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MATRIX ELEMENTS OF THE LAMBDA-NUCLEON POTENTIAL FROM PHASE SHIFTS*

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Experimental ΛN phase shifts are used to deduce matrix elements of the ΛN potential in a harmonic oscillator basis. An inconsistency between these matrix elements and experimental Λ separation energies for light hypernuclei is noted.

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Elliott, et al. [1] have described a method for obtaining matrix elements of the nucleon-nucleon potential in a relative harmonic oscillator basis directly from the nucleon-nucleon phase shifts. It is of interest to use this technique and the limited amount of ΛN scattering information to obtain matrix elements for use in calculations of the Λ separation B_Λ in hypernuclei.

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Following the technique of ref. [1], we construct an auxiliary potential of the form

$$V_0 = \frac{1}{2}\mu\omega^2 r^2 - \frac{\hbar^2}{2\mu}\alpha \qquad r < a$$
$$= 0 \qquad r > a$$

where μ is the reduced mass of the ΛN system. When

$$\frac{\hbar^2}{2\mu}(k^2 + \alpha) = (2n + \ell + 3/2)\hbar\omega \quad ,$$

the solution to the Schrödinger equation for scattering from the potential V_0 for $r < a$ has the form $BR_{n\ell}$ where $R_{n\ell}$ is the ordinary harmonic oscillator wave function. By matching this solution to a plane wave at $r = a$, we can calculate an auxiliary phase shift δ_0. Provided that the difference between the true potential V and the auxiliary potential is sufficiently small to be treated by Born approximation, the matrix elements of the true potential are given by

$$\langle n\ell|V|n\ell\rangle = \langle n\ell|V_0|n\ell\rangle - \frac{\hbar^2 k}{2\mu B^2}\tan(\delta-\delta_0) \tag{1}$$

Mavromatis, et al. [2] have shown that the matrix elements of eq. (1) should be the same as the matrix elements of V_{LONG} in a Scott-Moszkowski approach when V is attractive even though $(V-V_0)$ is too large for the Born approximation to be valid.

In practice, the parameters of the auxiliary potential are chosen separately for each value of n, ℓ, and $\hbar\omega$ in such a way that $\delta-\delta_0$ is small. The resulting values of $\langle n\ell|V|n\ell\rangle$ are found to be essentially independent of the choice of a and α even though the individual parameters vary over a wide range. It is this insensitivity of the matrix elements to the choice

of an auxiliary potential which leads Elliott, et al. to conclude that they are in some sense model independent.

The ΛN scattering data at laboratory energies of less than 45 MeV has been described in terms of scattering lengths and effective ranges by several groups [3,4,5] under the assumption that low energy cross-sections are dominated by $^1S_0$ and $^3S_1$ channels. The phase shifts are related to these parameters through the usual expression

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_{eff} k^2 \quad .$$

The scattering parameters used in this calculation of ΛN matrix elements are summarized in table 1. The parameters of ref. [3] were obtained from an analysis of 362 events. The more recent parameters of refs. [4] and [5] were obtained from analysis of the previous data and an additional 16 events. The resulting diagonal matrix elements for n = 0 are shown in fig. 1. It is clear that the existing scattering data are not sufficient to provide an unambiguous determination of the $^1S_0$ and $^3S_1$ phase shifts and matrix elements. In fig. 2 we note that the combination $\frac{1}{4} V_s + \frac{3}{4} V_t$ is much more stable with respect to variations in the scattering parameters. This combination achieves relative stability by exploiting the strong correlations between the errors on singlet and triplet phase shifts. Thus, Λ separation energies involving combinations other than $\left(\frac{1}{4} V_s + \frac{3}{4} V_t\right)$ cannot be calculated with confidence from present ΛN scattering data. Therefore $_\Lambda^5$He, $_\Lambda^9$Be, and $_\Lambda^{13}$C should provide the most reliable tests of these matrix elements.

In fig. 2 we also plot $\frac{1}{4} V_s + \frac{3}{4} V_t$ as deduced from the effective ranges and scattering lengths of a variety of potentials constructed to reproduce Λ separation energies of s-shell hypernuclei [3]. These potentials show

similar variations in the values of V_s and V_t, but the same stability in the combination shown. We note that in the vicinity of $\hbar\omega \sim 17$ MeV, a value appropriate for Λ separation energy calculations in light hypernuclei, the matrix elements deduced from phenomenological interactions are approximately 25% smaller than those obtained directly from phase shifts. Thus, independent of the technique used in the calculation of Λ separation energies, we would expect that the matrix elements deduced from scattering data would seriously overestimate separation energies. Although this conclusion has been reached by other authors [5] on the basis of specific potential models, we would suggest that it is independent of the model selected.

In calculating the Λ separation energies for ${}_{\Lambda}^{5}He$, ${}_{\Lambda}^{9}Be$, and ${}_{\Lambda}^{13}C$, we assume that the nucleons lie in their lowest shell model states. The values of the single particle oscillator length parameter b were picked in such a way as to reproduce the mean square charge radii of ${}^{4}He$, ${}^{8}Be$, and ${}^{12}C$. (The effects of a finite proton charge distribution were included.) The Λ was assumed to occupy the 0S oscillator state with the same value of $\hbar\omega$. The p-shell nucleons in ${}^{8}Be$ and ${}^{12}C$ are assumed to be in an LS-coupling state with L = 0 and S = 0. Matrix elements of the ΛN interaction in single particle oscillator states were converted into the relative oscillator matrix elements of fig. 1 by means of an unequal mass Moshinsky transformation [7]. The interaction was assumed to act in relative s-states only.

The results of this calculation are summarized in table 2. The errors on b reflect the experimental error in the determination of charge radii. The errors quoted for separation energy include the effects of the uncertainty of b and the spread of values obtained for the relative oscillator

matrix elements. Our expectation that the matrix elements deduced from scattering data will overestimate Λ separation energies is confirmed. This disagreement may be due to the importance of higher partial waves in low energy scattering or the importance of tensor forces in the $^3S_1$ channel. The latter explanation is appealing since nucleon-nucleon matrix elements derived from scattering length and effective range data are about 30% more attractive than matrix elements deduced from a proper coupled channel calculation as described in ref. [1].

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Table 1. ΛN scattering parameters.

	Alexander and Karshon [3]	Alexander, *et al.* [4]	Fast, *et al.* [5]
a_s	-1.36 fm	-1.8	-1.7
r_s	3.06 fm	2.8	2.5
a_t	-1.62 fm	-1.6	-1.5
r_t	2.93 fm	3.3	2.0

Table 2. Comparison of calculated and experimental Λ separation energies.

	b	Scattering data	Phenomenological potentials	Experiment [6]
${}_{\Lambda}^{5}He$	1.34±0.03 fm	9.1±1.8 MeV	3.5±1.5 MeV	3.00±0.02 MeV
${}_{\Lambda}^{9}Be$	1.66±0.05	10.5±0.9	4.6±0.6	6.52±0.05
${}_{\Lambda}^{13}C$	1.66±0.05	19.5±1.2	11.5±0.8	10.51±0.51

Figure Captions

Fig. 1. Matrix elements $\langle OS|V_s|OS\rangle$ and $\langle OS|V_t|OS\rangle$ as deduced from ΛN scattering parameters of a) Alexander and Karshon [3], b) Alexander, et al. [4], and c) Fast, et al. [5].

Fig. 2. Matrix elements $\frac{1}{4}\langle OS|V_s|OS\rangle + \frac{3}{4}\langle OS|V_t|OS\rangle$ of ΛN potential. Curves a) - c) as in Fig. 1. Curves d) are deduced from scattering parameters of potentials constructed to fit s-shell separation energies [3].

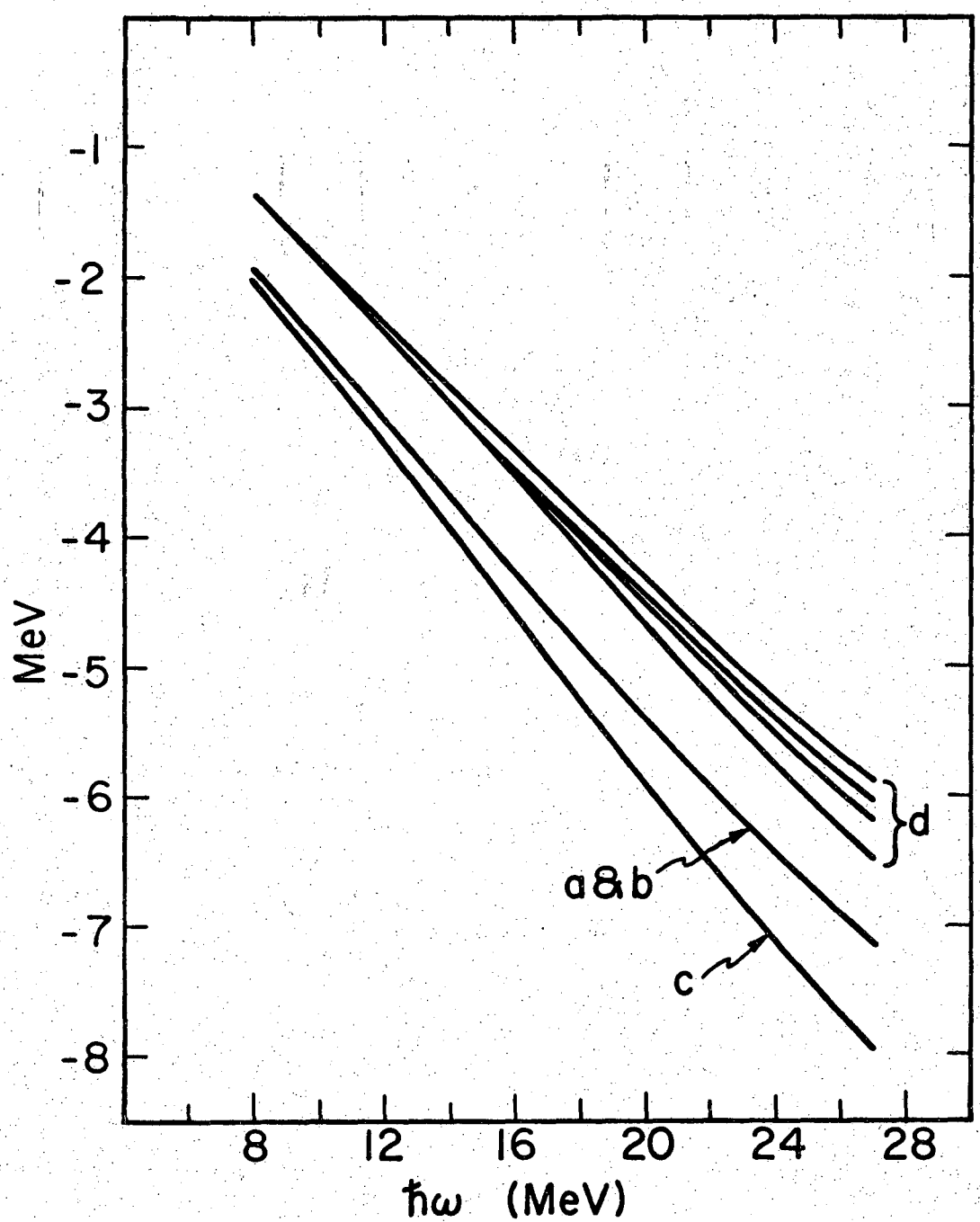

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Fig. 1

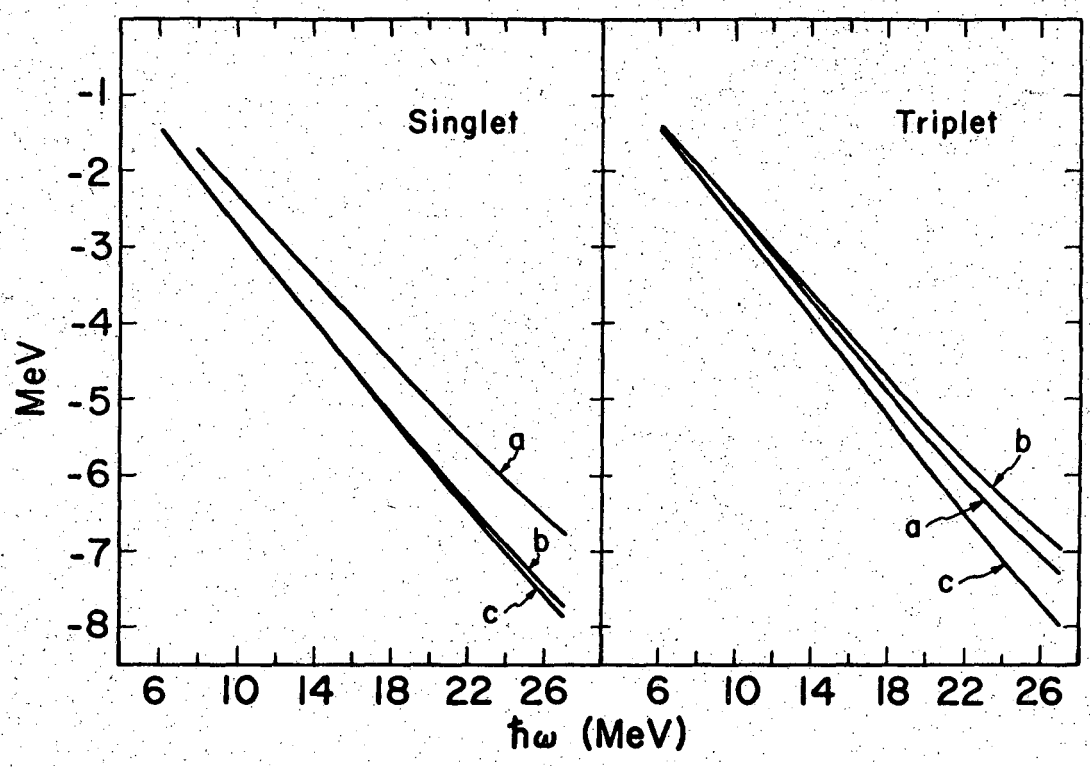

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Fig. 2

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