Submitted to Journal of Nuclear Physics

UCRL-18907
Preprint

Cy. 2

LINEAR RESPONSE THEORY IN NUCLEON-NUCLEUS SCATTERING

RECEIVED
LAWRENCE
RADIATION LABORATORY
JUL 10 1969
LIBRARY AND
DOCUMENTS SECTION

Manfred Weigel

May 1969

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

UCRL-18907

Cy. 2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LINEAR RESPONSE THEORY IN NUCLEON-NUCLEUS SCATTERING†

Manfred Weigel††

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

May 1969

Abstract:

It is shown that the transition amplitude for scattering of nucleons on a hole nucleus can be directly expressed by the linear response function. The correlations in the ground state have been included. The structure of the transition amplitude was studied for an extended schematic model. Furthermore a short summary of the linear response theory is given to clarify the assumptions about the effective interaction.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

††On leave from the Sektion Physik der Universität Munich, Munich, Germany.

1. Introduction

Two different approaches have been used in dealing with the problem of nucleon-nucleus scattering based on a microscopic nuclear description. In the coupled channel treatment Feshbach's projection operator technique is utilised to set down the relevant system of equations.[1,2,3] The microscopic picture of the nuclei is then contained in a set of amplitudes.[4] In the shell model approach one includes the one-particle continuum states in the treatment of nuclear reactions.[5-10] The details can be found in the cited references.

Several authors have shown that one can express the S-matrix for nuclear scattering processes in terms of Green's functions.[11-13] In the Green's function approach of Zhivopistsev the problem of calculating the T-matrix was reduced to the determination of the socalled reducible vertices ("effective scattering amplitudes"). The reducible vertex obeys an equation similar to the equation of the usual scattering amplitude. Instead of the normal interaction one has to insert the irreducible vertex, and the intermediate propagators are expressed in terms of dressed particles. It is possible to treat more complicated scattering processes with this method using appropriate approximations for the vertices.[14]

We limit ourselves to the description of nucleon scattering on a hole nucleus for two reasons: First, that problem has been treated most in the shell model approach; second, we are able to reduce this problem to the calculation of the linear response function, which is frequently used in nuclear structure calculations. Furthermore, the renormalisation procedure for the forces is better known in this simple case. We assume that the

state of the target nucleus is given by the ground state of the compound system plus one quasi-hole. The ground state-correlations of the compound nucleus are included in our treatment.

In the second section we derive the needed scattering formalism using Green's functions. In order to make plain the assumptions involved, we will give a short summary of linear response theory in terms of dressed one-particle propagators and effective interactions in the third part. The assumptions about the two-point functions and the effective interactions will be discussed, too. In the fourth section we use for the zero-order response the shell model response. In this case the T-matrix simplifies considerably. Also the reduction to MacDonald's treatment is given.[6]) In order to get some insight in the structure of the solutions, we treat the degenerated case in the fifth section using Brown-Bolsterli forces. In the appendix the inclusion of quasi-bound single-particle states is given.

2. Scattering Formalism

According to our assumption that the target nucleus can be described by the core plus one quasi-hole in the state α, we have for the target nucleon state:

$$|T\rangle = z_\alpha^{-1/2}\, \psi_\alpha\, |0\rangle \quad . \tag{2.1}$$

$|0\rangle$ denotes the normalised ground state of the compound nucleus ($H|0\rangle = E_0(A)|0\rangle$; A particles). Here, ψ_α and $\psi_\alpha^\dagger$ are the Schroedinger annihilation — and creation operators, respectively, of a nucleon with the quantum number set α fixed by an independent-particle Hamiltonian H_s. The spectrum of H_s is assumed to have a discrete part plus a continuum part. Since the ground state may contain correlations, we have to multiply the state $\psi_\alpha|0\rangle$ with Migdal's renormalisation constant[15]) ($z_\alpha = : |\langle \varepsilon_\alpha|\psi_\alpha|0\rangle|^2$) then getting a normalised state. If it is necessary to distinguish the continuum states from the bound states we will label continuum states by p, k, p' etc.

The independent-particle model scattering state can now be put in a convenient form:

$$|\phi_{p,\alpha}^{(+)}\rangle = (z_\alpha z_p)^{-1/2}\, \psi_p^\dagger(+)\, \psi_\alpha|0\rangle \quad . \tag{2.2}$$

By the index (±) we distinguish the solutions with ($^{out}_{in}$-) coming spherical waves, respectively. By well known methods[16]) we obtain for the scattering solution of the full Hamiltonian ($\eta \to 0$):

$$\langle \phi^{(-)}_{k,\beta} | \Psi^{(+)}_{p,\alpha} \rangle = \langle \phi^{(-)}_{k,\beta} | \frac{i\eta}{E-H+i\eta} | \phi^{(+)}_{p,\alpha} \rangle \quad , \tag{2.3}$$

or using eq. (2.2):

$$\langle \phi^{(-)}_{k,\beta} | \Psi^{(+)}_{p,\alpha} \rangle = (z_\beta z_k z_p z_\alpha)^{-1/2} \langle 0 | \psi_\beta \psi^\dagger_k(-) \frac{i\eta}{E-H+i\eta} \psi^\dagger_p(+) \psi_\alpha | 0 \rangle \quad . \tag{2.4}$$

Here, E is the energy of the scattering system ($E = : \varepsilon_p - \varepsilon_\alpha$). ε_p is the energy of the incident particle, ε_α is the energy of the hole.

The total T-matrix can be divided into two parts ($T = T^S + T^R$); T^S describes the pure shell model scattering and T^R the effect of residual interaction[16]). By means of the Gell-Mann-Goldberger identities and splitting off the Hamiltonian we obtain then for eqs. (2.3) and (2.4):

$$\langle \phi^{(-)}_{k,\beta} | \Psi^{(+)}_{p,\alpha} \rangle = S^S_{kp} \, \delta_{\alpha\beta} + \frac{1}{E_{p,\alpha} - E_{k,\beta} + i\eta} T^R_{k\beta,p\alpha} \quad . \tag{2.5}$$

S^S denotes the S-matrix in the independent-particle model:

$$S^S_{kp} = : \langle k^{(-)} | p^{(+)} \rangle \quad . \tag{2.6}$$

From eqs. (2.4) and (2.5) and introducing the same boundary conditions for the shell model states, we obtain the following equation for the unknown transition amplitude T^R:

$$\frac{1}{E_{p,\alpha} - E_{k,p} + i\eta} T^R_{k\beta,p\alpha} = \sum_{p'} \left\{ (z_\beta z_{p'} z_p z_\alpha)^{-1/2} \langle 0 | \psi^\dagger_\beta \psi_{p'}(+) \frac{i\eta}{E_{p,\alpha} - H + i\eta} \psi^\dagger_p(+) \psi_\alpha | 0 \rangle S^S_{k,p'} \right\}$$

$$- \delta_{\alpha\beta} S^S_{k,p} \tag{2.7}$$

$\sum_{p'}$ denotes integration as well as summation over the other quantum numbers.

By comparison with the definition of the response function $(E_0(A) = 0)$ [17]

$$L_{\kappa\mu\lambda\nu}(\omega) = : -\langle 0| \left\{ \psi^\dagger_\lambda \psi_\kappa (\omega - H + i\eta)^{-1} \psi^\dagger_\nu \psi_\mu + \psi^\dagger_\nu \psi_\mu (\omega + H - i\eta)^{-1} \psi^\dagger_\lambda \psi_\kappa \right\} |0\rangle \qquad (2.8)$$

and assuming the ground state of the compound nucleus — for instance, the O^{16}-core — does not contain components of the continuum states or equivalently all renormalisation constants for continuum states are equal to one, we obtain[†])

$$-i\eta \tilde{L}_{p'\alpha\beta p}(E) = (z_\beta z_{p'} z_p z_\alpha)^{-1/2} \langle 0| \psi^\dagger_\beta \psi_{p'} \frac{i\eta}{E - H + i\eta} \psi^\dagger_p \psi_\alpha |0\rangle \qquad (2.9)$$

with:

$$\tilde{L}_{\kappa\mu\lambda\nu}(\omega) = (z_\kappa z_\mu z_\lambda z_\nu)^{-1/2} L_{\kappa\mu\lambda\nu}(\omega) \quad . \qquad (2.10)$$

Anticipating a result from the next chapters $-i\eta\tilde{L}$ takes the form

$$-i\eta \tilde{L}_{p'\alpha\beta p}(E_{p,\alpha}) = \delta_{pp'} \delta_{\alpha\beta} + \frac{1}{E_{p,\alpha} - E_{p',\beta} + i\eta} \tilde{T}_{p'\beta p\alpha}(E_{p,\alpha}) \qquad (2.11)$$

[†] In the following the upper index "plus" is left out. Furthermore, we mark all quantities describing quasi-particles by a tilde.

giving the following result for the wanted amplitude

$$T^R_{k\beta,p\alpha}(E) = \sum_{p'} \left\{ S^S_{k,p'} \tilde{T}_{p'\beta p\alpha}(E) \right\} \tag{2.12}$$

or

$$T^R_{k\beta,p\alpha}(E) = \tilde{T}_{k\beta,p\alpha}(E) - 2\pi i \sum_{p'} \left\{ \delta(\varepsilon_{p'}-\varepsilon_k) T^S_{kp'} \tilde{T}_{p'\beta p\alpha}(E) \right\} \quad . \tag{2.13}$$

According to eq. (2.11) we have to find an equation for the linear response function, which is the topic of the next section.

3. Linear Response Theory

We use the formulation of the nuclear many-body problem in terms of Green's functions†. Here, for completion we recall the relevant system of equations using the following definition of Green's functions:

$$g_{a_1 \ldots a_n, a'_1 \ldots a'_n} =: (-i)^n \langle 0 | T \, \psi_{a_1} \ldots \psi_{a_n} \ldots \psi^\dagger_{a'_n} \ldots \psi^\dagger_{a'_1} | 0 \rangle \quad . \tag{3.1}$$

Here, ψ_a and $\psi^\dagger_a$ are the Heisenberg annihilation — or creation operators, respectively, of a nucleon with the quantum number set α at the time $t_a (a =: \alpha, t_a)$. T denotes the time ordering operator. $|0\rangle$ is the normalised A-particle ground state of the Hamiltonian

$$H =: \frac{1}{2m} \sum \psi^\dagger_\alpha p^2_{\alpha\beta} \psi_\beta + \frac{1}{2} \sum \psi^\dagger_\alpha \psi^\dagger_\beta v_{\alpha\beta\rho\sigma} \psi_\sigma \psi_\rho \quad . \tag{3.2}$$

The first functional derivative of $g_{k\ell}$ with respect to an external source u_{mn} is usually named as the generalised linear response function L [20]):

$$L_{km\ell n} =: i \left. \frac{\delta g_{k\ell}}{\delta u_{nm}} \right|_{u=0} = -i \, (g_{km\ell n} - g_{k\ell} g_{mn}) \quad , \tag{3.3}$$

since it determines the first-order change in the single-particle propagator due to an external perturbation:

$$\Delta g_{k\ell} = \left. \frac{\delta g_{k\ell}}{\delta u_{mn}} \right|_{u=0} u_{mn} + \ldots \quad . \tag{3.4}$$

---

†See, for instance, refs. [15,18] and [19]).

We employ, until further notice, the convention according to which summation or integration, respectively, is to be carried out over all doubly occuring Latin indices.

With the help of the functional method[20,21]) or by perturbation theory[18]) the following exact system of equations is obtained $(u = 0)$:

$$-\left\{ i \frac{\partial}{\partial t_k} \delta_{kn} + \frac{1}{2m} p^2_{kn} + v_{kn} \right\} g_{n\ell} = \delta_{k\ell} \quad , \tag{3.5}$$

$$v_{kj} = -2iv_{kajb}g_{ba} + iv_{kman}g_{ar}I_{rcjd}L_{dncm} \quad , \tag{3.6}$$

$$L_{kmjn} = L^0_{kmjn} - L^0_{kbjc}I_{cubv}L_{vmun} \quad , \tag{3.7}$$

with

$$L^0_{kmjn} = : -ig_{kn}g_{mj} \quad , \tag{3.8}$$

$$I_{kmjn} = : i \frac{\delta v_{kj}}{\delta g_{nm}} \quad . \tag{3.9}$$

I is the socalled effective particle-hole interaction ("irreducible vertex part"), v_{kj} is the effective one-particle potential ("irreducible mass operator"). Iterating (3.6) and (3.7) and using (3.9) yields the representation of v, L, and I in Feynman graphs containing only the two-particle interaction and the dressed one-particle propagators[15]). All graphs of I are irreducible in the particle-hole channel. Upon transition to energy representation and use of the energy conservation one gets for (3.7):

$$L_{\kappa\mu\lambda\nu}(\varepsilon,\varepsilon';\omega) = L^0_{\kappa\mu\lambda\nu}(\varepsilon,\varepsilon';\omega) - \sum_{\substack{\alpha\beta\\ \gamma\rho}} \int d\varepsilon'' d\varepsilon''' \Big(L^0_{\kappa\delta\lambda\alpha}(\varepsilon,\varepsilon'';\omega) I_{\alpha\beta\delta\rho}(\varepsilon'',\varepsilon''',\omega)$$

$$L_{\rho\mu\beta\nu}(\varepsilon''',\varepsilon';\omega)\Big) \quad . \tag{3.10}$$

Comparison of (2.8) and (3.3) leads to

$$L_{\kappa\mu\lambda\nu}(\omega) = \frac{1}{2\pi} \oint d\varepsilon d\varepsilon' \; L_{\kappa\mu\lambda\nu}(\varepsilon,\varepsilon';\omega) \quad . \qquad (\omega \neq 0) \tag{3.11}$$

Due to the structure of L^0

$$L^0_{\kappa\mu\lambda\nu}(\varepsilon,\varepsilon';\omega) = i g_{\kappa\nu}(\varepsilon+\tfrac{\omega}{2})\; g_{\mu\lambda}(\varepsilon-\tfrac{\omega}{2})\; \delta(\varepsilon-\varepsilon') \quad . \tag{3.12}$$

with

$$g_{\kappa\nu}(\omega) = \langle 0 | \left\{ \psi_\kappa(\omega-H+i\eta)^{-1}\psi^\dagger_\nu + \psi^\dagger_\nu(\omega+H-i\eta)^{-1}\psi_\kappa \right\} | 0 \rangle \tag{3.13}$$

one can carry out only one integration in (3.11):

$$L_{\kappa\mu\lambda\nu}(\varepsilon,\omega) = L^0_{\kappa\mu\lambda\nu}(\varepsilon,\omega) - 2\pi \sum \int d\varepsilon' L^0_{\kappa\gamma\lambda\alpha}(\varepsilon,\omega) I_{\alpha\beta\gamma\rho}(\varepsilon,\varepsilon';\omega) L_{\rho\mu\beta\nu}(\varepsilon';\omega) \quad . \tag{3.14}$$

We have not been able to reduce this equation without an assumption about the effective particle-hole force. Therefore, we neglect retardation effects (ε,ε'-dependence) in the considered energy region caused by diagrams in I with intermediate particle-hole type propagation. Our assumption is fulfilled, for instance, by restricting only to ladder graphs for I. In nuclear matter even the ω-dependence of the ladder approximation is weak[15,22]).

The neglect of the retardation effects leads to:

$$I_{\kappa\mu\lambda\nu}(\omega) = L^{0}_{\kappa\mu\lambda\nu}(\omega) - 2\pi \sum_{\substack{\alpha\gamma \\ \beta\rho}} L^{0}_{\kappa\gamma\lambda\alpha}(\omega)\, I_{\alpha\beta\gamma\rho}(\omega)\, L_{\rho\mu\beta\nu}(\omega) \tag{3.15}$$

with

$$L^{0}_{\kappa\mu\lambda\nu}(\omega) = : \frac{i}{2\pi} \int d\varepsilon\, g_{\kappa\nu}(\varepsilon+\frac{\omega}{2})\, g_{\mu\lambda}(\varepsilon-\frac{\omega}{2}) \quad . \tag{3.16}$$

About the density dependence of the complex effective particle-hole force very little is known. In most approaches this dependence has been disregarded explicitly; sometimes it was included in a phenomological manner[15,23]).

Introducing the renormalisation procedure

$$\tilde{I}_{\kappa\mu\lambda\nu}(\omega) = : (z_\kappa z_\lambda z_\mu z_\nu)^{1/2}\, I_{\kappa\mu\lambda\nu}(\omega) \quad , \tag{3.17}$$

$$\tilde{g}_{\kappa\lambda}(\omega) = :(z_\kappa z_\lambda)^{-1/2}\, g_{\kappa\lambda}(\omega) \quad , \tag{3.18}$$

$$\tilde{v}_{\kappa\lambda}(\omega) = :(z_\kappa z_\lambda)^{1/2}\, v_{\kappa\lambda}(\omega) \quad , \tag{3.19}$$

we obtain from eqs. (3.15) and (3.16):

$$\tilde{L}_{\kappa\mu\lambda\nu}(\omega) = \sum_{\gamma\rho} \left\{ \tilde{L}^{0}_{\kappa\gamma\lambda\rho}(\omega) \left[\delta_{\gamma\mu}\gamma_{\nu\rho} - 2\pi \sum_{\beta\sigma} \tilde{I}_{\rho\beta\gamma\sigma}(\omega)\, \tilde{L}_{\sigma\mu\beta\nu}(\omega) \right] \right\} \tag{3.20}$$

or equivalently:

$$\tilde{I}_{\kappa\mu\lambda\nu}(\omega) = \sum_{\beta\rho}\left\{\left[\delta_{\rho\kappa}\delta_{\beta\lambda} - 2\pi \sum_{\sigma\gamma} \tilde{L}_{\kappa\gamma\lambda\sigma}(\omega)\, \tilde{I}_{\sigma\beta\gamma\rho}(\omega)\right] \tilde{L}^{0}_{\rho\mu\beta\nu}(\omega)\right\} . \tag{3.21}$$

4. T-Matrix with Shell Model Response

As a first step we assume that in the considered energy region all terms in (3.6) are weakly energy-dependent and can be collected in a shell-model potential, for example, of Saxon-Wood shape. This assumption seems reasonable, since the single particle character of $v_{k\ell}$ is maintained if one neglects in the linearised equation for the effective scattering amplitude Γ all terms with intermediate particle-hole propagation. Then, one obtains:

$$v^S_{km} \approx - i\Gamma^{\ell}_{kbma} g_{ab} \quad . \tag{4.1}$$

Diagonalising the shell model Hamiltonian $\frac{1}{2m} p^2_{k\ell} + \tilde{v}^S_{k\ell}$ and using this basis one obtains for the two-point function:

$$\tilde{g}^S_{\sigma\mu}(\omega) = \delta_{\sigma\mu} \left\{ \frac{1-n_\sigma}{\omega-\varepsilon_\sigma+i\eta} + \frac{n_\sigma}{\omega-\varepsilon_\sigma+i\eta} \right\} =: \delta_{\sigma\mu} \tilde{g}^S_\sigma(\omega) \quad . \tag{4.2}$$

Here, n_σ is the quasi-particle occupation number. Using eq. (4.2) the zeroth-order response reduces to:

$$\tilde{L}^0_{\kappa\mu\lambda\nu}(\omega) = \delta_{\kappa\nu}\delta_{\mu\lambda} \tilde{L}^S_{\kappa\lambda}(\omega) \tag{4.3}$$

with

$$\tilde{L}^S_{\kappa\lambda}(\omega) =: \frac{n_\kappa(1-n_\lambda)}{\omega+\varepsilon_\lambda-\varepsilon_\kappa-2i\eta} - \frac{n_\lambda(1-n_\kappa)}{\omega-\varepsilon_\kappa+\varepsilon_\lambda+2i\eta} \quad . \tag{4.4}$$

For the wanted response function in eq. (2.9) we obtain from (3.21), (4.3), and (4.4) the equation:

$$\tilde{L}_{p'\alpha\beta p}(\omega) = L^{S}_{p\alpha}(\omega)\left\{\delta_{pp'}\delta_{\alpha\beta} - 2\pi\sum_{\gamma\nu}\tilde{L}_{p'\gamma\beta\nu}(\omega)\,\tilde{I}_{\nu\alpha\gamma p}(\omega)\right\} \quad . \tag{4.5}$$

Hence $\tilde{T}_{p'\beta p\alpha}$ is given by

$$\tilde{T}_{p'\beta p\alpha}(E) = 2\pi\, L^{S^{-1}}_{p'\beta}(E)\sum_{\gamma\nu}\tilde{L}_{p'\gamma\beta\nu}(E)\,\tilde{I}_{\nu\alpha\gamma p}(E) \quad , \tag{4.6}$$

where $\tilde{L}_{p'\gamma\beta\nu}$ is the solution of

$$\tilde{L}_{p'\gamma\beta\nu}(\omega) = \tilde{L}^{S}_{p'\beta}(\omega)\left\{\delta_{p'\nu}\delta_{\beta\gamma} - 2\pi\sum_{\rho\mu}\tilde{I}_{p'\rho\beta\mu}(\omega)\,\tilde{L}_{\mu\gamma\rho\nu}(\omega)\right\} . \tag{4.7}$$

$\sum$ means summation over the discrete states as well as integration over the continuum states.

For a general consideration it may be useful to symmetrise the expressions in the variable pair $(p'\beta;p\alpha)$. We achieve this by introducing the effective scattering amplitude $\tilde{\Gamma}$ $(\tilde{\Gamma}\,\tilde{L}^{0} = \tilde{I}\,\tilde{L})$. The equation for $\tilde{\Gamma}$ can be read off, for instance, from the formal solution of (4.7):

$$\tilde{\Gamma} = \tilde{I} - 2\pi\,\tilde{I}\,\tilde{L}^{S}\,\tilde{\Gamma} \quad , \tag{4.8}$$

or

$$\tilde{\Gamma} = \tilde{I} - 2\pi\,\tilde{I}\,\tilde{L}\,\tilde{I} \quad . \tag{4.9}$$

Insertion into eq. (4.5) leads to:

$$\tilde{L}_{p'\alpha\beta p}(\omega) = \tilde{L}^{S}_{p\alpha}(\omega) \left\{ \delta_{pp'}\delta_{\alpha\beta} - \tilde{L}^{S}_{p'\beta}(\omega) \left[2\pi\, \tilde{I}_{p'\alpha\beta p}(\omega) - (2\pi)^2 \sum_{\substack{\nu\mu \\ \rho\sigma}} \tilde{I}_{p'\mu\beta\nu}(\omega)\, \tilde{L}_{\nu\sigma\mu\rho}(\omega)\, \tilde{I}_{\rho\alpha\sigma p}(\omega) \right] \right\} . \tag{4.10}$$

Then $\tilde{T}$ takes the form:

$$\tilde{T}_{p'\beta\alpha p}(E) = (2\pi)\, \tilde{I}_{p'\alpha\beta p}(E) - (2\pi)^2 \sum_{\substack{\rho\mu \\ \sigma\nu}} \tilde{I}_{p'\mu\beta\nu}(E)\, \tilde{L}_{\nu\sigma\mu\rho}(E)\, \tilde{I}_{\rho\alpha\sigma p}(E) \quad . \tag{4.11}$$

The complex energy resonances are given by the zero-points of the Fredholm determinant[24]):

$$\mathrm{Det}\,(1 + 2\pi\, \tilde{I}\, \tilde{L}^{S}) = 0 \quad . \tag{4.12}$$

You may obtain these zero-points E_r by evaluating the Fredholm determinant:

$$0 \overset{!}{=} 1 + 2\pi \sum_{\nu\mu} \tilde{I}_{\nu\mu\mu\nu}(E_r)\, \tilde{L}^{S}_{\nu\mu}(E_r)$$

$$+ \frac{(2\pi)^2}{2!} \sum_{\substack{\nu\mu \\ \rho\sigma}} \left\{ \begin{vmatrix} \tilde{I}_{\nu\mu\mu\nu}(E_r) & \tilde{I}_{\nu\rho\mu\sigma}(E_r) \\ \tilde{I}_{\sigma\mu\rho\nu}(E_r) & \tilde{I}_{\sigma\rho\rho\sigma}(E_r) \end{vmatrix} \tilde{L}^{S}_{\nu\mu}(E_r)\, \tilde{L}^{S}_{\sigma\rho}(E_r) \right\} + \ldots \tag{4.13}$$

Equation (4.11) looks similar to the expression for the T-matrix given by MacDonald[6]). But he uses a response function $\tilde{L}^D$ in which only the discrete part $\tilde{L}^{SD}$ of $\tilde{L}^S$ is taken into account ($\tilde{L}^S = \tilde{L}^{SD} + \tilde{L}^{SC}$). Hence we have the following equation for $\tilde{L}^D$:

$$\tilde{L}^D = \tilde{L}^{SD} - 2\pi\, \tilde{L}^{SD}\, \tilde{I}^D\, \tilde{L}^D \quad . \tag{4.14}$$

Here, all quantum numbers are restricted to the discrete part of the spectrum and $\tilde{I}^D$ obeys the equation

$$\tilde{I}^D = \tilde{I} - (2\pi)\, \tilde{I}\, \tilde{L}^{SC}\, \tilde{I}^D \quad . \tag{4.15}$$

Utilising operator algebra one can transform eq. (4.10) in

$$\tilde{L} = \tilde{L}^S - \tilde{L}^S \left(2\pi\, \tilde{I}^D - (2\pi)^2\, \tilde{I}^D\, \tilde{L}^D\, \tilde{I}^D\right) \tilde{L}^S \quad . \tag{4.16}$$

Then $\tilde{T}$ takes the form

$$\tilde{T}_{p'\beta\alpha p}(E) = (2\pi)\, \tilde{I}^D_{p'\alpha\beta p}(E) - (2\pi)^2 \sum_{\substack{\nu\mu \\ \rho\nu}}{}' \tilde{I}^D_{p'\mu\beta\nu}(E)\, \tilde{L}^D_{\nu\sigma\mu\rho}(E)\, \tilde{I}^D_{\rho\alpha\sigma p}(E) \tag{4.17}$$

expressing $\tilde{T}$ in terms of $\tilde{I}^D$ and $\tilde{L}^D$ only. Insertion of a complete system $\{|m\rangle ;\ H|m\rangle = E_m|m\rangle\}$ of the compound system into the definition (2.8) of L leads to $\left(\tilde{A}_{\alpha\beta,m} = : (z_\alpha z_\beta^-)^{-1/2} \langle 0|\psi_\beta^\dagger \psi_\alpha|m\rangle\right):$

$$\tilde{L}_{\alpha\sigma\beta\rho}(\omega) = -\sum_m \left\{ \frac{\tilde{A}_{\alpha\beta,m}\tilde{A}^*_{\rho\sigma,m}}{\omega - E_m + i\eta} - \frac{\tilde{A}_{\sigma\rho,m}\tilde{A}^*_{\beta\alpha,m}}{\omega + E_m - i\eta} \right\} \quad . \tag{4.18}$$

Now, one obtains the result of MacDonald assuming that form remains valid for $\tilde{L}^D$ where $|m\rangle$ and E_m have got to be replaced by the solutions of the eigenvalue problem with the complex interaction $2\pi\tilde{I}_D$. The second term on the right-hand side of (4.18) is neglected in this approach.

5. Schematic Model

In order to show the structure of the solutions for the T-matrix more explicitly, we solve the problem for the Brown-Bolsterli forces[25]). We choose this force because its separable form leads to a degeneration of the Fredholm determinant (4.13). The explicit form of the force is:

$$2\pi\, \tilde{I}_{\kappa\mu\lambda\nu}(\omega) = \lambda\, f_{\mu\nu} f_{\kappa\lambda} \tag{5.1}$$

with

$$f_{\mu\nu} =: V_{\mu\nu}(1-n_\mu)n_\nu + V_{\nu\mu}n_\mu(1-n_\nu) \quad . \tag{5.2}$$

For the solution of (4.5) one obtains using (5.1):

$$\tilde{L}_{p'\alpha\beta p}(\omega) = \tilde{L}^S_{p\alpha}(\omega)\left\{ \delta_{pp'}\delta_{\alpha\beta} - \tilde{L}^S_{p'\beta}(\omega)\, \frac{\lambda V_{p\alpha} V_{p'\beta}}{1+\Delta(\omega)} \right\} \tag{5.3}$$

where

$$\Delta(\omega) =: 2\lambda \sum_{\substack{\mu>\lambda_F \\ \eta\leq\lambda_F}} |\langle \mu|V|\eta\rangle|^2 \frac{\varepsilon_\mu-\varepsilon_\eta}{(\varepsilon_\mu-\varepsilon_\eta)-\omega^2}$$

$$+ 2\lambda \sum_{\eta\leq\lambda_F} \sum_{\nu_p} P\int d\varepsilon_p |\langle \varepsilon_p\nu_p|V|\eta\rangle|^2 \frac{\varepsilon_p-\varepsilon_\eta}{(\varepsilon_p-\varepsilon_\eta)^2-\omega^2}$$

$$+ i\pi\lambda \sum_{\eta\leq\lambda_F} \sum_{\nu_p} \left\{ |\langle \omega+\varepsilon_\eta\nu_p|V|\eta\rangle|^2 + |\langle \varepsilon_\eta-\omega\nu_p|V|\eta\rangle|^2 \right\}$$

$$= \Delta^D(\omega) + \Delta^C(\omega) + i\gamma(\omega) \quad . \tag{5.4}$$

λ_F denotes the Fermi level. In (5.4) we used a more explicit notation labelling the hole states with η, the bound particle states with μ, and the continuum states with $(\varepsilon_p \nu_p \equiv p)$. We omit explicit angular momentum labels. Now, $\tilde{T}$ takes the form

$$\tilde{T}_{p'\beta p\alpha}(E) = \frac{\lambda\langle p|V|\alpha\rangle\langle p'|V|\beta\rangle}{1+\Delta(E)} \quad . \tag{5.5}$$

The complex energy resonances $E_r(E_r =: \varepsilon_r - i\Gamma_r)$ are given by the solutions of $1+\Delta(E_r) = 0$, therefore, we obtain near an isolated resonance the Breit-Wigner formula[16])

$$\tilde{T}_{p'\beta p\alpha}(E) \approx \frac{\lambda\langle p|V|\alpha\rangle\langle p'|V|\beta\rangle\Delta'(E_r)}{E-E_r} \quad . \tag{5.6}$$

In nuclear structure calculations only the first term in (5.4) is used; the last two terms give the influence of the continuum producing an energy-shift and a width. If the energy-shift, caused by the continuum, is small we obtain:

$$\tilde{T}_{p'\beta p\alpha}(E) \approx \frac{\lambda\langle p|V|\alpha\rangle\langle p'|V|\beta\rangle\Delta^{D'}(\varepsilon_r^D)}{E-\varepsilon_r^D + \dfrac{(\Delta^C(\varepsilon_r^D)+i\gamma(\varepsilon_r^D)}{\Delta^{D'}(\varepsilon_r^D)}} \quad . \tag{5.7}$$

ε_r^D is the solution of the nuclear structure calculation. A more detailed discussion can be found in the treatment of Lemmer[26]), who disregards the second term in (4.18).

6. Summary

A formalism for calculating nucleon scattering by one-hole nuclei using linear response theory has been set down. We have been able to reduce our treatment to earlier approaches by neglecting ground state correlations of the compound nucleus. Furthermore, we discussed the assumptions about the effective forces which enter in the nucleon scattering problem starting from the exact system of equations for the many-body problem. In the appendix the influence of resonant single-particle states is given.

Acknowledgments

The author would like to thank Drs. L. Garside and G. Wegmann for helpful discussions and Prof. G. Süssmann for stimulating my interest in this problem. He wishes to express his gratitude to the members of the LRL theoretical nuclear physics group for their hospitality. The Deutsche Forschungsgemeinschaft, Germany, supported this work by a grant.

Appendix

So far we have used the assumption that the one-particle motion could be described by a weakly energy-dependent potential. But the second term in eq. (3.6) may have a strong energy-dependent part, for instance, due to the coupling of the one-particle motion to resonant states of the compound nucleus. Furthermore, according to eqs. (2.8) and (2.9) the ground state correlations have been restricted to bound states only. But, in many nuclear structure calculations, using a realistic shell-model potential, one needs single-particle states that lie in the continuum. They cause a sharp single particle resonance, for instance, the $1d_{3/2}$ state in O^{16}. Such states give a strong energy-dependence in the last two terms of (5.4). Several methods are known to overcome this difficulty[5-10,28]), especially, the most important case of a single-particle resonance. We adopt the method of Garside and MacDonald[6]) introducing an additional one-particle potential $-v^R_{k\ell}$, which shifts the relevant state into a bound state. Then $v^R_{k\ell}$ causes the energy-shift and the width. This method can also be applied to the coupling of the one-particle states to resonant states. In this case $v^R_{k\ell}$ is the resonant part of the one-particle potential. In both cases the one-particle propagator obeys the following equation

$$\tilde{g}_{\mu\nu}(\omega) = \tilde{g}^S_{\mu\nu}(\omega) + \sum_\lambda \tilde{g}^S_\mu(\omega)\tilde{v}^R_{\mu\lambda}(\omega)\tilde{g}_{\lambda\nu}(\omega) \quad , \tag{A.1}$$

where the total effective one-particle potential is given by

$$\tilde{v}_{k\ell} = \tilde{v}^S_{k\ell} + \tilde{v}^R_{k\ell} \quad . \tag{A.2}$$

The resonant one-particle state is bound in the potential $\tilde{v}^S_{k\ell}$.

Introducing the effective one-particle scattering amplitude by

$$\tilde{t}_{\mu\nu}(\omega) = \tilde{v}^{R}_{\mu\nu}(\omega) + \sum_{\gamma} \tilde{v}^{R}_{\mu\gamma}(\omega)\, \tilde{g}^{S}_{\gamma}(\omega)\, \tilde{t}_{\gamma\nu}(\omega) \quad , \tag{A.3}$$

we obtain

$$\tilde{g}_{\mu\nu}(\omega) = \delta_{\mu\nu}\, \tilde{g}^{S}_{\nu}(\omega) + \tilde{g}^{S}_{\mu}(\omega)\, \tilde{t}_{\mu\nu}(\omega)\, \tilde{g}^{S}_{\nu}(\omega) \quad . \tag{A.4}$$

Due to the more complicated structure of (A.4) we obtain for the zeroth order response (3.16) instead of (4.3) the expression:

$$\tilde{L}^{0}_{\kappa\mu\lambda\nu}(\omega) = \tilde{L}^{S}_{\nu\mu}(\omega) \Big\{ \delta_{\kappa\nu}\delta_{\mu\lambda} - \delta_{\kappa\nu} n_{\nu}\, \tilde{L}^{S}_{\nu\lambda}(\omega)\, \tilde{t}_{\mu\lambda}(\varepsilon_{\nu}-\omega)\, \delta_{\lambda\bar{\nu}}$$
$$- \delta_{\mu\lambda} n_{\mu}\, \tilde{L}^{S}_{\kappa\mu}(\omega)\, \tilde{t}_{\kappa\nu}(\varepsilon_{\mu}+\omega)\, \delta_{\nu\bar{\nu}} \Big\} \quad . \tag{A.5}$$

In (A.5) we have restricted ourselves to the particle part of the $\tilde{t}$-matrix. Furthermore $\bar{\nu}$ denotes the quantum number set of the relevant one-particle state shifted by the additional potential. If $v^{R}_{k\ell}$ influences several particle-states, one has to drop $\delta_{\nu\bar{\nu}}$ and $\delta_{\lambda\bar{\nu}}$ [27]). The equation

$$\tilde{L} = \tilde{L}^{0} - 2\pi\, \tilde{L}\, \tilde{I}\, \tilde{L}^{0} \tag{A.6}$$

can now be reduced to a simpler one of the form (4.5):

$$\tilde{L}' = \tilde{L}^{S} - 2\pi\, \tilde{L}'\, \tilde{I}'\, \tilde{L}^{S} \tag{A.7}$$

by introducing the matrix

$$A_{\kappa\mu\lambda\nu}(\omega) = : \delta_{\kappa\nu}\delta_{\mu\lambda} + a_{\kappa\mu\lambda\nu} \tag{A.8}$$

where

$$a_{\kappa\mu\lambda\nu}(\omega) = :-\delta_{\kappa\nu}\delta_{\lambda\bar{\nu}}(1-n_\mu)n_\nu\ \tilde{L}^S_{\nu\lambda}(\omega)\ \tilde{t}_{\mu\lambda}(\varepsilon_\nu-\omega) - \delta_{\mu\lambda}\delta_{\nu\bar{\nu}}\ n_\mu(1-n_\nu)\cdot$$

$$\tilde{L}^S_{\kappa\mu}(\omega)\ \tilde{t}_{\kappa\nu}(\varepsilon_\nu+\omega) \quad , \tag{A.9}$$

$$\tilde{I}' =:\tilde{I}\ A = \tilde{I} + \tilde{I}a =:\tilde{I} + \Delta\tilde{I} \tag{A.9}$$

$\tilde{L}$ can be expressed by A and $\tilde{L}'$

$$\tilde{L} = A\ \tilde{L}' \quad . \tag{A.11}$$

The analogue of (4.11) takes now the form:

$$\tilde{T}_{p'\beta p\alpha}(E) = \delta_{\alpha\beta}\delta_{p\bar{\nu}}\tilde{t}_{p'p}(\varepsilon_p+i\eta) + 2\pi\sum_\sigma\Bigg[\ \tilde{\Gamma}'_{\sigma\alpha\beta p}(E)\Big(\delta_{p'\sigma} - \delta_{\sigma\bar{\nu}}\ (1-n_\sigma)\cdot$$

$$\tilde{t}_{p'\sigma}(\varepsilon_\beta+E+i\eta)\Big)\Bigg] \tag{A.12}$$

with:

$$\tilde{\Gamma}' = \tilde{I}' -2\pi\ \tilde{I}'\ \tilde{L}'\ \tilde{I}' \quad . \tag{A.13}$$

As in section 4, it is possible to restrict oneself to bound states by introducing a new effective interaction. The procedure can be taken over from section 4.

For some purposes it may be useful to calculate $\tilde{L}'$ with the help of perturbation theory. We obtain

$$\tilde{L}' = \tilde{L}_I - (2\pi)\, \tilde{L}_I\, \Delta\, \tilde{\Gamma}\, \tilde{L}_I \tag{A.14}$$

where $\tilde{L}_I$ is the solution of (A.7) with $\tilde{I}' = I$. $\Delta\, \tilde{\Gamma}$ obeys the equation

$$\Delta\, \tilde{\Gamma} = \Delta\, \tilde{I} - 2\pi\, \Delta\, \tilde{I}\, \tilde{L}_I\, \Delta\, \tilde{\Gamma} \quad . \tag{A.15}$$

Iteration of (A.15) and insertion in (A.14) yields the perturbative expansion of L'.

SCHEMATIC MODEL:

In the schematic model we obtain a result very similiar to (5.3) and (5.5):

$$\tilde{L}_{p'\alpha\beta p}(\omega) = \tilde{L}^S_{p\alpha}(\omega) \Big\{ \delta_{pp'}\delta_{\alpha\beta} - \tilde{L}^S_{p'\beta}(\omega) \Big[\delta_{\alpha\beta}\tilde{t}_{p'p}(\varepsilon_\alpha+\omega+i\eta)\, \delta_{p\bar{\nu}} + D^{-1}(\omega)\, C_{\alpha p}(\omega)\, C_{p'\beta}(\omega) \Big] \Big\} \tag{A.16}$$

or

$$\tilde{T}_{p'\alpha\beta'}(E) = \delta_{\alpha\beta}\tilde{t}_{p'p}(\varepsilon_p+i\eta)\delta_{p\bar{\nu}} + \lambda\, D^{-1}(\omega) C_{\alpha p}(E) C_{p'\beta}(E) \quad , \tag{A.17}$$

respectively.

Here $C(\omega)$ and $D(\omega)$ are defined by

$$C_{\mu\nu}(\omega) =: \sum_{\rho\eta} f_{\rho\eta} A_{\eta\mu\rho\nu} \tag{A.18}$$

and

$$D(\omega) = : 1 + \Delta(\omega) + \Delta^R(\omega) \tag{A.19}$$

with

$$\Delta^R(\omega) = : - \sum_{\kappa\lambda} \left\{ (1-n_\kappa) n_\lambda V_{\kappa\lambda} \tilde{L}^S_{\lambda\kappa}(\omega) \sum_\rho \left(V_{\rho\lambda}(1-n_\rho)[\tilde{L}^S_{\lambda\rho}(\omega)\delta_{\rho\bar{\nu}} \tilde{t}_{\kappa\rho}(\varepsilon_\lambda-\omega+i\eta) \right.\right.$$

$$\left.\left. + \delta_{\kappa\bar{\nu}} \tilde{L}^S_{\rho\lambda}(\omega) \tilde{t}_{\rho\kappa}(\varepsilon_\lambda+\omega+i\eta)] \right) \right\} . \tag{A.20}$$

References

1) H. Feshbach, Ann. Phys. (N.Y.) 5 (1958) 357; 19 (1962) 287.

2) N. F. Mott and H. S. W. Massey, "The Theory of Atomic Collisions", (Clarendon Press, Oxford, 1933); in 2nd Ed. see p. 144.

3) T. Tamura, Rev. Mod. Phys. 37 (1965) 679.

4) N. K. Glendenning, Proceedings of the International School of Physics "Enrico Fermi" Course XL (1967) (Academic Press, New York; to be published) and University of California, Lawrence Radiation Laboratory Report UCRL-17503.

5) R. H. Lemmer and C. M. Shakin, Ann. Phys. (N.Y.) 27 (1964) 13.

6) W. M. MacDonald, Nucl. Phys. 54 (1964) 393; 56 (1964) 636, W. M. Mac Donald and A. Mekjian, Phys. Rev. 160 (1967) 730, W. P. Beres and W. M. MacDonald, Nucl. Phys. A91 (1967) 529, L. Garside, Thesis, University of Maryland (1966).

7) W. W. Balashov et al., Sov. J. Nucl. Phys. 2 (1965) 461 [Engl. Transl.].

8) C. Bloch, Proceedings of the Enrico Fermi Summer School, Varenna, Course XXXVI, p. 394 (Academic Press, New York, 1966).

9) H. A. Weidenmüller, Nucl. Phys. 75 (1966) 189; H. A. Weidenmüller and K. Dietrich, ibid. 83 (1966) 332; W. Glöckle et al., ibid. A90 (1967) 481, W. Ebenhöh et al., Z. Physik 202 (1967) 301, J. Hüfner and R. H. Lemmer, Phys. Rev. 175 (1968) 1394.

10) M. Danos and W. Greiner, Phys. Rev. 146 (1966) 708.

11) J. S. Bell and E J. Squires, Phys. Rev. Letters 3 (1959) 96.

12) M. Naminiki, Progr. Theoret. Phys. 23 (1960) 629.

13) F. A. Zhivopistsev, Sov. J. Nucl. Phys. 1 (1965) 429 [Engl. Trans.].

14) S. G. Kadmenskii, Sov. J. Nucl. Phys. 5 (1967) 538 [Engl. Trans.].

15) A. B. Migdal and A. I. Larkin, Nucl. Phys. 51 (1964) 501, A. B. Migdal, "Theory of Finite Fermi Systems and Applications to Atomic Nuclei" (Interscience Publishers, New York, 1967).

16) M. L. Goldberger and K. M. Watson, "Collision Theory" (John Wiley and Sons, New York, 1964).

17) D. J. Thouless, Nucl. Phys. 22 (1961) 78.

18) A. A. Abvikosov, L. P. Gorkov, J. E. Dzyaloskinsky, "Methods of Quantum Field Theory in Statistical Physics" (Pergamon Press, Oxford, 1965).

19) P. C. Martin and J. Schwinger, Phys. Rev. 115 (1959) 1342.

20) G. Baym and L. P. Kadanoff, Phys. Rev. 124 (1961) 287; "Quantum Statistical Mechanics" (Benjamin, New York, 1962); G. Baym, Phys. Rev. 127 (1962) 1391.

21) Yu. V. Novozhilov and A. V. Tulub, "The Method of Functionals in the Quantum Theory of Fields" (Gordon and Breach, New York, 1961); S. Engelsberg, Phys. Rev. 126 (1962) 1251; W. Brenig and H. Wagner, Z. Physik 173 (1963) 484.

22) M. Weigel and G. Wegmann, Greensche Funktionen in der Theorie der Kernmaterie, preprint.

23) P. Haug, Thesis, University of Munich, 1969.

24) K. Gottfried and L. Picman, Mat. Fys. Dan. Vid. Selsk. 32, No. 13 (1960), F. G. Tricomi, "Integral Equations" (Interscience Publishers, New York, 1957) pp. 64-76.

25) J. E. Brown, "Unified Theory of Nuclear Models and Forces" (North-Holland Publishing Co. and J. Wiley, Amsterdam, New York, 1967; 2nd Ed.); G. E. Brown and M. Bolsterli, Phys. Rev. Letters 3 (1959) 472.

26) R. H. Lemmer, Phys. Letters 4 (1963) 205.

27) W. Brenig, Z. Physik 202 (1967) 340; M. Weigel, Nuovo Cimento 57B (1968) 527.

28) K. W. McVoy and W. J. Romo, Nucl. Phys. A126 (1969) 161.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720