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CLOSED COSMOLOGICAL SOLUTIONS TO EINSTEIN'S FIELD EQUATIONS*

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ABSTRACT

A prescription is made for "geometrizing" the space-time manifold as an extension of Wheeler's "wormhole" theory. In the method presented, a set of variables, termed quantal units, is introduced into the cosmological equations and, acting as an additional constraint, gives closed cosmological solutions. Recent experimental evidence is cited for a closed universe theory.

J. A. Wheeler[1] attempted to geometrize the space-time manifold in terms of a "quantum of length, "termed the Wheeler "wormhole," $\ell = (G\hbar/c^3)^{1/2}$. Wheeler pictures the metric of his space as fluctuations in a multiple-connected, foamlike structure in which the microcurvature has a scale "size" of ℓ characteristic of his topology. The microcurvature sets a lower limit on the meaningful intervals of length and time. He also discusses a quantum of mass, $m = (c\hbar/G)^{1/2}$ and a quantum of energy, $E = (\hbar c^5/G)^{1/2}$. Earlier, M. Planck[2] derived dimensioned quantities in terms of the fundamental universal constants $\hbar$ (Planck's constant), G (universal gravitational constant), c (velocity of light). The quantities Planck introduced were:

$$\ell = (G\hbar/c^3)^{1/2} = 1.60\times 10^{33} \text{ cm}, \tag{1a}$$

$$t = (G\hbar/c^5)^{1/2} = 5.36\times 10^{-44} \text{ sec}, \tag{1b}$$

$$m = (c\hbar/G)^{1/2} = 2.82\times 10^{-5} \text{ gm}, \tag{1c}$$

for length, time, and mass, respectively. The values of the universal constants used in evaluating the Planck quantities are: $c = 2.998\times 10^{10}$ cm/sec, $\hbar = 1.055\times 10^{-27}$ erg-sec, and $G = 6.673\times 10^{-8} \frac{\text{cm}^3}{\text{gm-sec}^2}$. These values are taken from the recent work of B. N. Taylor, W. H. Parker, and D. N. Langenberg[3] on the theoretical and experimental implications of the universal constants.

Planck discussed the universality of the expressions in (1a), (1b), (1c), which comes about through their unique expression in terms of the universal constants. The quantities in (1a), (1b), (1c), and all physical variables, can be uniquely expressed in terms of universal constants and, in this form, are here termed "quantal units."

Table I contains the set of quantal units and their numerical values relevant to the calculations in this paper. For a more detailed discussion of the geometrical interpretation of the quantal units, see E. A. Rauscher.[4] Briefly, it is pictured that the quantal units "quantize" the matter-energy and space-time in the form of physical variables in the manifold.

E. R. Harrison[5] discusses some aspects of what he terms "quantum cosmology" in terms of the Planck quantities or quantal units and their limiting values in the space-time manifold, and consequences for the early universe.

Also, the implications of the universal constants for cosmology have been investigated by P. A. M. Dirac[6] and more recently by R. A. Alpher and G. Gamow,[7] who have developed the atomic, nuclear, and

cosmological aspects of the universal constants, on a theoretical basis, as dimensionless combinations of universal constants. There has been much interest in the recent work of B. A. Taylor, W. H. Parker, and D. A. Langenberg[3] on the theoretical implications of the universal constants.

Now we shall show how the quantal force, $F = c^4/G$ is prominently manifest in Einstein's field equations and how the quantal units act as an additional constraint to give closed cosmological solutions.

We deal in this paper with an idealized universe that is isotropic and homogeneous. Consistent with this, we use the Robertson uniform line-element[8]

$$ds^2 = c^2dt^2 - \frac{\mathcal{R}^2(t)}{(1+1/4kr^2)^2}(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2). \qquad (2)$$

For this metric, the intervals of cosmic time, t are measured along world-lines orthogonal to a spatial hypersurface of uniform curvature which is mapped with r, θ, ϕ co-moving coordinates. The curvature constant of the metric, $k = 0, \pm 1$, corresponds respectively to Euclidian, closed, and open curvature.

Let us proceed from the general relatively equation, with the constraint that the cosmoligical constant, $\Lambda = 0$,

$$\mathcal{R}_{ik} - 1/2 g_{ik}\mathcal{R}_{kk} = -\frac{8\pi G}{c^4}T_{ik}. \qquad (3)$$

For $8\pi G/c^4 = 2.07\times 10^{-48}(\text{Gauss-cm})^{-2}$ and for $F = c^4/G = 1.22\times 10^{49}$ dynes, we have $8\pi G/c^4 = 8\pi/F$. The stress-energy tensor, T_{ik} for the idealized model is given as $T_{44} = \rho c^2$ for density, ρ and $T_{11} = T_{22} = T_{33} = \mathcal{P}$ for the isotropic pressure, $\mathcal{P}$. The term $\frac{8\pi G}{c^4}T_{ik}$ in (3) then becomes,

$$\frac{8\pi G T_{ik}}{c^4} = \frac{8\pi}{F}\,\frac{F^2}{\mathscr{t}'} = \frac{F}{\mathscr{t}'} \tag{4}$$

where $\mathscr{t}'$ is defined in Table I.

Using (2) and (3), the solution of this equation gives,

$$\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} = \frac{8\pi G\rho}{3} - \frac{kc^2}{\mathcal{R}^2} \tag{5a}$$

and

$$\frac{8\pi G\mathscr{P}}{c^2} = \frac{2\mathcal{R}\ddot{\mathcal{R}} + \dot{\mathcal{R}}^2 + kc^2}{\mathcal{R}^2}, \tag{5b}$$

where k is the curvature constant, and the dots denote differentiation with respect to time.

Let us now substitute the quantal unit of density, $\rho = F^2/c^2\mathscr{t}'$ in (5a) and the quantal unit of pressure, $\mathscr{P} = F^2/\mathscr{t}'$ in (5b). Also let us make the additional substitutions of $\mathcal{R} = \ell = (\mathscr{t}'/F)^{1/2}$, the quantal length; $\dot{\mathcal{R}} = c$, the quantal velocity, and $\ddot{\mathcal{R}} = (c^2F/\mathscr{t})^{1/2}$, the quantal acceleration.

First, considering (5a), we have $\dot{\mathcal{R}}^2/\mathcal{R}^2 = (c/\ell)^2 = c^5/\hbar G = F/\mathscr{t} = 1/t^2$, and $8\pi G\rho/3 = 8\pi cF/3\hbar = 8\pi F/3\mathscr{t}' = 2.67\times 10^{87}\ 1/\mathrm{sec}^2$ and $c^2/\mathcal{R}^2 = (c/\ell)^2$. Upon substitution of these quantities in (5a), we have

$$\left(\frac{c}{\ell}\right)^2 = \frac{8\pi F}{3\mathscr{t}} - k(c/\ell)^2, \tag{6}$$

and substituting $(c/\ell)^2 = 1/t^2 = F/\mathscr{t}$ in (6), we get $F/\mathscr{t} = 8\pi/3 \cdot F/\mathscr{t} - k(F/\mathscr{t})$, so that $1 - 8\pi/3 = -k$ or $k = 8\pi/3 - 1 \approx 8.4 - 1 \approx 7.4$ or $k \geqslant 1$. This is a positive curvature solution.

Considering (5b) and substituting the quantal form of the variables $\mathscr{P}$, $\mathcal{R}$, $\dot{\mathcal{R}}$, $\ddot{\mathcal{R}}$, we have $8\pi G\mathscr{P}/c^2 = (8\pi c^2/F)\cdot(\mathscr{P}) = (8\pi c^2/F)\cdot(F^2/\mathscr{t}') = 8\pi F/\mathscr{t}$ for the left of (5b). For the right side of (5b), we have $2\mathcal{R}\ddot{\mathcal{R}}/\mathcal{R}^2 = 2a/\ell = 2F/\mathscr{t}$ for the first term, $\dot{\mathcal{R}}^2/\mathcal{R}^2 = (c/\ell)^2 = F/\mathscr{t}$ for the second term, and

$c^2/\mathcal{R}^2 = (c/\ell)^2 = F/\ell$ for the third term. Thus, upon substitution (5b) becomes: $8\pi F/\ell = 2F/\ell + F/\ell + k(F/\ell)$ or $8\pi + 1 = k$ or $k \approx 26.3$. As we see, we have obtained a larger positive value of k from the second solution than from the first. Equation (5b) appears to be a stricter criterion on the curvature of space-time structure.

For both cosmological solutions to the general relativity equation, the extra constraint of the universal constants in quantal unit form give closed (positively curved) cosmological solutions.

This model could therefore be used to describe a continuously oscillating universe. For a recent discussion of such models see I. M. Khalatnikov and E. M. Lipshitz.[9] In order to have such a universe, one would have to avoid the singular state in the early universe.[10] Then contractions and expansions from a state of finite, large density, perhaps related to the quantal density $\rho = 6.50 \times 10^{93}$ gm/cm^3, could occur. It is possible that ℓ represents a lower limit, in the manifold, on the Schwarzschild radius, $\mathcal{R}_s$ which is defined as that radius of an object of mass, m undergoing gravitational collapse,

$$\mathcal{R}_s = \frac{2Gm}{c^2}. \tag{7}$$

For the quantal mass, $m = (c\hbar/G)^{1/2}$, we have

$$\mathcal{R}_s = 2\left(\frac{G\hbar}{c^3}\right)^{1/2} = 2\ell,$$

where ℓ is the quantal length. The "geometrical" structure of the space-time manifold may prevent a completely singular state from occurring at any particular point in the manifold. It may also be noted that the gravitational red shift, $z = \Delta\lambda/\lambda = Gm/\mathcal{R}c^2 = 1$ for $\mathcal{R} = \ell$, where m is the quantal mass. The quantity $\Delta\lambda$ is the shift in the emission wave length λ.

It is suggested that the quantal power, $\mathcal{P} = cF = 3.66\times 10^{59}$ erg/sec, has cosmological significance; for example, the energy required to expand a dynamical universe to the present state from a "big bang" origin is $E = p \cdot t_0 \approx 4\times 10^{78}$ ergs, where the age of the universe is $t_0 \approx 2\times 10^{17}$ sec. The quantal power times the age of the universe is 4×10^{76} ergs; this energy is a factor of 10^2 less than the energy required to expand the universe. Some quantal unit forms of physical variables, such as ℓ and t, have microscopic implications in the manifold, and others, such as F and $\mathcal{P}$, give the macroscopic curvature. For a discussion of the relationship between microcurvature and macrocurvature and its relevance to the geometry of the space-time manifold, see Ref. 11.

Recent experimental evidence supports closed universe models such as the continuously oscillatory ones. Previous experiments suggested that the average density of the universe was about 6×10^{-31} gm/cm^3. The critical density for a closed universe is $\rho_c = 3H^2/8\pi G = 2\times 10^{-29}$ gm/cm^3 (where Λ and k are zero). The ratio $\dot{\mathcal{R}}/\mathcal{R} = H$, is Hubble's constant, and for $t_0 \leq 1/H$, $H = 3\times 10^{-18}$ cm/sec/cm. Thus, the detected matter is only 3% of that necessary for a closed universe. Through x-ray studies, H. Friedman, R. C. Henry and others [12,13] have detected intergalactic gas amounting to the remaining required 97%. This lends much credence to the closed universal models as opposed to the open ones.

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FOOTNOTE AND REFERENCES

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Table I. Universal quantal units.

		Quantal unit in terms of force, $\mathscr{t}$ and $\mathscr{t}'$ [a]	Numerical value of quantal unit[b]	
$\ell = (\frac{G\hbar}{c^3})^{1/2}$	length	$\ell = (\frac{\mathscr{t}'}{F})^{1/2}$	1.60×10^{-33}	cm
$t = (\frac{G\hbar}{c^5})^{1/2}$	time	$t = (\frac{\mathscr{t}}{F})^{1/2}$	5.36×10^{-44}	sec
$m = (\frac{c\hbar}{G})^{1/2}$	mass	$m = (\frac{\mathscr{t}F}{c^2})^{1/2}$	2.82×10^{-5}	gm
$E = (\frac{c^5\hbar}{G})^{1/2}$	energy	$E = (\mathscr{t}'F)^{1/2}$	1.25×10^{16}	ergs
$p = (\frac{c^3\hbar}{G})^{1/2}$	momentum	$p = (\mathscr{t}F)^{1/2}$	4.16×10^{10}	$\frac{\text{gm-cm}}{\text{sec}}$
$L = \hbar$	angular momentum	$L = \hbar$	1.06×10^{-27}	erg-sec
$F = c^4/G$	force	$F = F$	1.22×10^{49}	dynes
$c = c$	velocity	$c = c$	3.00×10^{10}	cm/sec
$a = (\frac{c^7}{G\hbar})^{1/2}$	acceleration	$a = (\frac{c^2F}{\mathscr{t}})$	5.72×10^{53}	cm/sec^2
$\mathscr{P} = \frac{c^5}{G}$	power	$\mathscr{P} = cF$	3.66×10^{59}	dyne $\frac{\text{cm}}{\text{sec}}$
$\mathcal{P} = \frac{c^7}{G^2\hbar}$	pressure	$\mathcal{P} = \frac{F^2}{\mathscr{t}'}$	4.75×10^{114}	dyne/cm^2
$\rho = \frac{c^5}{G^2\hbar}$	density	$\rho = \frac{F^2}{c^2\mathscr{t}}$	6.50×10^{93}	gm/cm

[a] The quantal units are expressed in terms of the universal quantal force, $F = c^4/G$, $\mathscr{t}$, $\mathscr{t}'$, and c. The quantities, $\mathscr{t}$ and $\mathscr{t}'$ are defined as $\mathscr{t} = \hbar/c$ and $\mathscr{t}' = c\hbar$.

[b] In the evaluation of the quantal units, the values of $\mathscr{t} = 3.50\times10^{-38}$ gm-cm and $\mathscr{t}' = 3.15\times10^{-17}$ erg-cm have been used.

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