Journal of Applied Physics
40, No. 11, 4470 (1970)

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ORIGIN OF LOW-VOLTAGE STRUCTURE AND ASYMMETRY IN THE I-V CHARACTERISTICS OF MULTIPLY-CONNECTED SUPERCONDUCTING JUNCTIONS

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October 1969

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Reprinted from JOURNAL OF APPLIED PHYSICS, Vol. 40, No. 11, 4470–4476, October 1969
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Origin of Low-Voltage Structure and Asymmetry in the I–V Characteristics of Multiply-Connected Superconducting Junctions

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(Received 28 May 1969)

A self-magnetic field effect is shown to be responsible for much of the low-voltage structure and for asymmetries observed in the I–V curves of superconducting niobium-solder junctions (Slugs). Similar effects are to be expected in other multiply-connected junctions.

I. INTRODUCTION

The current–voltage characteristics of weak-link junctions between bulk superconductors, such as small metallic bridges, point contacts, or tunnel barriers usually display a variety of structure. That structure which occurs between the voltage onset at the critical current and voltages of a few millivolts, corresponding to the superconducting energy gap, is often attributed to a process involving the ac Josephson[1] effect via feedback of Josephson radiation to the junction. The resultant alternating voltage is superimposed on the applied voltage V and can produce a non-zero supercurrent contribution to the dc current I, so that the shape of the I–V curve in this case reflects the frequency dependence of the impedance seen by the junction.[2] A sharp feature in the I–V curve is then taken as being associated with electromagnetic resonances coupled to the junction. The well-known "Fiske steps" of tunnel junctions[3,4] are examples of this effect and correspond to cavity resonances in the stripline-like structure of the junction. Similar steps are commonly observed in many other junction geometries.

In this paper we show that for the multiple-junction devices developed by Clarke,[5] known as the Slug (Superconducting Low-inductance Undulatory Galvanometer), much of the I–V structure occurring in the low-voltage region, below about 200 μV, is often caused by a quite different mechanism, namely a self-field effect arising from the magnetic field of the measuring current I. A further result of the self-field effect is an asymmetry in both the critical current and the shape of the I–V curve.[6] Although the experimental results are for Slugs, the mechanism applies generally to multiply-connected weak-link junctions.

The self-field effect on the critical current of a singly connected Josephson (oxide tunneling barrier) junction has been described by several authors.[7–9] Finite voltage self-field effects in junctions with normal metal barriers have also been studied by Clarke.[10]

II. THE EXPERIMENTAL JUNCTION

A Slug (also called a drop-form or solder-blob junction) consists typically of a bead of lead–tin solder frozen about an oxidized niobium wire. A cross section is shown in Fig. 1. At temperatures for which both materials are superconducting weak links between the solder and niobium, probably metal-to-metal contacts, give rise to I–V curves such as the example in Fig. 1. The zero-voltage current has a maximum value I_c, beyond which the voltage becomes non-zero and increases thereafter. The curves frequently have considerable structure[5,11] in the voltage region up to 2 mV. At high values of critical current I_c the curves may contain discontinuous voltage jumps or hysterisis but for critical currents of a few milliamperes or less, such as are encountered in the present experiments, the I–V curves are generally continuous.

If the Slug possesses more than one solder-niobium contact, as is usually the case, the critical current and the shape of the I–V curve for $V \lesssim 200\,\mu$V may be modulated by a current I_H passed along the niobium wire (i.e., between terminals 2 and 3 of Fig. 1), the values of I_c and of V for $I > I_c$ oscillating as I_H increases. These oscillations occur because the current I_H generates a magnetic field in the space between the solder and niobium and within their penetration depths. The flux from this field threads the superconducting loops closed by any two contacts. A change in I_H modifies this flux and gives rise to quantum interference effects,[12]

[1] B. D. Josephson, Phys. Lett. **1**, 251 (1962); Advan. Phys. **14**, 419 (1965).
[2] D. E. McCumber, J. Appl. Phys. **39**, 297, 3113 (1968).
[3] M. D. Fiske, Rev. Mod. Phys. **36**, 221 (1964).
[4] D. N. Langenberg, D. J. Scalapino, and B. N. Taylor, Proc. IEEE **54**, 560 (1966).
[5] J. Clarke, Phil. Mag. **13**, 115 (1966); and Proc. Symp. Supercond. Devices, Charlottesville, Va., 1967.
[6] These effects were first reported in J. Clarke, thesis, University of Cambridge, England, Oct. 1967 (unpublished).
[7] T. Yamashita and Y. Onodera, J. Appl. Phys. **38**, 3523 (1967).
[8] C. S. Owen and D. J. Scalapino, Phys. Rev. **164**, 538 (1967).
[9] A. M. Goldman and P. J. Kreisman, Phys. Rev. **164**, 544 (1967).
[10] J. Clarke (unpublished).
[11] S. Shapiro, Phys. Lett. **25A**, 537 (1967).
[12] R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, Phys. Rev. Lett. **12**, 159 (1964).

in which the values of I_c and of V for $I>I_c$ depend on the flux threading the several loops periodically modulo Φ_0, the flux quantum, where $\Phi_0=h/2e=2.07\times10^{-15}$ Wb. In the case of the Slug there are usually several contacts so that the I_c or V against I_H plots consist of the superposition of several periodicities. The fastest oscillations have a typical period of a few hundreds of microamps. The dependence of V and I_c upon I_H has made the Slug useful as a superconducting low-inductance galvanometer, particularly for low-voltage measurements.[5,13,14] Sensitivities approaching 10^{-17} V have been achieved.[15]

III. SELF-FIELD MECHANISM FOR LOW-VOLTAGE STRUCTURE AND ASYMMETRIES

Consider first a Slug with just two identical solder-niobium contacts. The shape of the I–V curve depends periodically with period Φ_0 on the flux Φ threading the loop closed by the two contacts. Figure 2 shows an oblique view of a schematic three-dimensional plot of V against I and Φ. It is assumed for simplicity that for fixed Φ the I–V curves are structureless and symmetrical. For fixed $|I|>|I_c|$ V is periodic in Φ with period Φ_0

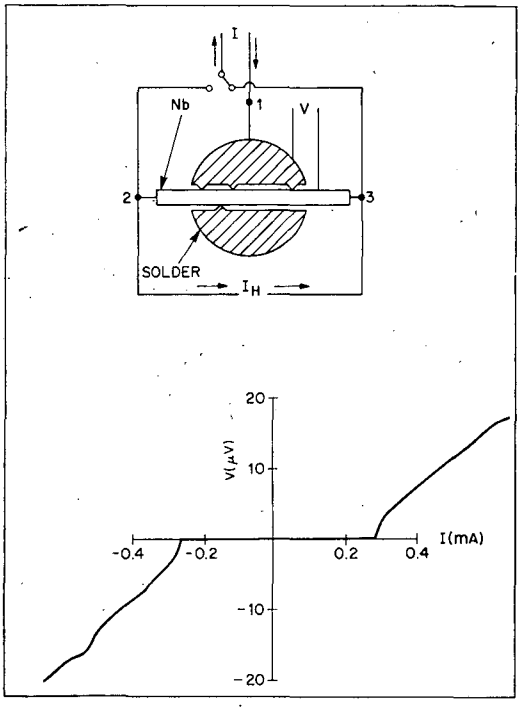

FIG. 1. A longitudinal cross section of a Slug and a typical I–V curve. The terminals I and V are used to measure the I–V curve and the terminals I_H are used for passing current along the niobium wire, thus applying a magnetic field to the Slug. The switch permits use of either 1 and 2 or 1 and 3 as current leads. The arrows show the directions of current flow selected for the positive senses of I and I_H.

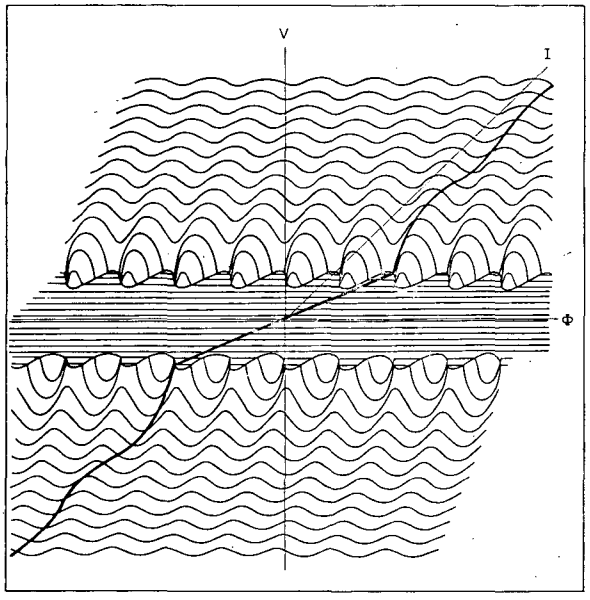

FIG. 2. A three-dimensional schematic plot of V against I and Φ. The heavy line shows the intersection of the V–I–Φ surface with the plane $\Phi=MI$ where $M\neq0$. The projection of this intersection onto an I–V plane gives the observed I–V curve.

and therefore in I_H with corresponding period I_p. The occurrence of the oscillations at finite V indicates that the ac Josephson currents in the several contacts are coherent with one another. The factors determining the ranges of I and V in which the coherence is maintained are not fully understood. Typically, however, the oscillations seem to persist with diminishing amplitude to values for I of several times I_c, corresponding to voltages $V\gtrsim200\,\mu$V.

The self-field effects we describe result from a *second* major contribution to Φ, namely that of the magnetic field generated by I. We may write $\Phi=MI_H+NI$, where the values of M and N depend on the Slug geometry and the dc current carried by the individual contacts. M and N consequently also vary somewhat with Φ, a complication which can be neglected without affecting the general argument. In the determination of the I–V characteristic it is I_H and therefore Φ–NI rather than Φ that is kept fixed. Consequently the observed I–V curve corresponds to the path traced by the heavy line in Fig. 2.

Several features of the resulting I–V curve can be deduced from the figure:

(i) The curve will contain a structure periodic in I (self-field structure) provided N is sufficiently large, i.e., $NI_c\gtrsim\Phi_0$. As M and N are generally of comparable magnitude for the Slug, this criterion is approximately $I_c\gtrsim I_p$. The period of the structure is $\Phi_0/N=(M/N)I_p\gtrsim I_p$.

(ii) As I_H is increased the self-field structure will appear to shift uniformly along the I–V curve, moving in the same direction for both positive and negative I.

[13] J. W. McWane, J. E. Neighbor, and R. S. Newbower, Rev. Sci. Instrum. **37**, 1602 (1966).
[14] D. A. Zych, Rev. Sci. Instrum. **39**, 1058 (1968).
[15] J. Clarke, Phys. Rev. Lett. **21**, 1566 (1968).

The amount of shift caused by a change ΔI_H in I_H is $(M/N)\Delta I_H$. The direction of the shift depends upon the signs of M and N, which are determined by the geometry once the arbitrary choices of the positive senses of Φ, I, and I_H are made. For example, if M and N have the same sign the structure moves in the negative I direction for increasing positive I_H.

(iii) A plot of V against I_H for fixed I will show periodic oscillations in the low-voltage region with period I_p. The phase of the oscillations depends on I, changing by 2π for an increase in I of Φ_0/N.

(iv) The derivatives $dV/dI)_{I_H}=\partial V/\partial I+N\partial V/\partial\Phi$ and $dV/dI_H)_I=M\partial V/\partial\Phi$ taken along the I–V curve both contain structure arising from the term $\partial V/\partial\Phi$. For $dV/dI)_{I_H}$, the differential resistance, both $\partial V/\partial I$ and $N\,\partial V/\partial\Phi$ oscillate with period Φ_0/N as I increases, the first term remaining positive while the second assumes both positive and negative values. If N is large enough that several oscillations occur in the I–V curve, then except for I very near I_c the second term will give the main contribution to the structure of $dV/dI)_{I_H}$. In this case the plots of $dV/dI)_{I_H}$ and $dV/dI_H)_I$ against I will show structure whose period and shape are similar, the amplitude of the former being smaller by the factor N/M.

(v) For I_H non-zero the I–V curve will be asymmetric with respect to a change in the sign of I. The same will be true for $I_H=0$ if there is some additional contribution to Φ such as stray trapped flux in the bulk superconductors.

(vi) The plot of I_c vs I_H will be asymmetric with respect to I_H, since $I_c(\Phi)=I_c(MI_H+NI_c)$. If M and N have the same signs, for example, $+I_c$ will alternately increase rapidly and decrease slowly in absolute value as I_H increases, while the opposite is true for $-I_c$.

(vii) The self-field properties (i)–(vi) are each dependent on the magnitude and sign of N, and will be sensitive to changes in N. For example, in the Slug with two identical contacts one can vary N continuously from $+M/2$ through zero to $-M/2$ by suitably dividing the measuring current I between the return leads 2 and 3 of Fig. 1. This would result, for instance, in first an increase and then a decrease of the period Φ_0/N of the self-field structure, the structure disappearing and reappearing as N passes through zero. Another result would be the change in the sense of the asymmetry of the plots of I_c against I_H, the plot becoming momentarily symmetric as N passes through zero. Similar changes occur in the other effects described above.

The generalization of these arguments to a multicontact structure is straightforward. For an $(n+1)$-contact slug V is a function $V(I,\Phi_1,\cdots,\Phi_n)$ where Φ_i is the magnetic flux linking the the area surrounded by the solder, niobium and two adjacent contacts. For fixed Φ_i, V is again by supposition a relatively smooth function of I. For fixed I, V in the low-voltage region varies periodically with each Φ_i with period Φ_0, the amplitude of the oscillations depending on the details of the Slug. Here again there are two contributions to Φ_i and we may write $\Phi_i=M_iI_H+N_iI$, the values of M_i and N_i depending upon the exact geometry and the dc current carried by the individual contacts and being taken as constant. In the measurement of the I–V curve it is I_H and therefore the quantities Φ_i–N_iI that are held constant. Self-field structure of several different periods can result, the structure associated with each Φ_i having a period Φ_0/N_i.

The general features of the periodic self-field structure and the asymmetries in the I–V, V–I_H, and I_c–I_H curves of a multi-contact structure are clearly similar to those described for the two-contact case. We expect to observe self-field effects if there is a period of oscillation in I_H which is comparable to or smaller than the critical current, provided the corresponding M_i and N_i are of comparable size. The following argument shows that the latter condition can always be satisfied for Slugs.

Let the arrows in Fig. 1 be the directions of current flow for positive I and I_H and suppose that the I–V curve is measured using terminal 3 as a current lead. If as I is varied the value of I_H is also adjusted such that $I_H=I$, then the current flow in the niobium wire on the "3" side of the Slug is zero, and the I–V plot is the same as if terminal 2 were being used as the current lead for I. Labeling the N_i by the subscript 2 or 3 according to which terminal is used, we deduce that $N_{i2}-N_{i3}=M_i$. Consequently either N_{i2} or N_{i3} or both will be comparable to M_i. Further, from the geometry one expects that N_{i2} and N_{i3} will have opposite signs in almost all cases. Consequently N_{i2} and N_{i3} are smaller than M_i in absolute value. A second consequence is that by an appropriate division of the current between the terminals the self-field structure associated with Φ_i can be eliminated.

IV. EXPERIMENTS AND RESULTS

The data of Figs. 3–11 are all taken at 4.2°K for one particular Slug, and contain the quantities V, $dV/dI)_{I_H}$, $dV/dI_H)_I$ and I_c plotted in various combinations against I or I_H. Much of the extensive structure and asymmetry in the I–V curves of this Slug will be seen to be the result of self-field effects, and each of the various aspects (i)–(vii) described in the previous section is illustrated in the data. These properties are each quite distinctive, and we regard the observation of any two or three in combination as sufficient to establish the presence of self-field effects.

The experiments use standard four-terminal measurements, although as discussed previously, the results depend on which terminals are used for current leads. The derivatives were taken by the usual method of superposing a small alternating current ($<5\,\mu$A in all cases) on the steady bias current I or I_H and measuring the resulting alternating voltage. I_c was monitored by use of a feedback loop locking the current I to the volt-

age onset. The figures are reproduced from x–y recorder plots. Slight differences in the plots from one figure to the next result from cycling the Slug to room temperature, or from spontaneous changes in the I–V characteristic at low temperatures. Both effects are probably due to small changes in the stray trapped flux in the Slug, as the self-field effects depend strongly on the magnetic field at the junction.

In Fig. 3 the differential resistance $dV/dI)_{I_H}$ is plotted vs I over a range corresponding to $0 \leq V \leq 1.7$ mV, displaying the structure in the I–V curve of this Slug. The low-voltage region is shown on an expanded scale in the lower curve. Two distinct self-field structures are present. The more prominent is the low-voltage oscillatory structure of some twenty peaks, having a period of 250–300 μA. Superimposed on this is a less

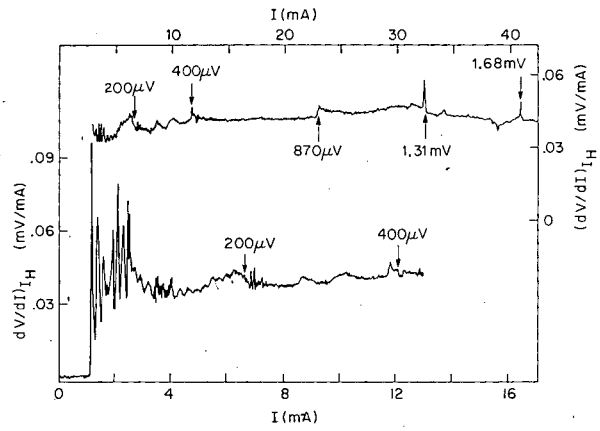

FIG. 3. Differential resistance dV/dI vs I of a slug for voltages up to 1.7 mV. The upper curve corresponds to the top and right scales while the lower is the low-voltage portion of the same curve and corresponds to the lower and left scales. The initial portion of the upper curve is deleted for clarity.

obvious oscillation of $\approx$5 mA period, of which about $1\frac{1}{2}$ cycles are visible. No self-field structure occurs above $\approx$300 μV in this Slug, the structure at higher voltages being due to some other cause, perhaps the ac Josephson mechanism described earlier. In the low-voltage region, aside from the dominant self-field structure, there are also several clusters of weak but sharp, closely-spaced peaks and dips in $dV/dI)_{I_H}$ of unknown origin. In what follows we will concentrate on the oscillatory structures, particularly the one of shorter period, showing that they possess the various properties (i)–(vii) characteristic of self-field effects.

The shape of the I–V curve of this Slug oscillates as I_H is swept, behavior that will be seen in several of the figures. Prominent is an oscillation having a period in I_H of 200 ± 10 μA (from Fig. 6), corresponding to an $M_i = 1.03 \leq 0.05 \times 10^{-11}$ H. As this period is smaller than the critical current of $\approx$1.0 mA we would expect from (i) an associated self-field structure in the I–V curve, which is in fact the shorter-period oscillatory structure of Fig. 3.

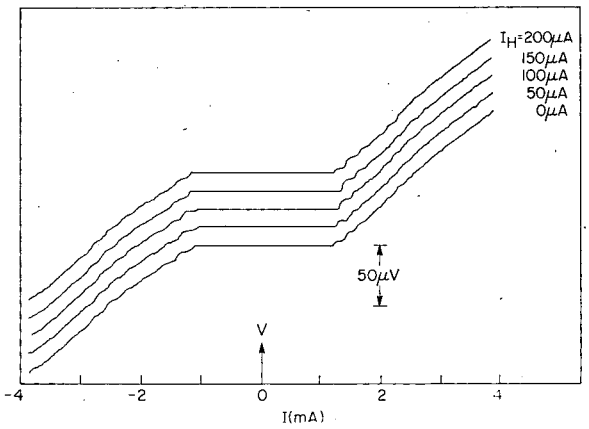

FIG. 4. I–V curves for the Slug of Fig. 3 for various indicated values of I_H. The curves are vertically displaced from one another.

The period of the latter is 270 ± 20 μA (from Fig. 8) giving a value for $N_i = 0.77 \pm 0.05 \times 10^{-11}$ H, smaller than that of M_i, as expected.

In Fig. 4 I–V plots of the low-voltage region are shown for a series of values of I_H. The series of rounded steps in these curves correspond to the shorter-period oscillations in Fig. 3. As I_H increases these appear to move along the curve in the direction of negative I both on the positive and negative I portions of the I–V curve as described in (ii). This behavior is shown rather more

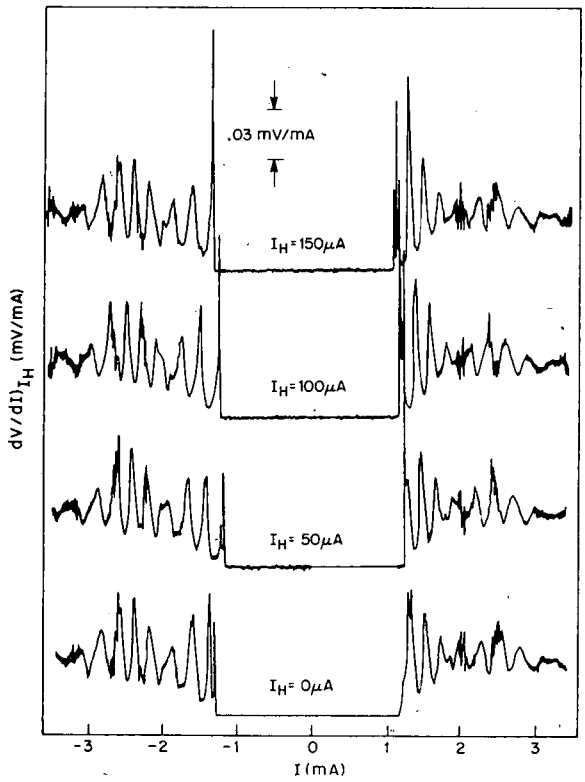

FIG. 5. $dV/dI)_{I_H}$ vs I for the Slug of Fig. 3 for various indicated values of I_H. The curves are vertically displaced from one another.

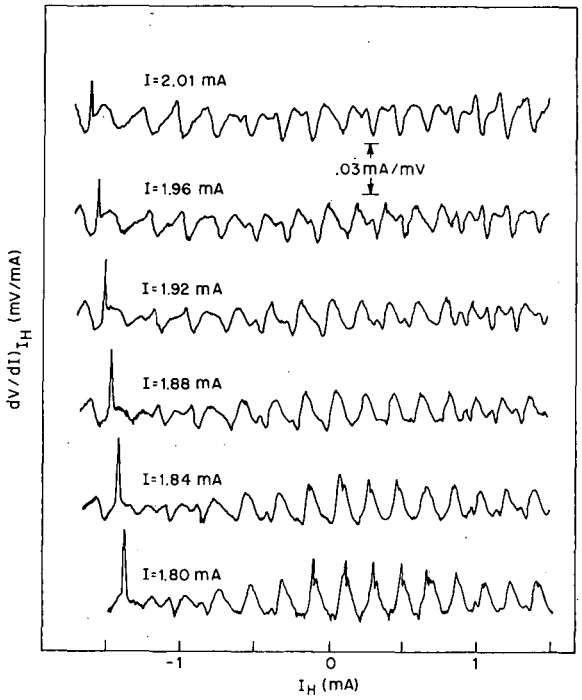

FIG. 6. $dV/dI)_{I_H}$ vs I_H for the Slug of Fig. 3 at various indicated values of I. The curves are vertically displaced from one another.

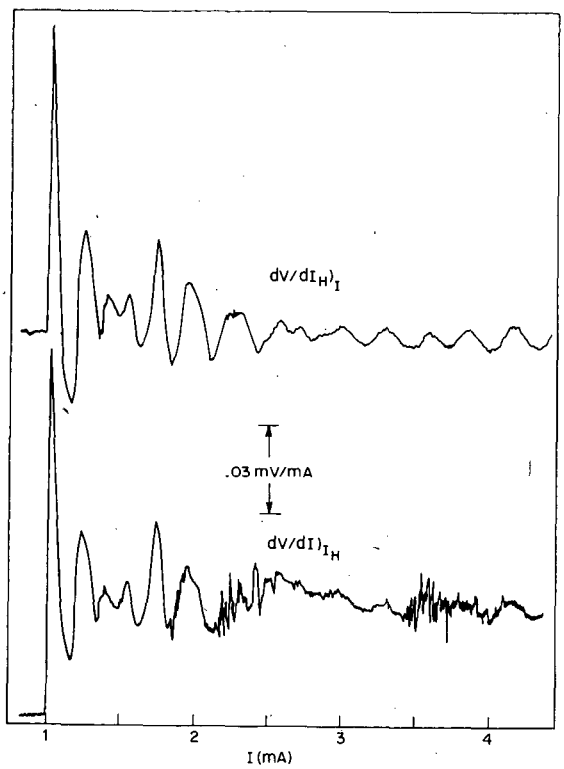

FIG. 7. $dV/dI)_{I_H}$ and $dV/dI_H)_I$ vs I for the Slug of Fig. 3. Both quantities are initially zero at the left (supercurrent region).

clearly in Fig. 5, which shows $dV/dI)_{I_H}$ vs I for a series of increasing values of I_H. The structure can be seen to shift nearly uniformly in I by an amount comparable to the change in I_H, while from (ii) the shift in I should be $M_i/N_i=1.35\pm0.1$ times the change in I_H. If we arbitrarily choose the sign of M_i as positive, then for the particular senses chosen for I and I_H the direction of shift corresponds to a positive value for N_i. Notice in contrast that the clusters of weak, sharp structure, such as the one at 2.0 mA in Fig. 5, do not move with I_H and are therefore not self-field structure.

While the data of Fig. 5 are sufficient to form a rough picture of the V–I–I_H surface, Figs. 6–8 display in a different fashion some of the features. In Fig. 6 we plot $dV/dI)_{IH}$ vs I_H for several increasing values of I, demonstrating both the $200\pm10\,\mu$A period oscillations

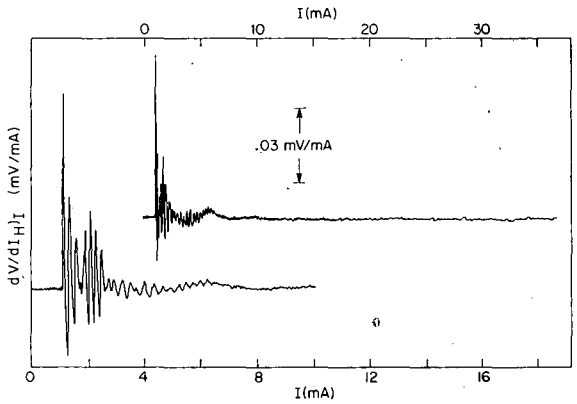

FIG. 8. $dV/dI_H)_I$ vs I for voltages up to 1.7 mV. The upper and lower curves correspond respectively to the upper and lower scales. The scales are the same as in Fig. 3. Both quantities are initially zero at the left (supercurrent region).

with I_H and the phase shift with I of these oscillations. As expected from (iii), the phase shift is approximately $(\Delta I/270\,\mu\text{A})\times2\pi$ for an increase ΔI in I, the shift being to negative I_H with increasing I, consistant with the positive sign of N_i.

In Fig. 7 we compare the differential resistance $dV/dI)_{I_H}$ with $dV/dI_H)_I$, both plotted against I. The close relation between the structure of the two quantities is in accord with (iv). Notice that $dV/dI_H)_I$ is alternately positive and negative while $dV/dI)_{I_H}$ is always positive, indicating that they differ by a smooth additive term, presumably $\partial V/\partial I$. Also note that the structure in $dV/dI_H)_I$ has a larger amplitude, as it should by the factor of $M_i/N_i=1.35\pm0.1$.

In Fig. 8 two plots of $dV/dI_H)_I$ against I on a compressed scale are shown, and may be compared with the differential resistance curves on the same scales in Fig. 3. These plots display clearly both the shorter

period and the longer, $\approx$5 mA period self-field structure. No other structure is present in $dV/dI_H)_I$, and consequently the remaining structure in the I–V curves is due to other causes. The curves of Fig. 8 also provide the cleanest measure of the period of the self-field structure.

The asymmetries produced by self-field effects, discussed in (v) and (vi), are well illustrated by the behavior of this Slug. The asymmetry of the I–V characteristic with respect to the change of sign of I and the variations in this asymmetry as I_H is changed, expected from (v), can be seen in Figs. 4 and 5. The asymmetry persists at $I_H=0$, an effect that probably reflects the presence of stray magnetic fields due to trapped flux in the bulk materials forming the Slug. As the fields caused at the junction by I_H in these experiments are of the order of milligauss, even a weak trapped field would cause substantial changes.

Further asymmetric behavior is illustrated in the plots of I_c vs I_H shown in Fig. 9(a). The asymmetry between $+I_c$ and $-I_c$ is another demonstration of (v). The alternate slow increase and rapid decrease in absolute value of $+I_c$ with increasing I_H, and the opposite behavior of $-I_c$ are as predicted in (vi), the sense of the asymmetry in I_H being consistant with the positive sign of N_i. The period of the variations in I_c is of order 200 μA, although the irregularity of the pattern indicates that several contacts of differing critical currents are involved. The longer-period oscillations in I_c which are associated with the $\approx$5 mA period self-field structure can be seen in the plot of I_c vs I_H on a compressed scale shown in Fig. 9(b).

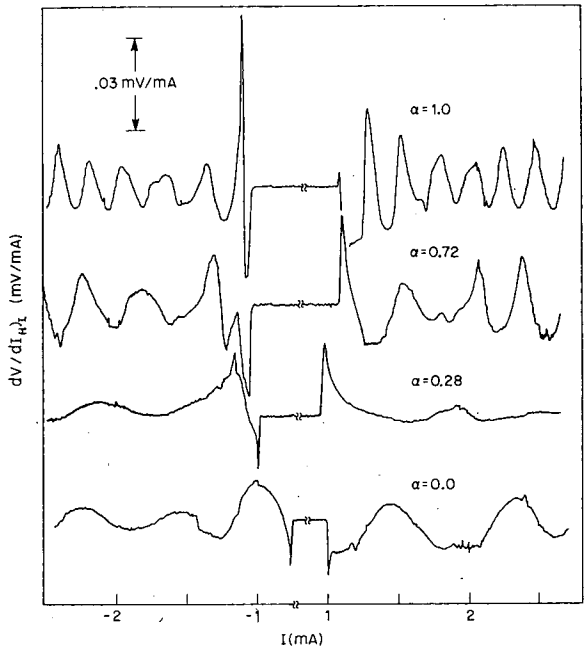

FIG. 10. $dV/dI_H)_I$ vs I for several values of α between 0 and 1.

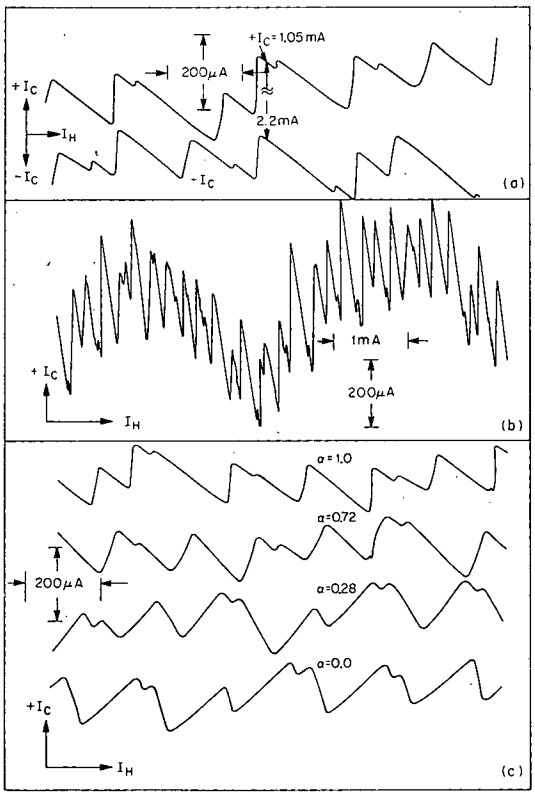

FIG. 9. I_c vs I_H (a) $+I_c$ and $-I_c$ vs I_H for $\alpha=1$ on a scale of 200 μA/div, (b) $+I_c$ vs I_H on a scale of 1 mA/div. (c) $+I_c$ vs I_H for several values of α between 0 and 1.

Perhaps the most convincing demonstration of the presence of self-field effects occurs upon varying the values of the M_i, with effects described in (vii). In the Slug this can be conveniently achieved by appropriately dividing the returning current I between terminals 2 and 3 of Fig. 1, a fraction α being diverted through terminal 2. The data of Figs. 3–9(b) were all taken using terminal 2 exclusively ($\alpha=1$). In Figs. 9(c)–11 some of the effects of varying α are shown.

Figure 9(c) shows a series of plots of $+I_c$ vs I_H for several indicated values of α between zero and one. As predicted in (vii), as α is decreased the slopes for decreasing I_c become steeper and those for increasing I_c shallower, the slopes becoming approximately equal at an intermediate value of α and attaining the opposite sense for $\alpha=0$. From the relation $N_{i3}=N_{i2}-M_i$ one has that $N_{i3}=-0.29\pm0.07\times10^{-11}$ H, so for $\alpha=0.28+0.07$ the value of N_i should become zero and the self-field effects disappear. From Fig. 9(c) the value of α at which the two slopes become equal actually occurs for α somewhat larger than 0.28.

Figure 10 shows $dV/dI_H)_I$ plotted against I, again for several values of α spanning the range between zero and

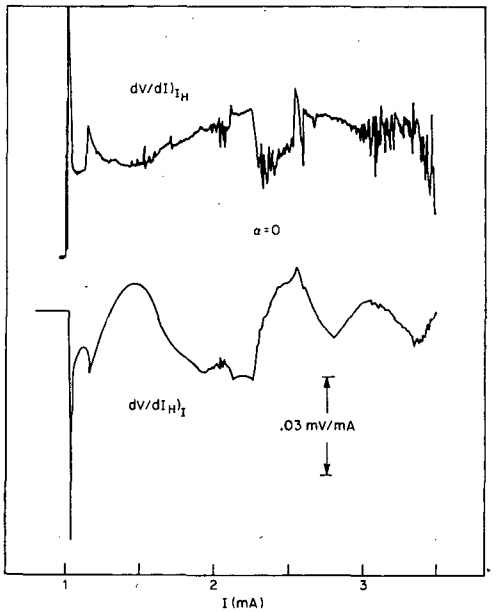

FIG. 11. $dV/dI)_{I_H}$ and $dV/dI_H)_I$ vs I for $\alpha=0$. Both derivatives are initially zero at the left (supercurrent region).

one. As also predicted in (vii) the period of the oscillations first increases, then decreases, again indicating that the value of N_i has been changed from positive to negative, passing through zero. The longest period of the oscillations occurs for $\alpha=0.28$ in accord with the expected zero of N_i.

Finally, in Fig. 11 we show both $dV/dI)_{I_H}$ and $dV/dI_H)_I$ for $\alpha=0$. Both quantities display an oscillation of long period, in approximate agreement with the predicted period $\Phi_0/N_{i3}=710\pm(230/135)\mu$A. Although both quantities show essentially the same period and phase of oscillation, the amplitude of the structure in $dV/dI)_{I_H}$ is weaker by the factor of $N_i/M_i\approx0.28\pm0.07$, and is partially obscured by the clusters of fine structure. Notice also that the signs of the two derivatives are inverted, in accord with the differing signs of N_{i3} and M_i.

A series of five randomly selected Slugs were investigated for self-field effects, using primarily a comparison of $dV/dI)_{I_H}$ and $dV/dI_H)_I$ for $\alpha=0$ and $\alpha=1$. For three, having variously $I_c=1.0$, 3.0, and 32 mA the self-field mechanism was found to cause almost all of the structure in the I–V curves below 100 μV. The Slug whose properties are shown in Figs. 3–11 is one of these, and is typical of the three except that the self-field structure was more regular in the others, indicating that they possessed fewer strong contacts. For a fourth Slug having $I_c=275$ μA the low-voltage structure was about 50% of self-field origin while for the fifth, of $I_c=85$ μA, only a weak, short-period self-field structure for I slightly larger than I_c was observed.

V. DISCUSSION

From the previous section it is apparent that much of the prominent low-voltage structure and asymmetries observed in the I–V curves of Slugs are caused by the self-field mechanism. One may expect to find similar effects in other multiply connected systems, provided that in the low-voltage region the measuring current I can contribute flux changes of order Φ_0 to the field sensitive areas. One expects this can occur whenever the self-inductance of the loop surrounding the field-sensitive area is of the order of or greater than Φ_0/I_c. This estimate is based on the assumption that either from geometry or from differences in the critical currents of the individual contacts the values of N_i will be roughly comparable to the self-inductance of the individual loops. Thus the effects will mostly be observed in multiple-junction interferometer devices rather than in, for example, nominal single-point contacts.

A second potential source of self-field effects in multiply-connected structures is the kinetic inductance. The quantum interference effects depend upon the integral $\int(2m\mathbf{v}+2e\mathbf{A})\cdot d\mathbf{l}$ taken around the various loops closed by the weak links. The kinetic inductance is related to the first term in the integral in the same way that the magnetic self-inductance is related to the second term. Josephson[1] has shown that if bulk materials (thick compared to the penetration depth) comprise both sides of the junction and if the integral $\int 2m\mathbf{v}\cdot d\mathbf{l}$ taken in the direction normal to the surface of these materials at the junction is negligible, then the unequal phase differences can be ascribed entirely to the magnetic effects. If, however, these contributions are not met, as would be the case for multiple junctions between films thin compared to the penetration depth, and as may also be the case for small-area metal-to-metal contacts, then the kinetic inductance may contribute to the self-field effects in a way analogous to that of the magnetic inductance.

Finally, on a practical note, use of the Slug as a galvanometer depends on the sensitivity of the I–V characteristics to changes in I_H. If the M_i and N_i are comparable, the same change in either I or I_H will affect the I–V characteristic to a comparable extent. Thus the magnitude of I or its distribution between connections 2 and 3 of Fig. 1 may be adjusted so as to maximize the sensitivity to changes in I_H, avoiding the necessity for a separate bias supply.

ACKNOWLEDGMENTS

We wish to thank D. E. McCumber for helpful conversations. One of us (T.A.F.) would like to thank A. Contaldo for technical assistance.

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