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SINGLE-PARTICLE RESONANCES IN THE RENORMALIZED RPA-TREATMENT OF NUCLEON-NUCLEUS SCATTERING*

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ABSTRACT

The method of Garside and MacDonald is used to treat the single-particle resonances in the RPA-treatment of scattering. The resonances are shifted by an additional potential to single-particle bound states, which can be handled in the usual manner. The coupling to the continuum is done analytically using separable matrix elements.

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I. INTRODUCTION

In a previous work[1] we have applied the general theory of nucleon-nucleus scattering[2] to a model, in which the so called bound-state problem or nuclear-structure problem has been treated with the full renormalized effective particle-hole force and the influence of the continuum has been taken approximately into account by a schematic force. In the bound-state problem one neglects all single-particle states belonging to the continuum. In our model the amplitudes of the bound-state problem are expected to be already a good approximation for the corresponding amplitudes with coupling to the continuum. But it is well known--for instance from RPA-calculations--that in many cases one has to include into the bound-state problem certain single-particle resonances in order to obtain a good agreement with the experimental situation.[3,4] An example is for instance the $1d_{3/2}$-resonance in ^{16}O necessary for the calculation of the excited negative parity-states of ^{16}O. Several methods are known to overcome this difficulty.[4] We are going to use the method of Garside and MacDonald[5] since we can obtain with it an explicit solution for the nucleon-nucleus scattering amplitude. In this approach the single-particle potential is split off in two parts

$$\tilde{v}_{\mu\nu} = \tilde{v}^{B}_{\mu\nu} + \Delta_{\mu\nu} \quad , \tag{I.1}$$

in such a way, that the single-particle resonances are shifted into bound-states if one uses $\tilde{v}^{B}_{\mu\nu}$ instead of $\tilde{v}_{\mu\nu}$.

In order to obtain the scattering amplitude and the (complex) energy-eigenvalues one had to solve a coupled system of equations for certain quasi-particle density matrix elements given in Ref. 1 (A; I.12; A; I.15-I.18).[††]

[††] In this paper we are using the same notations and definitions as in Ref. 1. We refer to Ref. 1 as A, so that (A;I.1) means Eq. (I.1) of Ref. 1.

If we use a single-particle representation according to the potential $\tilde{v}^B_{\mu\nu}$ instead of $\tilde{v}_{\mu\nu}$ one has to insert the following replacements:[5]

$$\varepsilon_\mu \tilde{\rho}_{\mu\nu,M} \to \varepsilon_\mu \tilde{\rho}_{\mu\nu} + \sum_\lambda \Delta_{\mu\nu} \{(1-n_\mu)(1-n_\lambda) + n_\mu n_\lambda\} \tilde{\rho}_{\lambda\nu,M} \quad , \tag{I.2}$$

$$\varepsilon_\nu \tilde{\rho}_{\mu\nu,M} \to \varepsilon_\nu \tilde{\rho}_{\mu\nu} + \sum_\lambda \Delta_{\lambda\nu} \{(1-n_\nu)(1-n_\lambda) + n_\nu n_\lambda\} \tilde{\rho}_{\mu\lambda,M} \quad , \tag{I.3}$$

where the quantities on the right-hand side now refer to the new basis defined by $\tilde{v}^B_{\mu\nu}$. In all practical cases the additional potential Δ is small compared with $\tilde{v}^B$.

In order to simplify the treatment, we will restrict ourselves first to the case of a resonance in one single-particle channel only (no extra bound state). The generalization to the case of several channels is straightforward and will will be given afterwards. Since we have the freedom of choosing a suitable analytical form for Δ we use a separable ansatz for Δ which allows us to read off easily the additional scattering amplitude[6] of the single-particle scattering in terms of the wavefunctions defined by $\tilde{v}^B$. For the separable potential

$$\Delta_{\mu\nu} = \gamma u_\mu u_\nu \quad , \tag{I.4}$$

one obtains for the scattering amplitude:

$$\tilde{t}_{\mu\nu}(\omega) = \ell^{i(\delta_\mu + \delta_\nu)} \frac{\gamma u_\mu u_\nu}{1 - \gamma \sum_\alpha \frac{u_\alpha^2}{\omega - \varepsilon_\alpha + i\eta}} \quad . \tag{I.5}$$

According to our assumption the additional potential can be assumed to be non zero only in a certain channel--for instance the $d_{3/2}$-states in ^{16}O. The explicit expression (I.5) can be used to fit the position and width of the experimental single-particle resonance.

II. ONE-CHANNEL CASE:

We label the corresponding bound state shifted from the continuum by ξ. Inserting the replacements (I.2) and (I.3) in (A; I.12) and (A; I.15) we obtain instead of these equations the following equations for the particle-hole amplitudes (p and j characterize the initial state; see A; I.1):

$$\tilde{\rho}_{\nu\mu,M} = \frac{(-i\eta)(n_\nu - n_\mu)\delta_{\nu p}\delta_{\mu j}}{E_M + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)}$$

$$+ \frac{1}{E_M + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)} \{(n_\mu - n_\nu)2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta}\tilde{\rho}_{\beta\alpha,M}$$

$$- \gamma \sum_\alpha u_\alpha(1-n_\alpha)[u_\mu(1-n_\mu)\tilde{\rho}_{\nu\alpha,M} - u_\nu(1-n_\nu)\tilde{\rho}_{\alpha\mu,M}]\} \quad . \qquad \text{(II.1)}$$

Here, $|M\rangle$ denotes the excited states of the system with inclusion of the single-particle continuum. Since we developed the general theory[2] by extending Migdal's approach[7] we have to use as in Ref. 1 renormalized quantities labelled by a tilde.

According to Ref. 1 the solutions of the bound-state problem $|n\rangle = C_n^\dagger|0'\rangle$ are given by the solutions of (II.1) where all matrix elements labelled by at least one quantum number set belonging to the continuum are set equal to zero. But instead of the energy ε_μ we are using in the bound-state problem new energies

defined by $\varepsilon_\mu + \delta_{\mu\xi}\gamma u_\xi^2$. With $|0'\rangle$ we denote the ground-state of the compound-system containing correlations of bound single-particle states only.

Repeating the procedure of Ref. 1 one obtains for the "deviation" matrix elements instead of (A; II.10; A; II.11) the following expressions

$$\langle 0|C_n^\dagger|M\rangle = \frac{1}{E_n+E_M}\{-\lambda F_n^\circ F_M - \gamma\, u_\xi \sum_i [\tilde{\rho}_{\xi i,n}\hat{R}_{M,i} + \tilde{\rho}_{i\xi,n}R_{M,i}]\} \quad , \tag{II.2}$$

$$\langle 0|C_n|M\rangle = \frac{1}{E_n-E_M}\{-\lambda F_n^{\circ *} F_M - \gamma\, u_\xi \sum_i [\tilde{\rho}^*_{i\xi,n}\hat{R}_{M,i} + \tilde{\rho}^*_{\xi i,n}R_{M,i}]\} \tag{II.3}$$

with

$$R_{M,i} := \sum_k{}'' u_k \tilde{\rho}_{ki,M} \quad , \tag{II.4}$$

$$\hat{R}_{M,i} := \sum_k{}'' u_k \tilde{\rho}_{ik,M} \quad . \tag{II.5}$$

F_n° and F_M are defined by (A; II.12) and (A; II.13).

The expressions for the amplitudes $\tilde{\rho}_{ki,B}$ and $\tilde{\rho}_{ik,B}$ (Bound states $|M\rangle$ are denoted by $|B\rangle$) are given by ($\nu\mu$ is here restricted to k,i or i,k):

$$\tilde{\rho}_{\nu\mu,B} = \frac{1}{E_B + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)}\{F_B\, M_{\nu\mu}(E_B)$$

$$+ \sum_i [A^i_{\nu\mu}(E_B)\hat{R}_{B,i} + B^i_{\nu\mu}(E_B)R_{B,i}]\} \quad , \tag{II.6}$$

with:

$$M_{\nu\mu}(E) := (n_\mu - n_\nu)\ \lambda\ w_{\nu\mu}\ [1 - \lambda \sum_n |F_n^o|^2\ \frac{2E_n}{E_n^2 - E^2}\ ]$$

$$+ \lambda\ \gamma\ u_\xi \sum_n \left(\frac{F_n^{o*} D_n^{\nu\mu}}{E_n - E} - \frac{F_n^o\ D_n^{\mu\nu*}}{E_n + E} \right), \qquad \text{(II.7)}$$

$$A_{\nu\mu}^i(E) := \gamma\ \{-u_\mu\ \delta_{i\nu} n_\nu (1-n_\mu)$$

$$- u_\xi \sum_n \left(\frac{\tilde{\rho}_{\xi i,n}}{E+E_n}\ [(n_\mu - n_\nu)\ \lambda\ w_{\nu\mu} F_n^{o*} + \gamma\ u_\xi\ D_n^{\mu\nu*}]\}\right.$$

$$\left. + \frac{\tilde{\rho}^*_{i\xi,n}}{E_n - E}\ [(n_\mu - n_\nu)\ \lambda\ w_{\nu\mu} F_n^o - \gamma\ u_\xi\ D_n^{\nu\mu}] \right) \} \qquad \text{(II.8)}$$

$$B_{\nu\mu}^i(E) := \gamma\ \{u_\nu\ \delta_{i\mu} n_\mu (1-n_\nu)$$

$$- u_\xi \sum_n \left(\frac{\tilde{\rho}_{i\xi,n}}{E+E_n}\ [(n_\mu - n_\nu)\ \lambda\ w_{\nu\mu} F_n^{o*} + \gamma\ u_\xi\ D_n^{\mu\nu*}]\right.$$

$$\left. + \frac{\tilde{\rho}^*_{\xi i,n}}{E_n - E}\ [(n_\mu - n_\nu)\ \lambda\ w_{\nu\mu} F_n^o - \gamma\ u_\xi\ D_n^{\nu\mu}] \right) \} \qquad \text{(II.9)}$$

$$D_n^{\nu\mu} := u_\mu (1-n_\mu) n_\nu \tilde{\rho}_{\nu\xi,n} - u_\nu (1-n_\nu) n_\mu \tilde{\rho}_{\xi\mu,n}\ . \qquad \text{(II.10)}$$

Eq. (II.6) is obtainable in the same manner as the expressions (A; II.14; II.15). In the derivation we expressed $\tilde{\rho}_{i\xi,B}$ and $\tilde{\rho}_{\xi i,B}$ in terms of $\langle 0|C_n^{\dagger}|B\rangle$ and $\langle 0|C_n|B\rangle$ with the help of Eq. (A; II.1). Use of (II.4), (II.5) and (II.6) gives the following system of equations for $R_{B,i}$ and $\hat{R}_{B,i}$:

$$R_{B,i} = F_B\, M_i\,(E_B) + \sum_{j'} (A_{ij'}\,(E_B)\,\hat{R}_{B,j'} + B_{ij'}\,(E_B)\,R_{B,j'}) \quad , \tag{II.11}$$

$$\hat{R}_{B,i} = F_B\, \hat{M}_i\,(E_B) + \sum_{j'} (\hat{A}_{ij'}\,(E_B)\,\hat{R}_{B,j'} + \hat{B}_{ij'}\,(E_B)\,R_{B,j'}) \quad , \tag{II.12}$$

with

$$M_i(E) := \sum_k{}'' \frac{u_k}{E + \varepsilon_i - \varepsilon_k + i\eta}\, M_{ki}(E) \quad , \tag{II.13}$$

$$A_{ij'}(E) := \sum_k{}'' \frac{u_k}{E + \varepsilon_i - \varepsilon_k + i\eta}\, A_{ki}^{j'}(E) \text{ etc.} \tag{II.14}$$

Since the system (II.11) and (II.12) represents a linear system of equations of the order 2N (N is the number of hole states) the explicit solution for $R_{B,i}$ and $\hat{R}_{B,i}$ is obtainable in terms of the solutions of the bound-state problem and the potential parameters alone. According to the structure of the equation-system the solutions have the following form:

$$R_{B,i}(E) = F_B\, r_{B,i}(E) \quad , \tag{II.15}$$

$$\hat{R}_{B,i}(E) = F_B\, \hat{r}_{B,i}(E) \quad , \tag{II.16}$$

where $r_{B,i}$ and $\hat{r}_{B,i}$ are known functions, after one has performed the integrations in (II.13) etc. Hence, (II.6) has the following solution:

$$\tilde{\rho}_{\nu\mu,B} = \frac{F_B}{E_B + \varepsilon_\mu - \varepsilon_\nu} K^B_{\nu\mu}(E_B) \quad , \tag{II.17}$$

with:

$$K^B_{\nu\mu}(E) := M_{\nu\mu}(E) + \sum_i \{A^i_{\nu\mu}(E) \ \hat{r}_{B,i}(E) \ \hat{r}_{B,i}(E) + B^i_{\nu\mu}(E) \ r_{B,i}(E)\} \quad . \tag{II.18}$$

The (complex) energy-eigenvalues E_B are given by the solutions of the dispersion relation, which we get by inserting (II.17) in the definition (A; II.13) of F_B. We obtain the following equation:

$$1 = \sum_k{}'' \sum_i w_{ki} \left\{ \frac{K^B_{ki}(E_B)}{E_B + \varepsilon_i - \varepsilon_k + i\eta} + \frac{K^B_{ik}(E_B)}{E_B + \varepsilon_k - \varepsilon_i} \right\} \quad . \tag{II.19}$$

Finally, the constant F_B can be determined by the normalization of the states (see Ref. 1).

The amplitudes $\tilde{\rho}_{ki,S}$ and $\tilde{\rho}_{ik,S}$ for the scattering states can be derived in the same manner (Scattering states $|M\rangle$ are denoted by $|S\rangle$). Instead of (II.6), (II.11) and (II.12) we obtain:

$$\tilde{\rho}_{\nu\mu,S} = \delta_{\nu p} \, \delta_{\mu j} + \frac{1}{E_S + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)} \{F_S \, M_{\nu\mu}(E_S)$$

$$+ \sum_i (A^i_{\nu\mu}(E_S) \ \hat{R}_{S,i} + B^i_{\nu\mu} \ R_{S,i})\} \quad , \tag{II.20}$$

$$R_{S,i} = u_p \, \delta_{ij} + F_S \, M_i \, (E_S) + \sum_{j'} \left(A_{ij'}(E_S) \, \hat{R}_{S,j'} + B_{ij'}(E_S) \, R_{S,j'} \right) \quad , \qquad \text{(II.21)}$$

$$\hat{R}_{S,i} = F_S \, \hat{M}_i \, (E_S) + \sum_{j'} \left(\hat{A}_{ij'}(E_S) \, \hat{R}_{S,j'} + \hat{B}_{ij'}(E_S) \, R_{S,j'} \right) \quad . \qquad \text{(II.22)}$$

The structure of (II.21) and (II.22) changes the solutions (II.15), (II.16) to the following form:

$$R_{S,i}(E) = F_S \, r_{S,i}(E) + u_p \, r^j_{S,i}(E) \quad , \qquad \text{(II.23)}$$

$$\hat{R}_{S,i}(E) = F_S \, \hat{r}_{S,i}(E) + u_p \, \hat{r}^j_{S,i}(E) \quad . \qquad \text{(II.24)}$$

Inserting this result in (II.20) gives the wanted amplitude $\tilde{\rho}_{ki,S}$:

$$\tilde{\rho}_{ki,S} = \delta_{kp} \, \delta_{ij} + \frac{1}{E_S + \varepsilon_i - \varepsilon_k + i\eta} \left[F_S \, K^S_{ki}(E_S) + u_p \, K^j_{ki}(E_S) \right] \quad , \qquad \text{(II.25)}$$

with:

$$K^j_{\nu\mu}(E) := \sum_i \left(\hat{r}^j_{S,i}(E) \, A^i_{\nu\mu}(E) + r^j_{S,i}(E) \, B^i_{\nu\mu}(E) \right) \quad . \qquad \text{(II.26)}$$

We get F_S by inserting (II.25) into the definition (A; II.13) of F_S:

$$F_S = \left\{ 1 - \sum_{k,i} w_{ki} \left(\frac{K^S_{ki}(E_S)}{E_S + \varepsilon_i - \varepsilon_k + i\eta} + \frac{K^S_{ik}(E_S)}{E_S + \varepsilon_k - \varepsilon_i} \right) \right\}^{-1} *$$

$$\left\{ w_{pj} + u_p \sum_{ki} w_{ki} \left(\frac{K^j_{ki}(E_S)}{E_S + \varepsilon_i - \varepsilon_k + i\eta} + \frac{K^j_{ik}(E_S)}{E_S + \varepsilon_k - \varepsilon_i} \right) \right\} . \qquad \text{(II.27)}$$

From the expressions for $\hat{R}_{M,i}$ and $R_{M,i}$ we can now also obtain the explicit form for the "deviation" matrix elements:

$$\langle 0|C_n^{\dagger}|M\rangle = \frac{1}{E_n + E_M} \{F_M[-\lambda F_n^o - \gamma\, u_\xi \sum_i (\tilde{\rho}_{\xi i,n}\, \hat{r}_{M,i}(E_M) + \tilde{\rho}_{i\xi,n}\, r_{M,i}(E_M))]$$

$$- \delta_{M,S}\, \gamma\, u_\xi\, u_p \sum_i (\tilde{\rho}_{\xi i,n}\, \hat{r}^j_{S,i}(E_S) + \tilde{\rho}_{i\xi,n}\, r^j_{S,i}(E_S))\} \qquad \text{(II.28)}$$

$$\langle 0|C_n|M\rangle = \frac{1}{E_n - E_M} \{F_M[-\lambda F_n^{o*} - \gamma\, u_\xi \sum_i (\tilde{\rho}^*_{i\xi,n}\, \hat{r}_{M,i}(E_M) + \tilde{\rho}^*_{\xi i,n}\, r_{M,i}(E_M))]$$

$$- \delta_{M,S}\, \gamma\, u_\xi\, u_p \sum_i (\tilde{\rho}^*_{i\xi,n}\, \hat{r}^j_{S,i}(E_S) + \tilde{\rho}^*_{\xi i,n}\, r^j_{S,i}(E_S))\} \quad . \qquad \text{(II.29)}$$

Finally, we can read off from the amplitude $\tilde{\rho}_{ki,S}$ the wanted extra-scattering amplitude, reinserting the single-particle phase factors taken out in Eq. (A; I.8):

$$T_{ki,pj}(E) = e^{i(\delta_p + \delta_k)} \{u_p\, K^j_{ki}(E_S) + F_S\, K^S_{ki}(E_S)\} \quad . \qquad \text{(II.30)}$$

Our formulas reduce to the corresponding ones of Ref. 1 for $\gamma \to 0$; if we neglect any particle-hole force we obtain the single-particle resonance scattering.

III. MANY-CHANNEL-CASE:

If one has a single resonance in different channels, respectively, one can treat the problem replacing (I.4) by

$$\Delta_{\mu\nu} = \sum_{\bar{\xi}} \Delta^{\bar{\xi}}_{\mu\nu} = \sum_{\bar{\xi}} \gamma^{\bar{\xi}}\, u^{\bar{\xi}}_\mu\, u^{\bar{\xi}}_\nu \quad . \qquad \text{(III.1)}$$

We make the assumption that in every relevant channel we have only one resonance and no bound states. This assumption is sufficient for light and medium nuclei. Then we can label the channels by $\bar{\xi}$ which is a subset of the quantum-number set ξ. For instance - $\bar{\xi}$ may be the $d_{3/2}$-channel whereas ξ includes also the principal quantum number and the magnetic quantum number of the $d_{3/2}$-bound state. Since $\gamma^{\bar{\xi}}$ and $u_{\mu}^{\bar{\xi}}$ are assumed to be non zero only in the channel $\bar{\xi}$ one can fit the resonance in each channel separately.

The changes against the one-channel case are obvious since one has only to replace the contribution of one channel in the formulas of Sec. II by the sum of the channel contributions. For the "deviation" matrix element (II.3)--for instance--we obtain now:

$$\langle 0|C_n|M\rangle = \frac{1}{E_n - E_M} \{-\lambda F_n^{o*} F_M - \sum_{\bar{\xi}} \gamma^{\bar{\xi}} u_{\xi}^{\bar{\xi}} \sum_i [\tilde{\rho}^*_{i\xi,n} \hat{R}^{\bar{\xi}}_{M,i} + \tilde{\rho}^*_{\xi i,n} R_{M,i}]\} , \quad \text{(III.2)}$$

with:

$$R^{\bar{\xi}}_{M,i} := \sum_k{}'' u_k^{\bar{\xi}} \tilde{\rho}_{ki,M} \text{ etc.} \quad \text{(III.3)}$$

For (II.6) we get:

$$\tilde{\rho}_{\nu\mu,B} = \frac{1}{E_B + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)} \{F_B M_{\nu\mu}(E_B)$$

$$+ \sum_{\bar{\xi}} \sum_i [A^{i,\bar{\xi}}_{\nu\mu} \hat{R}^{\bar{\xi}}_{B,i} + B^{i,\bar{\xi}}_{\nu\mu} R^{\bar{\xi}}_{B,i}]\} , \quad \text{(III.4)}$$

with:

$$M_{\nu\mu}(E) = (n_\mu - n_\nu)\ \lambda\ w_{\nu\mu}\ [1-\lambda \sum_n |F_n^o|^2 \frac{2E_n}{E_n^2 - E^2}]$$

$$+ \lambda \sum_{\bar{\xi}} \gamma^{\bar{\xi}} u_{\xi}^{\bar{\xi}} \sum_n \left(\frac{F_n^{o*} D_n^{\nu\mu,\bar{\xi}}}{E_n - E} - \frac{F_n^o D^{\mu\nu,\bar{\xi}*}}{E_n + E} \right) \quad . \qquad \text{(III.5)}$$

The definitions of $A_{\nu\mu}^{i,\bar{\xi}}$, $B_{\nu\mu}^{i,\bar{\xi}}$ and $D_n^{\nu\mu,\bar{\xi}}$ can be taken over from Sec. II by replacing $u \to u^{\bar{\xi}}$ and $\gamma \to \gamma^{\bar{\xi}}$. Instead of (II.12) and (II.13) we obtain now; respectively:

$$R_{B,i}^{\bar{\xi}} = F_B\ M_i^{\bar{\xi}}\ (E_B) + \sum_{\bar{\xi}'} \sum_{j'} (A_{ij'}^{\bar{\xi}\bar{\xi}'}\ (E_B)\ \hat{R}_{B,j}^{\bar{\xi}'} + B_{ij'}^{\bar{\xi}\bar{\xi}'}\ (E_B)\ R_{B,j}^{\bar{\xi}'}) \quad , \qquad \text{(III.6)}$$

$$\hat{R}_{B,i}^{\bar{\xi}} = F_B\ \hat{M}_i^{\bar{\xi}}\ (E_B) + \sum_{\bar{\xi}'} \sum_{j'} (A_{ij'}^{\bar{\xi}\bar{\xi}'}\ (E_B)\ \hat{R}_{B,j}^{\bar{\xi}\bar{\xi}'} + \hat{B}_{ij}^{\bar{\xi}\bar{\xi}'}(E_B)\ R_{B,j}^{\bar{\xi}'}) \quad . \qquad \text{(III.7)}$$

with:

$$M_i^{\bar{\xi}}\ (E) := \sum_k{}'' \frac{u_k^{\bar{\xi}}}{E + \varepsilon_i - \varepsilon_k + i\eta}\ M_{ki}\ (E) \qquad \text{(III.8)}$$

$$A_{ij'}^{\bar{\xi}\bar{\xi}'}\ (E) := \sum_k{}'' \frac{u_k^{\bar{\xi}}}{E + \varepsilon_i - \varepsilon_k + i\eta}\ A_{ki}^{j'\ \bar{\xi}'}\ (E) \text{ etc.} \qquad \text{(III.9)}$$

The only difference from the one-channel case is that we have to deal with a linear system of larger order. The order is now $2(N \times M)$ if we have M resonances.

But the structure of the system remains the same, therefore, the form of the solution does not change. We obtain as in (II.15) and (II.16):

$$R^{\bar{\xi}}_{B,i}(E) = F_B\, r^{\bar{\xi}}_{B,i}(E) \quad , \tag{III.10}$$

$$\hat{R}^{\bar{\xi}}_{B,i}(E) = F_B\, r^{\bar{\xi}}_{B,i}(E) \quad . \tag{III.11}$$

For the scattering states one can follow the same road. According to the procedure we have now only to add in the formulas of Sec. II to each summation over the hole-states a summation over the channels. Hence, we obtain instead of (II.18) and (II.26); respectively:

$$K^{M}_{\nu\mu}(E) := M_{\nu\mu}(E) + \sum_{\bar{\xi}} \sum_{i} \{A^{i,\bar{\xi}}_{\nu\mu}(E)\, \hat{r}^{\bar{\xi}}_{M,i}(E) + B^{i,\bar{\xi}}_{\nu\mu}(E)\, r^{\bar{\xi}}_{M,i}(E)\} \tag{III.12}$$

$$K^{j\bar{p}}_{\nu\mu}(E) := \sum_{\bar{\xi}} \sum_{i} (\hat{r}^{\bar{\xi},j\bar{p}}_{S,i}(E)\, A^{i\bar{\xi}}_{\nu\mu}(E) + r^{\bar{\xi},j\bar{p}}_{S,i}(E)\, B^{i\bar{\xi}}_{\nu\mu}) \quad . \tag{III.13}$$

For F_S we get:

$$F_S = \left\{ 1 - \sum_{k,i} w_{ki} \left(\frac{K^{S}_{ki}(E_S)}{E_S + \varepsilon_i - \varepsilon_k + i\eta} + \frac{K^{S}_{ik}(E_S)}{E_S + \varepsilon_k - \varepsilon_i} \right) \right\}^{-1}$$

$$* \left\{ w_{pj} + u^{\bar{p}}_{p} \sum_{ki} w_{ki} \left(\frac{K^{j\bar{p}}_{ki}(E_S)}{E_S + \varepsilon_i - \varepsilon_k + i} + \frac{K^{j\bar{p}}_{ik}(E_S)}{E_S + \varepsilon_k - \varepsilon_i} \right) \right\} \quad . \tag{III.14}$$

In (III.13) and (III.14) $\bar{p}$ denotes the channel of the incoming particle in the state p. If there is no resonance--necessary in the bound-state problem--in this channel one has to set $u_p^{\bar{p}}$ equal to zero. Now we can easily obtain the final expressions for the "deviation" matrix elements, the particle-hole amplitude $\tilde{\rho}_{ki,S}$ and the scattering amplitude in the multi-channel case:

$$\langle 0|C_n^\dagger|M\rangle = \frac{1}{E_n + E_M} \{F_M[-\lambda F_n^o - \sum_{\bar{\xi}} \gamma^{\bar{\xi}} u_{\xi}^{\bar{\xi}} \sum_i (\tilde{\rho}_{\xi i,n} \hat{r}_{M,i}^{\bar{\xi}}(E_M) + \tilde{\rho}_{i\xi,n} r_{M,i}^{\bar{\xi}}(E_M))]$$

$$- \delta_{M,S} \sum_{\bar{\xi}} \gamma^{\xi} u_{\xi}^{\bar{\xi}} u_p^{\bar{p}} \sum_i (\tilde{\rho}_{\xi i,n} \hat{r}_{S,i}^{\bar{\xi},j\bar{p}}(E_S) + \tilde{\rho}_{i\xi,n} r_{S,i}^{\bar{\xi},j\bar{p}}(E_S))\} , \quad \text{(III.15)}$$

$$\langle 0|C_n|M\rangle = \frac{1}{E_n - E_M} \{F_M[-\lambda F_n^{o*} - \sum_{\bar{\xi}} \gamma^{\bar{\xi}} u_{\xi}^{\bar{\xi}} \sum_i (\tilde{\rho}^*_{i\xi,n} \hat{r}_{M,i}^{\bar{\xi}}(E_M) + \tilde{\rho}^*_{\xi i,n} r_{M,i}^{\bar{\xi}}(E_M))]$$

$$- \delta_{M,S} \sum_{\bar{\xi}} \gamma^{\bar{\xi}} u_{\xi}^{\bar{\xi}} u_p^{\bar{p}} \sum_i (\tilde{\rho}^*_{i\xi,n} \hat{r}_{S,i}^{\bar{\xi},j\bar{p}}(E_S) + \tilde{\rho}^*_{\xi i,n} r_{S,i}^{\bar{\xi},j\bar{p}}(E_S))\} , \quad \text{(III.16)}$$

$$\tilde{\rho}_{ki,S} = \delta_{kp}\, \delta_{ij} + \frac{1}{E_S + \varepsilon_i - \varepsilon_k + i\eta} [F_S K_{ki}(E_S) + u_p^{\bar{p}} K_{ki}^{j\bar{p}}(E\ )] , \quad \text{(III.17)}$$

$$T_{ki,pj}(E) = e^{i(\delta_p + \delta_k)} \{F_S K_{ki}^S(E_S) + u_p^{\bar{p}} K_{ki}^{j\bar{p}}(E_S)\} . \quad \text{(III.18)}$$

IV. SUMMARY

A model for calculating nucleon scattering by one-hole nuclei using linear response theory has been set down. We used a separable particle-hole force for the treatment of the single-particle continuum starting from the solutions of the nuclear-structure problem with the full renormalized particle-hole interaction. The necessary inclusion of certain single-particle resonant states in the treatment of the nuclear structure problem has been achieved by the method of Garside and MacDonald. The additional single-particle potential was chosen in a separable form, too. We have been able to give the final results in terms of the solutions of the nuclear-structure problem and the potentials in a transparent manner. In order to obtain a full explicit result one has only to perform integrations and to solve linear systems of equations of finite dimension. Due to our used ansatzes for the treatment of the continuum we have avoided the original much more complicated Fredholm-problem.

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REFERENCES

1. M. Weigel, Subm. to Phys. Rev., Lawrence Radiation Laboratory Report UCRL-18994.
2. M. Weigel, Nucl. Phys. A137, 629 (1969); Lawrence Radiation Laboratory Report UCRL-18907.
3. V. Gillet, "Proceedings of the International School of Physics" Enrico Fermi, Course XXXVI (1966), (Academic Press, N. Y., 1966).
4. C. Mahaux and H. A. Weidenmüller, "Shell-Model Approach to Nuclear Reactions," (Wiley, N. Y., 1969).
5. L. Garside and W. M. MacDonald, Phys. Rev. 138, B582 (1965); W. P. Beres and W. M. MacDonald, Nucl. Phys. A91, 529 (1967).
6. K. Kikuchi and M. Kawai, "Nuclear Matter and Nuclear Reactions," (Wiley, N. Y., 1968) p. 181.
7. A. B. Migdal, "Theory of Finite Fermi Systems and Applications to Atomic Nuclei," (Interscience Publ., N. Y., 1967).

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