UCRL-18916
UC-34 Physics
TID-4500 (54th Ed.)

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

# TABLES AND GRAPHS OF FIRST ORDER PERTURBATION EFFECTS IN IRON DOMINATED TWO DIMENSIONAL SYMMETRICAL MULTIPOLES

Klaus Halbach and Ron Yourd

May 1969

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

This document is
PUBLICLY RELEASABLE
B Steele
Authorizing Official
Date: 9-20-06

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

## DISCLAIMER

**This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.**

**DISCLAIMER**

**Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.**

# Tables And Graphs Of First Order Perturbation Effects In Iron Dominated Two Dimensional Symmetrical Multipoles

Klaus Halbach and Ron Yourd

Because of space limitations, in a recently written paper[1)] only the most important tables and graphs could be included. This report contains more extensive tables and graphs than the original paper. When referring to figures or equations of that paper, the identification number is preceded by I.

To make the use of the tables more convenient, in this report the following quantities are tabulated for N = 2, 3, 4: $j_n$, $b_n$, $a_n$, $\rho_n$, $b_n \pm a_n$, $G_x$, $G_y$, $\rho_n/\cos(\frac{\pi}{2} \cdot \frac{n}{N})$, as well as these quantities multiplied by n/N. To cover any reasonable foreseeable need, but also realizing that tables for larger N will not be used very often, for N = 6, 8, 12 only the following basic quantities are tabulated: $j_n \cdot n/N$, $b_n \cdot n/N$, $a_n \cdot n/N$, $\rho_n \cdot n/N$. The notation and use of the tables is discussed in detail in I.

Since the above mentioned paper was written, the graphs in Fig. I8 - Fig. I11 were used extensively for shimming quadrupoles. They proved to be so useful that it seemed worthwhile to reproduce more specific graphs, combining, with some modifications, Fig. I8 with Fig. I9 - Fig. I11 for N = 1, 2, 3, 4.

Using, and slightly modifying, the equation following equ. I36, equ. I40f can be rewritten as follows:

$$\Delta F = \sum Z^{N(2m+1)} \cdot \Delta C_{N(2m+1)}, \qquad (1)$$

$$\Delta C_{N(2m+1)} = i \cdot 4N \cdot \mathrm{Re}(L_{2m+1}) \cdot \Delta V \cdot \frac{d\psi}{d\Phi} \cdot \Delta\Phi/\pi. \qquad (2)$$

Using $\Delta C_{N(2m+1)} \cdot n/N = (2m+1) \cdot \Delta C_{N(2m+1)}$ to describe the effect of the poleface modification in terms of its normalized contribution to the field (see equ. I3e), introducing

$$\alpha = N\Phi, \qquad (3)$$

and using equ.'s I36 and I20d, $(2m+1)\Delta C_{N(2m+1)}$ for an extended poleface modification can be expressed by:

$$(2m+1)\ \Delta C_{N(2m+1)} = i \cdot \int_0^{\pi/2} P_{N(2m+1)}(\alpha) \cdot h(\alpha) d\alpha \qquad (4)$$

$$P_{N(2m+1)} = \mathrm{Re}((2m+1)L_{2m+1}) \cdot \frac{2N}{\cos h(\frac{\pi}{2} \tan \alpha) \cdot (\cos \alpha)^{3-1/N}} \cdot \frac{\pi}{180}. \qquad (5)$$

Graphs of $P_{N(2m+1)}$ are shown in Fig. 1P, 2P, 3P, 4P for N = 1, 2, 3, 4, and for 2m+1 = 3, 5, 7. For convenience, the factor $\pi/180$ has been included in the definition of $P_{N(2m+1)}$ (equ. 5) so that when carrying out the integration in equ. (4) numerically, $d\alpha$ is measured in degrees. As in I, $h(\alpha)>0$ corresponds to addition of iron, normalized with the aperture radius of the magnet. While equ. 4 is very useful for localized poleface modifications, the integration becomes tedious for extended modifications. If $h(\alpha)$ is given by

$$\begin{aligned} h(\alpha) &= g_o + g_1 \cdot \alpha \quad &&\text{for} \quad \alpha_1 \leq \alpha \leq \alpha_2 \\ h(\alpha) &= 0 &&\text{for} \quad \alpha_2 < \alpha < \alpha_1, \end{aligned} \qquad (6)$$

and one introduces

$$Q_{N(2m+1)}(\alpha) = \int_0^{\alpha} P_{N(2m+1)}(\alpha) \cdot d\alpha \qquad (7)$$

and

$$R_{N(2m+1)}(\alpha) = \int_0^{\alpha} P_{N(2m+1)}(\alpha) \cdot \alpha \cdot d\alpha, \tag{8}$$

then $(2m+1) \cdot \Delta C_{N(2m+1)}$ resulting from a modification described by equ. (6) becomes:

$$(2m+1) \cdot \Delta C_{N(2m+1)} = i \left\{ g_o(Q_{N(2m+1)}(\alpha_2) - Q_{N(2m+1)}(\alpha_1)) + g_1(R_{N(2m+1)}(\alpha_2) - R_{N(2m+1)}(\alpha_1)) \right\} . \tag{9}$$

The graphs for $Q_{N(2m+1)}(\alpha)$ and $R_{N(2m+1)}(\alpha)$ are shown in the remaining figures.

In all equations and graphs, $\alpha$ is measured in degrees. Since every polygon - shaped poleface modification can be described by a series of equations in the form of equ. 6, equ. 9 together with the graphs allows a fairly simple evaluation of the effects of such modifications.[2)]

REFERENCES

1. K. Halbach, First Order Perturbation Effects in Iron-Dominated Two-Dimensional Symmetrical Multipoles, UCRL-18841, to be published in Nucl. Instr. Methods.

2. This work performed under the auspices of the U. S. Atomic Energy Commission.

N = 2

| n | $j_n = \frac{\Delta C_n(x)}{i\epsilon}$ | $b_n = \frac{\Delta C_n(rd)}{i\epsilon}$ | $a_n = \frac{\Delta C_n(ad)}{\epsilon}$ | $\rho_n = \frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 3.99E-01 | -8.51E-01 | 1.49E-01 | 3.52E-01 |
| 2 | 2.50E-01 | -5.16E-01 | 2.14E-01 | 5.00E-01 |
| 3 | 1.04E-01 | -1.92E-01 | 1.92E-01 | 4.40E-01 |
| 4 | 0. | 3.38E-02 | 1.15E-01 | 2.50E-01 |
| 5 | -8.20E-03 | 4.33E-02 | 4.33E-02 | 7.63E-02 |
| 6 | 0. | 1.48E-02 | 9.58E-03 | 0. |
| 7 | 4.60E-03 | -2.97E-03 | 2.97E-03 | -8.75E-03 |
| 8 | 0. | 3.20E-03 | 3.89E-03 | 0. |
| 9 | -4.22E-04 | 2.79E-03 | 2.79E-03 | 1.67E-03 |
| 10 | 0. | 1.27E-03 | 1.16E-03 | 0. |
| 11 | 5.72E-04 | -4.44E-04 | 4.44E-04 | -6.58E-04 |
| 12 | 0. | 4.44E-04 | 4.65E-04 | 0. |
| 13 | -3.76E-05 | 3.49E-04 | 3.49E-04 | 1.43E-04 |
| 14 | 0. | 1.80E-04 | 1.76E-04 | 0. |
| 15 | 8.92E-05 | -7.40E-05 | 7.40E-05 | -8.87E-05 |
| 16 | 0. | 7.19E-05 | 7.27E-05 | 0. |

N = 2

| n | $b_n - a_n$ | $b_n + a_n$ | N+n=even $G_x$ | N+n=even $G_y$ | N+n=odd $\frac{\rho_n}{\cos(\frac{\pi}{2}\cdot\frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -1.00E+00 | -7.02E-01 | 0. | 0. | 4.98E-01 |
| 2 | -7.30E-01 | -3.02E-01 | 3.02E-01 | -7.30E-01 | 0. |
| 3 | -3.84E-01 | 0. | 0. | 0. | -6.22E-01 |
| 4 | -8.16E-02 | 1.49E-01 | -4.78E-02 | -1.63E-01 | 0. |
| 5 | 0. | 8.66E-02 | 0. | 0. | -1.08E-01 |
| 6 | 5.24E-03 | 2.44E-02 | -1.35E-02 | -2.10E-02 | 0. |
| 7 | -5.94E-03 | 0. | 0. | 0. | -1.24E-02 |
| 8 | -6.88E-04 | 7.09E-03 | 4.53E-03 | 5.50E-03 | 0. |
| 9 | 0. | 5.58E-03 | 0. | 0. | 2.37E-03 |
| 10 | 1.13E-04 | 2.44E-03 | 1.64E-03 | 1.80E-03 | 0. |
| 11 | -8.89E-04 | 0. | 0. | 0. | 9.31E-04 |
| 12 | -2.05E-05 | 9.09E-04 | -6.28E-04 | -6.57E-04 | 0. |
| 13 | 0. | 6.98E-04 | 0. | 0. | -2.02E-04 |
| 14 | 3.98E-06 | 3.56E-04 | -2.49E-04 | -2.55E-04 | 0. |
| 15 | -1.48E-04 | 0. | 0. | 0. | -1.25E-04 |
| 16 | -8.04E-07 | 1.45E-04 | 1.02E-04 | 1.03E-04 | 0. |

N = 2

| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 1.99E-01 | -4.25E-01 | 7.46E-02 | 1.76E-01 |
| 2 | 2.50E-01 | -5.16E-01 | 2.14E-01 | 5.00E-01 |
| 3 | 1.57E-01 | -2.88E-01 | 2.88E-01 | 6.60E-01 |
| 4 | 0. | 6.76E-02 | 2.31E-01 | 5.00E-01 |
| 5 | -2.05E-02 | 1.08E-01 | 1.08E-01 | 1.91E-01 |
| 6 | 0. | 4.45E-02 | 2.87E-02 | 0. |
| 7 | 1.61E-02 | -1.04E-02 | 1.04E-02 | -3.06E-02 |
| 8 | 0. | 1.28E-02 | 1.56E-02 | 0. |
| 9 | -1.90E-03 | 1.25E-02 | 1.25E-02 | 7.53E-03 |
| 10 | 0. | 6.37E-03 | 5.81E-03 | 0. |
| 11 | 3.15E-03 | -2.44E-03 | 2.44E-03 | -3.62E-03 |
| 12 | 0. | 2.66E-03 | 2.79E-03 | 0. |
| 13 | -2.45E-04 | 2.27E-03 | 2.27E-03 | 9.28E-04 |
| 14 | 0. | 1.26E-03 | 1.23E-03 | 0. |
| 15 | 6.69E-04 | -5.55E-04 | 5.55E-04 | -6.66E-04 |
| 16 | 0. | 5.76E-04 | 5.82E-04 | 0. |

N = 2

| n | $\frac{n}{N}(b_n - a_n)$ | $\frac{n}{N}(b_n + a_n)$ | N+n=even $\frac{n}{N} \cdot G_x$ | N+n=even $\frac{n}{N} \cdot G_y$ | N+n=odd $\frac{n}{N} \cdot \frac{\rho_n}{\cos(\frac{\pi}{2} \cdot \frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -5.00E-01 | -3.51E-01 | 0. | 0. | 2.49E-01 |
| 2 | -7.30E-01 | -3.02E-01 | 3.02E-01 | -7.30E-01 | 0. |
| 3 | -5.76E-01 | 0. | 0. | 0. | -9.33E-01 |
| 4 | -1.63E-01 | 2.99E-01 | -9.56E-02 | -3.27E-01 | 0. |
| 5 | 0. | 2.16E-01 | 0. | 0. | -2.70E-01 |
| 6 | 1.57E-02 | 7.32E-02 | -4.06E-02 | -6.29E-02 | 0. |
| 7 | -2.08E-02 | 0. | 0. | 0. | -4.33E-02 |
| 8 | -2.75E-03 | 2.84E-02 | 1.81E-02 | 2.20E-02 | 0. |
| 9 | 0. | 2.51E-02 | 0. | 0. | 1.06E-02 |
| 10 | 5.63E-04 | 1.22E-02 | 8.22E-03 | 9.01E-03 | 0. |
| 11 | -4.89E-03 | 0. | 0. | 0. | 5.12E-03 |
| 12 | -1.23E-04 | 5.45E-03 | -3.77E-03 | -3.94E-03 | 0. |
| 13 | 0. | 4.54E-03 | 0. | 0. | -1.31E-03 |
| 14 | 2.79E-05 | 2.49E-03 | -1.74E-03 | -1.78E-03 | 0. |
| 15 | -1.11E-03 | 0. | 0. | 0. | -9.41E-04 |
| 16 | -6.43E-06 | 1.16E-03 | 8.14E-04 | 8.23E-04 | 0. |

N = 3

| n | $j_n = \frac{\Delta C_n(x)}{i\epsilon}$ | $b_n = \frac{\Delta C_n(rd)}{i\epsilon}$ | $a_n = \frac{\Delta C_n(ad)}{\epsilon}$ | $\rho_n = \frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 2.94E-01 | -9.41E-01 | 1.53E-01 | 2.54E-01 |
| 2 | 2.35E-01 | -7.43E-01 | 2.57E-01 | 4.26E-01 |
| 3 | 1.67E-01 | -5.15E-01 | 3.03E-01 | 5.00E-01 |
| 4 | 9.99E-02 | -2.92E-01 | 2.92E-01 | 4.79E-01 |
| 5 | 4.26E-02 | -1.04E-01 | 2.38E-01 | 3.86E-01 |
| 6 | 0. | 3.28E-02 | 1.59E-01 | 2.50E-01 |
| 7 | -5.75E-03 | 4.64E-02 | 8.38E-02 | 1.23E-01 |
| 8 | -4.02E-03 | 3.39E-02 | 3.39E-02 | 4.06E-02 |
| 9 | 0. | 1.39E-02 | 8.36E-03 | 0. |
| 10 | 2.74E-03 | -5.69E-04 | 5.69E-04 | -1.02E-02 |
| 11 | 2.65E-03 | -3.95E-03 | 1.50E-03 | -5.58E-03 |
| 12 | 0. | 2.63E-03 | 3.28E-03 | 0. |
| 13 | -2.33E-04 | 2.48E-03 | 3.13E-03 | 1.69E-03 |
| 14 | -2.52E-04 | 2.11E-03 | 2.11E-03 | 1.25E-03 |
| 15 | 0. | 1.01E-03 | 9.12E-04 | 0. |
| 16 | 3.05E-04 | -2.37E-04 | 2.37E-04 | -6.85E-04 |
| 17 | 3.65E-04 | -6.66E-04 | 2.09E-04 | -4.49E-04 |
| 18 | 0. | 3.36E-04 | 3.54E-04 | 0. |
| 19 | -1.77E-05 | 2.88E-04 | 3.34E-04 | 1.29E-04 |
| 20 | -2.55E-05 | 2.53E-04 | 2.53E-04 | 1.18E-04 |
| 21 | 0. | 1.33E-04 | 1.30E-04 | 0. |
| 22 | 4.43E-05 | -4.12E-05 | 4.12E-05 | -8.78E-05 |
| 23 | 6.07E-05 | -1.22E-04 | 3.21E-05 | -6.25E-05 |
| 24 | 0. | 5.16E-05 | 5.22E-05 | 0. |

N = 3

| n | $b_n - a_n$ | $b_n + a_n$ | N+n=even $G_x$ | N+n=even $G_y$ | N+n=odd $\frac{\rho_n}{\cos(\frac{\pi}{2}\cdot\frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -1.09E+00 | -7.88E-01 | -1.49E+00 | 7.00E-01 | 0. |
| 2 | -1.00E+00 | -4.86E-01 | 0. | 0. | 8.53E-01 |
| 3 | -8.18E-01 | -2.13E-01 | 6.05E-01 | -1.03E+00 | 0. |
| 4 | -5.84E-01 | 0. | 0. | 0. | -9.59E-01 |
| 5 | -3.42E-01 | 1.35E-01 | 3.63E-02 | -3.06E-01 | 0. |
| 6 | -1.26E-01 | 1.92E-01 | 0. | 0. | -2.50E-01 |
| 7 | -3.73E-02 | 1.30E-01 | -2.78E-02 | -1.02E-01 | 0. |
| 8 | 0. | 6.77E-02 | 0. | 0. | -8.12E-02 |
| 9 | 5.50E-03 | 2.22E-02 | -1.67E-02 | -2.77E-02 | 0. |
| 10 | -1.14E-03 | 0. | 0. | 0. | -2.03E-02 |
| 11 | -5.45E-03 | -2.46E-03 | -5.18E-03 | 2.70E-04 | 0. |
| 12 | -6.51E-04 | 5.91E-03 | 0. | 0. | 0. |
| 13 | -6.56E-04 | 5.61E-03 | 2.15E-03 | 3.46E-03 | 0. |
| 14 | 0. | 4.22E-03 | 0. | 0. | 2.49E-03 |
| 15 | 1.00E-04 | 1.92E-03 | 1.82E-03 | 2.02E-03 | 0. |
| 16 | -4.73E-04 | 0. | 0. | 0. | 1.37E-03 |
| 17 | -8.75E-04 | -4.57E-04 | 8.94E-04 | 1.93E-05 | 0. |
| 18 | -1.76E-05 | 6.90E-04 | 0. | 0. | 0. |
| 19 | -4.59E-05 | 6.22E-04 | -2.65E-04 | -3.57E-04 | 0. |
| 20 | 0. | 5.06E-04 | 0. | 0. | -2.35E-04 |
| 21 | 3.29E-06 | 2.62E-04 | -2.59E-04 | -2.66E-04 | 0. |
| 22 | -8.23E-05 | 0. | 0. | 0. | -1.76E-04 |
| 23 | -1.54E-04 | -8.97E-05 | -1.67E-04 | -1.27E-05 | 0. |
| 24 | -6.48E-07 | 1.04E-04 | 0. | 0. | 0. |

N = 3

| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 9.79E-02 | -3.14E-01 | 5.09E-02 | 8.47E-02 |
| 2 | 1.56E-01 | -4.95E-01 | 1.71E-01 | 2.84E-01 |
| 3 | 1.67E-01 | -5.15E-01 | 3.03E-01 | 5.00E-01 |
| 4 | 1.33E-01 | -3.90E-01 | 3.90E-01 | 6.39E-01 |
| 5 | 7.09E-02 | -1.73E-01 | 3.97E-01 | 6.43E-01 |
| 6 | 0. | 6.55E-02 | 3.18E-01 | 5.00E-01 |
| 7 | -1.34E-02 | 1.08E-01 | 1.95E-01 | 2.88E-01 |
| 8 | -1.07E-02 | 9.03E-02 | 9.03E-02 | 1.08E-01 |
| 9 | 0. | 4.16E-02 | 2.51E-02 | 0. |
| 10 | 9.13E-03 | -1.90E-03 | 1.90E-03 | -3.38E-02 |
| 11 | 9.72E-03 | -1.45E-02 | 5.49E-03 | -2.05E-02 |
| 12 | 0. | 1.05E-02 | 1.31E-02 | 0. |
| 13 | -1.01E-03 | 1.07E-02 | 1.36E-02 | 7.34E-03 |
| 14 | -1.18E-03 | 9.85E-03 | 9.85E-03 | 5.82E-03 |
| 15 | 0. | 5.06E-03 | 4.56E-03 | 0. |
| 16 | 1.63E-03 | -1.26E-03 | 1.26E-03 | -3.66E-03 |
| 17 | 2.07E-03 | -3.77E-03 | 1.18E-03 | -2.54E-03 |
| 18 | 0. | 2.02E-03 | 2.12E-03 | 0. |
| 19 | -1.12E-04 | 1.82E-03 | 2.12E-03 | 8.18E-04 |
| 20 | -1.70E-04 | 1.69E-03 | 1.69E-03 | 7.84E-04 |
| 21 | 0. | 9.30E-04 | 9.07E-04 | 0. |
| 22 | 3.25E-04 | -3.02E-04 | 3.02E-04 | -6.44E-04 |
| 23 | 4.65E-04 | -9.34E-04 | 2.46E-04 | -4.79E-04 |
| 24 | 0. | 4.13E-04 | 4.18E-04 | 0. |

N = 3

| n | $\frac{n}{N}(b_n-a_n)$ | $\frac{n}{N}(b_n+a_n)$ | N+n=even $\frac{n}{N}\cdot G_x$ | N+n=even $\frac{n}{N}\cdot G_y$ | N+n=odd $\frac{n}{N}\cdot\frac{\rho_n}{\cos(\frac{\pi}{2}\cdot\frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -3.65E-01 | -2.63E-01 | -4.96E-01 | 2.33E-01 | 0. |
| 2 | -6.67E-01 | -3.24E-01 | 0. | 0. | 5.69E-01 |
| 3 | -8.18E-01 | -2.13E-01 | 6.05E-01 | -1.03E+00 | 0. |
| 4 | -7.79E-01 | 0. | 0. | 0. | -1.28E+00 |
| 5 | -5.70E-01 | 2.24E-01 | 6.05E-02 | -5.09E-01 | 0. |
| 6 | -2.52E-01 | 3.83E-01 | 0. | 0. | -5.00E-01 |
| 7 | -8.71E-02 | 3.04E-01 | -6.48E-02 | -2.39E-01 | 0. |
| 8 | 0. | 1.81E-01 | 0. | 0. | -2.16E-01 |
| 9 | 1.65E-02 | 6.67E-02 | -5.02E-02 | -8.32E-02 | 0. |
| 10 | -3.79E-03 | 0. | 0. | 0. | -6.77E-02 |
| 11 | -2.00E-02 | -9.00E-03 | -1.90E-02 | 9.89E-04 | 0. |
| 12 | -2.60E-03 | 2.36E-02 | 0. | 0. | 0. |
| 13 | -2.84E-03 | 2.43E-02 | 9.32E-03 | 1.50E-02 | 0. |
| 14 | 0. | 1.97E-02 | 0. | 0. | 1.16E-02 |
| 15 | 5.02E-04 | 9.62E-03 | 9.12E-03 | 1.01E-02 | 0. |
| 16 | -2.52E-03 | 0. | 0. | 0. | 7.31E-03 |
| 17 | -4.96E-03 | -2.59E-03 | 5.07E-03 | 1.10E-04 | 0. |
| 18 | -1.05E-04 | 4.14E-03 | 0. | 0. | 0. |
| 19 | -2.91E-04 | 3.94E-03 | -1.68E-03 | -2.26E-03 | 0. |
| 20 | 0. | 3.37E-03 | 0. | 0. | -1.57E-03 |
| 21 | 2.31E-05 | 1.84E-03 | -1.81E-03 | -1.86E-03 | 0. |
| 22 | -6.04E-04 | 0. | 0. | 0. | -1.29E-03 |
| 23 | -1.18E-03 | -6.88E-04 | -1.28E-03 | -9.76E-05 | 0. |
| 24 | -5.18E-06 | 8.30E-04 | 0. | 0. | 0. |

N = 4

| n | $j_n = \frac{\Delta C_n(x)}{i\epsilon}$ | $b_n = \frac{\Delta C_n(rd)}{i\epsilon}$ | $a_n = \frac{\Delta C_n(ad)}{\epsilon}$ | $\rho_n = \frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 2.29E-01 | -9.73E-01 | 1.37E-01 | 1.97E-01 |
| 2 | 1.99E-01 | -8.42E-01 | 2.45E-01 | 3.52E-01 |
| 3 | 1.64E-01 | -6.83E-01 | 3.17E-01 | 4.54E-01 |
| 4 | 1.25E-01 | -5.13E-01 | 3.50E-01 | 5.00E-01 |
| 5 | 8.70E-02 | -3.46E-01 | 3.46E-01 | 4.92E-01 |
| 6 | 5.22E-02 | -1.94E-01 | 3.11E-01 | 4.40E-01 |
| 7 | 2.28E-02 | -6.74E-02 | 2.53E-01 | 3.54E-01 |
| 8 | 0. | 2.88E-02 | 1.81E-01 | 2.50E-01 |
| 9 | -3.93E-03 | 4.26E-02 | 1.13E-01 | 1.51E-01 |
| 10 | -4.10E-03 | 3.89E-02 | 6.12E-02 | 7.63E-02 |
| 11 | -2.29E-03 | 2.64E-02 | 2.64E-02 | 2.68E-02 |
| 12 | 0. | 1.20E-02 | 6.95E-03 | 0. |
| 13 | 1.72E-03 | 7.88E-04 | -7.88E-04 | -9.61E-03 |
| 14 | 2.30E-03 | -4.59E-03 | -1.34E-03 | -8.75E-03 |
| 15 | 1.64E-03 | -3.72E-03 | 9.54E-04 | -3.86E-03 |
| 16 | 0. | 2.11E-03 | 2.68E-03 | 0. |
| 17 | -1.39E-04 | 2.08E-03 | 2.96E-03 | 1.50E-03 |
| 18 | -2.11E-04 | 2.06E-03 | 2.52E-03 | 1.67E-03 |
| 19 | -1.57E-04 | 1.63E-03 | 1.63E-03 | 9.46E-04 |
| 20 | 0. | 8.01E-04 | 7.17E-04 | 0. |
| 21 | 1.81E-04 | -1.26E-04 | 1.26E-04 | -6.07E-04 |
| 22 | 2.86E-04 | -7.28E-04 | -2.57E-05 | -6.58E-04 |
| 23 | 2.37E-04 | -6.49E-04 | 1.27E-04 | -3.13E-04 |
| 24 | 0. | 2.60E-04 | 2.74E-04 | 0. |

N = 4

| n | $b_n - a_n$ | $b_n + a_n$ | N+n=even $G_x$ | N+n=even $G_y$ | N+n=odd $\frac{\rho_n}{\cos(\frac{\pi}{2}\cdot\frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -1.11E+00 | -8.36E-01 | 0. | 0. | 2.13E-01 |
| 2 | -1.09E+00 | -5.97E-01 | -1.74E+00 | 1.10E+00 | 0. |
| 3 | -1.00E+00 | -3.66E-01 | 0. | 0. | 1.19E+00 |
| 4 | -8.63E-01 | -1.63E-01 | 9.14E-01 | -1.34E+00 | 0. |
| 5 | -6.91E-01 | 0. | 0. | 0. | -1.29E+00 |
| 6 | -5.05E-01 | 1.16E-01 | 1.21E-01 | -4.25E-01 | 0. |
| 7 | -3.20E-01 | 1.85E-01 | 0. | 0. | -3.83E-01 |
| 8 | -1.52E-01 | 2.10E-01 | -3.12E-02 | -1.96E-01 | 0. |
| 9 | -7.06E-02 | 1.56E-01 | 0. | 0. | -1.64E-01 |
| 10 | -2.22E-02 | 1.00E-01 | -2.51E-02 | -8.32E-02 | 0. |
| 11 | 0. | 5.28E-02 | 0. | 0. | -7.00E-02 |
| 12 | 5.04E-03 | 1.89E-02 | -1.82E-02 | -3.13E-02 | 0. |
| 13 | 1.58E-03 | 0. | 0. | 0. | -2.51E-02 |
| 14 | -3.25E-03 | -5.94E-03 | -9.52E-03 | -5.99E-03 | 0. |
| 15 | -4.68E-03 | -2.77E-03 | 0. | 0. | -4.17E-03 |
| 16 | -5.63E-04 | 4.79E-03 | 2.29E-03 | 2.90E-03 | 0. |
| 17 | -8.75E-04 | 5.04E-03 | 0. | 0. | 1.62E-03 |
| 18 | -4.63E-04 | 4.58E-03 | 1.87E-03 | 3.08E-03 | 0. |
| 19 | 0. | 3.26E-03 | 0. | 0. | 2.47E-03 |
| 20 | 8.42E-05 | 1.52E-03 | 1.87E-03 | 2.09E-03 | 0. |
| 21 | -2.52E-04 | 0. | 0. | 0. | 1.59E-03 |
| 22 | -7.02E-04 | -7.53E-04 | 1.36E-03 | 6.04E-04 | 0. |
| 23 | -7.76E-04 | -5.21E-04 | 0. | 0. | 3.38E-04 |
| 24 | -1.44E-05 | 5.33E-04 | -2.81E-04 | -2.97E-04 | 0. |

N = 4

| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 5.73E-02 | -2.43E-01 | 3.43E-02 | 4.93E-02 |
| 2 | 9.97E-02 | -4.21E-01 | 1.23E-01 | 1.76E-01 |
| 3 | 1.23E-01 | -5.12E-01 | 2.38E-01 | 3.41E-01 |
| 4 | 1.25E-01 | -5.13E-01 | 3.50E-01 | 5.00E-01 |
| 5 | 1.09E-01 | -4.32E-01 | 4.32E-01 | 6.15E-01 |
| 6 | 7.83E-02 | -2.91E-01 | 4.66E-01 | 6.60E-01 |
| 7 | 3.99E-02 | -1.18E-01 | 4.42E-01 | 6.20E-01 |
| 8 | 0. | 5.76E-02 | 3.62E-01 | 5.00E-01 |
| 9 | -8.84E-03 | 9.58E-02 | 2.55E-01 | 3.40E-01 |
| 10 | -1.03E-02 | 9.73E-02 | 1.53E-01 | 1.91E-01 |
| 11 | -6.31E-03 | 7.25E-02 | 7.25E-02 | 7.37E-02 |
| 12 | 0. | 3.60E-02 | 2.08E-02 | 0. |
| 13 | 5.59E-03 | 2.56E-03 | -2.56E-03 | -3.12E-02 |
| 14 | 8.05E-03 | -1.61E-02 | -4.69E-03 | -3.06E-02 |
| 15 | 6.15E-03 | -1.40E-02 | 3.58E-03 | -1.45E-02 |
| 16 | 0. | 8.46E-03 | 1.07E-02 | 0. |
| 17 | -5.91E-04 | 8.86E-03 | 1.26E-02 | 6.37E-03 |
| 18 | -9.49E-04 | 9.27E-03 | 1.13E-02 | 7.53E-03 |
| 19 | -7.46E-04 | 7.74E-03 | 7.74E-03 | 4.49E-03 |
| 20 | 0. | 4.00E-03 | 3.58E-03 | 0. |
| 21 | 9.48E-04 | -6.61E-04 | 6.61E-04 | -3.19E-03 |
| 22 | 1.57E-03 | -4.00E-03 | -1.41E-04 | -3.62E-03 |
| 23 | 1.36E-03 | -3.73E-03 | 7.32E-04 | -1.80E-03 |
| 24 | 0. | 1.56E-03 | 1.64E-03 | 0. |

N = 4

| n | $\frac{n}{N}(b_n-a_n)$ | $\frac{n}{N}(b_n+a_n)$ | N+n=even $\frac{n}{N}\cdot G_x$ | N+n=even $\frac{n}{N}\cdot G_y$ | N+n=odd $\frac{n}{N}\cdot\frac{\rho_n}{\cos(\frac{\pi}{2}\cdot\frac{n}{N})}$ |
|---|---|---|---|---|---|
| 1 | -2.78E-01 | -2.09E-01 | 0. | 0. | 5.34E-02 |
| 2 | -5.43E-01 | -2.98E-01 | -8.71E-01 | 5.49E-01 | 0. |
| 3 | -7.50E-01 | -2.75E-01 | 0. | 0. | 8.90E-01 |
| 4 | -8.63E-01 | -1.63E-01 | 9.14E-01 | -1.34E+00 | 0. |
| 5 | -8.64E-01 | 0. | 0. | 0. | -1.61E+00 |
| 6 | -7.57E-01 | 1.75E-01 | 1.81E-01 | -6.38E-01 | 0. |
| 7 | -5.60E-01 | 3.24E-01 | 0. | 0. | -6.71E-01 |
| 8 | -3.05E-01 | 4.20E-01 | -6.24E-02 | -3.92E-01 | 0. |
| 9 | -1.59E-01 | 3.50E-01 | 0. | 0. | -3.68E-01 |
| 10 | -5.56E-02 | 2.50E-01 | -6.28E-02 | -2.08E-01 | 0. |
| 11 | 0. | 1.45E-01 | 0. | 0. | -1.93E-01 |
| 12 | 1.51E-02 | 5.68E-02 | -5.45E-02 | -9.40E-02 | 0. |
| 13 | 5.12E-03 | 0. | 0. | 0. | -8.16E-02 |
| 14 | -1.14E-02 | -2.08E-02 | -3.33E-02 | -2.10E-02 | 0. |
| 15 | -1.75E-02 | -1.04E-02 | 0. | 0. | -1.57E-02 |
| 16 | -2.25E-03 | 1.92E-02 | 9.15E-03 | 1.16E-02 | 0. |
| 17 | -3.72E-03 | 2.14E-02 | 0. | 0. | 6.90E-03 |
| 18 | -2.08E-03 | 2.06E-02 | 8.43E-03 | 1.39E-02 | 0. |
| 19 | 0. | 1.55E-02 | 0. | 0. | 1.17E-02 |
| 20 | 4.21E-04 | 7.59E-03 | 9.36E-03 | 1.05E-02 | 0. |
| 21 | -1.32E-03 | 0. | 0. | 0. | 8.33E-03 |
| 22 | -3.86E-03 | -4.14E-03 | 7.50E-03 | 3.32E-03 | 0. |
| 23 | -4.46E-03 | -3.00E-03 | 0. | 0. | 1.95E-03 |
| 24 | -8.64E-05 | 3.20E-03 | -1.69E-03 | -1.78E-03 | 0. |

N = 6

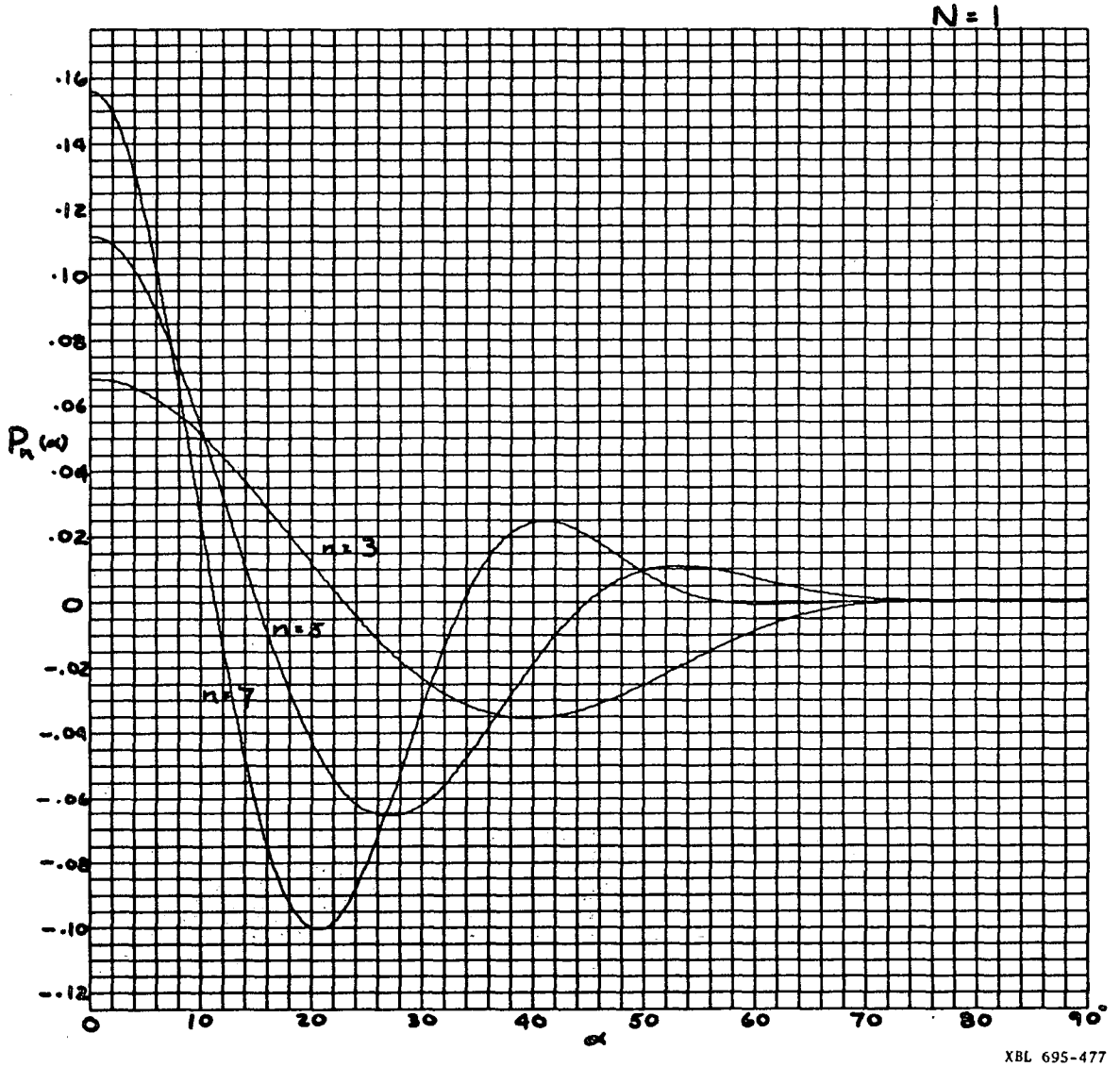
| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 2.64E-02 | -1.66E-01 | 1.79E-02 | 2.26E-02 |
| 2 | 4.89E-02 | -3.07E-01 | 6.73E-02 | 8.47E-02 |
| 3 | 6.65E-02 | -4.16E-01 | 1.40E-01 | 1.76E-01 |
| 4 | 7.82E-02 | -4.87E-01 | 2.26E-01 | 2.84E-01 |
| 5 | 8.38E-02 | -5.18E-01 | 3.16E-01 | 3.96E-01 |
| 6 | 8.33E-02 | -5.10E-01 | 3.99E-01 | 5.00E-01 |
| 7 | 7.73E-02 | -4.66E-01 | 4.66E-01 | 5.84E-01 |
| 8 | 6.66E-02 | -3.92E-01 | 5.11E-01 | 6.39E-01 |
| 9 | 5.22E-02 | -2.96E-01 | 5.29E-01 | 6.60E-01 |
| 10 | 3.55E-02 | -1.85E-01 | 5.17E-01 | 6.43E-01 |
| 11 | 1.76E-02 | -6.90E-02 | 4.76E-01 | 5.88E-01 |
| 12 | 0. | 4.45E-02 | 4.08E-01 | 5.00E-01 |
| 13 | -4.52E-03 | 7.36E-02 | 3.25E-01 | 3.94E-01 |
| 14 | -6.71E-03 | 8.62E-02 | 2.42E-01 | 2.88E-01 |
| 15 | -6.84E-03 | 8.42E-02 | 1.66E-01 | 1.91E-01 |
| 16 | -5.36E-03 | 7.10E-02 | 1.01E-01 | 1.08E-01 |
| 17 | -2.87E-03 | 5.05E-02 | 5.05E-02 | 4.41E-02 |
| 18 | 0. | 2.73E-02 | 1.51E-02 | 0. |
| 19 | 2.65E-03 | 5.51E-03 | -5.51E-03 | -2.51E-02 |
| 20 | 4.56E-03 | -1.12E-02 | -1.36E-02 | -3.38E-02 |
| 21 | 5.37E-03 | -2.04E-02 | -1.23E-02 | -3.06E-02 |
| 22 | 4.86E-03 | -2.04E-02 | -5.73E-03 | -2.05E-02 |
| 23 | 3.02E-03 | -1.13E-02 | 2.17E-03 | -8.67E-03 |
| 24 | 0. | 5.94E-03 | 7.64E-03 | 0. |

N = 8

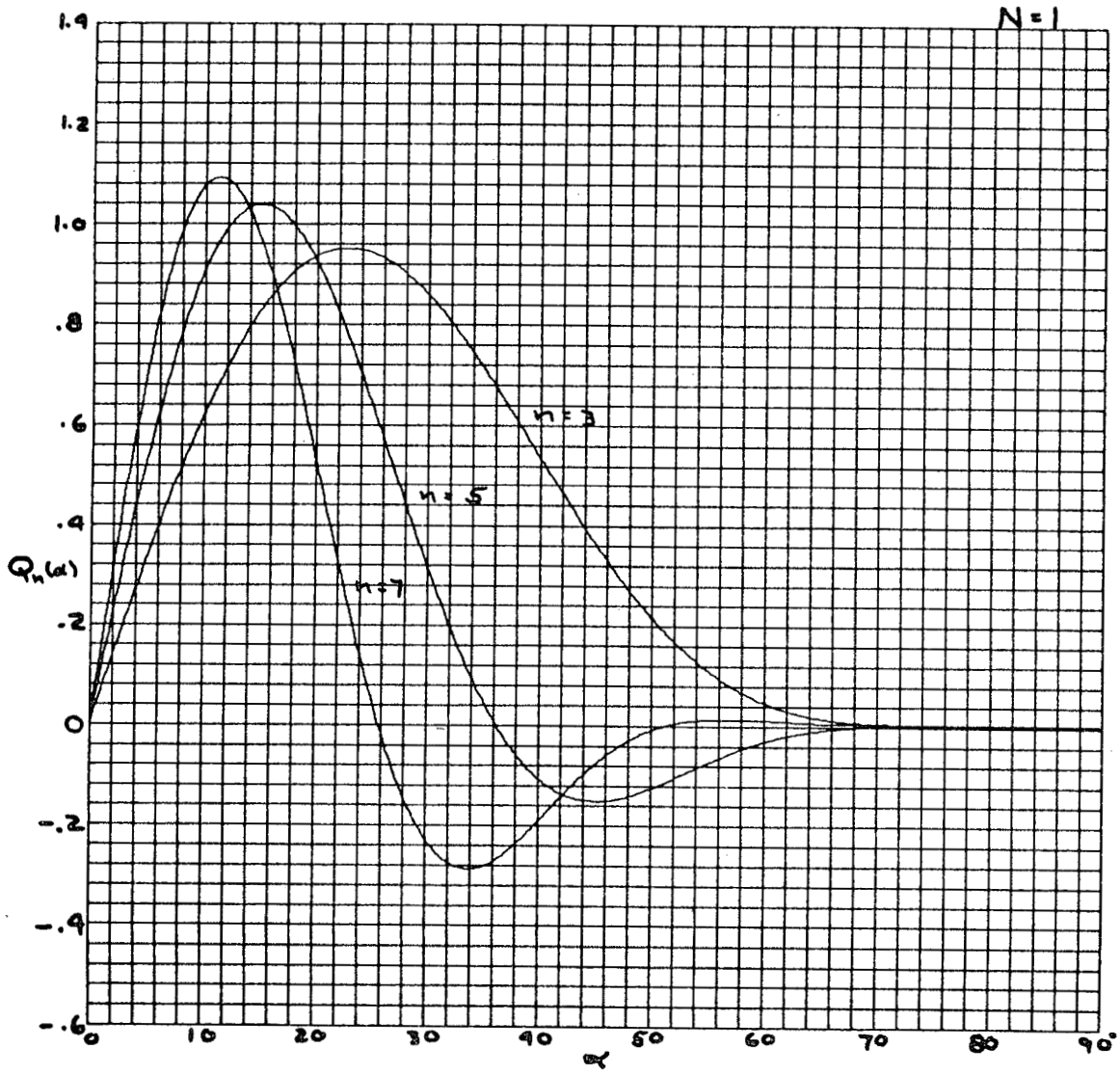
| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 1.51E-02 | -1.25E-01 | 1.09E-02 | 1.29E-02 |
| 2 | 2.87E-02 | -2.38E-01 | 4.16E-02 | 4.93E-02 |
| 3 | 4.04E-02 | -3.35E-01 | 8.90E-02 | 1.05E-01 |
| 4 | 4.99E-02 | -4.12E-01 | 1.49E-01 | 1.76E-01 |
| 5 | 5.69E-02 | -4.69E-01 | 2.17E-01 | 2.56E-01 |
| 6 | 6.13E-02 | -5.04E-01 | 2.88E-01 | 3.41E-01 |
| 7 | 6.32E-02 | -5.17E-01 | 3.58E-01 | 4.24E-01 |
| 8 | 6.25E-02 | -5.08E-01 | 4.23E-01 | 5.00E-01 |
| 9 | 5.95E-02 | -4.79E-01 | 4.79E-01 | 5.65E-01 |
| 10 | 5.44E-02 | -4.33E-01 | 5.22E-01 | 6.15E-01 |
| 11 | 4.75E-02 | -3.72E-01 | 5.50E-01 | 6.48E-01 |
| 12 | 3.92E-02 | -2.99E-01 | 5.61E-01 | 6.60E-01 |
| 13 | 2.99E-02 | -2.19E-01 | 5.54E-01 | 6.50E-01 |
| 14 | 2.00E-02 | -1.34E-01 | 5.30E-01 | 6.20E-01 |
| 15 | 9.87E-03 | -4.77E-02 | 4.88E-01 | 5.69E-01 |
| 16 | 0. | 3.57E-02 | 4.31E-01 | 5.00E-01 |
| 17 | -2.71E-03 | 5.88E-02 | 3.65E-01 | 4.21E-01 |
| 18 | -4.42E-03 | 7.27E-02 | 2.98E-01 | 3.40E-01 |
| 19 | -5.19E-03 | 7.80E-02 | 2.33E-01 | 2.63E-01 |
| 20 | -5.13E-03 | 7.57E-02 | 1.73E-01 | 1.91E-01 |
| 21 | -4.39E-03 | 6.72E-02 | 1.19E-01 | 1.27E-01 |
| 22 | -3.15E-03 | 5.41E-02 | 7.43E-02 | 7.37E-02 |
| 23 | -1.62E-03 | 3.84E-02 | 3.84E-02 | 3.12E-02 |
| 24 | 0. | 2.17E-02 | 1.18E-02 | 0. |

N = 12

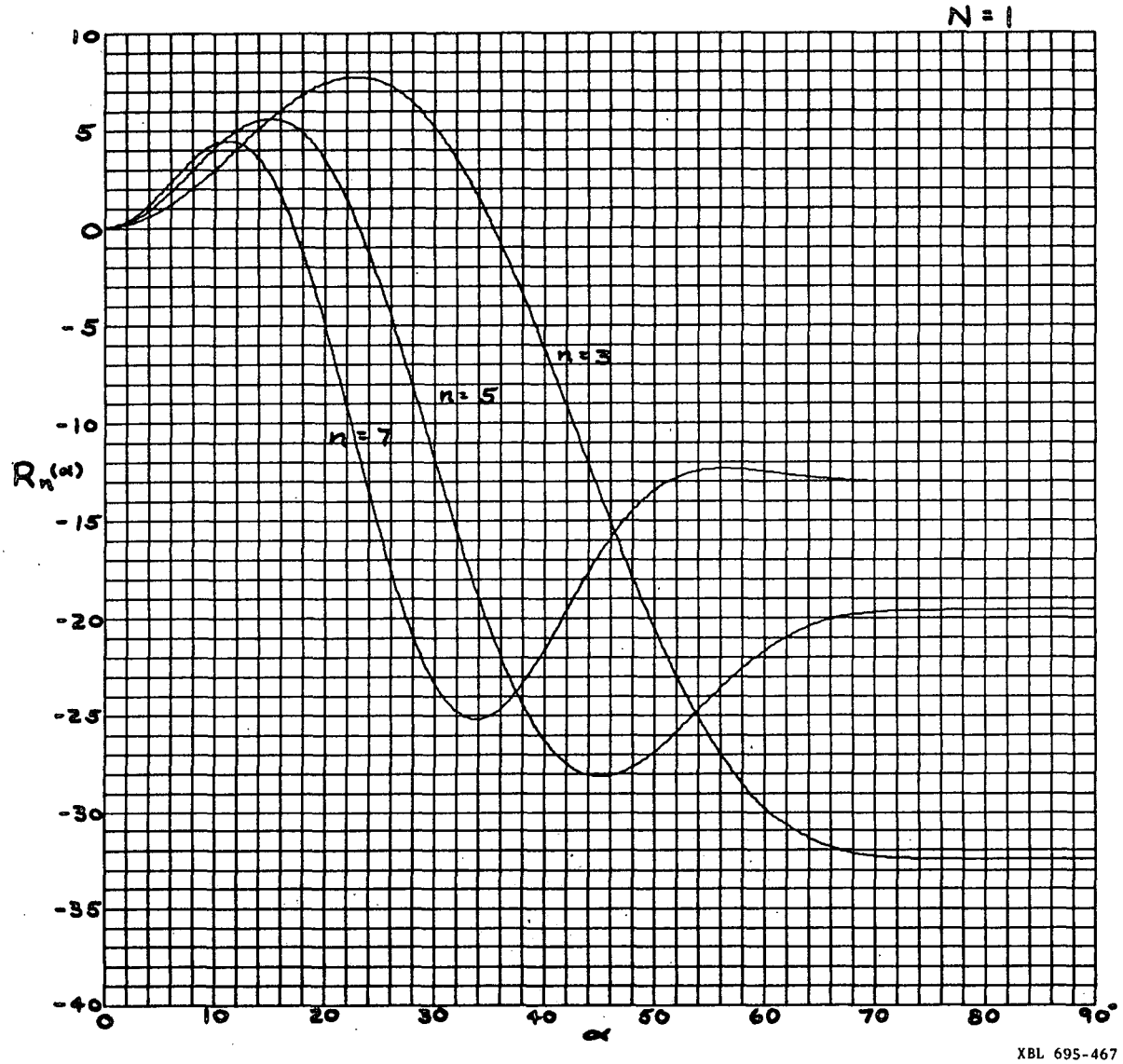
| n | $\frac{n}{N}\cdot j_n = \frac{n}{N}\cdot\frac{\Delta C_n(x)}{i\epsilon}$ | $\frac{n}{N}\cdot b_n = \frac{n}{N}\cdot\frac{\Delta C_n(rd)}{i\epsilon}$ | $\frac{n}{N}\cdot a_n = \frac{n}{N}\cdot\frac{\Delta C_n(ad)}{\epsilon}$ | $\frac{n}{N}\cdot \rho_n = \frac{n}{N}\cdot\frac{\Delta C_n(r)}{\epsilon}$ |
|---|---|---|---|---|
| 1 | 6.79E-03 | -8.36E-02 | 5.19E-03 | 5.79E-03 |
| 2 | 1.32E-02 | -1.63E-01 | 2.02E-02 | 2.26E-02 |
| 3 | 1.91E-02 | -2.35E-01 | 4.42E-02 | 4.93E-02 |
| 4 | 2.45E-02 | -3.01E-01 | 7.59E-02 | 8.47E-02 |
| 5 | 2.92E-02 | -3.59E-01 | 1.14E-01 | 1.27E-01 |
| 6 | 3.32E-02 | -4.08E-01 | 1.58E-01 | 1.76E-01 |
| 7 | 3.66E-02 | -4.49E-01 | 2.05E-01 | 2.29E-01 |
| 8 | 3.91E-02 | -4.79E-01 | 2.55E-01 | 2.84E-01 |
| 9 | 4.09E-02 | -5.00E-01 | 3.05E-01 | 3.41E-01 |
| 10 | 4.19E-02 | -5.11E-01 | 3.56E-01 | 3.96E-01 |
| 11 | 4.22E-02 | -5.13E-01 | 4.04E-01 | 4.50E-01 |
| 12 | 4.17E-02 | -5.05E-01 | 4.49E-01 | 5.00E-01 |
| 13 | 4.05E-02 | -4.89E-01 | 4.89E-01 | 5.45E-01 |
| 14 | 3.87E-02 | -4.65E-01 | 5.24E-01 | 5.84E-01 |
| 15 | 3.62E-02 | -4.33E-01 | 5.53E-01 | 6.15E-01 |
| 16 | 3.33E-02 | -3.95E-01 | 5.74E-01 | 6.39E-01 |
| 17 | 2.99E-02 | -3.52E-01 | 5.88E-01 | 6.54E-01 |
| 18 | 2.61E-02 | -3.03E-01 | 5.94E-01 | 6.60E-01 |
| 19 | 2.20E-02 | -2.51E-01 | 5.91E-01 | 6.56E-01 |
| 20 | 1.77E-02 | -1.97E-01 | 5.79E-01 | 6.43E-01 |
| 21 | 1.33E-02 | -1.41E-01 | 5.60E-01 | 6.20E-01 |
| 22 | 8.82E-03 | -8.47E-02 | 5.32E-01 | 5.88E-01 |
| 23 | 4.36E-03 | -2.89E-02 | 4.96E-01 | 5.48E-01 |
| 24 | 0. | 2.55E-02 | 4.54E-01 | 5.00E-01 |


N = 1
.16
.14
.12
.10
.08
.06
$P_n(\alpha)$
.04
.02
0
-.02
-.04
-.06
-.08
-.10
-.12
n = 3
n = 5
n = 7
0
10
20
30
40
50
60
70
80
90°
α

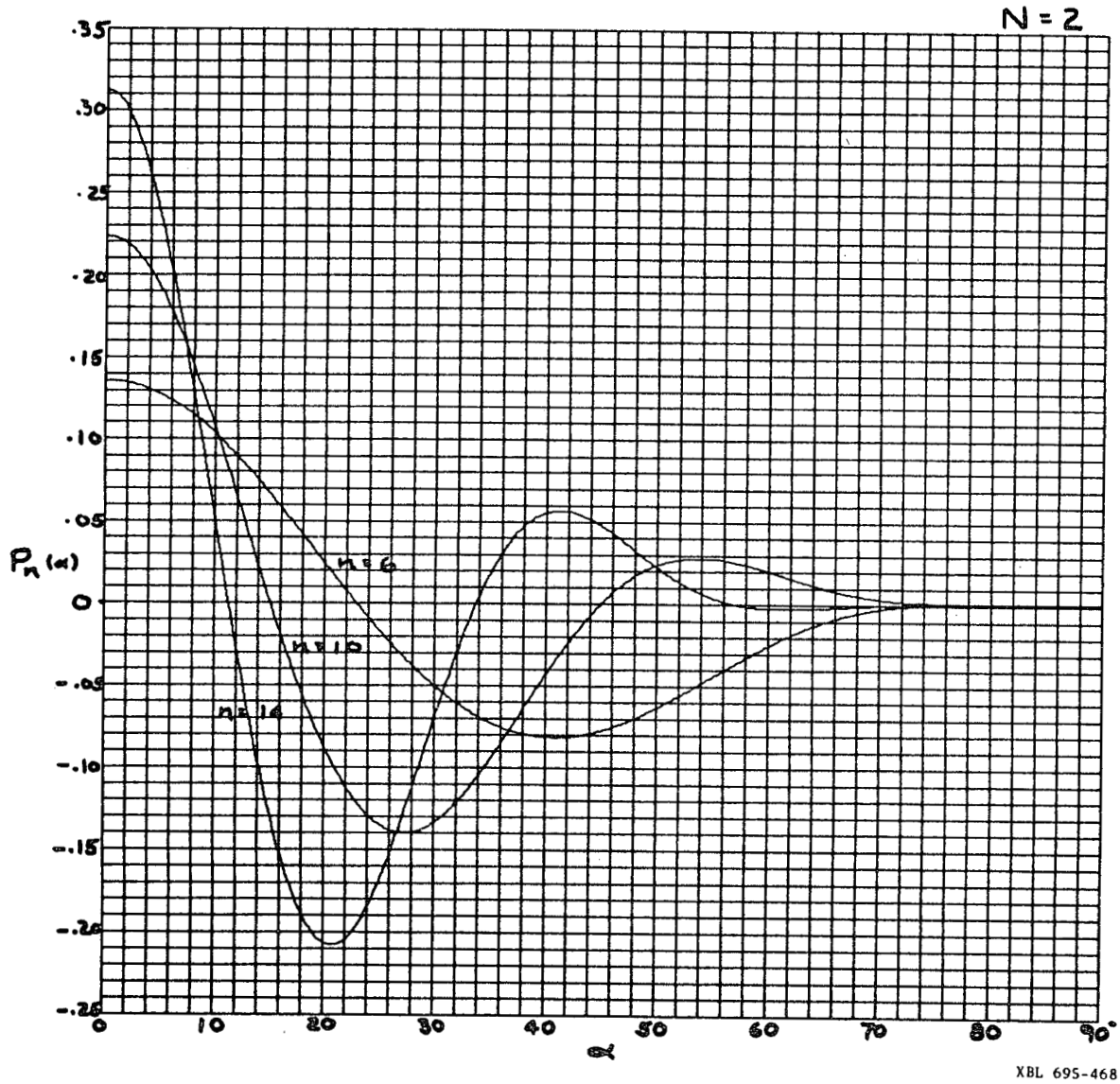
XBL 695-477


N = 1
1.4
1.2
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
$Q_n(\alpha)$
n = 3
n = 5
n = 7
0
10
20
30
40
50
60
70
80
90°
$\alpha$

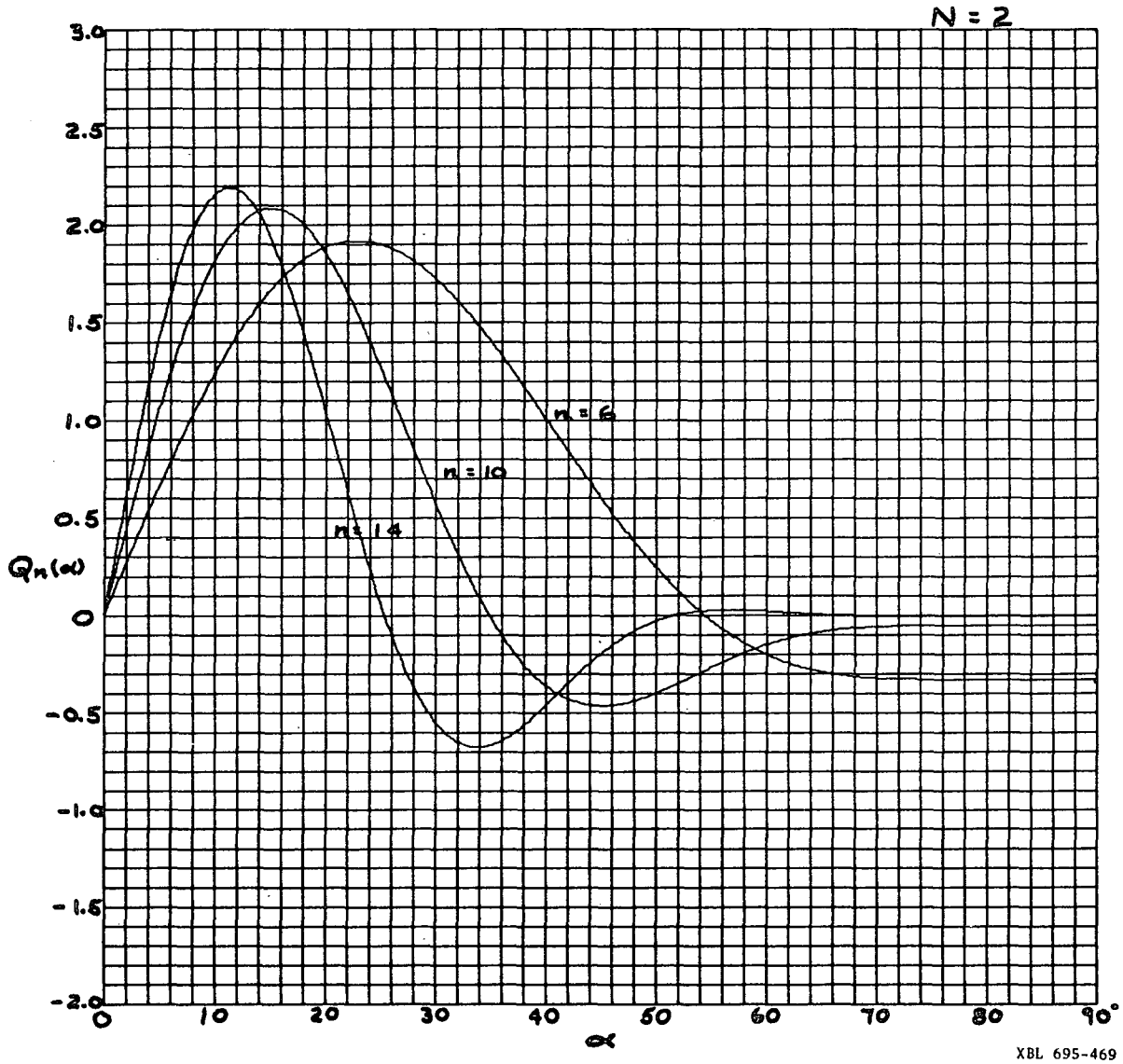
XBL 695-466


N = 1
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
$R_n(\alpha)$
n = 3
n = 5
n = 7
0
10
20
30
40
50
60
70
80
90°
$\alpha$

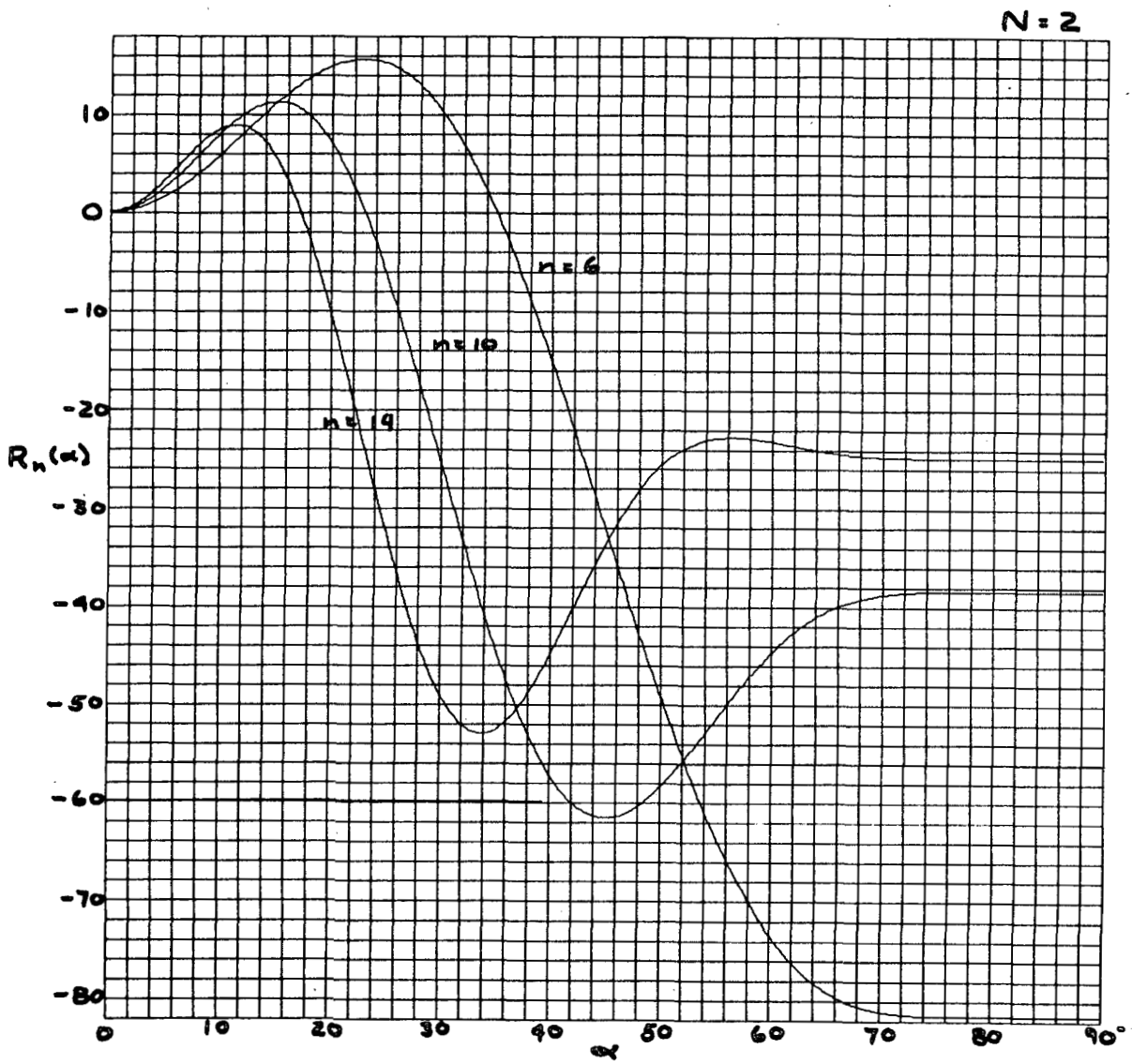
XBL 695-467


N = 2
.35
.30
.25
.20
.15
.10
.05
0
-.05
-.10
-.15
-.20
-.25
$P_n(\alpha)$
n=6
n=10
n=14
0
10
20
30
40
50
60
70
80
90°
$\alpha$

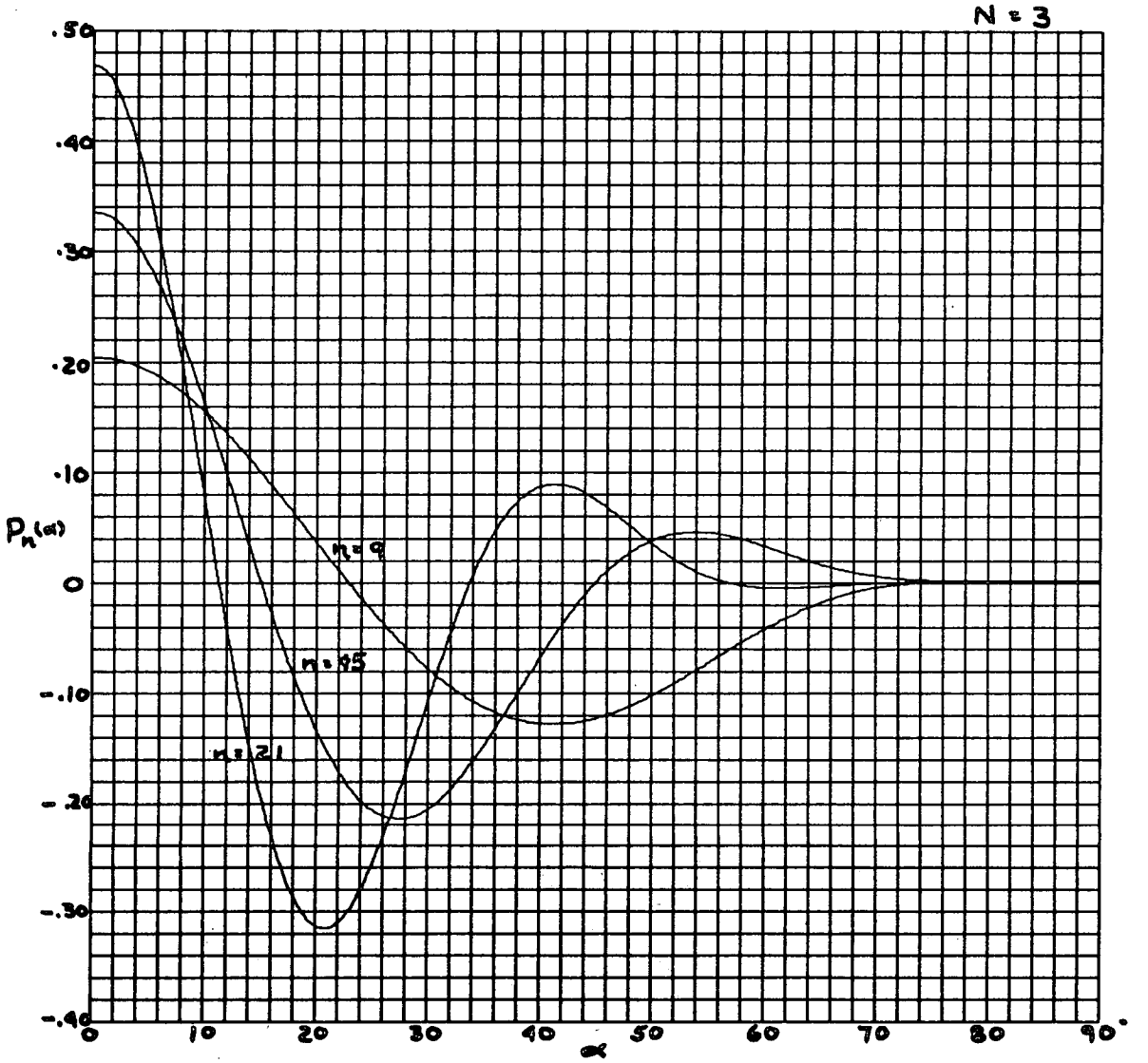
XBL 695-468


N = 2
3.0
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
$Q_n(\alpha)$
n = 6
n = 10
n = 14
0
10
20
30
40
50
60
70
80
90°
$\alpha$

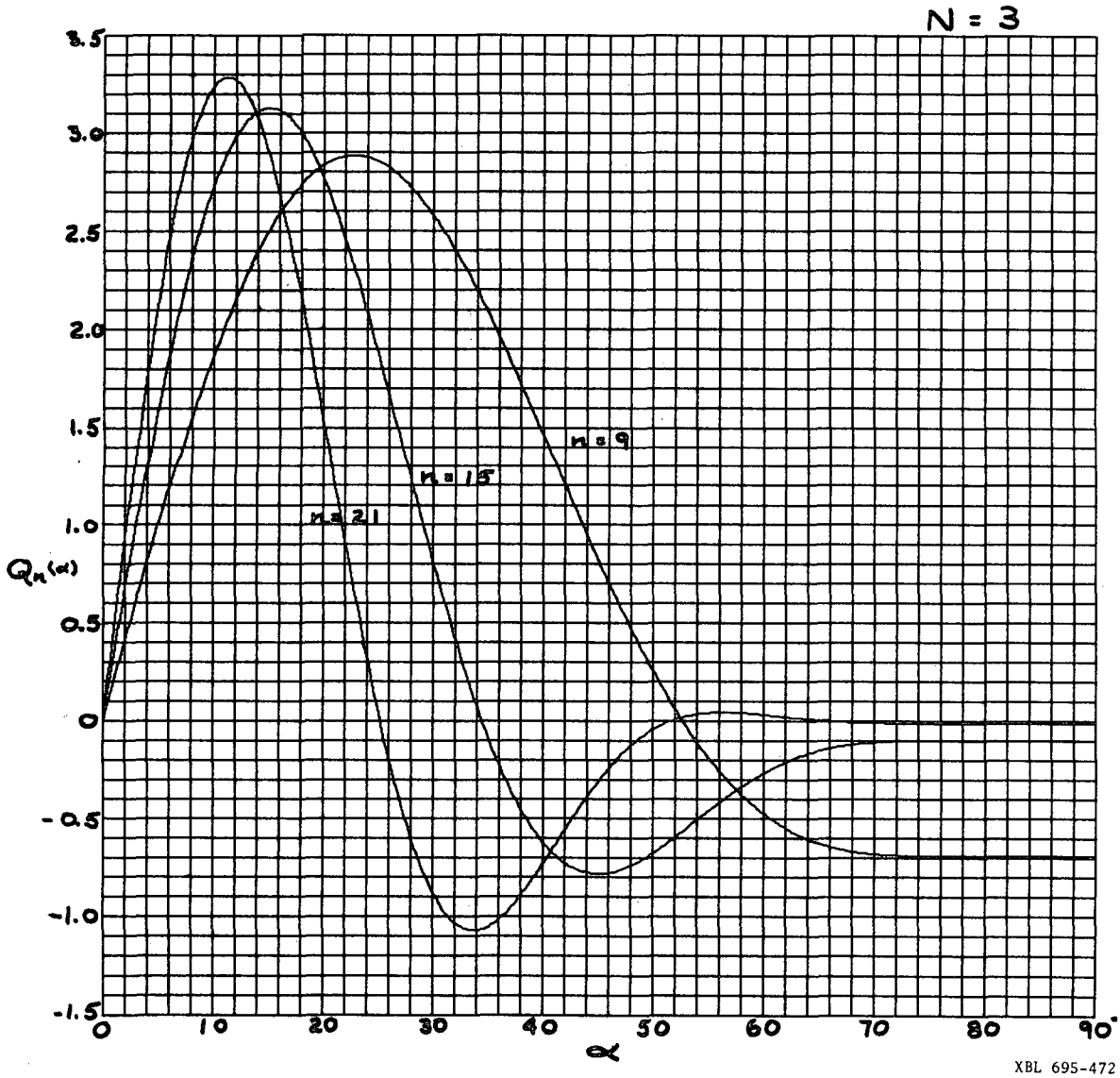
XBL 695-469


N = 2
10
0
-10
-20
-30
-40
-50
-60
-70
-80
$R_n(\alpha)$
n = 6
n = 10
n = 14
0
10
20
30
40
50
60
70
80
90°
$\alpha$

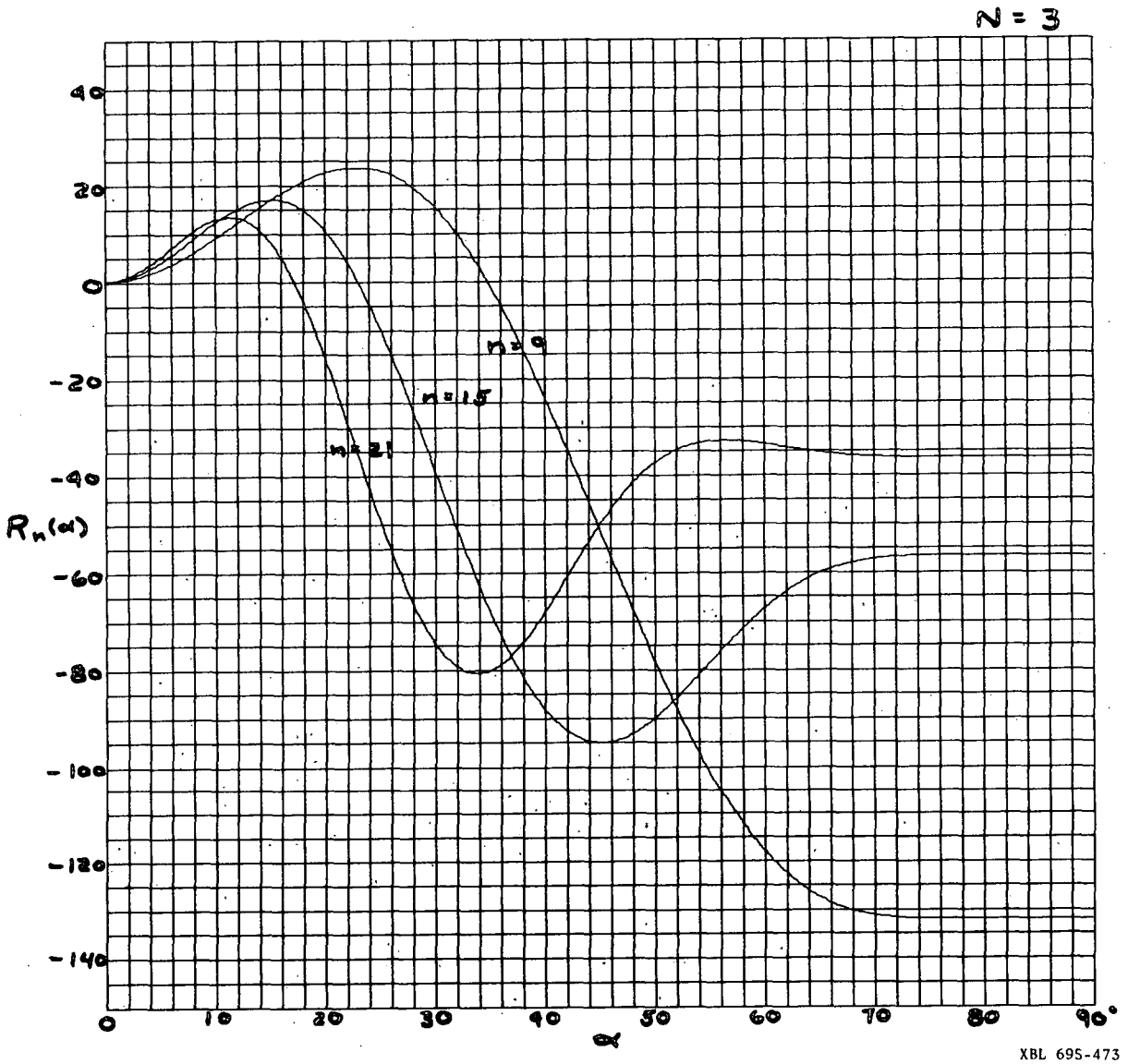
XBL 695-470


N = 3
.50
.40
.30
.20
.10
0
-.10
-.20
-.30
-.40
$P_n(\alpha)$
n = 9
n = 45
n = 21
0
10
20
30
40
50
60
70
80
90°
$\alpha$

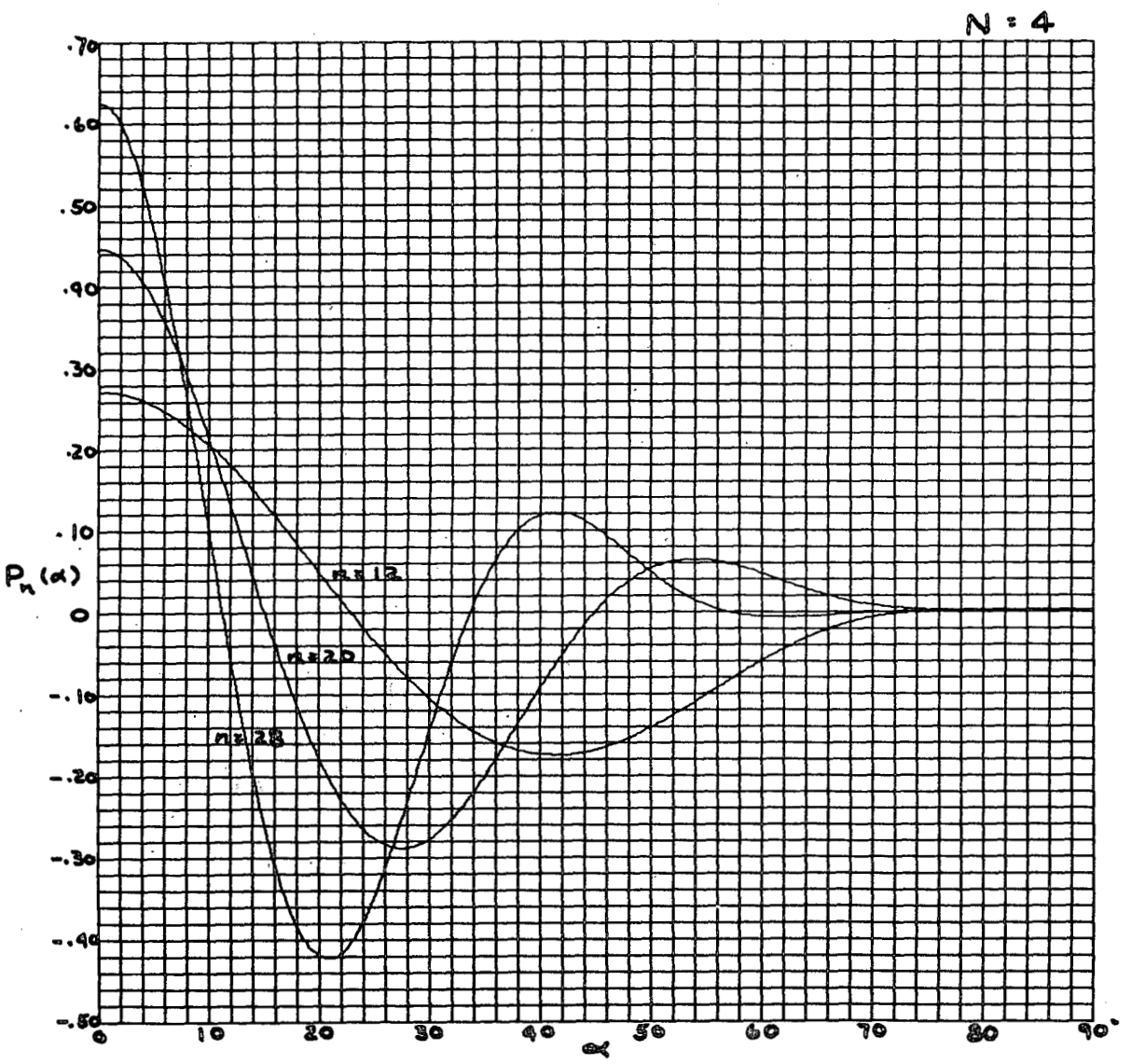
XBL 695-471


N = 3
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
$Q_n(\alpha)$
n = 9
n = 15
n = 21
0
10
20
30
40
50
60
70
80
90°
$\alpha$

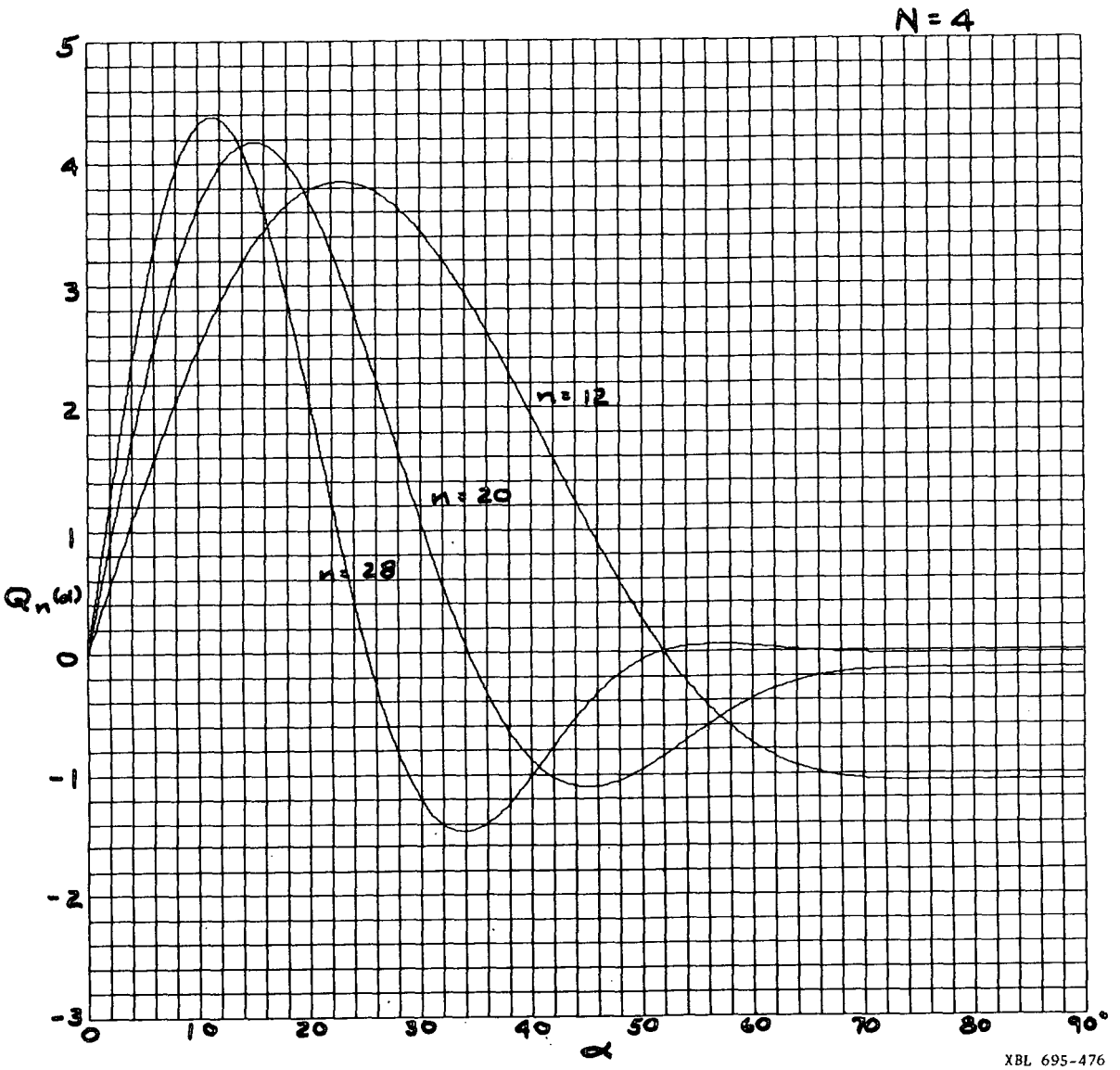
XBL 695-472


N = 3
40
20
0
-20
-40
-60
-80
-100
-120
-140
$R_n(\alpha)$
n=9
n=15
n=21
0
10
20
30
40
50
60
70
80
90°
$\alpha$

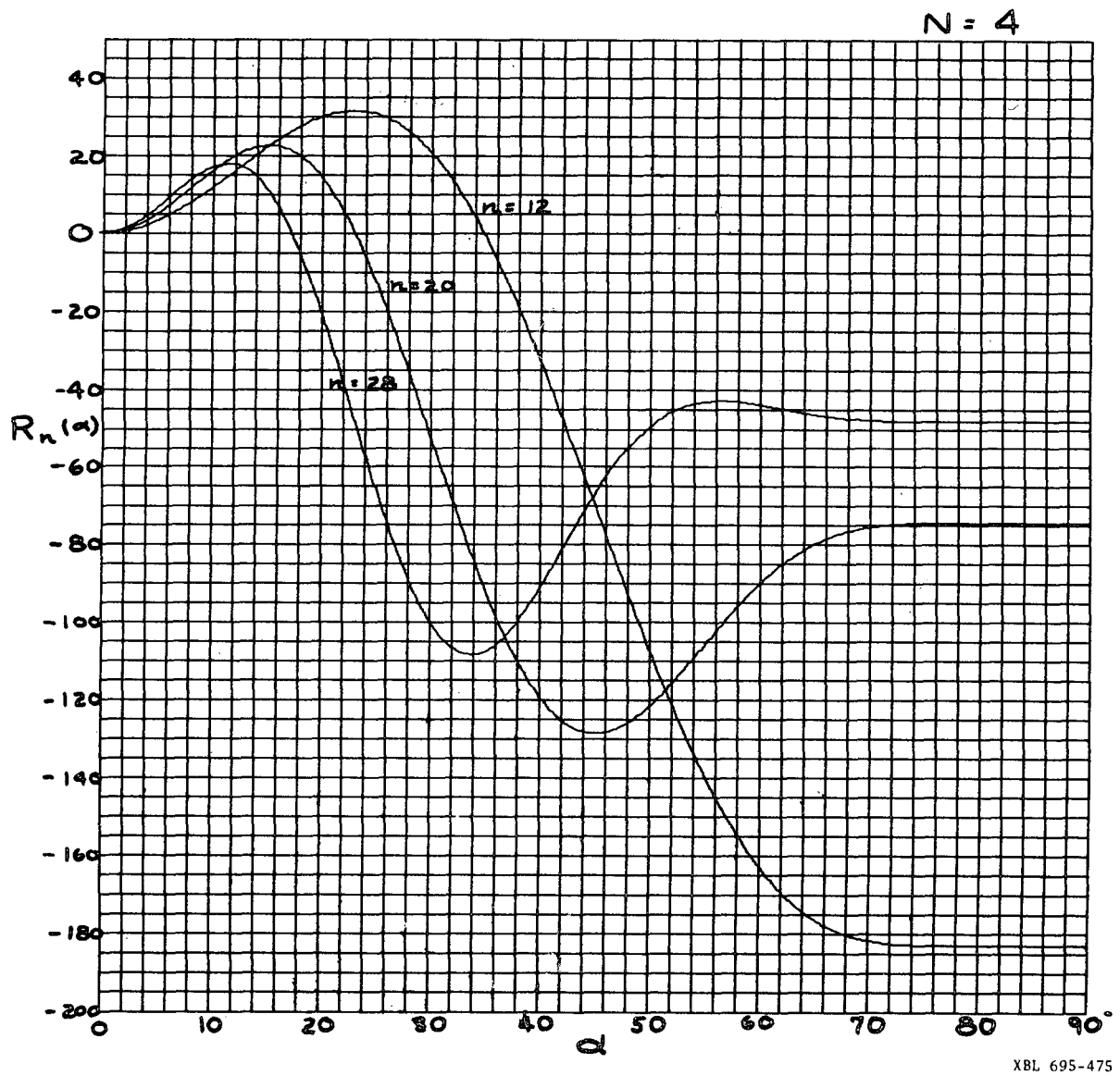
XBL 695-473


N = 4
.70
.60
.50
.40
.30
.20
.10
0
-.10
-.20
-.30
-.40
-.50
$P_n(\alpha)$
n = 12
n = 20
n = 28
0
10
20
30
40
50
60
70
80
90°
$\alpha$

XBL 695-474


N = 4
5
4
3
2
1
0
-1
-2
-3
$Q_n(\alpha)$
n = 12
n = 20
n = 28
0
10
20
30
40
50
60
70
80
90°
$\alpha$

XBL 695-476


N = 4
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
$R_n(\alpha)$
n = 12
n = 20
n = 28
0
10
20
30
40
50
60
70
80
90°
$\alpha$

XBL 695-475