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A MODEL OF THE POMERANCHUK POLE-CUT RELATIONSHIP*

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ABSTRACT

Using the multiperipheral integral equation at zero momentum transfer, we construct a model in which the dynamical interrelation of Regge poles and cuts can be studied. Chief attention is paid to the region near $J = 1$ in an elastic forward amplitude. A consistent solution is found in which the Pomeranchuk pole appears at $J = 1 - a$, with $a \gtrsim 0.01$, while the AFS branch point appears at $J = 1 - 2a$. To a good approximation the pole residue corresponds to the inelastic part of the total cross section, while the integral over the AFS cut corresponds to the elastic cross section.

I. INTRODUCTION

The relationship of Regge cuts and poles remains uncertain, with regard to both relative strength and relative location. Recently it was realized that the multiperipheral integral equation may be able to shed light on the matters,[1] and we here report a preliminary investigation of the Regge singularities in a forward amplitude, employing the model of Chew and Pignotti[2] (hereafter designated CP) to suggest a simplified kernel and inhomogeneous term for the integral equation. The chief emphasis here will be on the region near $J = 1$ in an elastic amplitude, but the model can be extended to lower J regions and to inelastic amplitudes.

II. THE FACTORIZABLE MODEL

The multiperipheral equation derived in Ref. 3, after projection onto angular momentum J,[4] takes the form

$$B_a^{\gamma'}(t',J) = B_{a,0}^{\gamma'}(t',J) + \sum_{\gamma} \int_{-\infty}^{0} dt\, B_a^{\gamma}(t,J)\, G^{\gamma\gamma'}(t,t',J), \tag{II.1}$$

with the absorptive part for the forward elastic process $ab \to ab$ given by

$$A_{ab}(J) = \sum_{\gamma'} \int_{-\infty}^{0} dt'\, B_a^{\gamma'}(t',J)\, G_b^{\gamma'}(t') . \tag{II.2}$$

The superscript γ (or γ') labels a particular "input" Regge pole, while t (or t') labels the squared momentum transfer associated with that pole.[5] The kernel $G^{\gamma\gamma'}(t,t',J)$ includes the internal coupling between adjacent poles γ and γ', together with the Regge "propagator" associated with γ'. For our model we assume the factored form

$$G^{\gamma\gamma'}(t,t',J) = \lambda^{\gamma\gamma'} \frac{g^{\gamma}(t)\, g^{\gamma'}(t')}{J - J_{\gamma'}(t')} , \tag{II.3}$$

where

$$J_{\gamma}(t) = 2\,\alpha_{\gamma}(t) - 1 , \tag{II.4}$$

$\alpha_{\gamma}(t)$ being the "input" Regge trajectory associated with γ. We are keeping in (II.3) only that J dependence associated with the leading pole resulting from the projection of Formula (D-5) of Ref. 3. This projection actually contains additional J singularities and a more complicated t' dependence,[6] but the most essential characteristics are represented by (II.3) if $g^{\gamma}(t)$ is taken to be a function that does not vanish at $t = 0$ and that decreases rapidly (e.g., exponentially) as $t \to -\infty$. The corresponding form to be assumed for the inhomogeneous term in (II.1) is

$$B^{\gamma'}_{a,0}(t',J) = \lambda_a^{\gamma'} \frac{g^{\gamma'}(t')}{J - J_{\gamma'}(t')} . \tag{II.5}$$

If (II.5) is substituted into (II.2) one gets the well-known AFS cut[7] as a "Born approximation" to the absorptive part. Evidently the solution to (II.1) may be written as

$$B_a^{\gamma'}(t',J) = b_a^{\gamma'}(J) \frac{g^{\gamma'}(t')}{J - J_{\gamma'}(t')} , \tag{II.6}$$

with

$$b_a^{\gamma'}(J) = \lambda_a^{\gamma'} + \sum_{\gamma} b_a^{\gamma}(J)\, \rho^{\gamma}(J)\, \lambda^{\gamma\gamma'} , \tag{II.7}$$

if

$$\rho^{\gamma}(J) = \int_{-\infty}^{0} dt\, \frac{[g^{\gamma}(t)]^2}{J - J_{\gamma}(t)} . \tag{II.8}$$

In order to construct the absorptive part, we also need the "end vertex" function $G_b^{\gamma'}(t')$, which for consistency with the above should be taken to be

$$G_b^{\gamma'}(t') = \lambda_b^{\gamma'} g^{\gamma'}(t') . \tag{II.9}$$

The final result is then

$$A_{ab}(J) = \sum_{\gamma'} \rho^{\gamma'}(J)\, b_a^{\gamma'}(J)\, \lambda_b^{\gamma'} , \tag{II.10}$$

with $b_a^{\gamma'}(J)$ the solution of the linear algebraic equation (II.7).

Since the functions $\rho^{\gamma}(J)$ have no poles, the poles of $A_{ab}(J)$ evidently coincide with those of $b_a^{\gamma}(J)$ and thus with zeros of the determinant $|\delta_{\gamma\gamma'} - \rho^{\gamma}(J)\, \lambda^{\gamma\gamma'}|$. Note that these pole locations are independent of the "external" indices a,b . Assuming linear input trajectories, the function $\rho^{\gamma}(J)$ may be rewritten as

$$\rho^{\gamma}(J) = \frac{1}{\pi} \int_{-\infty}^{J_\gamma(0)} dJ' \, \frac{\mathrm{Im}\, \rho^{\gamma}(J')}{J' - J} , \tag{II.11}$$

where

$$\mathrm{Im}\, \rho^{\gamma}(J) = -\frac{\pi}{2\alpha'_\gamma} \left[g^{\gamma}\left(\frac{J - J_\gamma(0)}{2\alpha'_\gamma} \right) \right]^2 , \tag{II.12}$$

exhibiting the branch point at $J = J_\gamma(0)$ and the associated cut along the negative real axis running to $J = -\infty$. For convenience we shall choose the normalization of the functions $g^{\gamma}(t)$ to be such that

$$\rho^{\gamma}(J) \underset{J \to \infty}{\sim} \frac{1}{J} , \tag{II.13}$$

or in other words, such that

$$-\frac{1}{\pi}\int_{-\infty}^{J_\gamma(0)} dJ' \ \mathrm{Im}\ \rho^\gamma(J') = 1 . \qquad \text{(II.14)}$$

Note that a rapid falloff of the functions $g^\gamma(t)$ as $t \to -\infty$ produces a corresponding rapid decrease of $\mathrm{Im}\ \rho^\gamma(J)$ as $J \to -\infty$.

A second important property of the function $\rho^\gamma(J)$ is the infinite logarithmic branch point at $J = J_\gamma(0)$:

$$\rho^\gamma(J) \underset{J \to J_\gamma(0)}{\sim} \frac{1}{\Delta_\gamma} \ \ell n \ \frac{\Delta_\gamma}{J - J_\gamma(0)} , \qquad \text{(II.15)}$$

where

$$\Delta_\gamma^{-1} \equiv -\frac{1}{\pi}\ \mathrm{Im}\ \rho^\gamma[J_\gamma(0)] , \qquad \text{(II.16)}$$

$$= \frac{1}{2\alpha'_\gamma}\ [g_\gamma(0)]^2 .$$

The general structure of $A_{ab}(J)$ then is that it is a real analytic function of J, with branch points at the various $J_\gamma(0)$ (associated cuts running to the left), poles at zeros of the aforementioned determinant, and an asymptotic behavior easily inferred from (II.7) and (II.10) to be that of the "Born approximation":

$$A_{ab}(J) \underset{J \to \infty}{\sim} \frac{1}{J} \sum_\gamma \lambda_a^\gamma \ \lambda_b^\gamma . \qquad \text{(II.17)}$$

Corresponding to (II.17) is the sum rule

$$- \frac{1}{\pi} \int_{-\infty}^{J_{max}(0)} \mathrm{Im}\, A_{ab}(J)\, dJ \; + \; \text{sum of residues of poles on physical sheet}$$

$$= \sum_{\gamma} \lambda_a^{\gamma} \; \lambda_b^{\gamma} \; . \qquad \text{(II.18)}$$

This rule will be helpful in assessing the relative importance of poles and cuts.

A final general remark concerns the factorizability of pole residues. From (II.7) we see that if a pole occurs at $J = \alpha_i$, such that

$$b_a^{\gamma'}(J) \underset{J \to \alpha_i}{\sim} \frac{{}^{i}r_a^{\gamma'}}{J - \alpha_i} \; , \qquad \text{(II.19)}$$

then the dependence of ${}^{i}r_a^{\gamma'}$ on the two indices a and γ' will factorize. It follows from (II.10) that in the corresponding residue of $A_{ab}(J)$ the dependence on the two indices a and b will factorize.

III. SINGLE INPUT POLE

With a single input pole the solution of Eq. (II.7) is

$$b_a(J) = \frac{\lambda_a}{1 - \lambda\rho(J)} \; , \qquad \text{(III.1)}$$

the superscript γ becoming superfluous. The corresponding absorptive part is

$$A_{ab}(J) = \frac{\lambda_a \; \lambda_b \; \rho(J)}{1 - \lambda \; \rho(J)} \; . \qquad \text{(III.2)}$$

Remembering (II.13) and (II.15), if $\lambda > 0$ there must be a pole of $A_{ab}(J)$ on the real J axis to the right of $J_\gamma(0) = J_{in}$; this can be shown to be the only pole on the physical sheet.

Two limiting situations are especially interesting:

1. λ so large that the pole falls into the region $J - J_{in} \gg \Delta$, where $\rho(J)$ can be approximated by

$$\rho(J) \approx \frac{1}{J - \bar{J}_{in}} , \tag{III.3}$$

where, evidently, $\bar{J}_{in} \lesssim J_{in}$. The absorptive part in this region then becomes

$$A_{ab}(J) \approx \frac{\lambda_a \lambda_b}{J - \bar{J}_{in} - \lambda} , \tag{III.4}$$

the pole occurring at

$$\alpha_{out} \approx \bar{J}_{in} + \lambda , \tag{III.5}$$

with residue $\lambda_a \lambda_b$. This residue exhausts the sum rule (II.16), leaving zero average weight for the cut discontinuity. The cut in the total absorptive part is thus much weaker than that in the "Born approximation."

2. λ so small that the pole falls into the region $J - J_{in} \ll \Delta$, where $\rho(J)$ can be approximated by (II.15). The pole residue here is approximately

$$\lambda_a \lambda_b \frac{\Delta}{\lambda^2} (\alpha - J_{in}) , \tag{III.6}$$

which approaches zero as $\alpha \to J_{in}$. In this limit, then the cut carries the full weight and the pole is negligible.

If λ is negative (as is possible for inelastic amplitudes) there are no poles on the physical sheet, but if $-\lambda$ is sufficiently large there will appear a complex pole on the next sheet near to the cut. In particular, if the pole occurs in the region where $J_{in} - \mathrm{Re}\, J \gg \Delta$, then in this region (on the upper side of the cut)

$$\rho(J) \approx \frac{1}{J - \bar{J}_{in}} - i\, \eta(J) ,$$

where $\eta(J)$ as given by (II.12) is very small compared with the first term. The condition $1 - \lambda\rho(\alpha) = 0$ requires that $\mathrm{Im}\, \rho(\alpha) = 0$, or

$$\begin{aligned} \mathrm{Im}\, \alpha &= - \mathrm{Re}\, \eta(\alpha)\, |\bar{J}_{in} - \alpha|^2 \\ &\approx - \eta(\mathrm{Re}\, \alpha)(\bar{J}_{in} - \mathrm{Re}\, \alpha)^2 , \end{aligned} \tag{III.7}$$

so the negative imaginary displacement of the pole is small. The real part of the pole position is given by

$$\mathrm{Re}\, \alpha_{out} \approx \bar{J}_{in} + \lambda \tag{III.8}$$

and the residue is approximately $\lambda_a \lambda_b$, exhausing the sum rule. The cut is then negligible except in the vicinity of the pole, where the discontinuity can be approximated by a delta function with integrated strength $\lambda_a \lambda_b$.

IV. THE CP TWO-INPUT-POLE MODEL

A more realistic model for forward elastic amplitudes, proposed by CP,[2] contains two input trajectories, α_P^{in} to represent the Pomeranchuk and α_M^{in} to represent all lower trajectories. The internal coupling matrix is positive definite (since each term in the iterated solution of the integral equation is a separate partial cross section) and has the form

$$\lambda^{\gamma\gamma'} = \begin{pmatrix} g_M^2 & g_P^2 \\ g_P^2 & 0 \end{pmatrix} , \qquad \text{(IV.1)}$$

leading to

$$A_{ab}(J) = \frac{\lambda_a^M \lambda_b^M \rho^M(J) + \lambda_a^P \lambda_b^P \rho^P(J)(1-g_M^2\rho^M(J)) + (\lambda_a^P\lambda_b^M + \lambda_a^M\lambda_b^P)g_P^2\rho^M(J)\rho^P(J)}{1 - g_M^2 \rho^M(J) - g_P^4 \rho^P(J) \rho^M(J)} . \qquad \text{(IV.2)}$$

Notice that at a zero of the denominator of (IV.2) the numerator takes the factored form,

$$[\lambda_a^M + \lambda_a^P g_P^2 \rho^P(\alpha)][\lambda_b^M + \lambda_b^P g_P^2 \rho^P(\alpha)]\rho^M(\alpha) . \qquad \text{(IV.3)}$$

Let us suppose that $2\alpha_M^{in}(0) - 1$ lies sufficiently below $J = 1$ so that near $J = 1$

$$\rho^M(J) \approx \frac{1}{J - \bar{J}_M} .$$

Multiplying numerator and denominator by $J - \bar{J}_M$ then brings (IV.2) to the form

$$A_{ab}(J) \approx \frac{N_{ab}(J)}{D(J)} , \tag{IV.3}$$

where

$$N_{ab}(J) \approx \lambda_a^M \lambda_b^M + \left[\lambda_a^P \lambda_b^P (J - \alpha_0) + (\lambda_a^M \lambda_b^P + \lambda_a^P \lambda_b^M) g_P^2 \right] \rho^P(J) \tag{IV.4}$$

and

$$D(J) \approx J - \alpha_0 - g_P^4 \rho^P(J) , \tag{IV.5}$$

if

$$\alpha_0 = \bar{J}_M + g_M^2 . \tag{IV.6}$$

Let us assume that the highest-lying zero of $D(J)$ occurs at a value $J = \alpha$, where $1 - \alpha \ll \Delta^P$. This point must be reexamined later for consistency, but if accepted it allows us to write

$$\rho^P(J) \approx \frac{1}{\Delta_P} \ln \frac{\Delta_P}{J - J_P} \tag{IV.7}$$

for J near α. Now $J_P = 2\,\alpha_P^{in}(0) - 1$, so if we require that $\alpha_P^{in}(0) = \alpha$, we have the determining equation for α,

$$0 = D(\alpha) \approx \alpha - \alpha_0 - \epsilon \ln \frac{\Delta_P}{1 - \alpha} , \tag{IV.8}$$

where $\epsilon = g_P^4/\Delta_P$. We may also note that $D'(\alpha) \approx 1 + \frac{\epsilon}{1 - \alpha}$. It follows that $\alpha_0 < \alpha < 1$.

Now it was shown by CP that $g_P^2 \ll 1$ (they estimated $g_P^2 \approx 0.02$ on the basis of measured diffractive dissociation cross sections as well

as Deck-model calculations), while an estimate for Δ_P can be obtained from a typical momentum transfer width Δt together with the Pomeranchuk slope α_P' :

$$\Delta_P \approx 2(\Delta t)\,\alpha'_P \,.$$

Thus, if the Pomeranchuk slope is anywhere near a "normal" value, the value of ϵ will be very small. For example, if $\alpha'_P \approx 1$ GeV^{-2}, $\Delta t \approx 0.2$ GeV^2, and $g_P^2 \approx 0.02$, then $\epsilon \approx 10^{-3}$. The value of α then has to be extremely close to 1 in order for the logarithm in (IV.8) to play an significant role.

Two characteristically different situations may be envisaged. The dynamically more "natural" situation is when $\epsilon \ll 1 - \alpha$. In that case $\alpha \approx \alpha_0$ and $D'(\alpha) = 1$. The pole residue is then

$$\left(\lambda_a^M + \lambda_a^P \frac{g_P^2}{\Delta_P} \ln \frac{\Delta_P}{1-\alpha_0}\right)\left(\lambda_b^M + \lambda_b^P \frac{g_P^2}{\Delta_P} \ln \frac{\Delta_P}{1-\alpha_0}\right) \approx \lambda_a^M \lambda_b^M \,, \tag{IV.9}$$

to be compared with the sum rule value $\lambda_a^M \lambda_b^M + \lambda_a^P \lambda_b^P$. The missing part $\lambda_a^P \lambda_b^P$ evidently resides in the cut. In this small g_P^2 situation, in fact, the absorptive part can be written

$$A_{ab}(J) \approx \frac{\lambda_a^M \lambda_b^M}{J - \alpha_0} + \lambda_a^P \lambda_b^P \rho^P(J) \,, \tag{IV.10}$$

since the M and P channels are almost decoupled. One can furthermore identify the pole contribution with the inelastic part of the total cross section and the cut contribution with the elastic part, a decomposition suggested some time ago by Freund and O'Donovan.[8]

Note that although we have assumed $1 - \alpha \gg \epsilon$, if $\epsilon \approx 10^{-3}$ this condition is satisfied for $1 - \alpha \gtrsim 10^{-2}$. Needless to say a deviation from unity of the order 0.01 would not have been noticed. Note further that at moderate energies (say 20 GeV lab) the cut, with an average position $\approx 1 - \Delta_P/2$, has a typical integrated strength of $\approx 1/4$ that of the pole. Thus a pure-pole phenomenological fit would place the effective pole at

$$\alpha_{eff} \approx \frac{3}{4}(1) + \frac{1}{4}\left(1 - \frac{\Delta_P}{2}\right)$$
$$= 1 - \frac{\Delta_P}{8} . \qquad \text{(IV.11)}$$

We recall the Cabibbo et al.[9] assignment of $1 - \alpha_{eff} \approx 0.07$, which would correspond to $\Delta_P \approx 0.6$, a reasonable value. Of course as the energy increases the ratio of elastic to inelastic cross section decreases and the effective pole position approaches that of the true pole.

A different possibility might seem to be $1 - \alpha \ll \epsilon$. In that case the residue of the pole is much less than the sum rule value, so the cut carries most of the load. Close examination, moreover, reveals the existence of a complex pole near $\mathrm{Re}\,\alpha = \alpha_0$ which carries a residue $\approx \lambda_a^M \lambda_b^M$. Hence this pole, which was not included as one of our input poles, dominates over the one at $\alpha \approx 1$, and we are in trouble with self-consistency.

One might try to avoid this inconsistency by supposing α_0 to lie so far below 1 as to have no connection with the Pomeranchuk phenomenon; instead the pole near α_0 could be identified with the

input meson pole. The difficulty with such an approach lies in the fact that the pole near α_0 has residue $\approx \lambda_a^M \lambda_b^M$, leaving $\lambda_a^P \lambda_b^P$ for the integrated weight of the singularities near $J = 1$ (recall the sum rule II.6). Now $\lambda_a^P \lambda_b^P$ corresponds to the elastic part of the total cross section (see Fig. 1). so the situation we are considering is an unrealistic one in that the total cross section becomes almost entirely elastic at high energies.

This last result can be understood much more directly. To see it in its simplest form let us return to the single-trajectory model of a self-consistent weakly coupled Pomeranchuk trajectory. With g_P^2 small we have a weak-coupling situation, in which the AFS diagram of Fig. 1 dominates over those with additional particles in the intermediate state. Hence the total cross section is almost entirely elastic. This argument is only a rewording of that given by CP,[2] that the single weakly coupled trajectory model conflicts with observed multiplicities.

The intermediate situation when $1 - \alpha$ is of order ϵ leads to the same sort of consistency problem in which the leading pole fails to represent most of the cross section. This difficulty is avoided only for the first case discussed, in which $\epsilon \ll 1 - \alpha$.

V. SUMMARY AND DISCUSSION

The most satisfactory of the models we have considered is a variation of the two-trajectory model of Chew and Pignotti, which we have shown to be a self-consistent solution of a factorizable model of the multiperipheral integral equation. It turns out that this solution is

practically identical to the weak-coupling limit where the internal coupling of the Pomeranchuk vanishes, $g_P^2 = 0$. The forward amplitude (see Eq. IV.10) takes the form of a Pomeranchuk pole, whose residue corresponds to the inelastic part of the total cross section, plus the AFS cut, which corresponds to the elastic part.[10] The most significant effect on this model of the small but nonvanishing $g_P^2 \approx 0.02$ is to prevent the Pomeranchuk intercept $\alpha_P(0)$ from being exactly equal to unity.

Many readers will be disturbed on esthetic grounds by a Pomeranchuk intercept that is not exactly at $J = 1$. The presence of a small parameter $1 - \alpha_P(0)$ is indeed surprising in hadronic physics, but when one recalls that we have already been forced to introduce one small parameter g_P^2, the second one comes as less of a surprise. Since it seems impossible for the internal Pomeranchuk coupling g_P^2 to be exactly zero,[2] theorists must look for a different kind of simplicity. Perhaps the most promising direction is to link a variety of phenomena which suggest the presence of small parameters.

One such phenomenon is the apparently small role of the Pomeranchuk trajectory in bootstrap models of the finite-energy sum rule or Veneziano type (this is obviously related to the smallness of g_P^2), as well as the zero-resonance-width approximation on which such models generally depend. It is an established and not at all understood fact, in other words, that hadron coupling constants are small. We assert that this smallness is no less mysterious than the smallness of $1 - \alpha_P(0)$, and we suggest that the two mysteries may be related.

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FOOTNOTES AND REFERENCES

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4. More precisely, after projection onto the Lorentz quantum numbers $\lambda = J + 1$ and $M = 0$.

5. In greater generality, t should be understood as a combination of two variables--the momentum transfer plus the helicity--and the integral over dt understood as including a helicity sum. Since the location of J singularities depends on momentum transfer but not on helicity, we have suppressed the latter variable in this paper.

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10. It is evidently necessary now to repeat the phenomenological Regge analysis of forward elastic scattering, since the cut contribution will alter the apparent residue of the P' trajectory.

FIGURE CAPTION

Fig. 1. The AFS elastic contribution to the unitarity sum.

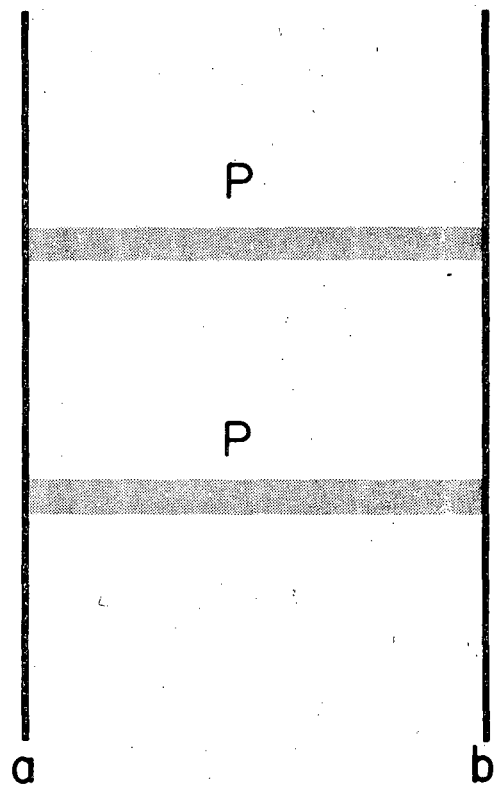

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Fig. 1

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