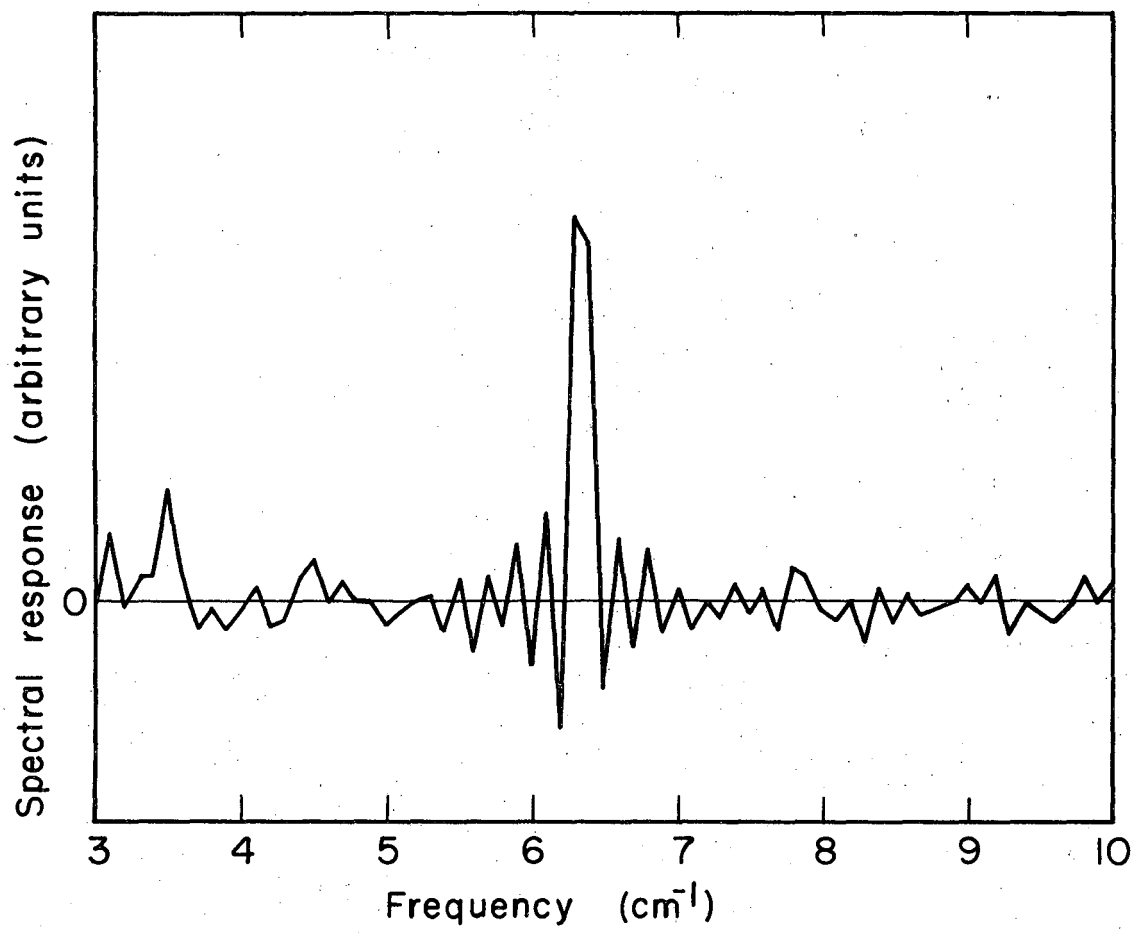

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Fig. 2

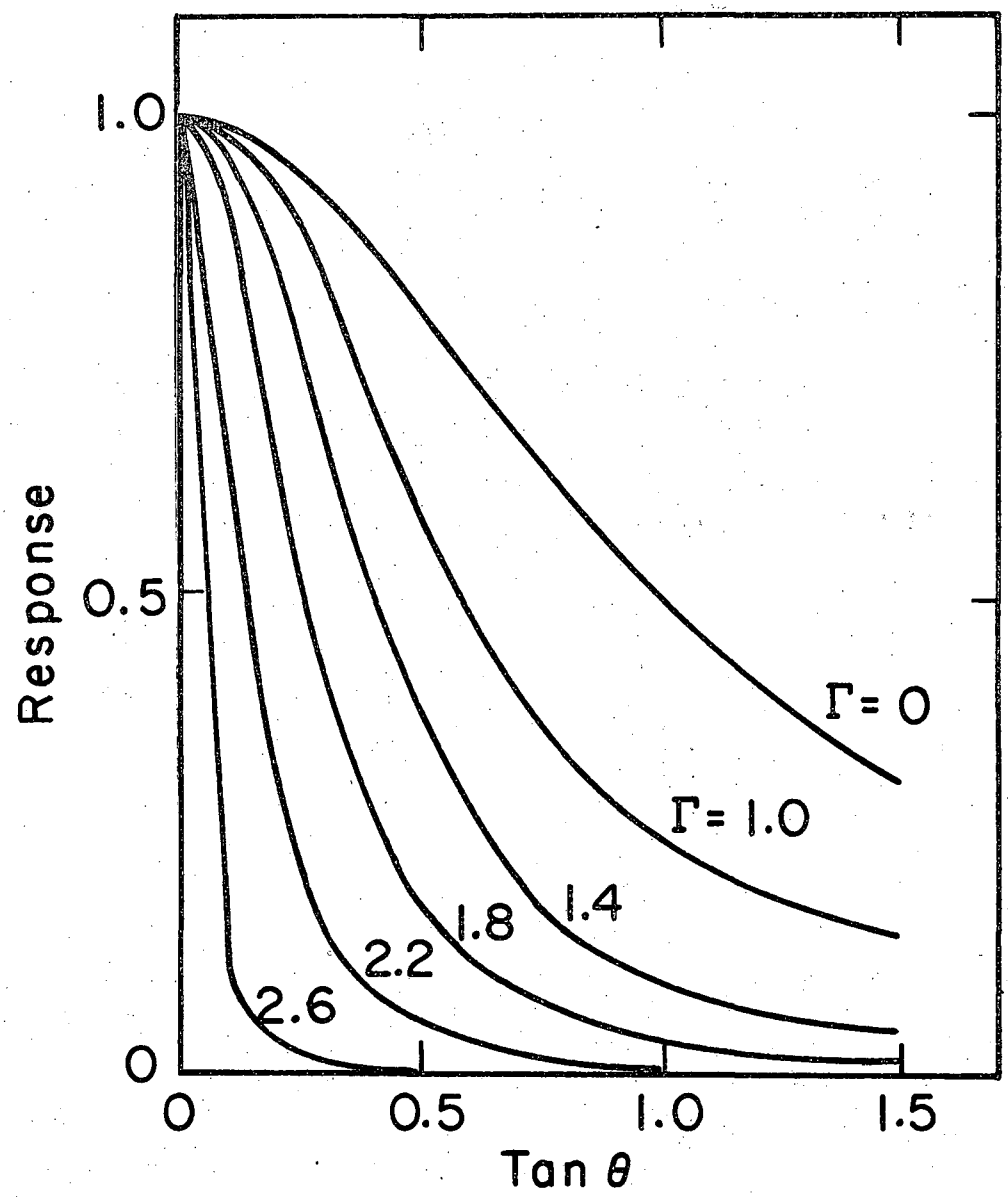

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Fig. 3

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