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ABSTRACT

The large s, small t pion photoproduction data and the corresponding continuous moment sum rules are fitted with evasive Regge poles and associated Regge cuts, instead of the commonly accepted pion conspiracy. The FESR actually yield the real parts of the photoproduction amplitudes, essentially independent of the model. A further assumption permits the prediction (in good agreement with existing data) of high energy cross sections directly from the FESR, without commitment to any particular model.

In this note we consider the photoproduction of charged pions at high energies and small momentum transfers: (a) We construct a model with evasive Regge poles and associated Regge cuts that fits both the high energy differential cross sections (including the data with linearly polarized photons) and the continuous moment finite energy sum rules. (b) We show that the normal finite energy sum rules in fact determine approximately the real parts of the photoproduction amplitudes at the upper limit of integration. (c) We develop a pseudomodel which directly relates the finite energy sum rules to the high energy data without the need for commitment to any particular theoretical model. Our most important conclusion is that, although finite energy sum rules for photoproduction allow an immediate connection between the low energy and

high energy regions, they make no distinction among a large class of models, and in particular, do not establish the existence of a pion conspiracy in photoproduction of pions.

The sharp forward peaks in n-p charge exchange scattering[1] and in charged pion photoproduction[2] are most commonly cited as evidence for a parity-doublet conspiracy of the pion (class III Lorentz pole with $M = 1$). Ball, Frazer, and Jacob[3] and Henyey[4] have given very satisfactory fits to the positive pion photoproduction data at small momentum transfers with such a conspiracy model. Apparently independent support for the idea of a pion conspiracy came from the general results of Drell and Sullivan,[5] from the application of normal finite energy sum rules (FESR) to the photoproduction amplitudes by Bietti et al[6] and by Roy and Chu,[7] and from the subsequent use of continuous moment finite energy sum rules (CMSR) to determine the trajectories and residues of the pion and its conspirator.[8,9] While the pion conspiracy is attractively economical in fitting photoproduction, it is certainly not unique[10,11] and it causes difficulties in other processes via factorization.[12]

Numerous models of hadronic processes have been made using Regge poles and ad hoc cuts[13,14] or cuts obtained from the absorptive optical model.[15-20] For photoproduction, models with poles plus cuts have been discussed by Amati et al,[11] Frøyland and Gordon[21] and Henyey et al.[22] All of this work shows that models with poles and cuts can avoid the factorization difficulties resulting from the conspiracy hypothesis, and provide explanations for the cross-over phenomena,[18] polarization,[17] and probably also the dip[20] in pion-nucleon charge exchange. The remaining question concerning the pion conspiracy, or lack of it, is whether the evidence deduced from CMSR can be regarded as conclusive. We show, by explicit construction of a counter-example, that

the CMSR results cannot be so regarded; in the process we show what can actually be learned from the sum rules in photoproduction.

Our model is similar to those of Refs. 16-20 in that it has evasive Regge poles and cuts generated by the recipe of the absorptive model. It differs in detail from all of them in its form for the Regge poles and in the assumption that the kernel in the cut-generating integral is energy-independent. Because the absorptive corrections are expressed in terms of s-channel amplitudes while the Regge exchanges and the amplitudes used in the sum rules are in the t-channel, it is necessary to specify amplitudes in both channels. In the t-channel we use the $F_i (i = 1,2,3,4)$ of Ref. 3. In the s-channel we choose the four amplitudes g_j, where $j = 1,2,3,4$ stands for the photon, initial nucleon, and final nucleon helicities $(\lambda_\gamma, \lambda_N; \lambda_{N'}) = (+,+;-), (+,-;-), (+,+;+), (-,+;-)$, respectively. The net helicity flip values $n(j) \equiv |\lambda_\gamma - \lambda_N + \lambda_{N'}|$ are 0, 1, 1, 2, respectively. At high energies and small momentum transfers the two sets of amplitudes are related by $g = (\nu/\sqrt{2})AF$, where $\nu = (s - m^2)/2m$ is the photon energy in the laboratory and the crossing matrix A is

$$A = \begin{pmatrix} -\tau/2m & -2m/(\mu^2 + \tau) & 1 & 0 \\ (\tau)^{\frac{1}{2}} & 0 & (\tau)^{\frac{1}{2}}/2m & 2m(\tau)^{\frac{1}{2}} \\ (\tau)^{\frac{1}{2}} & 0 & (\tau)^{\frac{1}{2}}/2m & -2m(\tau)^{\frac{1}{2}} \\ -\tau/2m & 2m/(\mu^2 + \tau) & 1 & 0 \end{pmatrix} . \quad (1)$$

In (1) we have introduced $\tau = -t$, and have neglected terms of relative order $(\tau/4m^2)$, as well as nonleading powers of ν.

The recipe of the absorptive model leads[19,22] to s-channel amplitudes of the form,

$$g_j(\nu,\tau) = g_j^{\text{Regge}}(\nu,\tau) - \frac{aC}{2}\int_0^\infty d\tau'\, e^{-\frac{a}{2}(\tau+\tau')} I_{n(j)}[a(\tau\tau')^{\frac{1}{2}}]\, g_j^{\text{Regge}}(\nu,\tau'), \tag{2}$$

where a and C are characteristic of the absorbing region (or of the Pomeranchuk pole, if the cut is viewed as arising from two-Regge-pole exchange), $I_n(x)$ is a modified Bessel function of the first kind of order n, and g^{Regge} is the amplitude made from one or more Regge poles. Equation (2) can be written in the compact operator form,

$$g_j = K_j \otimes g_j^{\text{Regge}} \quad . \tag{3}$$

Specification of the Regge amplitudes and the parameters a and C determines the model. For our present purposes, where we concentrate on the very small τ behavior, we ignore the ρ trajectory and keep only the pion and the A_2, contributing respectively to F_2 and to F_1 and F_3. Explicitly the s-channel Regge amplitudes are

$$g_1^{\text{Regge}} = g_\pi - g_A, \quad g_2^{\text{Regge}} = g_3^{\text{Regge}} = h_A, \quad g_4^{\text{Regge}} = -g_\pi - g_A \quad , \tag{4}$$

where

$$g_\pi = eg\, \alpha_\pi'\, \tau\, \Gamma\big(-\alpha_\pi(\tau)\big)\left[1 + e^{-i\pi\alpha_\pi(\tau)}\right](\nu/\nu_0)^{\alpha_\pi(\tau)} \quad ,$$

$$g_A = eg\, \frac{\tau}{4m^2}\,(\xi_1 + \xi_2)\, \Gamma\big(1 - \alpha_A(\tau)\big)\left[1 + e^{-i\pi\alpha_A(\tau)}\right](\nu/\nu_0)^{\alpha_A(\tau)} \quad ,$$

$$h_A = eg\, \frac{(\tau)^{\frac{1}{2}}}{2m}\,(\xi_1 - \tau\xi_2/4m^2)\, \Gamma\big(1 - \alpha_A(\tau)\big)\left[1 + e^{-i\pi\alpha_A(\tau)}\right](\nu/\nu_0)^{\alpha_A(\tau)} \quad . \tag{5}$$

The pion amplitude gives the correct residue in g_1 and g_4 at the pion pole; it has the τ-dependence normally associated with a Regge pole plus a

necessary factor of τ. The A_2 amplitudes correspond to the Gell-Mann mechanism at the nonsense point $\alpha = 0$;[23] the residues ξ_i are chosen as constants for simplicity.

In the forward direction the s-channel helicity amplitudes vanish as $(\tau)^{n/2}$. Thus only g_1 will survive at $\tau = 0$. Since both the pion and A_2 Regge amplitudes in (5) vanish as τ, the entire value of g_1 in the forward direction comes from the cut contribution. The mechanism for the sharp forward peak in the cross section is now clear. The cut contribution varies relatively slowly in the forward direction. The pion pole term, increasing linearly from zero at $\tau = 0$, interferes destructively with the cut contribution, causing a sharp drop in the cross section.

Before presenting a fit to the high energy cross section we turn to the finite energy sum rules. We use the general notation of Refs. 6 and 8. The relevant sum rules are those for the isovector photon amplitudes $F_2^{(-)}$ and $F_3^{(-)}$, which receive contributions from the pion and from the pion conspirator and the A_2 in models using only poles.[23] In terms of the s-channel amplitudes (3), we have

$$F_2^{(-)} = \frac{(\mu^2 + \tau)}{4m\nu} \left[(K_1 + K_4) \otimes g_\pi - (K_1 - K_4) \otimes g_A \right] ,$$

$$F_3^{(-)} = \frac{1}{2\nu} \left[-(K_1 - K_4) \otimes g_\pi + (K_1 + K_4) \otimes g_A - \frac{(\tau)^{\frac{1}{2}}}{m} (K_2 \otimes h_A) \right] . \quad (6)$$

The amplitude $F_3^{(-)}$ does not contain the pion pole, but does get a pion cut contribution. It is basically this cut amplitude that plays the role taken by the pion conspirator in other models. It is easy to verify from the behavior of K_j as $\tau \to 0$ that $F_2^{(-)}$ and $F_3^{(-)}$ in (6) satisfy the conspiracy relation, $F_2^{(-)} = -(\mu^2/2m)F_3^{(-)}$, at $\tau = 0$.

In the CMSR it is necessary to evaluate an integral of the form, $\varphi_i = -(\mu/\pi)\int_0^{\nu_{max}} d\nu(\nu/\nu_{max})^{\gamma}\,\mathrm{Im}\left[e^{-i\pi\gamma/2}\,F_i(\nu,\tau)\right]$, or to do something equivalent.[18] If we assume that K_j in (3) is independent of ν the integration over ν of the Regge cut amplitude becomes the same as for a Regge pole.[24] It is then straightforward to show that to go from $F_i^{(-)}$ in (6) to φ_i one merely substitutes in (6) according to

$$\frac{1}{\nu}\,g(\nu,\ \tau') \to G \ = \ \frac{\mu\ \sin[\pi(\gamma + \alpha(\tau'))/2]}{\pi[\gamma + \alpha(\tau')]}\,e^{i\pi\alpha(\tau')/2}\ g(\nu_{max},\tau') \quad . \tag{7}$$

Here g stands for g_π, g_A or h_A, and $\alpha(\tau')$ is the appropriate α. The high energy sides of the CMSR, [(1a) and (1b) of Ref. 8] are then

$$\varphi_2(\gamma,\ \nu_{max},\ \tau) \ = \ \frac{(\mu^2 + \tau)}{4m}\left[(K_1 + K_4)\otimes G_\pi - (K_1 - K_4)\otimes G_A\right] ,$$

$$\varphi_3(\gamma,\ \nu_{max},\ \tau) \ = \ \frac{1}{2}\left[-(K_1 - K_4)\otimes G_\pi + (K_1 + K_4)\otimes G_A - \frac{(\tau)^{\frac{1}{2}}}{m}(K_2\otimes H_A)\right] . \tag{8}$$

The simultaneous fitting of the CMSR and the high energy cross section for small τ values was done by trial and error. No attempt was made to use elaborate residues in order to obtain a precise fit over a wide range of τ values since we are interested mainly in $|\tau| < 0.1\ (\mathrm{GeV}/c)^2$. The results of one fit[25] are shown in Figs. 1 through 3. The cross sections for both unpolarized and linearly polarized[26] photons are given in Fig. 1. Figure 2 illustrates the fit to the γ dependence of the CMSR at $\tau = 0$, $\nu_{max} = 1.28$ GeV. The low energy sides of the CMSR are those of Fox,[27] based on the fit of Walker[28] to the low energy data on photoproduction. The comparison of Fox's results with our model as a function of τ (for $\gamma = 0$) is shown in Fig. 3. The overall fit to both high energy data and CMSR is seen

to be satisfactory for $|\tau| < 0.1\ (\mathrm{GeV}/c)^2$. This establishes that the CMSR for $F_2^{(-)}$ and $F_3^{(-)}$ are satisfied equally by a model with an evasive pion plus a Regge cut and models with a pion conspiracy.

We now examine the connection between the t-channel amplitudes given by (6) and the usual FESR[6,7] given by (8) with the G's, of Eq. (7), having $\gamma = 0$. To the extent that $\frac{1}{2}\left(1 + \cos \pi\alpha(\tau')\right) \approx \left(\sin \pi\alpha(\tau')\right)/\pi\alpha(\tau')$, the FESR and the amplitudes are related directly by

$$\varphi_i(\gamma = 0,\ \nu_{max},\ \tau) \approx (\mu\nu_{max}/2)\ \mathrm{Re}\ F_i^{(-)}(\nu_{max},\ \tau) \quad , \tag{9}$$

independent of the particular form of K_j in (3). In practice this means that if the major contributions to the integrals over τ' come from regions where all the Regge poles have $|\alpha(\tau')| < 1$, Eq. (9) is an essentially model-independent result. Verification of (9) for the particular model described above is shown in Fig. 3.

The direct connection between the FESR φ_i and the t-channel amplitudes F_i afforded by (9) allows us to connect the low-energy data and the high-energy data without an intervening model. We need one more assumption or empirical fact. Figure 1 shows that the power law behavior of the photoproduction amplitudes corresponds to $\nu^{\alpha_{eff}}$ with $\alpha_{eff} \approx 0$. Independent of the possible presence of logarithmic variation in ν, the Phragmén-Lindelöff theorem allows us to conclude that the phases of the amplitudes are given by $\left(1 + \exp(-i\pi\alpha)\right)$. With $\alpha \approx 0$, the amplitudes are predominantly real. The high energy cross sections for linearly polarized photons can thus be written for small τ directly in terms of the FESR φ_i:

$$\tilde{\sigma}_{\parallel} \equiv \frac{1}{2}(s - m^2)^2 \frac{d\sigma_{\parallel}}{dt} \approx (m^2/\pi\mu^2) \left[\frac{(\varphi_2)^2}{(\mu^2 + \tau)^2} + \tau(\varphi_4)^2\right] \quad , \tag{10}$$

$$\tilde{\sigma}_{\perp} \equiv \frac{1}{2}(s - m^2)^2 \frac{d\sigma_{\perp}}{dt} \approx (1/4\pi\mu^2) \left[(\varphi_3)^2 + \tau(\varphi_1)^2\right] . \tag{10}$$

Strictly speaking these are only the contributions from isovector photons, but the data on the π^-/π^+ ratio in deuterium[29] indicate that for $\tau < 0.1$ $(\mathrm{GeV}/c)^2$ these contributions dominate. The FESR results[27] show that for small τ the cross section is given entirely by the first terms in (10).

Comparison of the pseudomodel given by (10) with the data at small τ is shown in Fig. 1 by the dotted curve. The agreement is surprisingly good, in view of the rather small value of ν_{max} necessarily used in the FESR.[30] But our point has been made, namely finite energy sum rules relate low energy and high energy data, usually by means of models for the high energy behavior, but they do not discriminate among models. In the present example, essentially no model is necessary to generate the high energy cross sections.

We wish to thank Dr. Geoffrey C. Fox for kindly supplying us with his evaluations of the CMSR.

FOOTNOTES AND REFERENCES

* This work was supported in part by the U.S Atomic Energy Commission.

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2. See, for example, A. M. Boyarski et al., Phys. Rev. Letters 20, 300 (1968); Abstract 430, Proc. XIVth Int. Conf. on High-Energy Physics, Vienna, (1968)[see Fig. 2, p.3].
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25. The fit we display was obtained with the parameters: $a = 8\ (\mathrm{GeV}/c)^{-2}$, $C = 1.19$, $\xi_1 = -0.75$, $\xi_2 = -4.92$, $\alpha_\pi(\tau) = -0.65(\tau + \mu^2)$, $\alpha_{A_2}(\tau) = 0.45 - \tau$, $s_0 \equiv 2m\nu_0 = 1\ (\mathrm{GeV})^2$. The nonuniqueness of this fit is illustrated by the fact that an equally good fit to the FESR (Fig. 3) and an even better fit to the high energy data (Fig. 1) is obtained with just an elementary pion $(\alpha_\pi' \to 0)$ and cut $(a = 8,\ C = 1.3)$.

26. Chr. Geweniger et al., Abstract 336, Proc. XIVth Int. Conf. on High-Energy Physics, Vienna (1968). [See Fig. 9, p.7.]

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28. R. L. Walker, Phys. Rev. (to be published).

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30. A possible reason is that, near $\tau = 0$ at least, the Born terms almost saturate the FESR. H. Harari (private communication) has emphasized this point as an explanation of why the gauge-invariant electric Born terms give roughly the correct high energy cross section near $\tau = 0$ (see Ref. 2). The pseudomodel continues to fit the cross section for τ at least as large as 0.5 $(\mathrm{GeV}/c)^2$, while the Born cross section fails beyond $\tau = 0.1$ $(\mathrm{GeV}/c)^2$.

FIGURE CAPTIONS

FIG. 1. Cross sections for $\gamma p \to \pi^+ n$: $(s - m^2)^2\, d\sigma/dt = \tilde{\sigma}_\perp + \tilde{\sigma}_\parallel$ with unpolarized photons (LH scale, solid symbols), $\sum \equiv (\tilde{\sigma}_\perp - \tilde{\sigma}_\parallel)/(\tilde{\sigma}_\perp + \tilde{\sigma}_\parallel)$ for linearly polarized photons (RH scale, open symbols). Data points are from SLAC (Ref. 2)[●, 5 GeV; ◆, 8 GeV; ■, 11 GeV; ▲, 16 GeV] and from DESY (Ref. 26)[□, 3.4 GeV; ○, 5 GeV]. The dashed curve is the prediction of our model, Ref. 25 (actually evaluated at 11 GeV, but virtually energy-independent). The dotted curve is the pseudomodel prediction, (10). The solid curve for $\sum$ is the model result at 5 GeV; the pseudomodel result is essentially indistinguishable.

FIG. 2. CMSR, evaluated at $\tau = 0$, as a function of the moment parameter γ. The solid (dashed) line is φ_2 $(\mu^2\varphi_3/2m)$ from Fox (Ref. 27). The circles are from our model (Ref. 25).

FIG. 3. FESR (CMSR, $\gamma = 0$) versus τ. The solid (dashed) curves are from Fox, Ref. 27 (our model, Ref. 25). Agreement between circles (RHS of Eq. (9)) and dashed curves indicates the validity of (9) for our particular model.

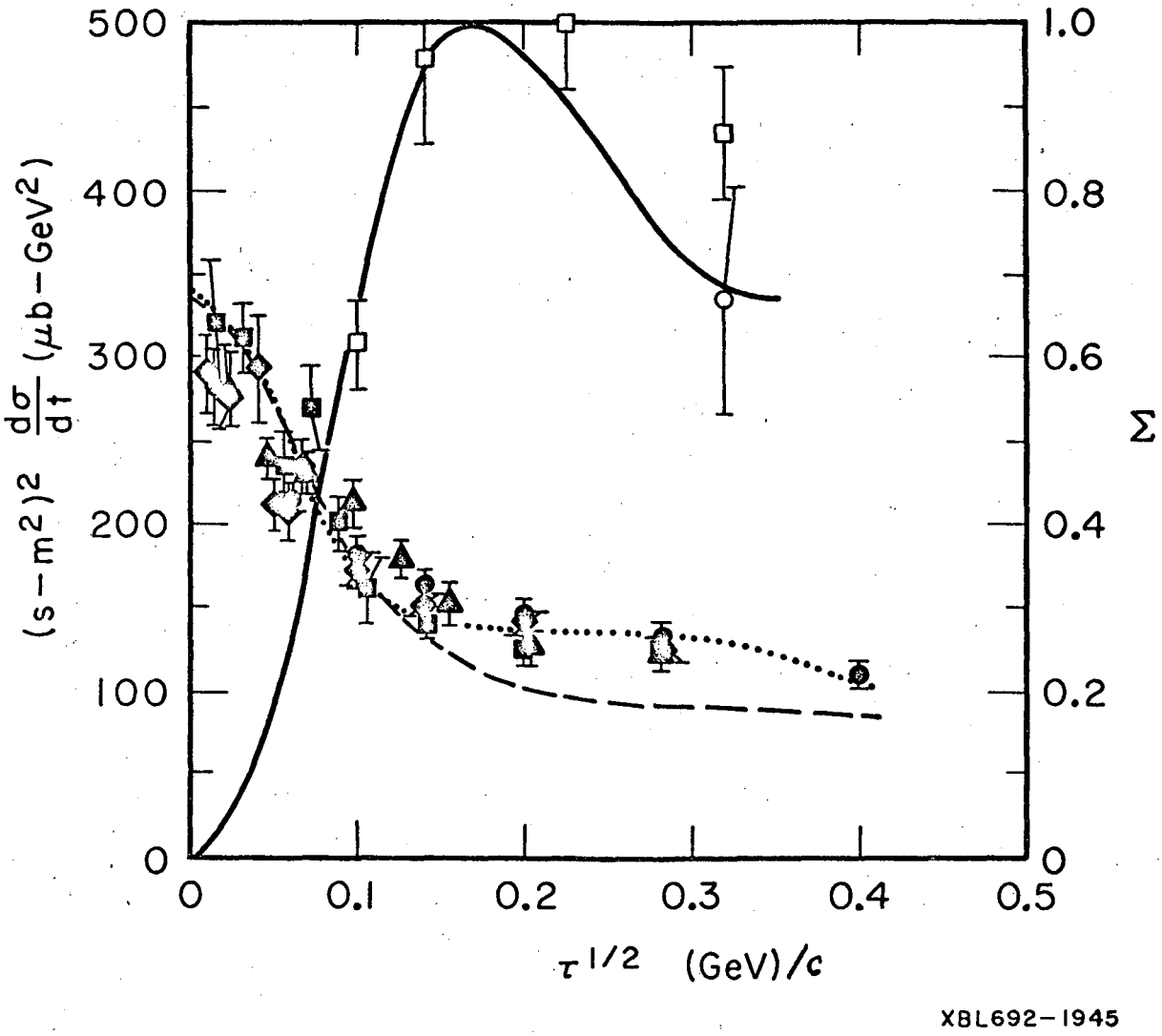

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Fig. 1

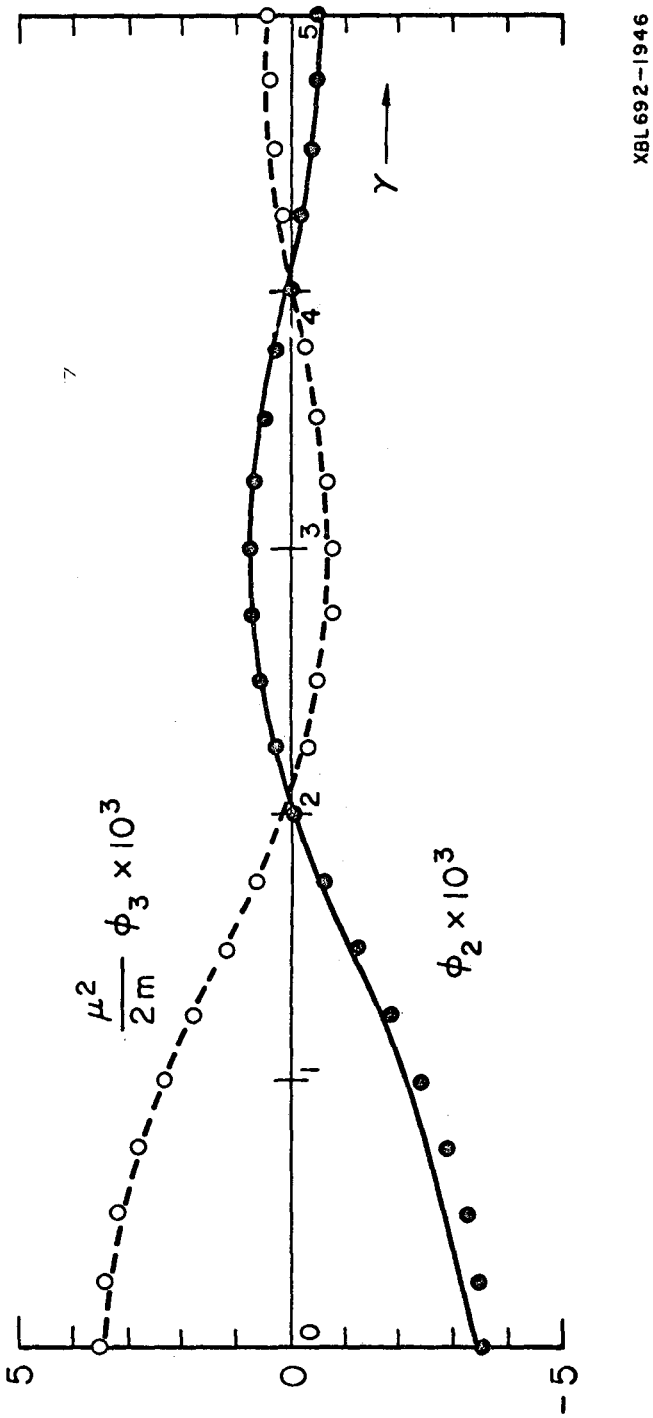
5
0
-5
0
1
2
3
4
5
γ
$\frac{\mu^2}{2m}\ \phi_3 \times 10^3$
$\phi_2 \times 10^3$
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Fig. 2

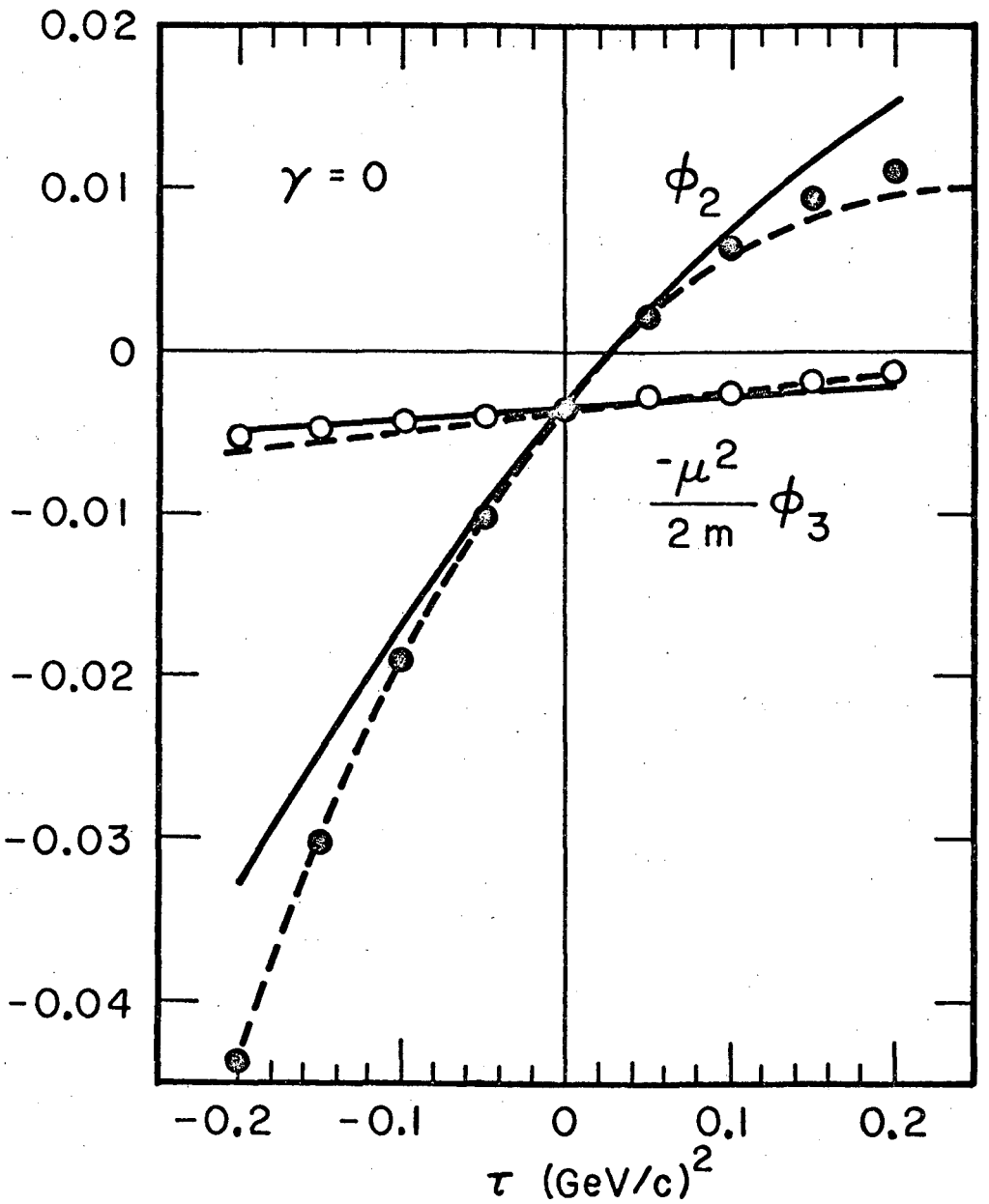

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Fig. 3.

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