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AEC RESEARCH AND
DEVELOPMENT REPORT

Asymptotic Behaviors of Electroproduction Amplitudes of One Hadron*

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ABSTRACT

Asymptotic behaviors for electroproduction of one hadron in the limit of $q^2 \to \infty$ (spacelike) with (laboratory energy)/q^2 fixed large are derived by means of the Bethe-Salpeter equation which takes account of the vector property of the photon.

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I. INTRODUCTION

The structure functions of inelastic electron-nucleon scattering have become of considerable experimental and theoretical interest. Bjorken[1] has put forth a conjecture called a scaling rule on the asymptotic behaviors of these functions in accord with the experimental measurement.[2] Combining his conjecture with Regge asymptotic behaviors, Abarbanel, Goldberger, and Treiman[3] have argued that the Pomeranchon in the Regge asymptotics also dominates in the Bjorken limit[4] with $2m\nu/q^2$ fixed large where q^2 and ν are the squared four-momentum and the laboratory energy of the photon, respectively. This line was subsequently pursued by Drell, Levy, and Yan,[5] and also by Altarelli and Rubinstein[6] in the ladder approximation to the generation of Regge particles. It was thus argued that the scaling rule is derived from the Pomeranchon contribution. It is, however, believed according to recent developments in hadron physics that the Pomeranchon is generated by a mechanism entirely different from that for the other (ordinary) Regge trajectories. The ladder approximation would be poor for the Pomeranchon so that some doubt may be cast on the q^2 dependence of the structure functions at high energies, while it will be a fair approximation to generating the ρ-, the f-, and other trajectories. On the experimental side, people are now planning to measure individual hadrons produced in electroproduction processes.

We are thus motivated to look at the two-body electroproduction processes (like $e + N \to e + N + \pi$) at high energies with a large "photon mass" in the ladder approximation to the ordinary Regge trajectories. We shall use the parametric representation of the solutions to the Bethe-Salpeter equation with a scalar particle exchange which takes proper

account of the vector property of the photon. In contrast to the virtual Compton scattering with highly virtual photons, we have no chance to make use of current algebra techniques in processes where only the incident is highly virtual. Our approach using the Bethe-Salpeter equation would therefore make much sense. We shall discuss the validity of the present model and make a few comments in a final section.

II. SOLUTION TO BETHE-SALPETER EQUATION

We begin with the Bethe-Salpeter equation of scalar rungs as shown in Fig. 1a,[7]

$$(m^2+k_1^2)(m^2+k_2^2)\, f(k_1^2,\, k_2^2,\, s,\, t) = \frac{1}{\mu^2 - s - i\epsilon}$$

$$+ \frac{\lambda}{\pi^2 i} \int d^4k_1' \, \frac{f(k_1'^2,\, k_2'^2,\, (p_1 + k_1')^2,\, t)}{\mu^2 + (k_1 - k_1')^2 - i\epsilon} \quad , \qquad (1)$$

where

$$p_1^2 = p_2^2 = -m^2 \quad , \quad s = -(p_1 + k_1)^2 \quad , \quad t = -(k_1 - k_2)^2 \quad . \qquad (2)$$

The solution is not obtained explicitly, but it is known to satisfy the following integral representation[8]

$$f(k_1^2,\, k_2^2,\, s,\, t) = \int_0^1 dy \int_{-1}^1 dz \int_{0-}^{\infty} d\gamma\, \phi(y,z,\gamma,t)$$

$$\left\{\gamma + (1-y)\left[\frac{1}{2}(1+z)(k_1^2+m^2) + \frac{1}{2}(1-z)(k_2^2+m^2)\right] - y(s-\mu^2) - i\epsilon\right\}^{-3} \quad , \qquad (3)$$

where the spectral function $\phi(y,z,\gamma,t)$ does <u>not</u> depend on any of s, k_1^2 and k_2^2, but on t. The function $\phi(y,z,\gamma,t)$ satisfies an integral equation

$$\phi(y,z,\gamma,t) = (1-y)\,\delta(\gamma) + \lambda \int_0^1 dy' \int_{-1}^{1} dz' \int_{0-}^{\infty} d\gamma'\, K(y,z,\gamma;\, y',z',\gamma';\, t)\, \phi(y',z',\gamma',t) \quad , \tag{4}$$

$$K(y,z,\gamma;\, y',z',\gamma') = \frac{1}{2}\, y(1-y)\, \theta(y'-y)\, \theta\left(R(z,z') - \frac{y(1-y')}{y'(1-y)}\right) \int_0^1 (1-x)^2\, dx\, \delta'\Big(x(1-x)\, y'\, \gamma - y \Big\{(1-x)\, \gamma' + \left[x + (1-x)^2\, y'\right] \mu^2 + (1-x)^2\, (1-y')^2\, \rho(z)\Big\}\Big) \quad , \tag{5}$$

where $\quad R(z,z') \equiv (1 \mp z)/(1 \mp z') \quad$ for $\quad z \gtrless z' \quad , \qquad (6)$

and $\quad \rho(z) \equiv m^2 - \frac{1}{4}(1 - z^2)\, t \quad . \qquad (7)$

We find from the analytic property of the kernel that:

(i) $\phi(y,z,\gamma,t)$ is singular at $y = 0$ like $\left[(1-z^2)/y\right]^{\alpha(t)+1}$, if a Regge pole exists at $\ell = \alpha(t)$;

(ii) it is regular elsewhere in y and z for $y \in [0,1]$ and $z \in [-1,1]$;

(iii) it falls off rapidly like γ^{-1} as $\gamma \to \infty$.

To construct the electropionproduction amplitude out of $f(k_1^2, k_2^2, s, t)$, we add one more rung, affix a photon line and a pion line to the extreme ends (see Fig. 1b), and supplement the Born term and a possible seagull term.

III. ELECTROPIONPRODUCTION AMPLITUDE

The two invariant matrix elements are defined as

$$M_\mu = M_1(q_1^2, \nu, t)\left(q_{2\mu} - \frac{(q_1 \cdot q_2)}{q_1^2}\, q_{1\mu}\right) + M_2(q_1^2, \nu, t)\left(P_\mu - \frac{(q_1 \cdot P)}{q_1^2}\, q_{1\mu}\right) \quad , \tag{8}$$

where $t = -(q_1 - q_2)^2$, $\nu = -(p_1 \cdot q_1)/m$, and $q_2^{\,2} = -m_\pi^{\,2}$. The ladder diagram for the electropionproduction leads us to

$$M_1(q_1^{\,2},\nu,t) = -\int_0^1 dx \int_0^1 dy \int_{-1}^1 dz \int_{0-}^{\infty} d\gamma \left(\tfrac{1}{2}\right) (1-x)^3 \left(1 - z(1-y)\right) \phi(y,z,\gamma,t)\, (Q - i\epsilon)^{-2} \quad , \tag{9}$$

$$M_2(q_2^{\,2},\nu,t) = \int_0^1 dx \int_0^1 dy \int_{-1}^1 dz \int_{0-}^{\infty} d\gamma\, y(1-x)^3\, \phi(y,z,\gamma,t)\, (Q - i\epsilon)^{-2} \quad , \tag{10}$$

$$\begin{aligned} Q \equiv\ & (1-x)\,\gamma + x\, m^2 + (1-x)^2\, y\, \mu^2 \\ & + (1-x)^2\,(1-y)^2 \left[m^2 - \tfrac{1}{4}\,(1-z^2)\, t\right] - x(1-x)\left[\tfrac{1}{2}\,(1-y)(1-z)(q_1^{\,2} - m^2)\right. \\ & \left. + \tfrac{1}{2}\,(1-y)(1-z)(m_\pi^{\,2} - m^2) + y\,(s - \mu^2)\right] \quad , \end{aligned} \tag{11}$$

where x is a new parameter introduced through Feynman parametrization. The crossed ladder diagram is obtained simply by interchanging p_1 and p_2, flipping the signs of q_1 and q_2, and replacing $i\epsilon$ by $-i\epsilon$.

In accordance with the remark in Ref. 7, these amplitudes may be thought of as electroproduction amplitudes of the nucleon target averaged over spin. Another remark is that because of the approximation we choose, the amplitudes, as they stand, do not satisfy the gauge invariance in general. We have therefore picked up the gauge invariant terms alone.

In the high-energy limit of $\nu \to \infty$ with $q_1^{\,2}$ fixed, M_1 and M_2 exhibit the usual Regge asymptotic behaviors of $\nu^{\alpha(t)}$ and $\nu^{\alpha(t)-1}$, respectively. In fact, by replacing $sy/(1-z)$ by a new variable, we find that the Regge behaviors are factored out of M_1 and M_2. Now we go to the limit of $q^2 \to \infty$ with $2m\nu/q_1^{\,2}$ fixed large. Watching carefully the function Q and keeping in mind the analytic properties of $\phi(y,z,\gamma,t)$

listed in the previous section, we replace the variables $(s/q_1^2)y/(1-z^2)$ and xq_1^2 by new variables η and ξ, respectively. We then find the Regge behaviors in the variable s/q_1^2 or $2m\nu/q_1^2$. Their leading terms are given as

$$M_1 \sim \beta_1(t) \frac{1}{q_1^2} \left(-\frac{2m\nu}{q_1^2}\right)^{\alpha(t)} , \tag{12}$$

$$M_2 \sim \beta_2(t) \frac{1}{q_1^2} \left(-\frac{2m\nu}{q_1^2}\right)^{\alpha(t)-1} , \tag{13}$$

$$\beta_1(t) = -\int_0^\infty d\xi \int_0^\infty d\eta \int_{-1}^{1} dz \int_{0-}^{\infty} d\gamma \left(\frac{1}{2}\right) (1-z(1-y))\, X(z,\gamma,t)\, (R-i\epsilon)^{-2} , \tag{14}$$

$$\beta_2(t) = \int_0^\infty d\xi \int_0^\infty d\eta \int_{-1}^{1} dz \int_{0-}^{\infty} d\gamma\, X(z,\gamma,t)\, (R-i\epsilon)^{-2} , \tag{15}$$

$$R = \gamma + \left(m^2 - \frac{1}{4}(1-z^2)\, t\right) + \xi \left[\frac{1}{2}(1+z) - (1-z)\,\eta\right] , \tag{16}$$

where $X(z,\gamma,t)$ is defined by the relation

$$\phi(y,z,\gamma,t) \sim \left[(1-z^2)/y\right]^{\alpha(t)+1} X(z,\gamma,t) . \tag{17}$$

Adding the crossed ladder gives the correct signatures to M_1 and M_2. Non-leading Regge terms are also obtained in the same way.

The asymptotic behaviors M_1 and M_2 given in Eqs. (12) and (13) are rewritten in terms of the transverse and longitudinal cross sections defined by Hand[9] as

$$\frac{d}{dt}\, \sigma_T(q_1^2,\nu,t) \to \left(\frac{1}{q_1^2}\right)^4 \left(\frac{2m\nu}{q_1^2}\right)^{2\alpha(t)-2} , \tag{18}$$

$$\frac{d}{dt}\, \sigma_L(q_1^2,\nu,t) \to \left(\frac{1}{q_1^2}\right)^3 \left(\frac{2m\nu}{q_1^2}\right)^{2\alpha(t)-2} , \tag{19}$$

as $q_1^2 \to \infty$ with $2m\nu/q_1^2$ fixed large. The trajectory $\alpha(t)$ may be any ordinary trajectory, but not the Pomeranchuk trajectory, for which the whole calculation would be doubtful.

If one connects our amplitudes by unitarity, one would obtain the lower bound of the two structure functions W_1 and W_2 in the inelastic electroproduction. With the usual definitions of W_1 and W_2, we find

$$W_1(q^2,\nu) \gtrsim \left(\frac{1}{q_1^2}\right)^3 \left(\frac{2m\nu}{q_1^2}\right)^{2\alpha(0)-1} \left[\log \frac{2m\nu}{q_1^2}\right]^{-1} , \tag{20}$$

$$W_2(q^2,\nu) \gtrsim \left(\frac{1}{q_1^2}\right)^3 \left(\frac{2m\nu}{q_1^2}\right)^{2\alpha(0)-3} \left[\log \frac{2m\nu}{q_1^2}\right]^{-1} , \tag{21}$$

where the right-hand sides are the lower bounds obtained by unitarizing the electropionproduction amplitudes. Experimentally, the scaling rule seems to hold for W_1 and νW_2 so that our M_1 and M_2 do not contradict with them.

IV. COMMENTS

In the present model we have not taken account of momentum dependences of the vertex function. It may change our conclusion on the q_1^2 dependence by the amount determined by the momentum dependence of the vertex functions. There may be some other reasons to choose constant vertex functions,[10] but we shall present here a new argument in favor of a constant vertex in the case of scalar hadrons. The three-body scalar vertex is defined by a vacuum expectation value as

$$\int d^4x\, d^4y\, e^{ip_a x + ip_b y} \langle 0 | T(\phi_a(x)\, \phi_b(y)\, \phi_c(0)) | 0 \rangle$$

$$= \Delta_a(p_a^2)\, \Delta_b(p_b^2)\, \Delta_c(p_c^2)\; \Gamma(p_a^2, p_b^2, p_c^2) , \tag{22}$$

where $p_a + p_b + p_c = 0$, and $\Delta(p^2)$ is a scalar-meson propagator. To avoid complications, let us assume that $\Gamma(p_a^2, p_b^2, p_c^2)$ is totally symmetric in p_a^2, p_b^2, and p_c^2. The argument goes through in the asymmetric cases too. In the limit of $(p_c)_4 = -(p_a + p_b)_4 \to \infty$ with $\vec{p}_a + \vec{p}_b$ and $(p_a)_\mu$ fixed, we can use Bjorken's technique. The left-hand side of Eq. (22) turns out to be of the fourth power of $(p_{c4})^{-1}$. In fact, the coefficient of $(p_{c4})^{-1}$ vanishes because of canonical commutation relations, the coefficient of $(p_{c4})^{-2}$ vanishes on the assumption that $\langle 0 \,|\phi(x)|\, 0\rangle = 0$ for any field describing a physical particle, and the coefficient of $(p_{c4})^{-3}$ drops also as a consequence of equations of motion in which the source functions do not involve a derivative of ϕ. Therefore, comparing both sides of Eq. (22), we deduce

$$\Gamma(p_a^2, p_b^2, p_c^2) \to \text{const.} \tag{23}$$

as $p_c^2 \to \infty$ with p_a^2 fixed. Repeating the same argument in different limits, we find that $\Gamma(p_a^2, p_b^2, p_c^2)$ behaves like a constant in all limits. This implies that a constant vertex is a good approximation as far as the asymptotic behaviors are concerned. The q^2 dependence in the low q^2 region will be naturally unreliable. The argument given above indicates that multiplying damping form factors at each vertex may be quite misleading. The electromagnetic form factor would not fall off so rapidly as $(q^2)^{-2}$ when one or two of the hadron lines attaching the photon line are off the mass shell. In more general cases where hadrons have spins and derivatives of boson fields enter in the interaction Lagrangian, the argument may not go through exactly in the same way. We have presented this argument to make less unreasonable the assumption of constant vertex functions.

The other comment is in connection with finite energy sum rules. We have found that the contributions from ordinary trajectories do not fall off so rapidly as the electromagnetic form factor as $q_1^2 \to \infty$. It was argued by Harari[11] in the case of the virtual Compton scattering that the contributions from ordinary trajectories must fall off rapidly to be consistent with finite energy sum rules. The idea was as follows: The finite energy sum rules connect the low-lying resonance contribution in the s- (and u-) channel to the Regge contributions in the t-channel. Since the electromagnetic form factors of transitions between low-lying resonances are expected to fall off rapidly, the Regge residue in the t-channel also must fall off in conformity with them. However, we are able to avoid this difficulty in the following way. When we write a finite energy sum rule with the upper bound of the integral being ν_{cut}, we choose $2m\nu_{cut}/q_1^2$ to be a large finite number, say L, to assure the dominance of a few leading Regge trajectories. For a small value of q_1^2, $\nu_{cut} = q_1^2 L/2m$ is not very large. But, as q_1^2 increases, we must choose larger and larger values of ν_{cut} in order to maintain the same accuracy for the finite energy sum rule. Since $s \sim 2m\nu$, higher resonances contribute to the sum rules more and more as ν_{cut} increases. The transition form factor from the nucleon to higher resonances may be well expected to damp mildly enough to be consistent with our predictions. In the other way around, if one increases q^2 alone with ν_{cut} fixed in the sum rules, exchanges of lower Regge trajectories become more and more important so that the q^2 dependence of a single Regge trajectory does not describe that of the whole amplitude.

In conclusion, we feel that our results of the asymptotic behaviors of electroproduction amplitudes may hold in more realistic cases although they are derived rather in a simplified model. They will be tested by experiment quite soon.

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10. In the foregoing Refs. 3, 5, and 6, the derivation of the scaling rule is always based on a constant vertex function.
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FIGURE CAPTIONS

Fig. 1a: Ladder diagram of Feynman amplitude for hadron-hadron scattering.

Fig. 1b: Electropionproduction process.

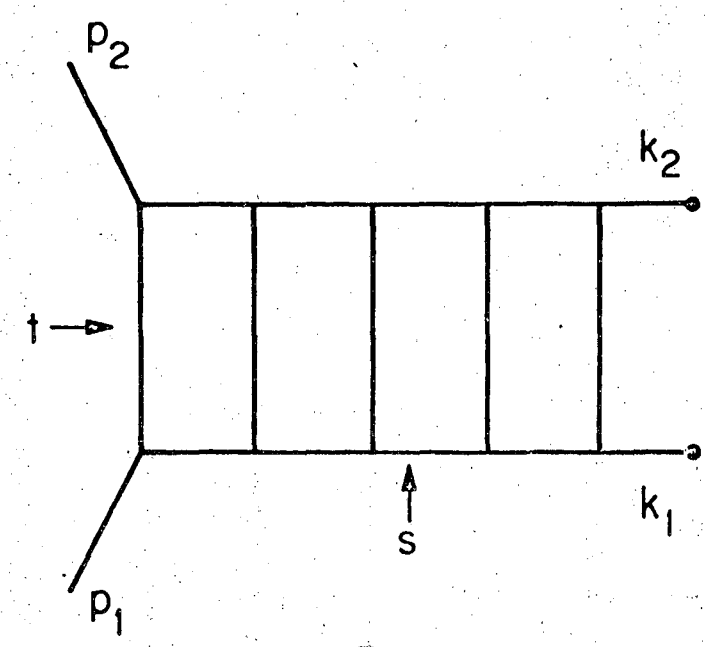

Fig. 1a

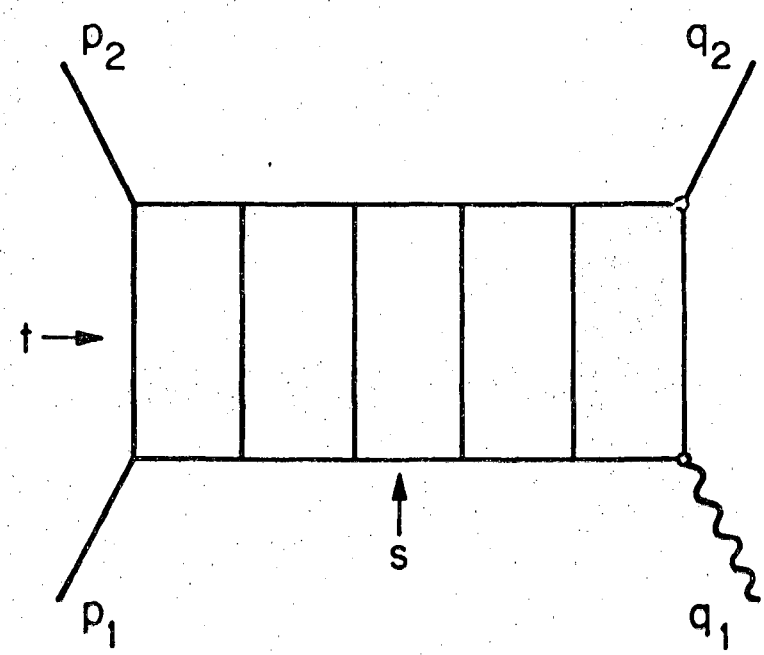

Fig. 1b

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