UCRL 1876

cy 2.

# UNIVERSITY OF CALIFORNIA

# Radiation Laboratory

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545*

BERKELEY, CALIFORNIA

UCRL-1876 c.2

## DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UCRL-1876
Unclassified-Physics Distribution

UNCLASSIFIED

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

# NUCLEAR FORCES FROM PSEUDOSCALAR MESON THEORY

M. Ruderman

July 1, 1952

Berkeley, California

# NUCLEAR FORCES FROM PSEUDOSCALAR MESON THEORY

M. Ruderman

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

July 1, 1952

## ABSTRACT

Radiative corrections are calculated for the interaction of two nucleons through pseudoscalar coupled mesons. After renormalization these are very large for the usual magnitude of coupling constant but do not change the range of the potential or its shape at large distances.

The tensor part of the nuclear force is relatively insensitive to these corrections. For the self action terms the relevant expansion parameter in the perturbation theory is $g^2/4\pi$ with none of the factors of $\mu/2M$ which result when such terms are neglected.

# NUCLEAR FORCES FROM PSEUDOSCALAR MESON THEORY

M. Ruderman

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

July 1, 1952

## Introduction

The terms in the perturbation expansion of the S-matrix describing the interaction of two nucleons through a meson field are of two types:

(a) In the Feynman diagrams representing the process the only mesons present are those which are exchanged between the two nucleons, viz., graphs (1), (4), (5), and (7) of Fig. 1.

(b) The graphs include radiative corrections which involve either closed nucleon loops or meson lines which begin and end on the same nucleon line, viz., (2), (3), (4abc), and (6) of Fig. 1. These lead to divergences which must be handled by the standard renormalization procedures.

The pseudoscalar meson theory with pseudoscalar coupling requires a large coupling constant; $g^2/4\pi\hbar c$ is usually taken between 4 and 40. It has been pointed out[1,2] that even for a coupling constant as large as 10 terms of type (a) should converge.

After a canonical transformation[3,4] charge symmetric pseudoscalar coupling becomes

$$\psi^{+}\left\{\frac{g}{2M}\sigma_{\mu}\frac{\partial}{\partial x_{\mu}}\tau_i\phi_i + \frac{g^2}{2M}\phi^2 - \frac{1}{M}\left(\frac{g^2}{2M}\right)^2\phi^4 - \frac{g^3}{4M^3}\sigma_{\mu}\left[\phi^2, \frac{\partial}{\partial x_{\mu}}\tau_i\phi_i\right] \cdots\right\}\psi . \tag{1}$$

$\phi^2 = \sum_i \phi_i^{\,2}$ ; $\tau_i$ is the nucleon isotopic spin operator and $M$ is the nucleon mass.

The first term of (1) gives rise to the interaction represented in graph (1) of Fig. 1:

$$V_2 = -\left(\frac{\mu}{2M}\right)^2\left(\frac{g^2}{4\pi}\right)\frac{\tau^{(1)}\cdot\tau^{(2)}\,\sigma^{(1)}\cdot\nabla\,\sigma^{(2)}\cdot\nabla}{\mu^2}\left(\frac{e^{-\mu r}}{r}\right) \tag{2}$$

The second term yields the major part of the potential represented by graphs (4) and (5) of Fig. 1. According to Lepore[4] it is

$$V_4 \approx -\left(\frac{g^2}{4\pi}\right)^2\left(\frac{\mu}{2M}\right)^2\frac{3\mu}{\pi(\mu r)^2}\int_{\mu r}^{\infty} K_0(2\lambda)\,d\lambda . \tag{3}$$

A contact term has been omitted. Graph (4) is a reducible Feynman diagram; it contains higher order corrections to the $g^2$ potential and the contribution to the scattering from the second Born approximation of the potential $V_2$ from graph (1). The latter is automatically included when $V_2$ is put into the Schroedinger equation and should not be included in $V_4$. However it is easily shown that for $r \sim \hbar/\mu c$ the iteration of $V_2$ in the Born approximation is

$\sim(\mu/M)^2 V_4$. For sufficiently large $r$ it therefore should not be necessary to make use of the Bethe-Salpeter[5] equation to extract the contribution to the potential from graph (4). Similarly the effect of successive interactions of the potentials from graphs (1) and (3) is $\sim(\mu/M)^4$ smaller than the scattering represented in the reducible graph (4c) if the effect of the various potentials is restricted to $r \gtrsim \hbar/2\mu c$. For the low energy properties of the nuclear force a good approximation is obtained for the contributions of graphs (1) - (6) merely by taking for the potential the Fourier Transform of the S-matrix. This is not true for graph (7) but it has a negligible effect unless $r \sim 2\hbar/Mc$.

Radiative Corrections

The $g^4$ radiative corrections to (2) (graphs (2) and (3) of Fig. 1))[6] have been shown[7] to be small next to the (a) type terms of graphs (4) and (5) of Fig. 1. As long as the transfer of momentum is small the matrix element for the nucleon line $p_1 - p_2'$ may be written[6]

$$\left\langle p_2' \left| \tau_i \gamma_5 \right| p_1 \right\rangle \left[ \left( -\frac{g^2}{4\pi} \right) \frac{1}{2\pi} \int_0^1 \ln \left\{ 1 + \frac{q^2 x (1-x)}{M^2} \right\} dx + O\left(\frac{\mu}{M}\right)^2 \right] \tag{4}$$

$\underset{\sim}{q} = \underset{\sim}{p}_2' - \underset{\sim}{p}_1$.

This matrix element is of order

$$\frac{1}{2\pi} \left( -\frac{g^2}{4\pi} \right) \left( \frac{q}{M} \right) \gamma_5 \sim \left( -\frac{g^2}{4\pi} \right) \left( \frac{\mu}{M} \right)^2 \frac{1}{2\pi} \gamma_5$$

since we are concerned only with the low energy properties of the nuclear force for large $r$. The insertion of a closed nucleon loop into a meson line of momentum $q$ multiplies the meson propagation

function by minus the bracket in (4). Its contribution is therefore small.

The expression (4) is correct only if $p_2'$ and $p_2$ refer to free nucleons. If $\langle p_2' |$ does not refer to a free nucleon state one must add to (4)

$$\langle p_2' | (p_2 - M) A \gamma_5 \gamma_j | p_1 \rangle \tag{5}$$

where

$$A \approx -\frac{1}{4\pi}\left(\frac{g^2}{4\pi}\right) \int_0^1 ds \int_0^{1-s} dt \left[ \frac{(t-1)M - s\underset{\sim}{q}}{(1-t)^2 M^2 + (1-t)s\underset{\sim}{p}_1 \cdot \underset{\sim}{q} - 2\underset{\sim}{p}_1 \cdot \underset{\sim}{q}s} \right] \tag{6}$$

For radiative corrections to higher order meson exchange diagrams the expression (5) is large. For example in graph (4c) of Fig. 1, if $\underset{\sim}{k}_2$ is small,

$$\langle p_2' | = \langle p_2 | \gamma_5 (\underset{\sim}{p}_2 - \underset{\sim}{k}_2 - M)^{-1} \approx \langle p_2 | \gamma_5 (-2M)^{-1} \tag{7}$$

and $(\underset{\sim}{p}_2 - M)$ of (5) becomes $-2M$ instead of zero. In addition there exist important radiative corrections from diagrams which can not be described as an iteration of lower order processes[8], viz., Fig. 1 graph (4a).

Let $\underset{\sim}{k}_1$ and $\underset{\sim}{k}_2$ be the 4-momentum for the two mesons which are exchanged in graph (4); $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$ are the momenta of one of the nucleons of graph (4) and (5) before and after the exchange of the meson pair.

The matrix element for this nucleon line is

$$- i \langle p_2 \mid \tau_j \; \gamma_5 (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - M)^{-1} \gamma_5 \; \tau_i \mid p_1 \rangle \tag{8}$$

$$\gamma_5^2 = -1 \; ; \qquad \underset{\sim}{p} - M \mid p_1 \rangle = 0 .$$

For those mesons which contribute to the low energy properties of the nuclear force at large distances. $\underset{\sim}{k}_1$ is small next to $\underset{\sim}{p}_1 \sim \gamma_4 M$ and (8) becomes

$$- \frac{i}{2M} \langle p_2 \mid \tau_j \, \tau_i \mid p_1 \rangle \; . \tag{9}$$

With this approximation the sum of the matrix elements from graphs (4) and (5), Fig. 1 gives just the nuclear force which arises from the second term of (1) alone, i.e., $V_4$ of (3). In computing the potential to terms in $g^4$ the error introduced by neglecting $\underset{\sim}{k}_1$ in the nucleon line is equivalent to the neglect of a repulsive term in the potential[2]

$$\left( \frac{g^2}{4\pi} \right)^2 \frac{\mu^2}{2M} \left( \frac{\mu}{2M} \right)^2 \frac{(1 + \mu r)^2}{(\mu r)^4} \; e^{-2\mu r} \tag{10}$$

which is $\mu/2M$ smaller than the dominant attractive term.

The correction to the nuclear force from Fig. 1, graph (4a) is finite. The matrix element for the nucleon line corresponding to (8) is

$$\sum_{m} g^2 \langle p_2 | \int \frac{d^4k}{(2\pi)^4} \gamma_5 \gamma_m (\underset{\sim}{p}_2 - \underset{\sim}{k} - M)^{-1} \gamma_j \gamma_5 (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - \underset{\sim}{k} - M)^{-1}$$

$$\times \gamma_i (\underset{\sim}{p}_1 - \underset{\sim}{k} - M)^{-1} \gamma_m (k^2 - \mu^2)^{-1} | p_1 \rangle . \tag{11}$$

Since the sum of graphs (4) and (5) yields a charge independent interaction even if $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$ do not refer to free nucleons, the sum of graph (4a) and the analogous (5a) is also charge independent. The sum on m gives a factor three. The integration in (11) can be performed in a straight forward manner using the parametric representation of Feynman to give [9]

$$\frac{i\,3g^2}{8\pi} \langle p_1 | \int_0^1 dt \int_0^{1-t} dx \int_0^{1-t-x} dy \; \frac{\underset{\sim}{p}_1 + \underset{\sim}{k} + 3\underset{\sim}{Q} - 2M}{\Delta}$$

$$+ \frac{Q(\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - \underset{\sim}{Q} + M)\underset{\sim}{Q}}{\Delta^2} | p_2 \rangle \tag{12}$$

$$\underset{\sim}{Q} = t\,\underset{\sim}{p}_1 + t\,\underset{\sim}{k}_1 + y\,\underset{\sim}{p}_1 + (1 - t - x - y)\underset{\sim}{p}_2$$

$$\Delta = Q^2 - (2p_1 \cdot k_1 + k_1^2)t + \mu^2 x .$$

For small $\underset{\sim}{k}_1$, and neglecting $\mu$ next to M, (12) becomes

$$\frac{1}{4\pi M}\left(\frac{3g^2}{4\pi}\right) \quad p_2 \left| \tau_j \tau_i \right| p_1 \rangle \ . \tag{13}$$

Comparing (13) and (9) and taking into account that the radiative correction contained in graph (4a) must be applied to both nucleon lines, we have

$$V_6^a \sim \left(-\frac{3}{\pi}\right)\left(\frac{g^2}{4\pi}\right) V_4 \ . \tag{14}$$

The sixth order potential from graph (4a) is repulsive and has the same range as the fourth order terms from graphs (4) and (5). No factors of $\mu/M$ enter into the ratio $V_6^a$ to $V_4$.

The radiative correction from (4b) is

$$(3g^2) \left\langle p_2 \right| \int \frac{d^4k}{(2\pi)^4} \gamma_5 \tau_j (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - M)^{-1} \gamma_5 (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - \underset{\sim}{k} - M)^{-1}$$

$$\times \gamma_5 (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - M)^{-1} \gamma_5 \tau_i (k^2 - \mu^2)^{-1} \Big/ p_1 \Big| \rangle \tag{15}$$

which is finite after charge and nucleon mass renormalization. For small $\underset{\sim}{k}_1$ the renormalized matrix element is

$$\frac{3i}{4\pi}\left(\frac{g^2}{4\pi}\right) \left\langle p_2 \right| \gamma_5 \tau_j (\underset{\sim}{p}_1 + \underset{\sim}{k}_1 - M)^{-1} \gamma_5 \tau_i \left| p_1 \right\rangle \left[ 1 - \eta \ln \eta_+ - \eta_- \ln \eta_- \right] \tag{16}$$

$$\eta_\pm = \frac{2p\cdot k}{M^2} \pm 2 \left( \left[\frac{p\cdot k}{M^2}\right]^2 - \left[\frac{\mu}{M}\right]^2 \right)^{\frac{1}{2}} \ .$$

Terms in $(\mu/M)^2 \ln(\mu/M)$ have been omitted. Comparing (16) and (8) the contribution of graph (4b) applied to both nucleon lines gives

$$V_6^{\,b} \sim \left(-\frac{3}{2\pi}\right)\left(\frac{g^2}{4\pi}\right) V_4 \quad . \tag{17}$$

The contribution of the vertex term of graph (4c) is

$$+g^2 \sum_m \left\langle p_2 \middle| \int \frac{d^4k}{(2\pi)^4}\, \tau_j\, \gamma_5 (p_2 - k - M)^{-1} \gamma_5\, \tau_m (p_1 + k_1 - k - M)^{-1} \right.$$

$$\times\ \gamma_5\, \tau_i (p_1 + k_1 - M)^{-1} \gamma_5\, \tau_m (k^2 - \mu^2)^{-1} \left| p_1 \right\rangle . \tag{18}$$

For small $\underset{\sim}{k}_1$ and $\underset{\sim}{k}_2$ Eq. (18), by virtue of (5) and (6), becomes

$$-\frac{i}{2M} \left\langle p_2 \middle| \tau_j\, \tau_i \middle| p_1 \right\rangle \left(\frac{g^2}{4\pi}\right) \quad . \tag{19}$$

Multiplying by four for the four vertices and comparing with (9)

$$V_6^{\,c} \sim \frac{2}{\pi} \left(\frac{g^2}{4\pi}\right) V_4 \quad . \tag{20}$$

The replacement of one of the meson lines in graphs (4) or (5) by graph (2) gives an interaction proportional to $(\mu/2M)^2 (g^2/4\pi) V_4$ . Therefore radiative corrections give a potential[10]

$$V_6 \sim -\frac{5}{2\pi} \left(\frac{g^2}{4\pi}\right) V_4 \quad . \tag{21}$$

Even for $g^2/4\pi \sim 1$ this correction is large. In fact since the radiative correction gives a repulsion, the usual fourth order perturbation calculations overestimate the nuclear force and a larger coupling constant would be needed so that the perturbation calculation would be even less sensible.

$V_6$ has also been estimated from the Bethe-Salpeter equation. For large $r$ the effective non-relativistic sixth order potential agrees with (21).

Conclusion

A small coupling constant $(g^2/4\pi \ll 1)$ for pseudoscalar coupling is in violent disagreement with experiment. For a large coupling radiative corrections from the self field of the nucleon, as given in Eq. (21)[11] are big enough to invalidate the perturbation theory expansion even for $g^2/4\pi \approx 1$ [12]. For $r \sim \hbar/\mu c$ the effect of these corrections is to multiply $V_4$ by some unknown constant while leaving $V_2$ unchanged. This is equivalent to redefining the coefficient of the second term of (1) so that the effective Hamiltonian for perturbation theory calculations involving mesons whose momentum is not large compared to $\mu c$ is

$$\psi^+ \left\{ \frac{g}{2M} \sigma_\mu \frac{\partial}{\partial x_\mu} \tau_i \phi_i + \frac{f^2}{2M} \phi^2 \right\} \psi \qquad (22)$$

where $f$ is an unknown function of $g$ which determines the ratio of ordinary to tensor force in the deuteron and the magnitude of low energy $\pi$- nucleon scattering.

This work was performed under the auspices of the Atomic Energy Commission.

REFERENCES

1. M. Levy, Phys. Rev. 86, 806 (1952).

2. R. P. Feynman, California Institute of Technology Lecture Notes (1951).

3. L. I. Foldy, Phys. Rev. 84, 168 (1951).

4. J. V. Lepore, Bull. Am. Phys. Soc. 27, No. 3, 33 (1952).

5. E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).

6. A nucleon self energy loop on a free nucleon leg is completely cancelled by mass and charge renormalization.

7. K. M. Watson and J. V. Lepore, Phys. Rev. 76, 1157 (1949).

8. The conclusions of this paper differ from those of M. Levy (see reference 1). The origin of the discrepency seems to lie in the absence of graphs like (4a) in the Bethe-Salpeter equation and the neglect of large terms like (5).

9. The integrals (11), (15), and (18) also occur in meson nucleon scattering and have been calculated by J. Ashkin, A. Simon, and R. Marshak (Prog. Theor. Phys. 5, 634 (1950)).

10. A correction of this sign and magnitude might be expected from a consideration of the second term of (1). Wentzel (Helv. Phys. Acta. 15, 111 (1942)) has shown that this term alone leads to a potential $-3/4 \left[4\pi(g^2/M)^{-1} + A\right]^{-2} \mu r^{-2} \left| H_1^{(1)}(2i\mu r)\right|$ if nucleon recoil is neglected. A is the cutoff momentum.

10. (Cont.)

Assuming $g^2/4\pi$ small and $A \sim Mc$,

$$V_4 = -2\left(\frac{g^2}{4\pi}\right) A/M\, V_2 \approx -(g^2/4\pi)\, V_2 .$$

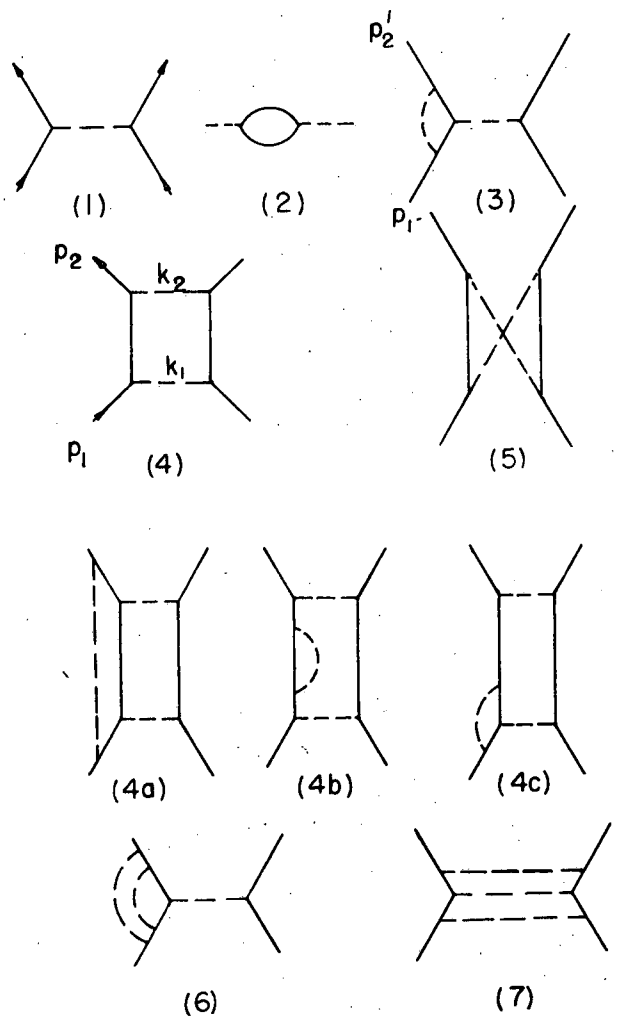
However, the pair interaction given in (1), when taken by itself, does not lead to a renormalizable meson theory.

11. It is easy to show that there exist radiative corrections to (3) of the same range that involve $(g^2/4\pi)$ to any power but do not introduce factors of $(\mu/2M)$.

12. The major part of the integrals represented in graphs (4a, b, c) are due to contributions when two nucleon pairs are present at at the same time. If these are neglected (P. T. Matthews and A. Salam, Phys. Rev. 86, 715 (1952)), the perturbation theory gives a reasonably convergent description for $g^2/4\pi$ as large as 10.

FIGURE CAPTION

Figure 1 Representative Feynman diagrams for the two nucleon interaction.




MU 4112

**Fig. 1**