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GLOSSARY FOR REGGE POLE THEORY

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UCRL-19351
UC-34 Physics
TID-4500 (54th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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Printed in the United States of America
Available from
Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151
Price: Printed Copy $3.00; Microfiche $0.65

GLOSSARY FOR REGGE POLE THEORY*

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September 24, 1969

ABSTRACT

Part A contains a glossary for Regge pole theory for the use of readers of papers in this area of high energy theoretical physics. Part B contains definitions of invariant amplitudes and t-channel helicity amplitudes for meson-baryon elastic scattering and photoproduction of pions, as well as the connection between crossing symmetry and signature and a list of Regge trajectories.

* This work was supported in part by the U. S. Atomic Energy Commission.

INTRODUCTORY NOTE

This short glossary and accompanying notes were prepared as an appendix to the version of my Lund review paper, "Models for High Energy Processes" (UCRL-19205), that is to appear in the January, 1970 issue of Reviews of Modern Physics. It seems worthwhile to make the material available separately. The text is unaltered, apart from the addition of a bibliography necessary to make the present document almost self-contained. Phrases such as "Chapters of this review", "see Eq. (VI.13)", "in Fig. 24", etc. refer to the text of UCRL-19205.

For a general introduction to Regge theory the authoritative reference is now the book by Collins and Squires (1968). Some of the more advanced technicalities are described in detail by Bertocchi (1967) in his review at the Heidelberg Conference.

APPENDIX

The appendix is designed to aid the reader in understanding this and other papers concerned with Regge pole theory. Part A is a modest glossary of some of the commoner terms used by the experts. If some of the definitions sound like gibberish worthy of Lewis Carroll or Edward Lear (for example, see Compensation), I apologize. My hope is that a careful use of the glossary will, by stages, lead to an explanation of any term in words that a mere mortal (or experimenter) can understand. Part B consists of a number of definitions and examples of invariant amplitudes, crossing, Regge trajectories, etc. that provide background material for several of the Chapters of this review.

A. Glossary

Ancestor states. States with $L > N$ that arise when the Veneziano amplitude (VI.2) is mutilated by replacing the real trajectory function by a complex function in order to obtain resonances with finite widths. When $\mathrm{Re}\,\alpha(s) = N$, the numerator in Eq. (VI.2) is no longer a real polynomial of degree N in t, and so the partial wave content is no longer restricted to $L \leqslant N$.

Chew mechanism. See ghost-killing.

Choosing sense or nonsense. See Nonsense-choosing or Sense-choosing.

Compensation (mechanism). The use of a compensating trajectory of opposite parity to cancel a pole occurring in a nonsense-nonsense amplitude at a right-signature nonsense point for a Regge pole that chooses nonsense. If a pole chooses nonsense, the leading terms in the helicity amplitudes behave as $(\cos\theta_t)^{\alpha}$ times 1,1,1 for A_{ss}, A_{sn}, A_{nn}, respectively, at a right-signature nonsense point. For A_{nn} alone there is a lower order term, $(\cos\theta_t)^{-\alpha-1}(\alpha - J)^{-1}$, that has a pole at $\alpha = J$ $(J \geqslant 0)$. The compensating trajectory is such that $\alpha_c = -J - 1$ when $\alpha = J$ and its residue is such as to cancel the pole. The situation is symmetrical around $J = -\frac{1}{2}$ with poles occurring when either J or $-(J + 1)$ is a nonsense point. (See Gell-Mann et al., 1964, Appendix B).

Conspiracy (conspiratorial solution). Relations at $t = 0$ among the trajectories and residues of a finite number of different Regge poles such that certain conspiracy relations are satisfied.

Conspiracy relation. An equation, usually holding at $t = 0$, relating t-channel helicity amplitudes which receive contributions from different parity sequences (different Regge poles). For an example, see Part B(e) of this Appendix. For details and references, see Collins and Squires (1968), p.109 ff and Hite (1969).

Crossing (symmetry). The replacement of 4-momenta by their negatives (line reversal) such that $s \longleftrightarrow u$ is called s-u crossing. An amplitude that is even or odd under this transformation is said to have a definite crossing symmetry (crossing symmetric or crossing antisymmetric, or just even or odd under crossing). The transformation $s \longleftrightarrow u$ generates $\nu \rightarrow -\nu$. Other crossings for the 4-particle amplitude are $s \longleftrightarrow t$ and $t \longleftrightarrow u$.

Cut (in the J-plane). See Regge cut.

Daughter trajectory. See Trajectory, daughter.

Evasion (Evasive solution). The placing of restrictions on the residues, but not on the trajectories, of a number of different Regge poles in order to satisfy conspiracy relations.

Exchange degeneracy. If two Regge poles, coupling to the same particles, and of opposite signature, have the same trajectory function, they are said to be exchange degenerate. This equality of trajectories is sometimes called "weak" exchange degeneracy. "Strong" exchange degeneracy requires equality of corresponding residue functions as well as trajectories.

Factorization. An almost intuitive property of Regge residues that follows from the definition of a simple pole of definite quantum numbers and unitarity. If $\beta_{ba}(t)$ is the residue of such a pole in an amplitude describing a transition from state a to state b (a,b specify particles and their helicities), then factorization of residues requires that $\beta_{ba}^{\ 2} = \beta_{bb}\,\beta_{aa}$ for all values of t.

Fixed pole. A pole in the complex angular momentum plane that does not move with the energy. In relativistic Regge theory there are fixed poles in the angular momentum plane at nonsense, wrong-signature points if the third double spectral function is nonvanishing (see, for example, Mandelstam and Wang, 1967, and Oehme, 1968).

Froissart bound. A bound on the rate of increase of the total cross section, first proved by M. Froissart (see Eden, 1967, p.168),

$$\sigma_t < C[\ell n(s/s_0)]^2, \quad \text{as} \quad s \to \infty.$$

This has the consequence in Regge pole theory that no Regge trajectory can have an intercept greater than unity at $t = 0$.

Gell-Mann mechanism. Same as nonsense-choosing mechanism.

Ghost-killing (factor, mechanism). The standard Regge pole amplitude for spinless particles has its singularity structure determined by $\frac{(\pm 1 + e^{-i\pi\alpha})}{\Gamma(\alpha + 1)\sin \pi\alpha}$ times a reduced residue. For odd signature this set of factors gives the desired poles at $\alpha = 1,3,5,\cdots$ and no other singularities. For even signature, however, there are poles at $\alpha = 0,2,4,\cdots$ and the point $\alpha = 0$ may lie at negative t, corresponding to imaginary mass and often in the physical region of

some process where poles are obviously not allowed. To eliminate this "ghost" pole it is necessary to make the reduced residue proportional to $\alpha(t)$. This factor of $\alpha(t)$ is called the ghost-killing factor. If the amplitude describes spin-flip and the residue "chooses sense" there will already be one or two factors of α present in the residue. Then it becomes a matter of taste (at this state of the art) whether any additional factor of α should be postulated, provided the requirements of factorization are satisfied. Chew's ghost-killing mechanism inserts a factor of α in <u>all</u> even signature Regge amplitudes, independently of whether or not they need them to prevent a pole at $\alpha = 0$.

<u>Kinematic singularity</u>. A square-root or power of a square-root singularity, generally in a helicity amplitude, at a threshold or pseudothreshold determined by the masses of the external particles, or at s, t, or u equal zero.

<u>Kinematic variables (s,t,u)</u>. See Mandelstam variables.

<u>Line reversal</u>. The replacement of the 4-momentum of an incoming or outgoing particle by its negative. Line reversal corresponds to replacing an incoming (outgoing) particle by its outgoing (incoming) antiparticle.

<u>Lorentz pole</u>. In equal mass scattering at zero total energy ($t = 0$) the scattering amplitudes possess the symmetry of the Lorentz group [O(3,1)]. Lorentz poles are the O(3,1) analog of Regge poles (which occur in the context of O(3) invariance and ordinary angular momentum). Lorentz poles are sometimes called Toller poles. They possess quantum numbers of the

same type as Regge poles, plus an additional quantum number M (see M quantum number). A Lorentz pole is equivalent to an infinite sequence of Regge poles (one or more leading poles, plus all their daughters). See Bertocchi (1967), p.202-215, and references given there.

<u>M quantum number</u>. A quantum number characteristic of a Lorentz pole, and important in determining the Regge pole content. A Lorentz pole with $M = 0$ corresponds to one ordinary leading Regge pole of definite parity and signature, plus its daughters. A Lorentz pole with $M = 1$ is equivalent to a parity-doublet of leading Regge poles of the same signature, having the same α values at $t = 0$, plus all their daughters. See, for example, Bertocchi (1967), p.202-215.

<u>Mandelstam variables (s,t,u)</u>. Lorentz invariant kinematic variables. In a process, $a + b \to c + d$,

$$s = -(p_a + p_b)^2 = (\text{total c.m. energy})^2$$

$$t = -(p_a - p_c)^2 = -(\text{4-momentum transfer})^2$$

$$u = -(p_a - p_d)^2 = -(\text{crossed 4-momentum transfer})^2 .$$

For the t-channel process, $a + \bar{c} \to \bar{b} + d$, t becomes the energy variable and s and u are momentum transfer variables. For the u-channel process, $\bar{c} + b \to \bar{a} + d$, u is the energy variable. The variables are related by

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2 .$$

A useful variable in dispersion relations is $\nu = (s - u)/4m_b$ where particle b is the target. For forward elastic scattering ν is the total energy of particle a in the rest frame of b.

No-compensation mechanism. An alternative to compensation. The pole in A_{nn} at a right-signature nonsense point $\alpha = J$ for a Regge trajectory that chooses nonsense can be eliminated and still satisfy factorization by postulating that all the residues have an additional factor $(\alpha - J)$. No additional compensating trajectories are necessary. The leading terms of all the nonsense-choosing Regge amplitudes then behave as $(\alpha - J)$ at a right-signature nonsense point, independent of helicities. The terms of relative order $(\cos \theta_t)^{-2\alpha-1}$ are proportional to $(\alpha - J)$, $(\alpha - J)$, 1 for sense-sense, sense-nonsense, and nonsense-nonsense amplitudes, respectively, $(J \geqslant 0)$. See Bertocchi (1967), p.219.

Nonsense-choosing. The residue of a Regge pole is said to choose nonsense when the α-dependences of the sense-sense and nonsense-nonsense amplitudes are reversed (their product must equal the square of the sense-nonsense dependence, by factorization). Specifically, near $\alpha = J$ the residues are proportional to $(\alpha - J)$, $(\alpha - J)^{\frac{1}{2}}$, 1 for sense-sense, sense-nonsense, and nonsense-nonsense amplitudes, respectively. The leading terms of the helicity amplitudes then behave as 1, 1, 1, respectively, at a right-signature nonsense point and as $(\alpha - J)$, $(\alpha - J)$, $(\alpha - J)$ at a wrong signature point $(J \geqslant 0)$. For $J < 0$ and/or for lower order terms in $\cos \theta_t$, the nonsense-nonsense amplitude has a pole at $\alpha = J$. See compensation and no-compensation mechanisms.

Nonsense-nonsense amplitude. See Sense-sense, etc.

Nonsense point. An integer (or half-integral for channels with odd baryon number) value of angular momentum that is unphysical for a particular set of helicities, i.e., $J < |J_z|$. Some authors define only non-negative integers as possible nonsense points (for example, Gell-Mann et al., 1964, Appendix B).

Pole (in the J-plane). See Fixed pole, Regge pole.

Regge cut. A branch cut in the complex angular momentum plane, a possible singularity of a relativistic partial wave amplitude. Regge cuts arise naturally by the exchange of two or more Regge poles, and occur in perturbation theory from the summing up of certain sets of crossed graphs (see Mandelstam, 1963; Collins and Squires, 1968).

Regge pole. A pole in a partial wave amplitude $A(J,t)$ considered as a function of complex angular momentum J. The position of the pole, $J = \alpha(t)$, depends on the energy variable t. In potential theory, with some restrictions on the form of the potential, Regge poles are the only singularities possible in the complex angular momentum plane.

Residue. The residue (in a Cauchy sense) of a Regge pole in a partial wave amplitude. Loosely, the true residue multiplied by a number of kinematic and α-dependent factors designed to give the product reasonable analytic properties in t. This last is sometimes called a reduced residue.

Right and wrong signature points. A Regge trajectory usually has a definite signature. A boson trajectory with even (odd) signature will have physical particles at $J = 0,2,4,\cdots$ ($J = 1,3,5,\cdots$). For a trajectory with even signature the points $J = 0,\pm 2,\pm 4,\cdots$ are called right-signature points, while $J = \pm 1,\pm 3,\cdots$ are called wrong-signature points. For a trajectory with odd signature the two sequences are reversed. For fermion trajectories the J values are shifted by half a unit.

Satellite term. Veneziano amplitude whose leading trajectory is one or more units of angular momentum below the leading Veneziano amplitude. For an example see Eq. (VI.13).

Sense-choosing. The residue of a Regge pole is said to be sense-choosing (or chooses sense) if it vanishes when the trajectory passes through nonsense points. In particular, the residue is proportional to 1, $(\alpha - J)^{\frac{1}{2}}$, $(\alpha - J)$ for sense-sense, sense-nonsense, and nonsense-nonsense amplitudes, respectively, near $\alpha = J$. These α-dependences of the residues imply that at a right-signature point the leading terms (highest powers of $\cos \theta_t$) in the helicity amplitudes are proportional to $(\alpha - J)^{-1}$, 1, $(\alpha - J)$, respectively, at least for $J \geqslant 0$. At wrong-signature points all the amplitudes are less singular by one power of $(\alpha - J)$, provided there are no multiplicative fixed poles (see Oehme, 1968, and Mandelstam and Wang, 1967). For more details, consult Bertocchi (1967), p.215 ff.

Sense point. A physically allowed value of angular momentum, i.e., an integer (half integer for fermions) value of J with $J \geqslant |J_z|$.

Sense-sense, sense-nonsense, and nonsense-nonsense amplitudes. Consider the process $ab \to cd$ and a partial wave or Regge amplitude in which the initial helicities are λ_a, λ_b with difference $\lambda = \lambda_a - \lambda_b$, and final helicities λ_c, λ_d with difference $\mu = \lambda_c - \lambda_d$. Let the angular momentum be J. If $J \geqslant |\lambda|, |\mu|$ the amplitude is said to be a sense-sense amplitude with respect to J (J is physical for both initial and final state). If $|\lambda| > J \geqslant |\mu|$ or $|\mu| > J \geqslant |\lambda|$, the amplitude is called sense-nonsense. If $J < |\lambda|, |\mu|$, the amplitude is said to be a nonsense-nonsense amplitude. When discussing the α-dependence of amplitudes near a particular value of $\alpha = J$ the helicity amplitudes are often labelled A_{ss}, A_{sn}, A_{nn} for the three categories of helicities.

Signature. An amplitude possesses definite signature, even or odd, if it is even or odd under the transformation, $\cos\theta \to -\cos\theta$. A Regge pole has definite signature if its contribution to an amplitude possesses definite signature. Such a pole gives physical particles at only every second integer in J (see right and wrong signature points).

Signature factor. $(\pm 1 + e^{-i\pi\alpha})$. This factor arises from the representation of an even- or odd-signatured amplitude by a Regge pole expansion involving $[P_\alpha(-z) \pm P_\alpha(z)] \propto (-z)^\alpha \pm (z)^\alpha$.

Threshold, normal and pseudo-. The normal and pseudothresholds are energies where the c.m. momentum in the initial or final state vanishes. If the masses are m_1 and $m_2 \neq m_1$ the normal threshold in the s-channel is at $s = (m_1 + m_2)^2$ and the pseudothreshold is at $s = (m_1 - m_2)^2$. For $m_1 = m_2$, the momentum vanishes only at the normal threshold.

Toller pole. Same as Lorentz pole.

Trajectory, daughter. A trajectory with an $\alpha(t)$ differing from its parent or leading trajectory by a negative integer, and possessing quantum numbers related to those of the parent. Daughter trajectories were first found important at $t = 0$ for unequal mass kinematics. Then they turned up in the expansion of Lorentz poles into a series of Regge poles. Most recently they appeared in Veneziano amplitudes.

Trajectory, leading. The highest lying Regge trajectory in a family or entering a given process, i.e., the trajectory corresponding to the largest value of angular momentum at a given energy.

Trajectory, parent. Same as leading trajectory.

Trajectory, Regge. The function $\alpha(t)$ that gives the location of a Regge pole in the complex angular momentum plane, or sometimes the real part of $\alpha(t)$ viewed as a function of t.

Trajectory, secondary. A Regge trajectory that has a smaller $\mathrm{Re}\,\alpha(t)$ than some other trajectory. A secondary trajectory is not necessarily a daughter trajectory, nor spaced below by an integer.

Wrong signature point. See Right and Wrong signature point.

B. Miscellaneous Definitions, Formulas, and Relations

(a) Invariant amplitudes and t-channel helicity amplitudes for meson-baryon elastic scattering

Let the meson mass be μ, the baryon mass m, and q, p, q', p' be the 4-momenta of the initial meson, initial baryon, final meson, and final baryon, respectively. The invariant Feynman amplitude for scattering with initial and final helicities λ and λ', respectively, is written as

$$\mathcal{M} = \bar{u}_{\lambda'}(p')[-A + i\gamma\cdot\left(\frac{q + q'}{2}\right) B]u_{\lambda}(p) \tag{A1}$$

where $u(p)$ is a Dirac spinor normalized to $\bar{u}u = 2m$. The amplitudes A and B are functions of the Mandelstam variables s and t [or alternatively $\nu = (s - u)/4m$ and t]. They are Lorentz scalars and are free from kinematic singularities. A convenient normalization via the optical theorem is

$$\mathrm{Im}[A(\nu,0) + \nu B(\nu,0)] = (\nu^2 - \mu^2)^{\frac{1}{2}} \sigma_t(\nu) . \tag{A2}$$

The invariant amplitude $A'(s,t)$ is defined by

$$A'(s,t) = A(s,t) + \frac{m(s - u)}{4m^2 - t} B(s,t) \tag{A3}$$

or

$$A'(\nu,t) = A(\nu,t) + \frac{\nu B(\nu,t)}{\left(1 - \frac{t}{4m^2}\right)} . \tag{A3'}$$

The use of A' and B, instead of A and B, stems from their relation to the nonflip and spinflip helicity amplitudes in the t channel. Explicitly, the t-channel helicity amplitudes [(A1) evaluated in the t-channel c.m.] are

$$f^{t}_{++;00} = -(t - 4m^2)^{\frac{1}{2}} A' \tag{A4}$$

$$f^{t}_{+-;00} = 2(t)^{\frac{1}{2}} p_t q_t \sin \theta_t B/(t - 4m^2)^{\frac{1}{2}} \tag{A5}$$

where p_t and q_t are the t-channel c.m. momenta of the baryon-antibaryon pair and the pair of mesons, respectively.

(b) <u>Crossing and even and odd amplitudes for K^-p and K^+p elastic scattering</u>

Let the K^-p elastic scattering amplitudes be $A(\nu,t)$ and $B(\nu,t)$ and the K^+p amplitudes be $\bar{A}(\nu,t)$ and $\bar{B}(\nu,t)$. The concept of line reversal and the accompanying s-u crossing $(\nu \to -\nu)$ requires that

$$\bar{A}(\nu,t) = A(-\nu,t) \tag{A6}$$

$$\bar{B}(\nu,t) = -B(-\nu,t) \quad . \tag{A7}$$

Note that if ν has a small positive imaginary part (above the real axis on the right in Fig. 20) then $-\nu$ has a small negative imaginary part (below the real axis on the left in Fig. 20). The minus sign in (A7) arises because of the way B enters (A1). Line reversal of the mesons changes $(q + q') \to -(\bar{q} + \bar{q}')$. The relations (A6) and (A7) are what gives meaning to the display of the K^-p and K^+p total cross sections in Fig. 24.

Amplitudes that are even or odd under s-u crossing are

$$A^{(\pm)}(\nu,t) = \frac{1}{2}[A(\nu,t) \pm \bar{A}(\nu,t)] \tag{A8}$$

$$B^{(\pm)}(\nu,t) = \frac{1}{2}[B(\nu,t) \pm \bar{B}(\nu,t)] \quad . \tag{A9}$$

Note the $A^{(+)}$ and $\nu B^{(+)}$ are even, while $A^{(-)}$ and $\nu B^{(-)}$ are odd under $\nu \to -\nu$. The amplitudes defined by (A8) and (A9), and analogous ones for π^+p and π^-p scatterings, are what are used in Chapter V on finite energy sum rules.

(c) Connection between s-u crossing symmetry and t-channel signature

Crossing involves $s \longleftrightarrow u$ or $\nu \to -\nu$, while t-channel signature (usually of a Regge pole amplitude) is associated with $\cos\theta_t \to -\cos\theta_t$. A heuristic derivation of the relation between these two symmetries follows immediately from the kinematic relation (for elastic scattering in the s channel),

$$4p_t\, q_t \cos\theta_t = s - u \quad . \tag{A10}$$

An s-channel amplitude that is even (odd) under crossing will involve only t-channel Regge poles of even (odd) signature at high energies.

(d) Short catalogue of Regge trajectories

(i) Boson trajectories

P: Pomeranchon, named after I.Ya. Pomeranchuk because of his theorem on asymptotic behavior of cross

sections (see Eden, 1967, p.213). This trajectory has even signature and carries the quantum numbers of the vacuum (the unit representation of all known and yet to be discovered internal symmetries). It has $\alpha_P(0) = 1$ if total cross sections are asymptotically constant. Exchange of the Pomeranchon supposedly describes diffraction scattering as $s \to \infty$. It does not seem to have any particles associated with it.

P': This even-signature trajectory apparently has the same quantum numbers as the P (at least it has even parity, charge conjugation, and G-parity, $I = Y = 0$). The $f^0(1260)$ meson with $J^P = 2^+$ is the only well established member on the P' trajectory. The intercept of the P' trajectory is $\alpha_{P'}(0) \sim 0.5\text{-}0.6$.

A_2 or R: An even-signature trajectory whose lowest physical state is the A_2 meson at 1315 MeV with $J^P = 2^+$. The internal quantum numbers of the trajectory (and the A_2) are $I = 1$, $Y = 0$, $G = -1$, and $P = +1$. Its intercept is $\alpha_{A_2}(0) \sim 0.3\text{-}0.5$

ρ: The trajectory of odd signature associated with the $\rho(765)$ meson has $I = 1$, $Y = 0$, $G = +1$, and $P = -1$. The g(1650) meson is probably the first

Regge recurrence, with $J^P = 3^-$. A popular parameterization of the ρ trajectory is $\alpha_\rho(t) \simeq 0.5 + 0.9t$.

ω: The ω trajectory has $I = 0$, $Y = 0$, $G = -1$, $P = -1$, odd signature, passes through the $\omega(783)$ meson, and is roughly equal to the ρ trajectory.

K^*: This trajectory (and its antiparticle partner) has $I = \frac{1}{2}$, $Y = 1\ (-1)$, $P = -1$, odd signature, and is associated with the $K^*(890)$ meson with $J^P = 1^-$. Its intercept is very poorly known: $\alpha_{K^*}(0) \simeq 0.1\text{-}0.5$.

K^{**}: This is a trajectory of even signature with $I = \frac{1}{2}$, $Y = 1$, $P = +1$. It is associated with the $K^*(1420)$ meson with $J^P = 2^+$. Its intercept is also very poorly determined: $\alpha_{K^{**}}(0) \sim 0.1\text{-}0.5$.

All of the above trajectories are associated with states of natural parity $\left(P = (-1)^J\right)$. They are relatively well-known because they occur in pseudoscalar meson-spin $\frac{1}{2}$ baryon processes. The unnatural parity states (π, η, A_1, etc.) presumably also lie on Regge trajectories. Only the pion trajectory has received much study in high energy processes, and even there the proximity of the pion pole to the physical region means that its Regge pole nature is relatively unimportant, apart from questions of conspiracy.

(ii) Baryon trajectories

The large number of baryon states precludes a systematic listing of all the baryon trajectories. A common notation to systematize the sequences of states is the use of subscripts α, β, γ, δ to signify the spin-parity sets,

$$\alpha : \frac{1}{2}^+, \frac{5}{2}^+, \frac{9}{2}^+, \cdots \text{ (even parity, even signature)}$$

$$\beta : \frac{1}{2}^-, \frac{5}{2}^-, \frac{9}{2}^-, \cdots \text{ (odd parity, even signature)}$$

$$\gamma : \frac{3}{2}^-, \frac{7}{2}^-, \frac{11}{2}^-, \cdots \text{ (odd parity, odd signature)}$$

$$\delta : \frac{3}{2}^+, \frac{7}{2}^+, \frac{11}{2}^+, \cdots \text{ (even parity, odd signature)}$$

Note that for the baryons signature is associated with $(j - \frac{1}{2})$. Examples of baryon trajectories and associated particles are

$$N_\alpha\left(N(938), J^P = \frac{1}{2}^+ ;\ N^*(1688), J^P = \frac{5}{2}^+ ;\ I = \frac{1}{2},\ Y = 1\right) ,$$

$$\Delta_\delta\left(\Delta(1236), J^P = \frac{3}{2}^+ ;\ \Delta(1940), J^P = \frac{7}{2}^+ ;\ I = \frac{3}{2},\ Y = 1\right) ,$$

$$\Lambda_\gamma\left(Y_0^*(1520), J^P = \frac{3}{2}^- ;\ Y_0^*(2100), J^P = \frac{7}{2}^- ;\ I = 0,\ Y = 0\right) .$$

For more details on the baryon trajectories, consult Collins and Squires (1968), p. 198 ff, or Hite (1969).

(e) Invariant amplitudes, t-channel helicity amplitudes, and conspiracy equations for pion photoproduction

Let k, p, q, p' be the 4-momenta of the photon, initial nucleon, pion, and final nucleon, respectively, and define $P = (p + p')/2$. The nucleon mass is m; the pion mass is μ. The invariant Feynman amplitude for a photon of polarization ϵ incident on a nucleon of helicity λ leading to the production of a pion and a nucleon of helicity λ' is written as

$$\mathcal{M} = \bar{u}_{\lambda'}(p')[A_1 \mathcal{O}_1 + A_2 \mathcal{O}_2 + A_3 \mathcal{O}_3 + A_4 \mathcal{O}_4] u_\lambda(p) \qquad \text{(A11)}$$

where the A_i are four invariant amplitudes and the four $\mathcal{O}_i$ are gauge-invariant Dirac operators,

$$\left.\begin{aligned}
\mathcal{O}_1 &= i\,\gamma_5 \gamma\cdot\epsilon\ \gamma\cdot k \\
\mathcal{O}_2 &= 2i\gamma_5(P\cdot\epsilon\ q\cdot k - p\cdot k\ q\cdot\epsilon) \\
\mathcal{O}_3 &= \gamma_5(\gamma\cdot\epsilon\ q\cdot k - \gamma\cdot k\ q\cdot\epsilon) \\
\mathcal{O}_4 &= 2\gamma_5(\gamma\cdot\epsilon\ P\cdot k - \gamma\cdot k\ P\cdot\epsilon - im\ \gamma\cdot\epsilon\ \gamma\cdot k)
\end{aligned}\right\} . \qquad \text{(A12)}$$

The invariant amplitudes A_i are Lorentz scalars, functions of s and t, and free of kinematic singularities.

The t-channel helicity amplitudes $f^t_{\lambda_\gamma 0;\bar{\lambda}\lambda}$ are customarily related to the invariant amplitudes A_i through the intermediary of the so-called parity-conserving t-channel amplitudes F_i, defined to be free of kinematic singularities. The F_i are related to the helicity amplitudes according to

$$F_1 = \frac{-2i}{(t - \mu^2)\sin\theta_t}\left(f^t_{10;++} + f^t_{-10;++}\right) \tag{A13}$$

$$F_2 = \frac{-2i(t)^{\frac{1}{2}}}{(t - 4m^2)^{\frac{1}{2}}\sin\theta_t}\left(f^t_{10;++} - f^t_{-10;++}\right) \tag{A14}$$

$$F_3 = \frac{-2i(t)^{\frac{1}{2}}}{t - \mu^2}\left(\frac{f^t_{10;+-}}{\cos\theta_t + 1} + \frac{f^t_{-10;+-}}{\cos\theta_t - 1}\right) \tag{A15}$$

$$F_4 = \frac{2i}{(t - \mu^2)(t - 4m^2)^{\frac{1}{2}}}\left(\frac{f^t_{10;+-}}{\cos\theta_t + 1} - \frac{f^t_{-10;+-}}{\cos\theta_t - 1}\right). \tag{A16}$$

The amplitudes F_1 and F_2 receive contributions from natural and unnatural parity sequences in the t-channel, respectively, and for charged pion photoproduction further restricted to even signature and odd G-parity, or odd signature and even G-parity. In leading order (in powers of $\cos\theta_t$) F_3 and F_4 involve natural and unnatural parity sequences, respectively. The amplitude F_3 has the same signature-G parity relation as F_1 and F_2, while for F_4 it is opposite. In terms of well-known particles pion exchange contributes only to F_2, ρ, and A_2 exchanges to F_1 and F_3.

The connections of the F_i to the A_i are

$$F_1 = -A_1 + 2mA_4 \tag{A17}$$

$$F_2 = (t - \mu^2)(A_1 + tA_2) \tag{A18}$$

$$F_3 = 2mA_1 - tA_4 \tag{A19}$$

$$F_4 = -A_3 \ . \tag{A20}$$

These equations imply certain relations among the F_i at special values of t and all s values. For example, at $t = 4m^2$ Eqs. (A17) and (A19) imply that $F_3 = -2mF_1$. Because this relation holds at $t = 4m^2$ ($p_t = 0$) it is called a threshold relation. Since F_1 and F_3 both involve natural parity sequences with the same quantum numbers, the threshold relation can generally be satisfied for all s by each exchanged trajectory separately. Of more interest are relations at $t = 0$. Equations (A18) and (A19) require that at $t = 0$, $F_2 = -\mu^2 F_3/2m$. Since F_2 and F_3 involve different parity sequences this relation must be satisfied either by "evasion" or "conspiracy." Suppose that the pion Regge pole and the ρ-meson Regge pole contribute respectively to F_2 and to F_3. Since $\alpha_\pi(0) \neq \alpha_\rho(0)$ there is no possibility of satisfying the relation $F_2 = -\mu^2 F_3/2m$ for all s, except by requiring $F_2 = F_3 = 0$ at $t = 0$. This is called the evasive solution and leads to a dip in the forward direction in the cross section for pion photoproduction. One way to avoid requiring $F_2 = 0$ is to postulate a natural parity partner for the pion (a conspirator) with a trajectory $\alpha_c(t)$ such that $\alpha_c(0) = \alpha_\pi(0)$ and a residue adjusted to satisfy $F_2 = -\mu^2 F_3/2m$. This last relation is called a conspiracy equation for obvious reasons. Such a conspiratorial solution to the observed forward peaking in high energy photoproduction is referred to in Chapter III,

Sec. 1 and in Chapter V, Sec. 2. Regge cuts offer another conspiratorial solution since the Regge cut amplitude corresponds to a mixture of parity sequences and will in general contribute to both F_2 and F_3.

The reader may wonder at the physical significance of the conspiracy equation, $F_2 = -\mu^2 F_3/2m$. Certainly the derivation from Eqs. (A18) and (A19) does not exhibit much physics. A satisfactory understanding can be found in terms of the connection with the s-channel helicity amplitudes, at least in the limit of $s \to \infty$. Examination of the s-t crossing matrix (see Jackson and Quigg, 1969) shows that to leading order in s the helicity double-flip amplitude in the s-channel is

$$g_{0-\frac{1}{2};-1,\frac{1}{2}} = \frac{s}{\sqrt{2}}\left(\frac{tF_1}{4m^2} + \frac{F_2}{\mu^2 - t} + \frac{F_3}{2m}\right) \quad . \tag{A21}$$

This s-channel amplitude must vanish as t in the forward direction because of conservation of the component of angular momentum along the beam direction. Thus independent of any dynamical assumptions the t-channel conspiracy relation, $F_2 = -\mu^2 F_3/2m$, at $t = 0$ is required by conservation of angular momentum in the s-channel.

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