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MULTIFACTORIZATIONS AND THE FOUR-REGGEON VERTEX FUNCTION IN THE DUAL RESONANCE MODELS. I.

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I. INTRODUCTION

Koba and Nielsen[1] have shown that the N-point dual amplitude can be represented in a projectively invariant manner by associating with each external particle one Koba-Nielsen variable x_i, $i=1,2\cdots N$ (Fig. 1). The N-point amplitude takes the form[2]

$$B_N(p_1,p_2,\cdots p_N) = \int \prod_{\substack{i\neq a\\ b\\ c}} dx_i \frac{1}{\left|\prod_{\substack{i\neq a\\ a-2}} (x_i - x_{i+2})\right|} \prod_{1\leq i<j}^{N} u_{ij}^{-\alpha_{ij}-1} , \tag{1}$$

where $x_a = \infty$, $x_b = 0$, $x_c = 1$, and

$$u_{ij} \equiv (x_i,x_{i-1},x_j,x_{j+1}) \equiv \frac{(x_i - x_j)(x_{i-1} - x_{j+1})}{(x_i - x_{j+1})(x_{i-1} - x_j)} . \tag{2}$$

The usual multiperipheral parameterization of the n-point function

$$B_N(p_1,\cdots p_N) = \int_0^1 \prod_{i=2}^{N-2} du_{1i} \prod_{i=2}^{N-2} u_{1i}^{-\alpha_{1i}-1} \prod_{i=2}^{N-1} (1 - u_{1i})^{a-1} \cdot \prod_{2=i\leq j\leq N-2} (1 - u_{1i}u_{1i+1}\cdots u_{1j})^{-2bp_i\cdot p_{j+1}} \tag{3}$$

is then in the particular projective "frame" $x_1 = \infty$, $x_2 = 1$, $x_N = 0$. Another useful form can be obtained[2] by choosing $x_1 = \infty$, $x_b = 0$, $x_c = 1$, thus the N-point function has the form

$$B_N(p_1,\cdots p_N) = \int \prod_{\substack{i\neq 1\\ b\\ c}} dx_i \left\{ \prod_{2\leq i<j}^{N} (x_i - x_j)^{-2bp_ip_j} \cdot \right.$$

$$\cdot \prod_{2\leq i}^{N-1} \left[\theta(x_i - x_{i+1})(x_i - x_{i+1})^{-a-b(p_i^2+p_{i-1}^2)-1} \right] \cdot$$

$$\left. \cdot \prod_{\substack{i=2\\ i\neq N-1}}^{N} (x_i - x_{i+2})^{a+bp_{i+1}^2} \right\} . \qquad (4)$$

For later convenience, we denote the integrand of Eq. (4) by

$$\{X_N\} \equiv \left\{ \prod_{2\leq i<j}^{N} (x_i - x_j)^{-2bp_i\cdot p_j} \cdot \prod_{2\leq i}^{N-1} \left[\theta(x_i - x_{i+1}) \cdot \right.\right.$$

$$\left.\left. \cdot (x_i - x_{i+1})^{-a-b(p_i^2+p_{i+1}^2)-1} \right] \cdot \prod_{\substack{i=2\\ i\neq N-1}}^{N} (x_i - x_{i+2})^{a+bp_{i+1}^2} \right\} . \qquad (5)$$

II. SINGLE FACTORIZATION

To do the single factorization of the N-point function, we divide the Koba-Nielsen variables x_i, $i=1,2,\cdots,N$ into two sets, y_i, $i=1,2,\cdots,M$, and z_j, $j=M+1,M+2,\cdots,N$, (Fig. 2). We explicitly choose three projective "frames" X, Y, and Z. They are defined by

$$\begin{aligned}
&X : x_1 = \infty, \quad x_2 = 1, \quad x_N = 0; \quad x_{N+i} \equiv x_i, && i=1,2,\cdots,N,\\
&Y : y_{M+1} = \infty, \quad y_M = 1, \quad y_1 = 0; \; y_{M+1+i} \equiv y_i, && i=1,2,\cdots,M, \qquad (6)\\
&Z : z_M = \infty, \quad z_{M+1} = 1, \quad z_N = 0; \; z_{N+i} \equiv z_{M-1+j}, && j=1,2,\cdots,N-M+1.
\end{aligned}$$

The Y frame specifies the left-hand M+1-point amplitude, the Z frame specifies the right-hand N-M+1-point amplitude, and the X frame specifies the N point amplitude as a whole. The following equations relate the X frame to the Y,Z frames,

$$\begin{aligned}
&x_i = \frac{y_2}{y_i}, && i=1,2,\cdots,M,\\
&x_j = y_2 t z_j, && j=M+1,\cdots,N, \qquad (7)\\
&t = \frac{x_{M+1}}{x_M};
\end{aligned}$$

t is the variable associated with the propagator of the factorization channel. The y_2 is equal to the product of all the usual multiperipheral parameterization variables belonging to the Y frame, i.e., $y_2 = u_{12}u_{13}\cdots u_{1M+1}$. Substituting Eq. (7) in (4), we obtain[2]

$$B_N(p_1,\cdots,p_N) = \int \prod_{\substack{i\neq M+1\\ M\\ 1}} dy_i \ \{Y_{M+1}\} \cdot \int \prod_{\substack{j\neq M\\ M+1\\ N}} dz_j \ \{Z_{N-M+1}\} \cdot$$

$$\cdot \int_0^1 dt\ t^{-\alpha(s_2)-1} (1-t)^{a-1} \cdot \prod_{i=1}^{M} \prod_{j=M+1}^{N} (1 - y_i t z_j)^{-2bp_i\cdot p_j} , \tag{8}$$

where $s_1 = (p_1 + p_2 + \cdots + p_M)^2 = (p_{M+1} + \cdots + p_N)^2$, and $a + bp_i^{\ 2} = 0$, $i=1,2,\cdots,N$. From now on, we will drop the $2b$ factor in $p_i \cdot p_j$. To factorize Eq. (8), we expand $\prod_{i=1}^{M} \prod_{j=M+1}^{N} (1 - y_i t z_j)^{-p_i\cdot p_j}$ as the exponent of a logarithm[3] and introduce the harmonic oscillator operators[4] a_n^+, a_n. Hence

$$\prod_{i=1}^{M} \prod_{j=M+1}^{N} (1 - y_i t z_j)^{-p_i\cdot p_j}$$

$$= \langle 0| \exp\left[\sum_n \frac{a_n}{(n)^{\frac{1}{2}}} \left(\sum_{j=M+1}^{N} p_j z_j^{\ n}\right)\right] \cdot t^{R_a}$$

$$\cdot \exp\left[\sum_n \frac{a_n^+}{n^{\frac{1}{2}}} \left(\sum_{i=1}^{M} p_i y_i^{\ n}\right)\right] |0\rangle . \tag{9}$$

Substituting (9) into (8) and defining

$$B_N(p_1\cdots p_N) = \langle 0| G^{(1)}_{(Z)}(a) \cdot D(R_a, s_1) \cdot G^{(1)}_{(Y)}(a^+)|0\rangle ,$$

we then find the single factorization result (Fig. 3),

$$\langle 0|G^{(1)}_{(Z)}(a)|\lambda_a\rangle = \int \pi dz_j \{Z\}\langle 0|\exp\left[\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left(\sum_{j=M+1}^{N} p_j z_j^{\,n}\right)\right]|\lambda_a\rangle , \tag{10.1}$$

$$D(R_a,s_1) = \int_0^1 dt\; t^{-\alpha(s_1)-1}(1-t)^{a-1}\cdot t^{R_a} , \tag{10.2}$$

$$\langle\lambda_a|G^{(1)}_{(Y)}(a^+)|0\rangle = \int \pi dy_i \{Y\}\langle\lambda_a|\exp\left[\sum_n \frac{a_n^+}{n^{\frac{1}{2}}}\left(\sum_{i=1}^{M} p_i\, y_i^{\,n}\right)\right]|0\rangle . \tag{10.3}$$

To record the ordering of the particles on the side which has been removed, we keep a dot on each factorized reggeon in such a way that the dot lies between x_M and x_{M+1}.

III. SECOND FACTORIZATION

A. Planar Case

We start from Eq. (10.1), let $M = 0$, and divide z_i, $i=1,2,\cdots,N$ into w_i, η, v_j so that

$$w_{L+1} = \infty, \qquad w_L = 1, \qquad w_1 = 0,$$

$$v_L = \infty, \qquad v_{L+1} = 1, \qquad v_N = 0,$$

$$z_1 = \infty, \qquad z_2 = 1, \qquad z_N = 0,$$

and so,

$$\begin{aligned} z_i &= \frac{w_2}{w_i}, & i&=1,2,\cdots,L, \\ z_j &= w_2 \eta v_j, & j&=L+1,\cdots,N, \\ \eta &= \frac{z_{L+1}}{z_L} \quad . \end{aligned} \tag{11}$$

Substituting Eq. (11) into Eq. (10.1), we find

$$\begin{aligned} \langle 0|G^{(1)}_{(Z)}(a)|\lambda_a\rangle &= \int \pi dw_i \{W_{L+1}\} \cdot \int \pi dv_j \cdot \{V\} \cdot \int_0^1 d\eta\, \eta^{-\alpha(s_2)-1} \cdot \\ &\cdot (1-\eta)^{a-1} \prod_{i=1}^{L} \prod_{j=L+1}^{N} (1 - w_i \eta v_j)^{-p_i p_j} \cdot \langle 0|\exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left[\sum_{i=2}^{L} p_i \left(\frac{w_2}{w_i}\right)^n \right.\right. \\ &\left.\left. + \sum_{j=L+1}^{N} p_j (w_2 \eta v_j)^n\right]\right\}|\lambda_a\rangle \ . \end{aligned} \tag{12}$$

To introduce the operator a_n^+, we observe, from Eq. (12), that

$$\prod_{i=L}^{L} \prod_{j=L+1}^{N} (1 - w_i \eta v_j)^{-p_i \cdot p_j} \cdot$$

$$\langle 0 | \exp \left\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} \left[\sum_{i=2}^{L} p_i \left(\frac{w_2}{w_i} \right)^n + \sum_{j=L+1}^{N} p_j (w_2 \eta v_j)^n \right] \right\} \cdot | \lambda_a \rangle$$

$$= \langle 0 | \exp \left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \left(\sum_{j=L+1}^{N} p_j \, (v_j)^n \right) \right] \cdot \eta^{R_a} \cdot \left\{ \exp \left[\sum_n \frac{a_n^+}{n^{\frac{1}{2}}} \right.\right.$$

$$\left.\left. \left(\sum_{i=1}^{L} p_i w_i^{\,n} \right) \right] \cdot w_2^{R_a} \cdot \exp \left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{i=2}^{L+1} p_i \left(\frac{w_2}{w_i} \right)^n \right] \right\} | \lambda_a \rangle \, . \tag{13}$$

Therefore, if we define, from Eqs. (12) and (13),

$$\langle 0 | G_{(Z)}^{(1)}(a) | \lambda_a \rangle = \langle 0 | G_{(V)}^{(1)}(a) \cdot D(R_a, s_2) \cdot G_{(W)}^{(2)}(a^+, a) | \lambda_a \rangle$$

$$\equiv \langle \mu_a | G_{(W)}^{(2)}(a^+, a) | \lambda_a \rangle \, ,$$

we find the second factorization result (Fig. 4)

$$\langle \mu_a | G_{(W)}^{(2)}(a^+, a) | \lambda_a \rangle = \int \pi dw_i \, \{W_{L+1}\} \langle \mu_a | \exp \left[\sum_n \frac{a_n^+}{n^{\frac{1}{2}}} \sum_{i=1}^{L} p_i w_i^{\,n} \right] \cdot w_2^{R_a} \cdot$$

$$\cdot \exp \left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{i=2}^{L+1} p_i \left(\frac{w_2}{w_i} \right)^n \right] | \lambda_a \rangle \, , \tag{14}$$

where $w_{L+1} = \infty$, $w_L = 1$, $w_1 = 0$. The expression Eq. (14) is identical to the formula of Fubini et al.,[4] since, in terms of Chan variables, $w_i = u_{1i} u_{1i+1} \cdots u_{1L-1}$, and $w_2 = \prod_{i=2}^{L-1} u_{1i}$.

By employing a technique similar to that used in Eq. (13), we can also introduce b_m-operator instead of a_n^+, and we find the second factorization expression

$$\langle 0|G^{(2)}_{(W)}(a,b)|^{\lambda_a}_{\lambda_b}\rangle = \int \pi dw_i \{W_{L+1}\} \langle 0|\exp(ab)_0 \cdot$$

$$\times \exp\left[\sum_m \frac{b_m}{m^{\frac{1}{2}}} \sum_{i=1}^{L} p_i\left(\frac{w_i}{w_2}\right)^m\right] w_2^{R_a} \exp\left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{i=2}^{L+1} p_i\left(\frac{w_2}{w_i}\right)^n\right] |^{\lambda_a}_{\lambda_b}\rangle \tag{15}$$

where $(ab)_0 = \sum_{n=1}^{\infty} a_n b_n$,

and

$$\langle 0|G^{(1)}_{(Z)}(a)|\lambda_a\rangle = \langle 0|G^{(2)}_{(W)}(a,b) \cdot D(R_b, s_2) \cdot G^{(1)}_{(V)}(b)|^{\lambda_a}_{0_b}\rangle .$$

B. Nonplanar Case

From Eq. (10.1), with $M = 0$, we change to the projective frame (Fig. 5),

$$z'_L = \infty, \quad z'_{L+1} = 1, \quad z'_{L-1} = 0 , \qquad (1 < L < N - 1) .$$

and relabel z'_i's so that

$$z''_1 = z'_L = \infty, \quad z''_2 = z'_{L+1} = 1, \quad z''_N = z'_{L-1} = 0$$

or

$$z_i' = z_{N+i}' = z_{N+1-L+i}'' = z_{1-L+i}'' .$$

Then Eq. (10.1) becomes

$$\langle 0|G^{(1)}_{(Z'')}(a)|\lambda_a\rangle = \int \pi dz_i''\{Z_N''\}\langle 0|\exp\left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{\substack{i=1\\(i\neq N-L+2)}}^{N} p_i \right.$$

$$\left.\left(\frac{z''_{N+2-L} - z''_{N-L+3}}{z''_{N+1-L} - z''_{N+3-L}}\right)^n \left(\frac{z''_{N+1-L} - z''_i}{z''_{N+2-L} - z''_i}\right)^n \right. |\lambda_a\rangle . \tag{16}$$

Carrying out the factorization as before, we find the second-factorized M + 1-point tree (Fig. 6)

$$\langle 0|\tilde{G}^{(2)}_{(W)}(a,b)|^{\lambda_a}_{\lambda_b}\rangle = \int \pi dw_i\{W_{M+1}\}\cdot\langle 0|\exp\left[\sum_{nm} \frac{a_n b_m}{(nm)^{\frac{1}{2}}}\right.$$

$$\left. \times \left(\frac{w_{L+1} - w_L}{w_{L+1} - w_{L-1}}\right)^n B_{nm}\left(\frac{w_{L-1}}{w_L}\right)\cdot w_L{}^m\right] \exp\left[\sum_m \frac{b_m}{m^{\frac{1}{2}}}\left(\sum_{i=1}^{M} p_i w_i{}^m\right)\right.$$

$$\left. + \sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{\substack{i=1\\(i\neq L)}}^{M+1} p_i \left(\frac{w_{L+1} - w_L}{w_{L+1} - w_{L-1}}\right)^n \left(\frac{w_{L-1} - w_i}{w_L - w_i}\right)^n\right] |^{\lambda_a}_{\lambda_b}\rangle , \tag{17}$$

where

$$B_{nm}(x) = \sum_{\ell=0}^{m} \binom{n}{\ell}\binom{-n}{m-\ell} (-)^m m\, x^\ell , \tag{18}$$

and

$$w_{M+1} = \infty, \quad w_M = 1, \quad w_1 = 0.$$

As a consistency check, we may remove all bottom legs in Eq. (17) by setting $L = 1$ in Eq. (17); we find

$$B_{nm}\left(\frac{w_{L-1}}{w_L}\right) \cdot w_L^{\,m} \left(\frac{w_{L+1} - w_L}{w_{L+1} - w_{L-1}}\right)^n \longrightarrow \delta_{nm} \cdot m \cdot w_2^{\,n} ,$$

$$\left(\frac{w_{L+1} - w_L}{w_{L+1} - w_{L-1}}\right)\left(\frac{w_{L-1} - w_i}{w_L - w_i}\right) \rightarrow \frac{w_2}{w_i} .$$

Hence Eq. (17) does reduce to the planar case, Eq. (15). However, we can equally well remove all "top" legs in Eq. (17) by setting $L = M$ in Eq. (17), we then find an expression with both dots on the other sides of the two excited legs (Fig. 7), as compared with (Fig. 4),

$$\langle 0|\tilde{\tilde{G}}^{(2)}_{(W)}(a,b)|^{\lambda_a}_{\lambda_b}\rangle = \int \pi dw_i \{W_{M+1}\} \cdot \langle 0|\exp\left[\sum_{nm} \frac{a_n b_m}{(nm)^{\frac{1}{2}}} B_{nm}(w_{M-1})\right]$$

$$\times \exp\left[\sum_m \frac{b_m}{m^{\frac{1}{2}}} \sum_{i=1}^{M} p_i w_i^{\,m} + \sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{\substack{i=1 \\ (i\neq M)}}^{M+1} p_i \left(\frac{w_{M-1} - w_i}{1 - w_i}\right)^n\right]|^{\lambda_a}_{\lambda_b}\rangle , \tag{19}$$

where $w_{M+1} = \infty$, $w_M = 1$, $w_1 = 0$. We can verify that Eq. (19) is related to Eq. (15) simply by a projective transformation $w_i = 1 - w_i'$ in Eq. (19) plus two "twisting" operations on both of the excited legs. To see this, we first change frame in Eq. (19) from $w_{M+1} = \infty$, $w_M = 1$, $w_1 = 0$ to $w'_{M+1} = \infty$, $w'_1 = 1$, $w'_M = 0$; i.e., $w_i = 1 - w_i'$. We then use the following identities[5,6]

$$B_{nm}(1 - x) = \sum_{\ell=0}^{m} \binom{n}{\ell}\binom{-n}{m-\ell}(-)^m \, m \, (1 - x)^{\ell}$$

$$= \sum_{\ell} \binom{n}{\ell}\binom{m}{\ell} \, \ell \, x^{\ell} , \tag{20.1}$$

$$\sum_{n=0}^{\infty} \binom{i}{n}\binom{n}{\ell}(-)^{n+\ell} = \delta_{i\ell} , \tag{20.2}$$

$$\sum_{nm} \frac{a_n b_m}{(nm)^{\frac{1}{2}}} B_{nm}(1 - x) = \sum_{n} \bar{a}_n \bar{b}_n x^n , \tag{20.3}$$

$$\bar{a}_n \equiv (-1)^n \sum_{m=n}^{\infty} a_m \binom{m}{n}\left(\frac{n}{m}\right)^{\frac{1}{2}} , \tag{20.4}$$

$$\bar{a}_n = (T^+)^{-1} a_n T^+ , \tag{20.5}$$

$$\Omega(\pi) = \exp\left(\sum_{n} \frac{\pi \cdot a_n^+}{n^{\frac{1}{2}}}\right) \cdot T ,$$

$$T = \left\{\prod_{i=1}^{\infty} \exp\left[\sum_{j>i} \left(\frac{i}{j}\right)^{\frac{1}{2}} \binom{j}{i} a_j^+ a_i\right]\right\}(-)^R , \tag{20.6}$$

$$\langle 0|(T^+)^{-1} = \langle 0| , \tag{20.7}$$

$$T^{\dagger}|\lambda\rangle = |\bar{\lambda}\rangle . \tag{20.8}$$

Then, clearly, Eq. (19) becomes

$$\langle 0|\tilde{\tilde{G}}^{(2)}_{(W)}(a,b)|^{\lambda_a}_{\lambda_b}\rangle = \int \pi dw'_i \{W'_{M+1}\}\langle 0|\exp(ab)_0 \cdot \exp\left[\sum_m \frac{b_m}{m^{\frac{1}{2}}} \cdot \sum_{i=1}^{M} p_i w_i'^m\right]$$

$$\cdot\ {}^{R_a}_{w_{M-1}} \cdot \exp\left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{\substack{i=1 \\ (i\neq M)}}^{M+1} p_i \left(\frac{w'_{M-1}}{w'_i}\right)^n\right] \Big| \frac{\bar{\lambda}_a}{\bar{\lambda}_b}\rangle \ , \qquad (21)$$

which is analogous to Eq. (15), except that the states $|\lambda_a\rangle$, $|\lambda_b\rangle$ are twisted. This is the expected result, since excited-particle lines with the dots on opposite sides are related to each other by the twisting operator.

IV. THIRD FACTORIZATION

We start from the second-factorized tree, Eq. (14), and change from the frame $w_{M+1} = \infty$, $w_M = 1$, $w_1 = 0$ to the new frame, $w'_{M+1} = \infty$, $w'_1 = 1$, $w'_M = 0$ (Fig. 8). From Eq. (14), we get

$$\langle \mu_a | G^{(2)}_{(W')}(a^+,a) | \lambda_a \rangle = \int \pi dw'_i \{W'\} \; \langle \mu_a | \exp\left[\sum_n \frac{a_n^+}{n^{\frac{1}{2}}} \sum_{i=1}^{M} p_i \left(1 - w'_i\right)^n\right]$$

$$\times \left(1 - w'_2\right)^{R_a} \exp\left[\sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{i=2}^{M+1} p_i \left(\frac{1 - w'_2}{1 - w'_i}\right)^n\right] \lambda_a \rangle \quad . \tag{14'}$$

We then divide w'_i into r_i, s_j, and η as before:

$$r_{L+1} = \infty, \quad r_L = 1, \quad r_0 = 0 \; ,$$

$$s_L = \infty, \quad s_{L+1} = 1, \quad s_M = 0 \; ,$$

$$w'_{M+1} = w'_0 = \infty, \quad w'_1 = 1, \quad w'_M = 0$$

and

$$w'_i = \frac{r_1}{r_i}, \quad i=1,2,\cdots,L \; ,$$

$$w'_j = r_1 \eta s_j, \quad j = L+1 \cdots M,$$

$$\eta = \frac{w'_{L+1}}{w'_L} \quad .$$

Therefore, in terms of r_i, s_j, and η, Eq. (14') becomes

$$\left\langle 0 \middle| G^{(3)}_{(R)}(a,b,c) \middle| \begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix} \right\rangle = \int \pi dr_i \{R_{L+2}\} \left\langle 0 \middle| \exp\left[\sum_{\ell=1}^{\infty} \frac{c_\ell}{\ell^{\frac{1}{2}}}\left(\sum_{i=1}^{L} p_i r_i^{\ell}\right)\right]\right.$$

$$\times \exp\left[\sum_{n=1}^{\infty} a_n b_n \left(1 - \frac{r_1}{r_2}\right)^n\right.$$

$$\times \exp\left\{\sum_m \frac{b_m}{m^{\frac{1}{2}}}\left[\sum_{i=1}^{L+1} p_i \left(1 - \frac{r_1}{r_i}\right)^m + \sum_{\ell=1}^{m} \binom{m}{\ell}(-)^{\ell} \cdot \ell (r_1)^{\ell} \frac{c_\ell}{\ell^{\frac{1}{2}}}\right]\right\}$$

$$\times \exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left(1 - \frac{r_1}{r_2}\right)^n \left[\sum_{i=2}^{L+1} p_i \left(1 - \frac{r_1}{r_i}\right)^{-n} + \sum_{\ell=1}^{\infty} \binom{-n}{\ell}(-)^{\ell}\, \ell\right.\right.$$

$$\left.\left.\left.\times\, r_1^{\ell} \frac{c_\ell}{\ell^{\frac{1}{2}}}\right]\right\} \left| \begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix} \right\rangle, \tag{23}$$

where, again, $r_{L+1} = \infty$, $r_L = 1$, $r_0 = 0$.

It is interesting to see how Eq. (23) can reduce to Sciuto's[7] three-reggeon vertex. Since we are unable to factorize the c_ℓ leg without assuming the factor $\left(1 - \frac{r_1}{r_2}\right)$ to be in the R frame (opposite the S frame), we therefore consider the case $L = 2$ (Fig. 10). Substituting $r_3 = \infty$, $r_2 = 1$, $r_0 = 0$ in Eq. (23), we get

$$\left\langle 0 \middle| G^{(3)}_{(R)}(a,b,c) \middle| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c\end{matrix} \right\rangle = \int_0^1 dr_1 \; r_1^{-\alpha_{01}-1} (1 - r_1)^{-\alpha_{12}-1}$$

$$\times \left\langle 0 \middle| \exp\left[\sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}}(p_1 r_1^{\ell} + p_2)\right] \cdot \exp\left[\sum_n a_n b_n (1 - r_1)^n\right]\right.$$

$$\times \exp\left\{\sum_m \frac{b_m}{m^{\frac{1}{2}}}\left[\left(p_3 + p_2(1 - r_1)^m\right) + \sum_{\ell=1}^{m}\binom{m}{\ell}(-)^\ell \ell \; r_1^{\ell} \frac{c_\ell}{\ell^{\frac{1}{2}}}\right]\right\}$$

$$\times \exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}(1 - r_1)^n\left[p_3 + p_2(1 - r_1)^{-n} + \sum_{\ell=1}^{\infty}\binom{-n}{\ell}(-)^\ell \ell \; r_1^{\ell} \frac{c_\ell}{\ell^{\frac{1}{2}}}\right]\right\}$$

$$\times \left. \middle| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c\end{matrix} \right\rangle . \qquad (24)$$

In order to remove the external scalar leg (i.e., r_2) in Fig. 10, we then go to the pole position of the α_{01} variable and look at the residue given by Eq. (24). We can rewrite Eq. (24) in the following form:

$$\left\langle \begin{matrix}\lambda_a\\ \lambda_b\\ 0_c\end{matrix}\right| G^{(3)}_{(R)}(a^+,b^+,c) \left| \begin{matrix}0_a\\ 0_b\\ \lambda_c\end{matrix}\right\rangle = \left\langle \begin{matrix}\lambda_a\\ \lambda_b\\ 0_c\end{matrix}\right| \left[\exp\left(\sum_\ell \frac{c_\ell^+}{\ell^{\frac{1}{2}}} p_2 \right) \exp\left(\sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} p_2 \right) \right.$$

$$\times \left[\int_0^1 dr_1\, r_1^{-\alpha_{01}-1} (1 - r_1)^{-a-b(p_1^2+p_2^2)-1}\, r_1^{R_c} (1 - r_1)^{R_a} \cdot (1 - r_1)^{-2bp_1\cdot p_2} \right]$$

$$\times \exp\left\{ \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} p_1 + (a^+,b^+)_0 + \sum_m \frac{b_m^+}{m^{\frac{1}{2}}} \left[p_3 + p_2(1 - r_1)^m \right] + (b^+,c)_- \right.$$

$$\left. + \sum_n \frac{a_n}{n^{\frac{1}{2}}} [p_3 + p_2(1 - r_1)^{-n}] + (a^+,c)_+ \right\} \left| \begin{matrix}0_a\\ 0_b\\ \lambda_c\end{matrix}\right\rangle . \tag{25}$$

We observe that the last factor { } in Eq. (25) is a three-reggeon vertex if $p_2 \to 0$. To get the three-reggeon vertex $W^{(3)}(a,b,c)$ from Eq. (25), we thus take the limit $p_2 \to 0$, and write

$$\lim_{p_2\to 0} \left\langle \begin{matrix}\lambda_a\\ \lambda_b\\ 0_c\end{matrix}\right| G^{(3)}(a^+,b^+,c) \left| \begin{matrix}0_a\\ 0_b\\ \lambda_c\end{matrix}\right\rangle$$

$$= \lim_{p_2\to 0} \left\langle \begin{matrix}\lambda_a\\ \lambda_b\\ 0_c\end{matrix}\right| V(p_2)\cdot D'(R_a,R_c)\cdot W^{(3)}(a^+,b^+,c) \left| \begin{matrix}0_a\\ 0_b\\ \lambda_c\end{matrix}\right\rangle . \tag{26}$$

But under this limit the discussion of the factorization must then be modified, since the mass of leg 2 is no longer equal to that of the other legs. The relevant modification is given in the appendix. As $p_2 \to 0$, the ordinary vertex function $V(p_2)$ becomes the unit operator and so $D'W^{(3)}(a^+,b^+,c)$ becomes a well-defined quantity. From Eq. (25) and Eq. (26), we obtain the c_ℓ-leg propagator

$$D'(R_c,s_3) = \int_0^1 dr_1\, r_1^{-\alpha_{01}-1}(1-r_1)^{-bp_1^2-1}\; r_1^{R_c}(1-r_1)^{R_a}\,, \quad (27)$$

and Sciuto's three-reggeon vertex (Fig. 11)

$$\left\langle 0|W^{(3)}(a,b,c)\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix}\right.\right\rangle$$

$$= \left\langle 0|\exp[p_1 c + (a,b)_0 + p_2' b + (b,c)_- + p_2' a + (a,c)_+]\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix}\right.\right\rangle\,, \quad (28)$$

where $p_2' \equiv p_3$.

Another interesing thrice-factorized tree (Fig. 12) can also be obtained easily by directly factorizing Eq. (15) in a general projective frame, similarly to what we did in Eq. (16) and Eq. (17). It takes the following form (Fig. 12):

$$\left\langle 0|\tilde{G}_{(R)}(a,b,c)\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix}\right.\right\rangle = \int \prod_{\substack{i\neq R+1\\ R\\ 1}} dr_i\,\{R_{R+1}\}\left\langle 0|\exp\left[\sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}}\sum_{i=1}^{R} p_i\, r_i^{\ell}\right]\right.$$

$$\times \exp(a,b)_0 \;\exp\left\{\sum_m \frac{b_m}{m^{\frac{1}{2}}}\left(\frac{r_{L-1}-r_L}{r_{L-1}-r_{L+1}}\right)^m \left[\sum_{\substack{i=1\\ (i\neq L)}}^{R+1} p_i\left(\frac{r_{L+1}-r_1}{r_L-r_i}\right)^m\right.\right.$$

$$\left.\left. + \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} B_{m\ell}\left(\frac{r_{L+1}}{r_L}\right)\cdot r_L^{\ell}\right]\right\} \;\exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left(\frac{r_{L-1}-r_L}{r_{L-1}-r_{L+1}}\right)^{-n}\right. \quad (29)$$

(cont.)

(cont.)

$$\left[\sum_{\substack{i=1\\(i\neq L+1)}}^{R+1} p_i \left(\frac{r_{L+1} - r_i}{r_L - r_i}\right)^{-n} + \sum_{\ell=1}^{\infty} \frac{c_\ell}{\ell^{\frac{1}{2}}} B_{-n\ell}\left(\frac{r_{L+1}}{r_L}\right) r_L{}^{\ell}\right]\Bigg\}$$

$$\times \left[\left(\frac{r_{L-1} - r_L}{r_{L-1} - r_{L+1}}\right)\left(\frac{r_{L+2} - r_{L+1}}{r_{L+2} - r_L}\right)\right]^{R_a} \Bigg| \begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\end{matrix} \Bigg\rangle \quad , \tag{29}$$

where

$$B_{nm}(x) = \sum_{i=0}^{m} \binom{n}{i}\binom{-n}{m-i} (-)^m \, m \, x^i = \sum_i \binom{n}{i}\binom{m}{i} \, i \, (1 - x)^i \quad . \tag{30}$$

The reason why we are interested in Eq. (29) will be clear in the next section. As a consistency check, we let $R = L - 2$ in Eq. (29). We get

$$\left(\frac{r_{L-1} - r_L}{r_{L-1} - r_{L+1}}\right)\left(\frac{r_i - r_{L+1}}{r_i - r_L}\right) \longrightarrow \left(1 - \frac{r_{L+1}}{r_i}\right) \quad ,$$

$$r_L{}^m \, B_{nm}\left(\frac{r_{L+1}}{r_L}\right) \longrightarrow \binom{n}{m} \, (-)^m \, m \, r_{L+1}{}^m \, ,$$

therefore Eq. (29) does reduce to Eq. (23), as it should.

V. FOURTH FACTORIZATION

Beginning with Eq. (23) (Fig. 9b), we change to the new frame

$$r_1' = \infty, \quad r_2' = 0, \quad r_0' = 1 \qquad (N \equiv L+1),$$

i.e.

$$r_i = \left(\frac{r_N' - r_{N-1}'}{r_N' - r_i'}\right)\left(\frac{1 - r_i'}{1 - r_{N-1}'}\right) .$$

Substituting this in Eq. (23), and factorizing as before, we then obtain the four-times-factorized tree (Fig. 13)

$$\left\langle 0 \middle| G^{(4)}_{(W)}(a,b,c,d,) \middle| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d\end{matrix} \right\rangle = \int \pi dw_i \{W_{M+1}\} \left\langle 0 \middle| \exp\left[\sum_k \frac{d_k}{k^{\frac{1}{2}}} \sum_{i=1}^{M} p_i w_i^{\,k}\right]\right.$$

$$\times \exp\left[\sum a_n b_n \left(1 - \frac{w_2}{w_3}\right)^n\right] \exp\left\{\sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}}\left[\sum_{\substack{i=1\\(i\neq 3)}}^{M+1} p_i \left(\frac{w_3 - w_4}{w_2 - w_4}\right)^\ell \left(\frac{w_2 - w_i}{w_3 - w_i}\right)^\ell\right.\right.$$

$$\left.\left. + \left(\frac{w_3 - w_4}{w_2 - w_4}\right)^\ell \sum_{k=1}^{\ell} \frac{d_k}{k^{\frac{1}{2}}} B_{\ell k}\left(\frac{w_2}{w_3}\right) w_3^{\,k}\right]\right\} \exp\left\{\sum_m \frac{b_m}{m^{\frac{1}{2}}}\right.$$

$$\left[\sum_{\substack{i=1\\(i\neq 2)}}^{M+1} p_i \left(\frac{w_2 - w_3}{w_2 - w_i}\right)^m \left(\frac{w_i}{w_3}\right)^m + \left(1 - \frac{w_2}{w_3}\right)^m \sum_{k=1}^{\infty} \binom{-m}{k} (-)^k k\, w_2^{\,k} \frac{d_k}{k^{\frac{1}{2}}}\right.$$

$$\left.\left. + \sum_{\ell=1}^{M} \binom{m}{\ell} (-)^\ell \ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \left(\frac{w_3 - w_4}{w_2 - w_4}\right)^\ell \left(\frac{w_2}{w_3}\right)^\ell\right]\right\} \exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left(1 - \frac{w_2}{w_3}\right)^n\right.$$

$$\left[\sum_{i=2}^{M+1} p_i \left(\frac{w_2 - w_3}{w_2 - w_i}\right)^{-n} \left(\frac{w_i}{w_3}\right)^{-n} + \left(1 - \frac{w_2}{w_3}\right)^{-n} \sum_k \binom{n}{k} (-)^k k\, w_2^{\,k} \frac{d_k}{k^{\frac{1}{2}}}\right.$$

(31)
(cont.)

(cont.)

$$+ \sum_{\ell=1}^{\infty} \binom{-n}{\ell} (-)^{\ell} \ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \left(\frac{w_2}{w_3}\right)^{\ell} \left(\frac{w_3 - w_4}{w_2 - w_4}\right)^{\ell} \Bigg]\Bigg\} \left| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle \quad , \tag{31}$$

where $w_{M+1} = \infty$, $w_M = 1$, $w_1 = 0$. We can simplify Eq. (31) by changing to the projective frame $w'_1 = 0$, $w'_2 = \infty$, $w'_M = 1$, i.e.,

$$w_i = (w'_{M+1}, w'_1, w'_M, w'_i) = \frac{(1 - w'_{M+1})w'_i}{(w'_i - w'_{M+1})} \quad .$$

Then Eq. (31) becomes the following expression (Fig. 13)

$$\left\langle 0 \middle| G^{(4)}_{(W')}(a,b,c,d) \middle| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle = \int \pi dw'_i \{W'_{M+1}\} \left\langle 0 \middle| \exp\left[\sum_k \frac{d_k}{k^{\frac{1}{2}}} \sum_{i=1}^{M} p_i \right.\right.$$

$$\left. \times \left(\frac{1 - w'_{M+1}}{w'_i - w'_{M+1}}\right)^k w_i'^k \right] \quad \exp\left[\sum_n a_n b_n \left(\frac{w'_{M+1}}{w'_i}\right)^n\right]$$

$$\times \exp\left\{\sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \left[\sum_{\substack{i=1 \\ (i \neq 3)}}^{M+1} p_i \left(\frac{w'_3 - w'_4}{w'_3 - w'_i}\right)^\ell + \left(\frac{w'_3 - w'_4}{w'_3 - w'_{M+1}}\right)^\ell \sum_{k=1}^{\ell} \frac{d_k}{k^{\frac{1}{2}}}\right.\right.$$

$$\left.\left. \times B_{\ell k}\left(1 - \frac{w'_{M+1}}{w'_3}\right) \left(\frac{(1 - w'_{M+1})w'_3}{w'_3 - w'_{M+1}}\right)^k \right]\right\} \quad \exp\left\{\sum_m \frac{b_m}{m^{\frac{1}{2}}} \left[\sum_{\substack{i=1 \\ (i \neq 2)}}^{M+1} p_i \left(\frac{w'_i}{w'_3}\right)^m \right.\right.$$

$$+ \left(\frac{w'_{M+1}}{w'_3}\right)^m \sum_{k=1}^{\infty} \binom{-m}{k} (-)^k k \, (1 - w'_{M+1})^k \frac{d_k}{k^{\frac{1}{2}}}$$

(32)
(cont.)

(cont.)

$$+ \sum_{\ell=1}^{m} \binom{m}{\ell} (-)^{\ell} \ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \left(1 - \frac{w_4'}{w_3'}\right)^{\ell} \Bigg]\Bigg\} \cdot \exp\Bigg\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} \left(\frac{w'_{M+1}}{w_3'}\right)^n$$

$$\times \Bigg[\sum_{i=2}^{M+1} p_i \left(\frac{w_i'}{w_3'}\right)^{-n} + \left(\frac{w'_{M+1}}{w_3'}\right)^{-n} \sum_{k=1}^{n} \binom{n}{k} (-)^k k \frac{d_k}{k^{\frac{1}{2}}} \cdot (1 - w'_{M+1})^k$$

$$+ \sum_{\ell=1}^{\infty} \binom{-n}{\ell} (-)^{\ell} \ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \left(1 - \frac{w_4'}{w_3'}\right)^{\ell} \Bigg]\Bigg\} \left| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle \quad . \tag{32}$$

Since the minimum number of legs for the fourth factorization to be feasible is five, i.e., $M = 4$ in Eq. (31), to get the four-reggeon vertex, we consider the case $M = 4$ in Eq. (31) (Fig. 14). Set $M = 4$ in Eq. (31); we get

$$\left\langle 0 \middle| G^{(4)}_{(W)}(a,b,c,d) \middle| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle = \int dw_2 dw_3 \Bigg\{ w_2^{-\alpha_{12}-1} \; w_3^{a+bp_2^2-2bp_1\cdot p_3}$$

$$\times (w_3 - w_2)^{-\alpha_{23}-1} \; (1 - w_2)^{a+bp_3^2-2bp_2\cdot p_4} \; (1 - w_3)^{-\alpha_{34}-1} \Bigg\}$$

$$\times \Bigg\langle 0 \Bigg| \exp\Bigg\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} \Bigg[p_3 \left(1 - \frac{w_2}{w_3}\right)^n + p_5 + p_4 \left(1 - \frac{w_2}{w_3}\right)^n \left(\frac{w_3 - w_2}{1 - w_2}\right)^{-n} w_3^n \Bigg]$$

$$+ \sum_k \frac{d_k}{k^{\frac{1}{2}}} (p_4 + p_3 w_3^k + p_2 w_2^k) + \sum_m \frac{b_m}{m^{\frac{1}{2}}}$$

$$\times \Bigg[p_3 + p_4 \left(\frac{1}{w_3}\right)^m \left(\frac{w_2 - w_3}{w_2 - 1}\right)^m + p_5 \left(1 - \frac{w_2}{w_3}\right)^m \Bigg] \tag{33}$$

(cont.)

(cont.)

$$+ \sum_{\ell} \frac{c_\ell}{\ell^{\frac{1}{2}}} \left[p_4 + p_5 \left(\frac{1 - w_3}{1 - w_2} \right)^{\ell} + p_1 \left(\frac{1 - w_3}{1 - w_2} \right)^{\ell} \left(\frac{w_2}{w_3} \right)^{\ell} \right] \Bigg\}$$

$$\times \exp \left\{ \sum_{n} a_n b_n \left(1 - \frac{w_2}{w_3} \right)^{n} + \sum_{n\ell} \frac{a_n c_\ell}{(n\ell)^{\frac{1}{2}}} \left(1 - \frac{w_2}{w_3} \right)^{\ell} \binom{-n}{\ell} (-)^{\ell} \ell \right.$$

$$\times \left(\frac{w_2}{w_3} \right)^{\ell} \left(\frac{1 - w_3}{1 - w_2} \right)^{\ell} + \sum_{nk} \frac{a_n d_k}{(nk)^{\frac{1}{2}}} \binom{n}{k} (-)^{k} k \; w_2^{\;k}$$

$$+ \sum_{m\ell} \frac{b_m c_\ell}{(m\ell)^{\frac{1}{2}}} \binom{m}{\ell} (-)^{\ell} \ell \left[\frac{w_2}{w_3} \left(\frac{1 - w_3}{1 - w_2} \right)^{\ell} \right.$$

$$+ \sum_{mk} \frac{b_m d_k}{(mk)^{\frac{1}{2}}} \binom{-m}{k} (-)^{k} k \; w_2^{\;k} \left(1 - \frac{w_2}{w_3} \right)^{m}$$

$$\left. + \sum_{\ell k} \frac{c_\ell d_k}{(\ell k)^{\frac{1}{2}}} \left(\frac{1 - w_3}{1 - w_2} \right)^{\ell} B_{\ell k} \left(\frac{w_2}{w_3} \right) \; w_3^{\;k} \right\} \left| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle \quad . \tag{33}$$

Notice, in terms of Chan variables, $w_2 = u_{12}u_{13}$, $w_3 = u_{13}$. We are interested in the poles of α_{13} variables. Analogously to the three-reggeon case, we therefore take the limit $p_4 \to 0$ in Eq. (33) and write

$$\lim_{p_4 \to 0} \left\langle 0 \middle| G^{(4)}_{(W)}(a,b,c,d) \middle| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle = \lim_{p_4 \to 0} \left\langle 0 \middle| W^{(4)}(a,b,c,d) D''(R_d, R_c, s_4) \right.$$

$$\left. \times \; V^{(d)}_{(p_4)} \middle| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle \quad . \tag{34}$$

We find

$$D''(R_d,R_c,s_4)$$

$$= \int_0^1 du_{13}\, u_{13}^{-\alpha_{123}-1+R_d}\,(1-u_{13})^{-bp_3^{\,2}-1+R_c}\,(1-u_{12}u_{13})^{\alpha(p_3)-R_c}\,, \tag{35}$$

and the four-reggeon vertex function

$$\left\langle 0\left|W^{(4)}(a,b,c,d)\right|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_d\end{matrix}\right\rangle = \int_0^1 du_{12}\, u_{12}^{-\alpha_{12}-1}(1-u_{12})^{-\alpha_{23}-1}$$

$$\times \left\langle 0\left|\exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}\left[p_4' + p_3(1-u_{12})^n\right] + \sum_m \frac{b_m}{m^{\frac{1}{2}}}\left[p_3 + p_4'(1-u_{12})^m\right]\right.\right.\right.$$

$$\left.+ \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}}\left(p_4' + p_1 u_{12}^{\ \ell}\right) + \sum_k \frac{d_k}{k^{\frac{1}{2}}}\left(p_3 + p_2 u_{12}^{\ k}\right)\right\}$$

$$\times \exp\left\{\sum_n a_n b_n (1-u_{12})^n + \sum_{n\ell} \frac{a_n c_\ell}{(n\ell)^{\frac{1}{2}}}(1-u_{12})^n \binom{-n}{\ell}(-)^\ell \ell\, u_{12}^{\ \ell}\right.$$

$$+ \sum_{nk} \frac{a_n d_k}{(nk)^{\frac{1}{2}}}\binom{n}{k}(-)^k k\, u_{12}^{\ k} + \sum_{m\ell} \frac{b_m c_\ell}{(m\ell)^{\frac{1}{2}}}\binom{m}{\ell}(-)^\ell \ell\, u_{12}^{\ \ell}$$

$$+ \sum_{mk} \frac{b_m d_k}{(mk)^{\frac{1}{2}}}(1-u_{12})^m \binom{-m}{k}(-)^k k\, u_{12}^{\ k}$$

$$\left.\left.+ \sum_{\ell k} \frac{c_\ell d_k}{(\ell k)^{\frac{1}{2}}} B_{\ell k}(u_{12})\right\}\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_d\end{matrix}\right\rangle\,, \tag{36}$$

where $p_4' = p_5$, and $u_{12} = \frac{w_2}{w_3}$. We can then re-introduce Koba-Nielsen variables $w_4 = \infty$, $w_3 = 1$, $w_2 = u_{12}$, $w_1 = 0$ into Eq. (36) (Fig. 15). Equation (36) looks all right, but we have to keep in mind the factor $\left(\frac{1 - u_{13}}{1 - u_{12}u_{13}}\right)^{R_c - \alpha(p_3)}$ in the propagator corresponding to the fourth leg (d_k leg).

Another interesting four-reggeon vertex (Fig. 16b) can be derived directly from Eq. (29). Directly factorizing Eq. (29) again, we obtain the four-times-factorized tree (Fig. 16a)

$$\left\langle 0 \middle| G^{(4)}_{(W)}(a,b,c,d) \middle| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d\end{matrix} \right\rangle = \int \pi dw_i \{W_{S+1}\} \left\langle 0 \middle| \exp\left\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} \sum_{\substack{i=1 \\ (i \neq L)}}^{S} p_i \right.\right.$$

$$\times \left(\frac{w_{L+2} - w_{L+1}}{w_{L+1} - w_L}\right)^{-n} \left(\frac{w_L - w_i}{w_{L+1} - w_i}\right)^{-n}$$

$$+ \sum_m \frac{b_m}{m^{\frac{1}{2}}} \sum_{\substack{i=1 \\ (i \neq L+1)}}^{S+1} p_i \left(\frac{w_{L+2} - w_{L+1}}{w_{L+2} - w_L}\right)^{m} \left(\frac{w_L - w_i}{w_{L+1} - w_i}\right)^{m}$$

$$\left. + \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} \sum_{i=1}^{S} p_i \cdot w_i^{\ell} + \sum_k \frac{d_k}{k^{\frac{1}{2}}} \sum_{i=2}^{S+1} p_i \left(\frac{w_2}{w_i}\right)^{k} \right\}$$

$$\times \exp\left\{ (a,b)_0 + \sum_{n\ell} \frac{a_n c_\ell}{(n\ell)^{\frac{1}{2}}} \left(\frac{w_{L+2} - w_{L+1}}{w_{L+2} - w_L}\right)^{-n} B_{-n\ell}\left(\frac{w_L}{w_{L+1}}\right) w_{L+1}^{\ell} \right.$$

(37)
(cont.)

$$+ \sum_{nk} \frac{a_n d_k}{(nk)^{\frac{1}{2}}} \left(\frac{w_{L+2} - w_{L+1}}{w_{L+2} - w_L}\right)^{-n} \left(\frac{w_L}{w_{L+1}}\right)^{-n} B_{-nk}\left(\frac{w_{L+1}}{w_L}\right) \cdot \left(\frac{w_2}{w_{L+1}}\right)^k$$

$$+ \sum_{n\ell} \frac{b_m c_\ell}{(m\ell)^{\frac{1}{2}}} \left(\frac{w_{L+2} - w_{L+1}}{w_{L+2} - w_L}\right)^{m} B_{m\ell}\left(\frac{w_L}{w_{L+1}}\right) w_{L+1}{}^{\ell}$$

$$+ \sum_{mk} \frac{b_m d_k}{(mk)^{\frac{1}{2}}} \left(\frac{w_{L+2} - w_{L+1}}{w_{L+2} - w_L}\right)^{m} \left(\frac{w_L}{w_{L+1}}\right)^{m} B_{mk}\left(\frac{w_{L+1}}{w_L}\right) \left(\frac{w_2}{w_{L+1}}\right)^k$$

$$+ \sum_{\ell k} \frac{c_\ell d_k}{(\ell k)^{\frac{1}{2}}} w_2{}^{\ell} \cdot \ell \Bigg\} \left[\left(\frac{w_{L+1} - w_{L+2}}{w_L - w_{L+2}}\right) \left(\frac{w_{L-1} - w_L}{w_{L-1} - w_{L+1}}\right)\right]^{R_a} \left| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle , \tag{37}$$

where $w_{S+1} = \infty$, $w_S = 1$, $w_1 = 0$. By procedures similar to those used in Eqs. (33), (34), (35), and (36), we obtain the four-reggeon vertex (Fig. 16a),

$$\left\langle 0 \left| W^{(4)}(a,b,c,d) \right| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle = \int_0^1 dw_2 \, w_2{}^{-\alpha_{12}-1} (1 - w_2)^{-\alpha_{23}-1}$$

$$\times \left\langle 0 \left| \exp \left\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} (p_1 + p_4 w_2{}^n) + \sum_m \frac{b_m}{m^{\frac{1}{2}}} (p_4 + p_1 w_2{}^m) \right.\right.\right.$$

$$\left.\left.\left. + \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}} (p_3 + p_2 w_2{}^\ell) + \sum_k \frac{d_k}{k^{\frac{1}{2}}} (p_2 + p_3 w_2{}^k) \right\} \right. \tag{38 (cont.)}$$

(cont.)

$$\times \exp\left\{\sum_n a_n b_n w_2^{\,n} + \sum_\ell c_\ell d_\ell w_2^{\,\ell} + \sum_{n\ell} \frac{a_n c_\ell}{(n\ell)^{\frac{1}{2}}} B_{-n\ell}(w_2)\, w_2^{\,n}\right.$$

$$+ \sum_{nk} \frac{a_n d_k}{(nk)^{\frac{1}{2}}} B_{nk}(w_2) + \sum_{m\ell} \frac{b_m c_\ell}{(m\ell)^{\frac{1}{2}}} B_{m\ell}(w_2) +$$

$$\left. + \sum_{mk} \frac{b_m d_k}{(mk)^{\frac{1}{2}}} B_{-mk}(w_2)\; w_2^{\,m}\right\} \left| \begin{matrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d \end{matrix} \right\rangle \quad . \tag{38}$$

Again, we must keep in mind the factor $\left(\frac{1 - u_{13}}{1 - u_{12}u_{13}}\right)^{R_b - \alpha(p_3)}$,

where $u_{12} = w_2$, in the propagator corresponding to the fourth leg (c_ℓ leg). The c_ℓ-leg propagator is

$$D''(R_c, R_b, s_4)$$

$$= \int_0^1 du_{13}\; u_{13}^{-\alpha_{123}-1+R_c} (1 - u_{13})^{-\alpha(p_3)-1+R_b+a} \; (1 - u_{12}u_{13})^{\alpha(p_3)-R_b} . \tag{39}$$

It is interesting to observe that if we directly twist[6] c_ℓ and d_k legs in Eq. (36) without worrying about the asymmetrical fourth leg (d_k leg in Fig. 15), we get, from Eq. (36),

$$\left\langle 0 \middle| W^{(4)}(a,b,c,d) \middle| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d\end{matrix} \right\rangle = \int_0^1 dw_2\, w_2^{-\alpha_{12}-1} (1 - w_2)^{-\alpha_{23}-1}$$

$$\times \left\langle 0 \middle| \exp\left\{ \sum_n \frac{a_n}{n^{\frac{1}{2}}} [p_4 + p_3(1 - w_2)^n] + \sum_m \frac{b_m}{m^{\frac{1}{2}}} [p_3 + p_4(1 - w_2)^m] \right.\right.$$

$$\left.\left. + \sum_\ell \frac{\bar{c}_\ell}{\ell^{\frac{1}{2}}} [p_2 + p_1(1 - w_2)^\ell] + \sum_k \frac{\bar{d}_k}{k^{\frac{1}{2}}} [p_1 + p_2(1 - w_2)^k] \right\}\right.$$

$$\times \exp\left\{ \sum_n a_n b_n (1 - w_2)^n + \sum_\ell \bar{c}_\ell \bar{d}_\ell (1 - w_2)^\ell \right.$$

$$+ \sum_{n\ell} \frac{a_n \bar{c}_\ell}{(n\ell)^{\frac{1}{2}}} B_{-n\ell}(1 - w_2) \quad (1 - w_2)^n + \sum_{nk} \frac{a_n \bar{d}_k}{(nk)^{\frac{1}{2}}} B_{nk}(1 - w_2)$$

$$\left. + \sum_{m\ell} \frac{b_m \bar{c}_\ell}{(m\ell)^{\frac{1}{2}}} B_{m\ell}(1 - w_2) + \sum_{mk} \frac{b_m \bar{d}_k}{(mk)^{\frac{1}{2}}} B_{mk}(1 - w_2) \quad (1 - w_2)^m \right\} \left| \begin{matrix}\lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_d\end{matrix} \right\rangle . \tag{40}$$

Comparing Eq. (40) with Eq. (38), we see that they are identical expressions, if p_1, p_2, p_3, p_4 and $(1 - w_2)$ in Eq. (40) are substituted by p_2, p_3, p_4, p_1 and w_2, respectively; as required for different labeling of w_i's in Eq. (38). The reason why we can ignore the factor

$\left(\frac{1 - u_{13}}{1 - u_{12}u_{13}}\right)^{R_c - \alpha(p_3)}$ in passing the twisting operator $\Omega_c^+(-p_3)$ to

the left side of Eq. (36) is the same as in Caneschi and Schwimmer's[8] treatment on the three-reggeon vertex case, since their derivation doesn't depend on the form of the z variable. In our case,

$$z = \frac{u_{13}(1 - u_{12})}{1 - u_{12}u_{13}} \quad .$$

Finally, we apply the twisting operators $\Omega_b^{+}(-p_3)$, $\Omega_d^{+}(-p_1)$ to the b_m and d_k legs in Eq. (38), then we end up with the completely symmetrical expression of the four-reggeon vertex function (Fig. 17),

$$\left\langle 0|W^{(4)}(a,b,c,d)\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_d\end{matrix}\right.\right\rangle = \int_0^1 dw_2\, w_2^{-\alpha_{12}-1}(1 - w_2)^{-\alpha_{23}-1}$$

$$\times \left\langle 0|\exp\left\{\sum_n \frac{a_n}{n^{\frac{1}{2}}}(p_1 + p_4 w_2^{\,n}) + \sum_m \frac{\bar{b}_m}{m^{\frac{1}{2}}}[p_2 + p_1(1 - w_2)^m]\right.\right.$$

$$\left.\left. + \sum_\ell \frac{c_\ell}{\ell^{\frac{1}{2}}}(p_3 + p_2 w_2^{\,\ell}) + \sum_k \frac{\bar{d}_k}{k^{\frac{1}{2}}}[p_4 + p_3(1 - w_2)^k]\right\}\right.$$

$$\times \exp\left\{\sum_{nm} \frac{a_n \bar{b}_m}{(nm)^{\frac{1}{2}}}\binom{m}{n}(-)^n n\, w_2^{\,n} + \sum_{m\ell} \frac{\bar{b}_m c_\ell}{(m\ell)^{\frac{1}{2}}}\binom{\ell}{m}(-)^m m(1 - w_2^{\,m})\right.$$

$$\left. + \sum_{\ell k} \frac{c_\ell \bar{d}_k}{(\ell k)^{\frac{1}{2}}}\binom{k}{\ell}(-)^\ell \ell\, w_2^{\,\ell} + \sum_{nk} \frac{\bar{d}_k a_n}{(kn)^{\frac{1}{2}}}\binom{n}{k}(-)^k k(1 - w_2)^k\right\}$$

$$\times \exp\left\{\sum_{n\ell} \frac{a_n c_\ell}{(n\ell)^{\frac{1}{2}}} B_{-n\ell}(w_2)\; w_2^{\,n} + \sum_{mk} \frac{\bar{b}_m \bar{d}_k}{(mk)^{\frac{1}{2}}} B_{-km}(1 - w_2)\,(1 - w_2)^k\right\}\left.\left|\begin{matrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_d\end{matrix}\right.\right\rangle . \tag{41}$$

To conclude, we see that the four-reggeon vertex function has a rather simple form. However, the fourth-leg propagator has an asymmetrical expression relative to the other three legs. Similar to the three-reggeon vertex case, this asymmetrical fourth leg again indicates the problem of linear dependence.[3] Unless we can find an N-point dual amplitude which automatically takes care of the problem of linear dependence, we will not be able to get a completely factorizable four-reggeon vertex function. On the other hand, the vertex composed of at least one external scalar particle together with an arbitrary number of factorized reggeons is an exact expression. This might give another view of the problem of linear dependence.

The generalization to N-reggeon amplitude has been carried out, but is still to be published. After completion of this work, we understand Carbone and Sciuto have also obtained the four-reggeon amplitude independently.

With an exactly similar technique, we obtain the completely symmetrical N-reggeon amplitude. It takes the following form:

$$W^{(N)}\left(a^{(i)}\right) = \int \prod_{\substack{i \neq N \\ N-1 \\ 1}} dw_i \{W_N\}$$

$$\times \exp \left\{ \sum_{i=1}^{N} \left[\sum_n \frac{a_n^{(i)}}{n^{\frac{1}{2}}} \left(\sum_{j=1}^{N-2} p_{N+i-j} \, P^n(i,i+1,i-1,N+i-j) \right) \right] \right.$$

(cont.)

(cont.)

$$+ \sum_{i=1}^{N} \left[\frac{a_n^{(i+1)} \; a_m^{(i)}}{(nm)^{\frac{1}{2}}} \binom{n}{m} (-1)^m m \; P^m(i,i+1,i-1,i+2) \right]$$

$$+ \sum_{i=1}^{N} \left[\frac{a_n^{(i+1)} \; a_m^{(i-1)}}{(nm)^{\frac{1}{2}}} P^n(i-1,i,i+2,i+1) B_{nm}[P(i-1,i,i+1,i+2)] \right.$$

$$\left. \times \; P^m(i-1,i,i-2,i+1) \right]$$

$$+ \sum_{i=1}^{N} \left[\frac{a_n^{(i+1)} \; a_m^{(i-2)}}{(nm)^{\frac{1}{2}}} P^n(i-2,i,i+2,i+1) \; B_{nm}[P(i-2,i-1,i+1,i+2)] \right.$$

$$\left. \times \; P^m(i-2,i-1,i-3,i+1) \right]$$

$$+ \cdots$$

$$+ \sum_{i=1}^{N} \left[\frac{a_n^{(i+1)} \; a_m^{(i-k)}}{(nm)^{\frac{1}{2}}} P^n(i-k,i,i+2,i+1) \; B_{nm}[P(i-k,i-k+1,i+1,i+2)] \right.$$

$$\left. \times \; P^m(i-k,i-k+1,i-k-1,i+1) \right]$$

$$+ \cdots$$

$$+ \sum_{i=1}^{N} \left[\frac{a_n^{(i+1)} \; a_n^{(i-\frac{N}{2}+1)}}{2(nm)^{\frac{1}{2}}} P^n(i-\tfrac{N}{2}+1,i,i+2,i+1) \; B_{nm}[P(i-\tfrac{N}{2}+1,i-\tfrac{N}{2}+2,i+1,i+2)] \right.$$

$$\left. \times \; P^m(i-\tfrac{N}{2}+1,i-\tfrac{N}{2}+2,i-\tfrac{N}{2},i+1) \right] \Bigg\}$$

where $w_N = \infty$, $w_{N-1} = 1$, $w_1 = 0$,

$$P(x,y,z,w) \equiv \frac{(x - z)(y - w)}{(x - w)(y - z)} ,$$

and N is assumed to be even. The superscript index of the operator a_n coincides with the Koba-Nielsen labeling.

ACKNOWLEDGMENTS

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APPENDIX

When the four-momentum goes to zero, say $p_M \to 0$, the bootstrap condition $a + bp_M^2 = 0$ is no longer satisfied, and the factor $\prod_{\substack{i=2 \\ i \neq N-1}}^{N} (x_i - x_{i+2})^{a+bp_{i+1}^2}$ in Eq. (4) gives an extra contribution[9] of $(1 - y_{M-1}t)^{a+bp_M^2}$ to Eq. (8). This extra factor will make the factorization of y_{M-1} and t unfeasible. To insure that this factor is absent from Eq. (8), we have to modify the spectrum of the trajectories in Eq. (8). We therefore go back to Eq. (4) and concentrate on the troublesome factor $(x_{M-1} - x_{M+1})$, which is raised to the power $\alpha_{M-1,M} - \alpha_{M-1,M+1} + \alpha_{M,M+1}$. If we assume the trajectories $\alpha_{M-1,M}$, $\alpha_{M-1,M+1}$, and $\alpha_{M,M+1}$ have the intercepts $a_{M-1,M}$, $a_{M-1,M+1}$, and $a_{M,M+1}$ respectively, then

$$\alpha_{M-1,M} - \alpha_{M-1,M+1} + \alpha_{M,M+1}$$

$$= a_{M-1,M} + a_{M,M+1} - a_{M-1,M+1} + bp_M^2 - 2bp_{M-1} \cdot p_{M+1} \quad . \qquad \text{(A)}$$

It is the extra power $a_{M-1,M} + a_{M,M+1} - a_{M-1,M+1} + bp_M^2$ which complicates the factorization. Therefore, we can demand

$$a_{M-1,M} + a_{M,M+1} - a_{M-1,M+1} + bp_M^2 = 0 \qquad \text{(B)}$$

for all values of p_M^2. In particular, we are interested in the case $p_M^2 = 0$. Thus, we obtain the condition

$$a_{M-1,M} + a_{M,M+1} - a_{M-1,M+1} = 0 \ . \qquad \text{(C)}$$

Further conditions in Eq. (C) can be obtained by requiring the propagators associated with the factorized channels $\frac{x_M}{x_{M-1}}$ and $\frac{x_{M+1}}{x_M}$ to have the ordinary form $p_M^2 \to 0$. This leads to the requirement

$$a_{M-1,M} = a_{M,M+1} = a_{M-1,M+1} = 0 \quad . \tag{D}$$

Once we assign the intercept 0 to the trajectory $\alpha_{M,M+1}$, the next factor whose power contains the trajectory $\alpha_{M,M+1}$,

$$(x_M - x_{M+2})^{\alpha_{M,M+1}-\alpha_{M,M+2}+\alpha_{M+1,M+2}} \quad , \tag{E}$$

will have an extra power $-a_{M,M+2}$ in Eq. (4). We can eliminate this extra power by assigning an intercept 0 to the trajectory $\alpha_{M,M+2}$ while keeping the value a for the intercept of the trajectory $\alpha_{M+1,M+2}$ in Eq. (E). Then the next factor which contains the trajectory $\alpha_{M,M+2}$ is

$$(x_M - x_{M+3})^{\alpha_{M,M+2}-\alpha_{M,M+3}-\alpha_{M+1,M+2}+\alpha_{M+1,M+3}} \quad . \tag{F}$$

We therefore again require $\alpha_{M,M+3}$ to have zero intercept. Continuing such reasoning, we conclude that the trajectories (assuming $x_N = 0$, $x_1 = \infty$)

$$\alpha_{i,j}, \qquad \begin{matrix} i=2,3,\cdots M, \\ (i \neq j) \end{matrix} \qquad j=M,M+1,\cdots,N-1 \tag{G}$$

must have the intercept zero, while all other trajectories have the intercept a. The requirement of Eq. (G) does not affect the final result. Then our procedure of letting $p_M \to 0$ is justified.

FOOTNOTES AND REFERENCES

* This work was supported in part by the U.S. Atomic Energy Commission.

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9. This is pointed out by Professor S. Mandelstam.

FIGURE CAPTIONS

Fig. 1. The N-point amplitude.

Fig. 2. The Y,Z frames.

Fig. 3. The first factorization.

Fig. 4. The second factorization (planar).

Fig. 5. The second factorization (nonplanar).

Fig. 6. The second factorization (nonplanar).

Fig. 7. Removing top legs.

Fig. 8. The third factorization (planar).

Fig. 9a. The third factorization (planar).

Fig. 9b. The third factorization (planar).

Fig. 10. Removing the r_2 leg.

Fig. 11. Sciuto's three-reggeon vertex.

Fig. 12. The third factorization (nonplanar).

Fig. 13. The fourth factorization (planar).

Fig. 14. Removing the w_4 leg.

Fig. 15. The four-reggeon amplitude.

Fig. 16a. The fourth factorization (nonplanar).

Fig. 16b. The four-reggeon amplitude.

Fig. 17. The symmetrical four-reggeon amplitude.

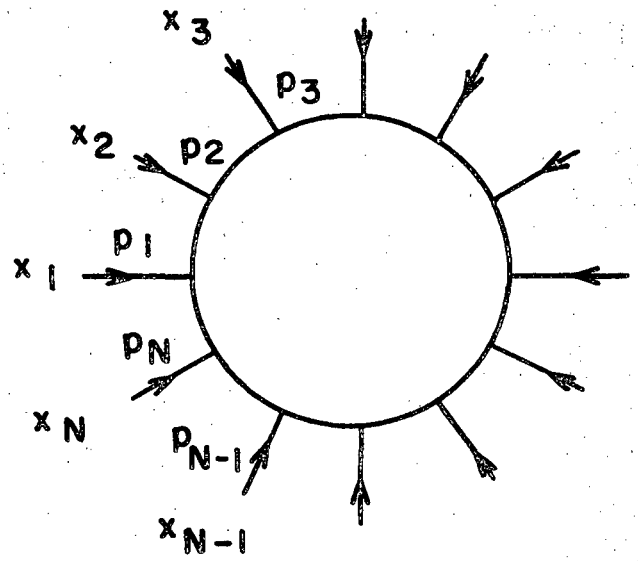

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Fig. 1

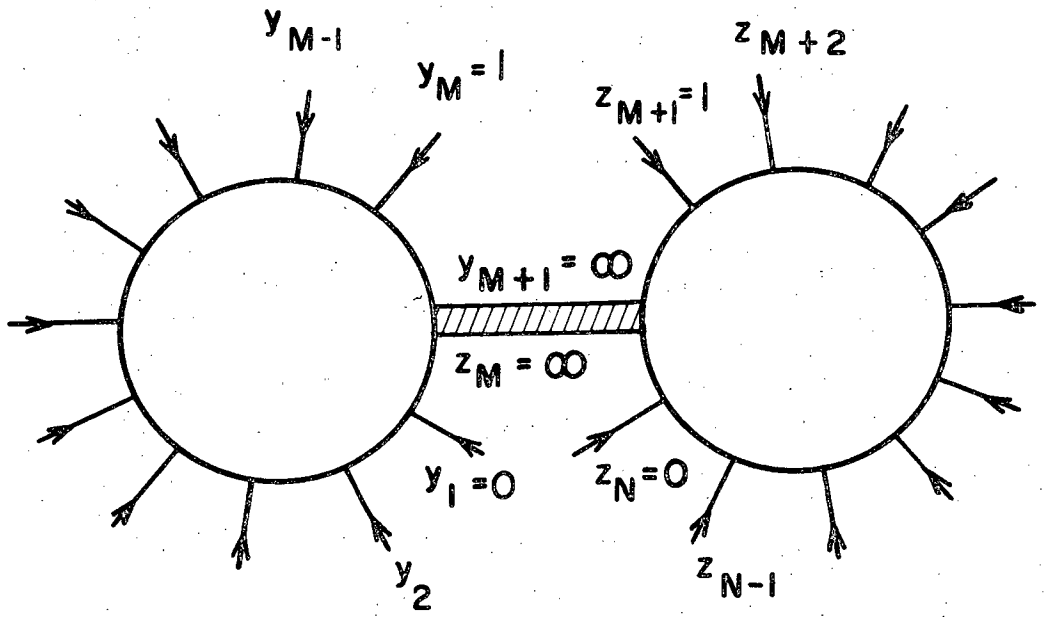

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Fig. 2

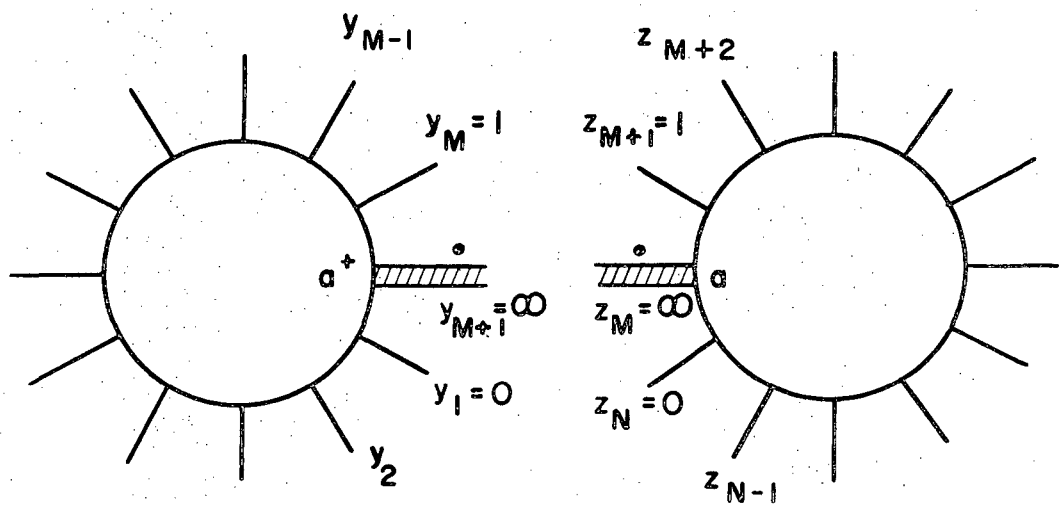

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Fig. 3

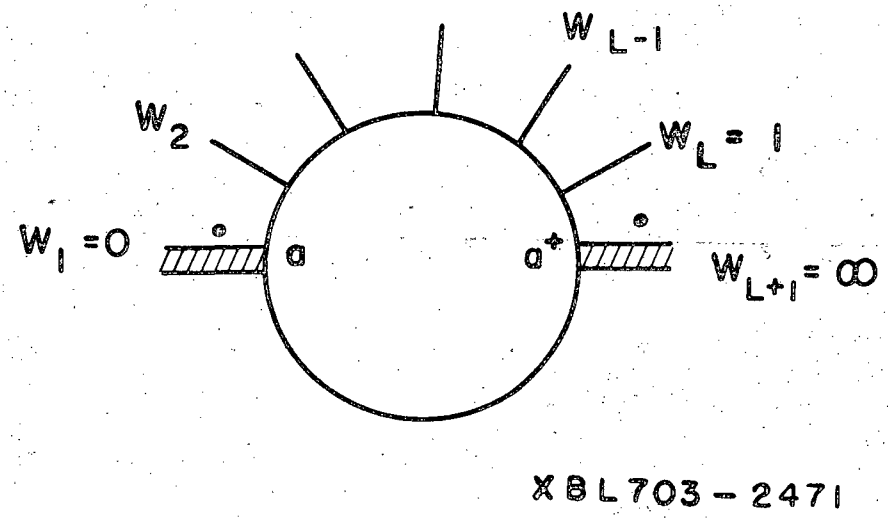
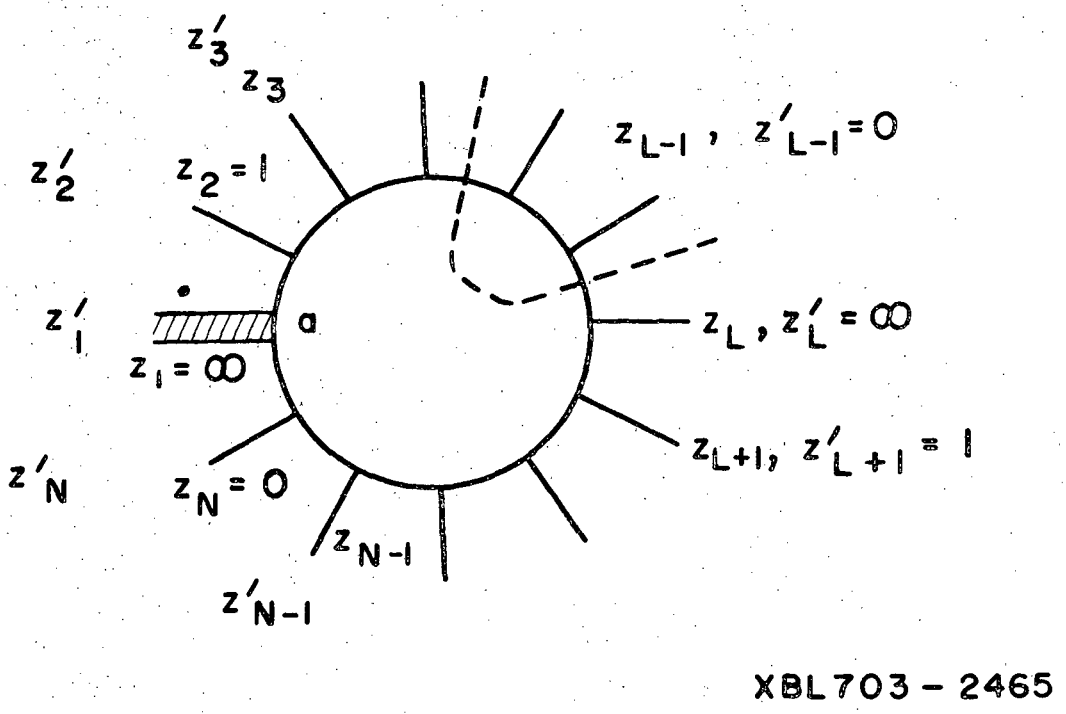


Fig. 4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z'1
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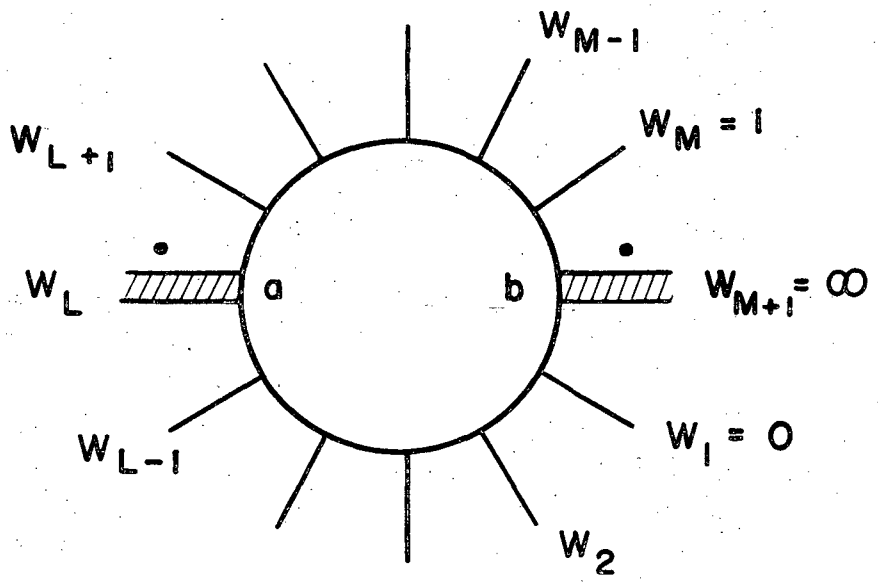
z'N
zN = 0
zN-1
z'N-1
zL-1 , z'L-1 = 0
zL , z'L = ∞
zL+1, z'L+1 = 1
XBL703 - 2465

Fig. 5



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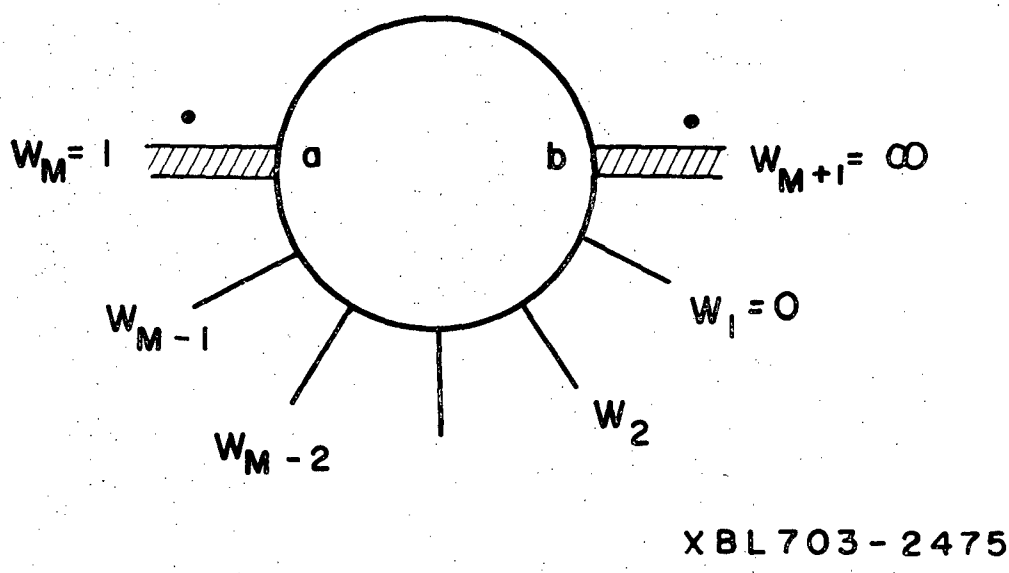

Fig. 7

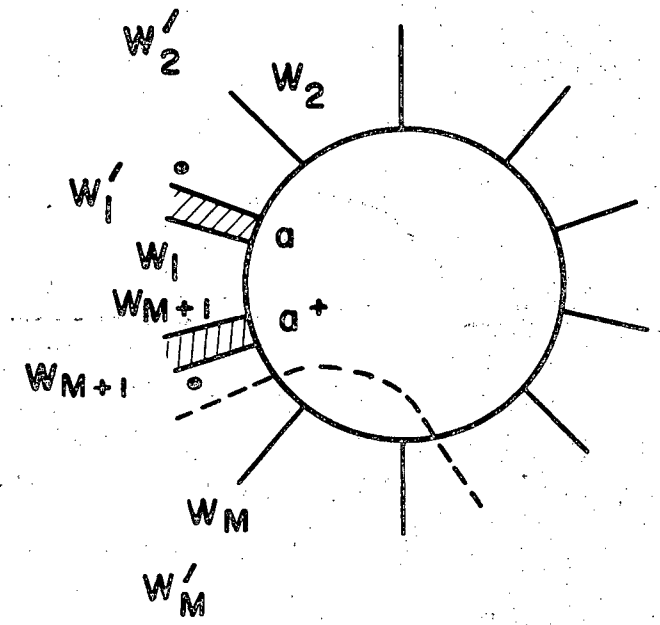

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Fig. 8

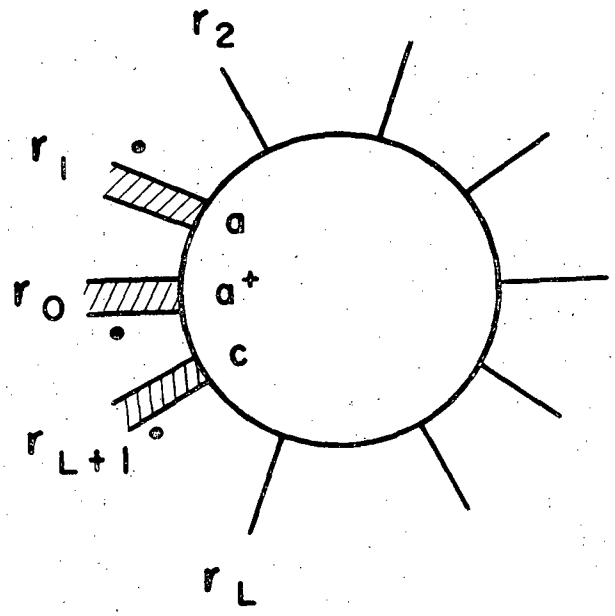

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Fig. 9a

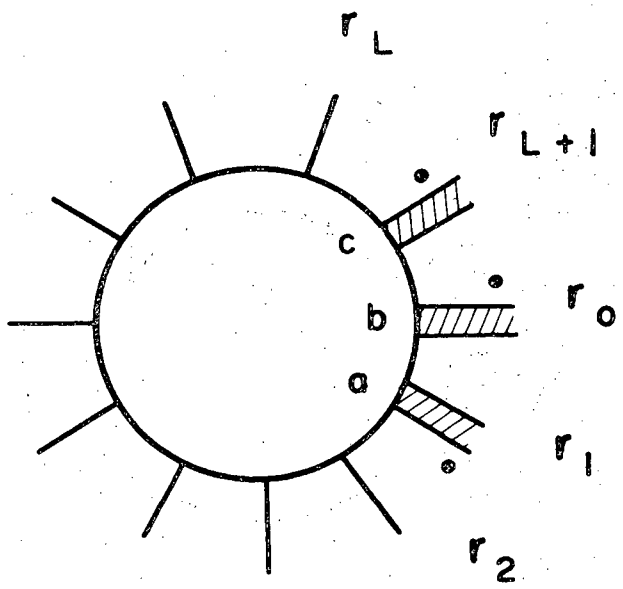

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Fig. 9b

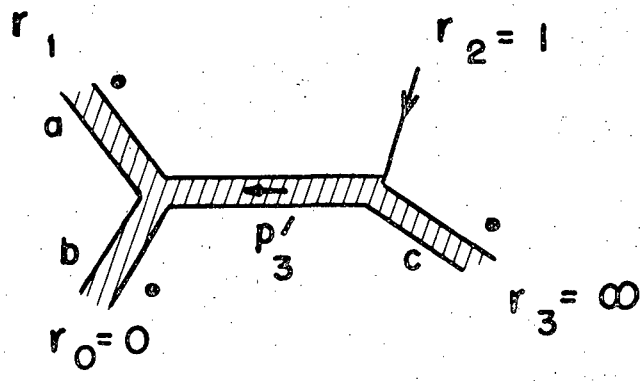

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Fig. 10

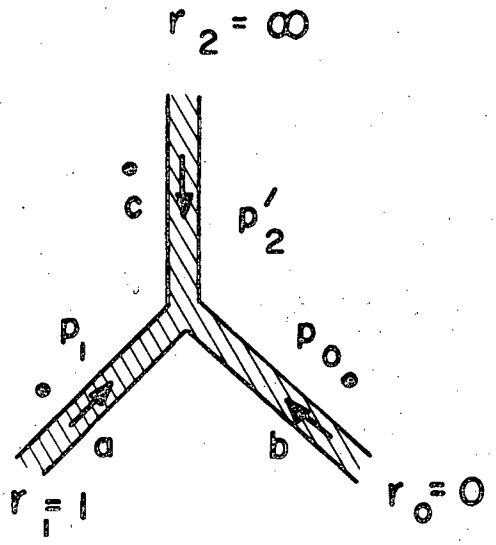

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Fig. 11

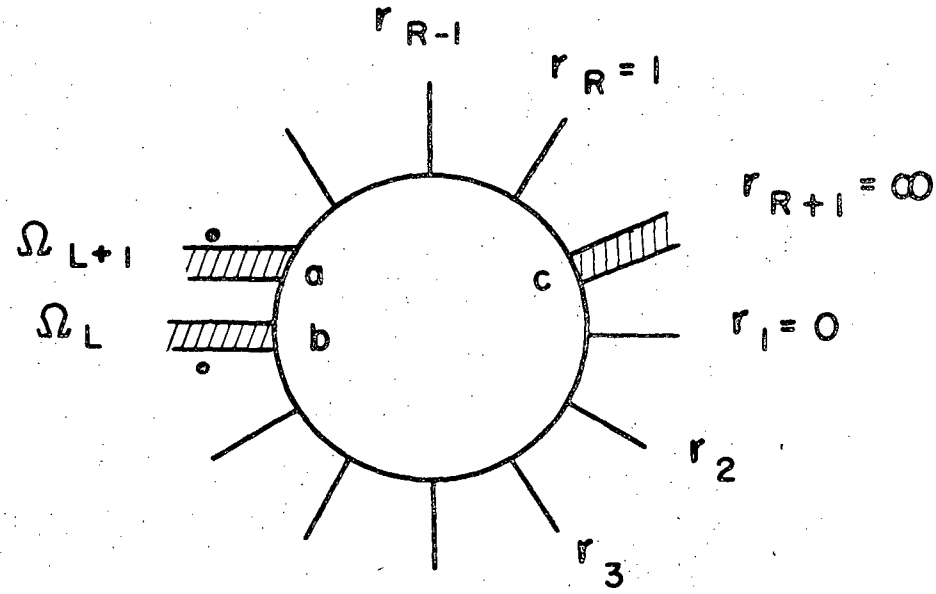

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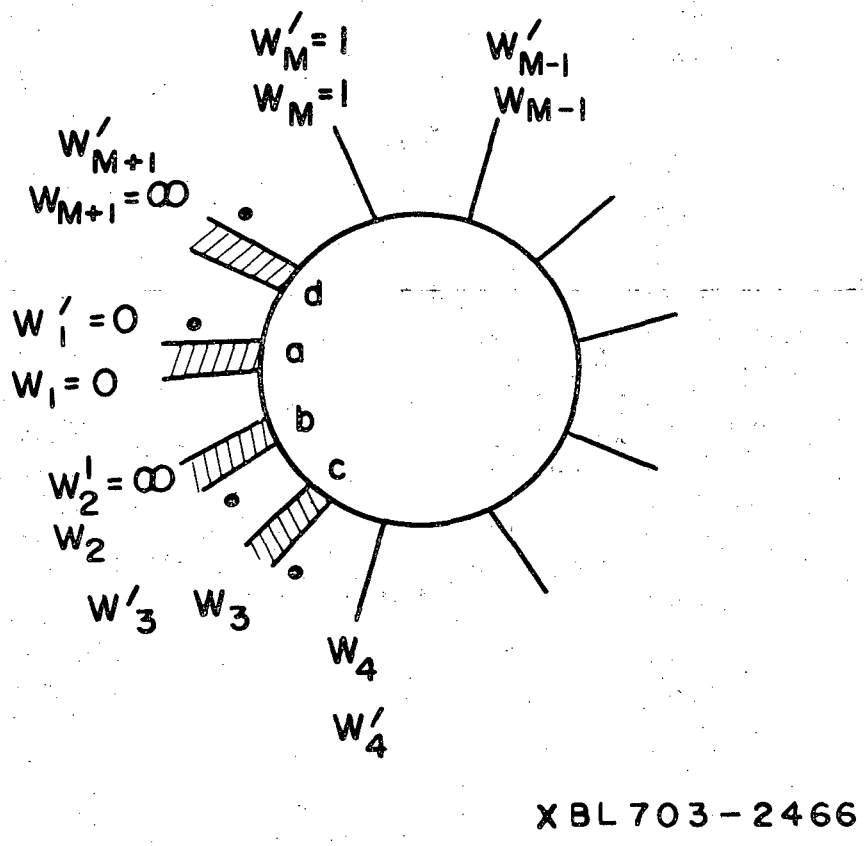
Fig. 12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WM-1
W'M+1
WM+1 = ∞
d
W'1 = 0
a
W1 = 0
b
W'2 = ∞
c
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W'3
W3
W4
W'4
XBL703-2466

Fig. 13

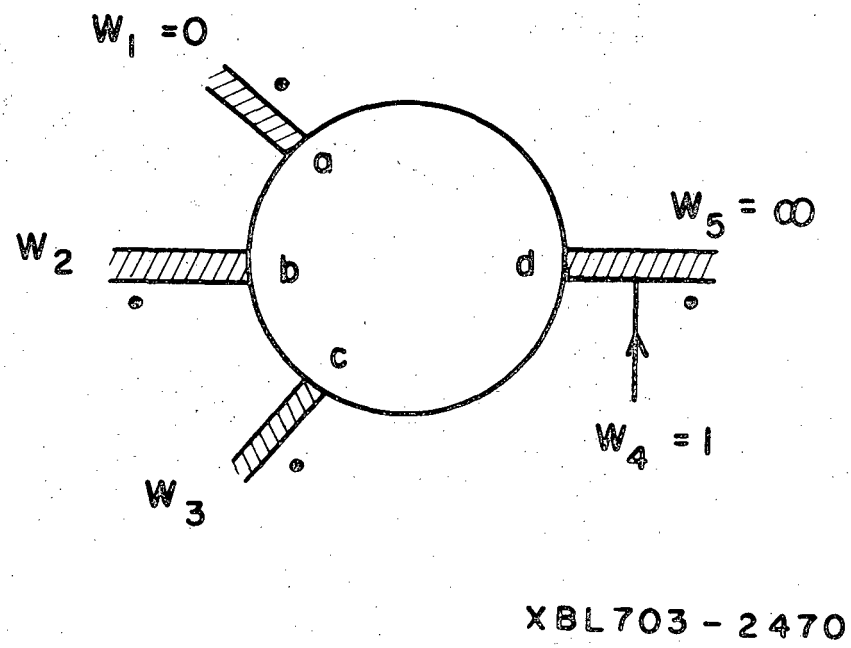

Fig. 14

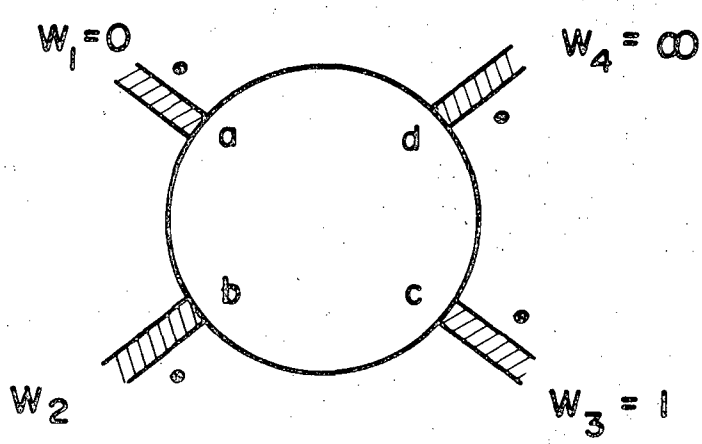

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Fig. 15

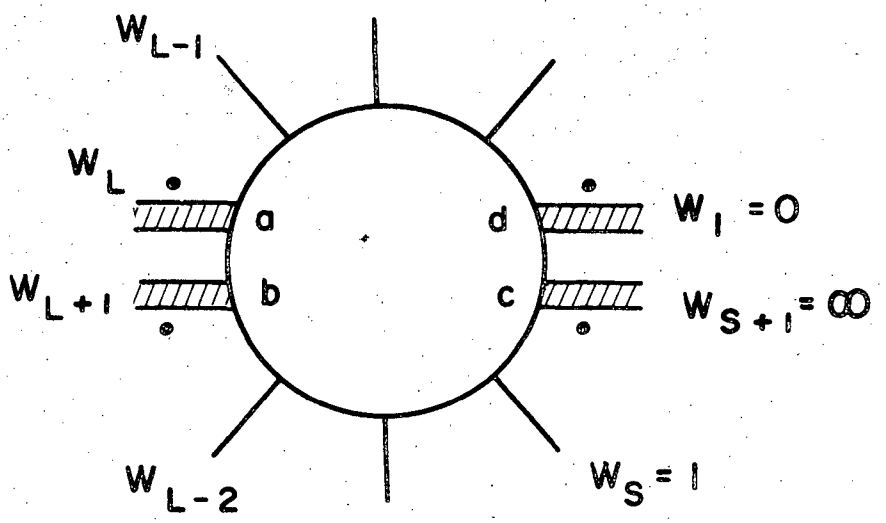

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Fig. 16a

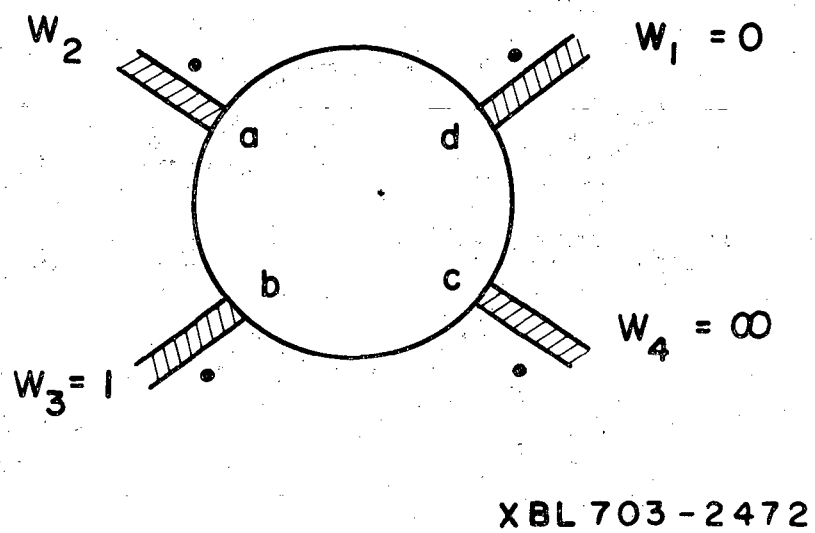
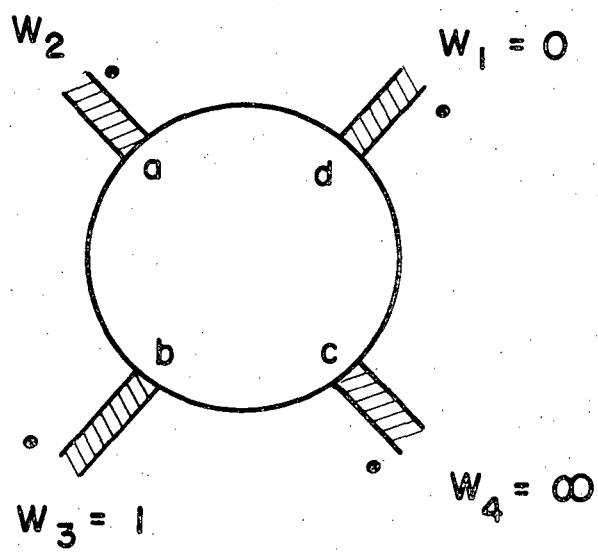


Fig. 16b



XBL703-2467

Fig. 17.

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