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EFFECT OF DRIFT TUBE TOLERANCES ON THE ELECTRIC FIELD DISTRIBUTION ALONG THE LENGTH OF AN ALVAREZ CAVITY*

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ABSTRACT

An analysis is presented that shows how small random errors of drift tube diameters, lengths, and positions perturb the electric field distribution in an Alvarez cavity. The field distribution due to the tolerances is calculated from the change of the resonance frequency of individual cells, taking the correlation of the detuning of adjacent cells into account. For all cells, the rms field errors and their correlations are calculated. From this is deduced the maximum rms tilt and the most probable distribution of the error fields throughout the cavity. The change of the resonance frequency of the whole cavity is also calculated. Using a different representation, the rms values of the Fourier coefficients of the field, and their correlations, are given.

Introduction

In order to obtain a sufficiently constant synchronous phase of the particles in an Alvarez cavity, one can allow only very small deviations of the electric field strength from its design value along the length of the cavity. We describe here the procedure that was used to calculate first order effects of errors in drift tube dimensions and locations on the field strength distribution in the eight cavities comprising the SuperHilac.[1] Since the effects of systematic errors follow trivially from the formulas given below, we discuss only the effects of random errors. The effects of the tolerances are expressed in two different ways: First we calculate directly the rms distribution of the field strength errors, and then, following a suggestion by F. Selph and L. Smith, the rms values of the Fourier coefficients describing the error field distribution. Although the second method of description could be considered more adequate for the specific problem discussed here, the first method is outlined in detail since it is also applicable to many other problems.

Model and Notation

Our cavities operate in the 0-mode and are terminated at each end by half a drift tube. We divide the cavity of length L into cells labelled 1, 2 · · · N. Cell n has length L_n and extends from x_{n-1} to x_n, where x_{n-1} and x_n are the distances of the centers of two adjacent drift tubes from one end of the cavity ($x_0=0$; $x_N=L$).

The drift tube tolerances lead to local errors of the resonance frequency, and the relative deviation of the electric field from its average value, $y(x) = \Delta E(x)/E$, is related to the local relative error in resonance frequency $u(x) = \Delta\omega(x)/\omega$ through[2]

$$d^2y(x)/dx^2 = K^2\left(u(x) - \tilde{u}\right). \tag{1}$$

$$\tilde{u} = \int_0^L u(x)dx/L,$$

and for our lightly loaded cavities we use for K^2 the value[3]

$$K^2 = 100/\lambda^2, \tag{2}$$

where λ is the vacuum wave length corresponding the frequency. Eq. (1) is of course only an approximation, but describes very well the long range effects of interest here. From Eq. (1) follows for y, satisfying $\int_0^L y(x)dx = 0$ and $y'(0) = 0$ (and consequently also $y'(L) = 0$):

$$y(x) = K^2\left\{\int_0^x u(t)\cdot(x-t)dt - \tilde{u}\,\frac{x^2}{2} - \left(\int_0^L u(t)\,\frac{(L-t)^2}{2}\,dt - \frac{L^3}{6}\,\tilde{u}\right)/L\right\} \tag{3}$$

Within each cell, the relative error of the resonance frequency is assumed constant and will be expressed in terms of the dimensional and positional errors of the drift tubes. It is convenient to express u(x) through

$$u(x) = L\cdot\sum_1^N v_n(x)\cdot a_n \tag{4a}$$

$$v_n(x) \begin{cases} = 1/L_n & \text{for } x_{n-1} \le x \le x_n \\ = 0 & \text{for } x_{n-1} > x > x_n \end{cases} \tag{4b}$$

$$a_n = \frac{\Delta\omega_n}{\omega}\cdot\frac{L_n}{L} \tag{4c}$$

This gives for $\tilde{u}$:

$$\tilde{u} = \sum_1^N a_n. \tag{5}$$

Method Of Direct Calculation Of Field Errors

We calculate y for all x_n and define a column vector y through

$$y_m = y(x_m). \quad m = 0, 1, 2 \cdots N. \tag{6}$$

Since the model does not describe the fine structure of y(x) well, it would not be meaningful to calculate y(x) at more than the N+1 chosen points. Defining also a

column vector a with components a_n, y can be expressed by

$$y = M\,a. \tag{7}$$

The elements of the matrix M are obtained from Eq. (3):

$$M_{mn} = \left(\frac{L^2}{6} - \frac{(L-c_n)^2}{2} - \frac{L_n^2}{24} - \frac{x_m^2}{2} + L\cdot(x_m - c_n)\cdot S(x_m - c_n)\right) K^2 \tag{8a}$$

$$c_n = (x_{n-1} + x_n)/2 \tag{8b}$$

$$S(x) \begin{cases} = 1 \text{ for } x > 0 \\ = 0 \text{ for } x < 0 \end{cases} \tag{8c}$$

Indicating the transpose of a vector or matrix by the superscript T, we now form $y\,y^T$ and take the statistical average, indicated by a bar:

$$\overline{y\,y^T} = M\,A\,M^T, \tag{9a}$$

where

$$A = \overline{a\,a^T}.$$

The convenience of this formulation lies in the fact that the statistical aspects of the problem are clearly separated from the system properties described by M; that Eq. (9) is easily evaluated by a computer; and that the results are easily interpreted.

Calculation Of A

Because of the large ratio (up to 1:18) of our drift tube radius to cavity radius, the existing fixed mesh computer programs for computation of the resonance frequency of individual cells converged much too slowly to be practical. A new program, RFISH, was developed by R. Holsinger, R. Yourd and the author to solve this eigenvalue problem. From the axis to an intermediate radius an overrelaxation method is used in the irregular triangular mesh used for evaluation of magnetostatic fields,[4] and the rest of the cavity is evaluated with analytical methods. After the problem has converged the program also integrates H^2 over the cavity volume and E^2-H^2 over the surfaces of interest so that Slater's formula[5] can be used to calculate the shift of the resonance frequency due to changes of the cavity geometry. With the sensitivity coefficients, a_n is a known linear function of the changes of length, radius, and position of drift tubes n-1 and n. Conversely, the geometrical changes of drift tube n appear only in the expression for a_n and a_{n+1}. Since we are dealing here only with random geometrical errors, $\overline{a_n a_m} = 0$ for $|n-m| > 1$, and $\overline{a_n a_m}$ is known linear function

of the variances of the geometrical tolerances for $n - m = 0, \pm 1$.

Interpretation Of Results

To simplify the interpretation of the results, we do not print out the matrix $\overline{y\,y^T}$, but the matrix Y, defined as follows:

The diagonal elements of Y are the rms value of y:

$$Y_{nn} = (\overline{y_n^2})^{1/2}. \tag{10a}$$

The elements to the left of the diagonal are the correlation coefficients

k_{mn} ($|k_{mn}| \leq 1$):

$$Y_{mn} \equiv k_{mn} = \overline{y_m y_n} \cdot (\overline{y_m^2} \cdot \overline{y_n^2})^{-1/2}; \; m > n \tag{10b}$$

The elements to the right of the diagonal give the rms value of the difference between y at the two locations:

$$Y_{mn} = \left(\overline{(y_m - y_n)^2}\right)^{1/2} = \left(\overline{y_m^2} + \overline{y_n^2} - 2\,\overline{y_m y_n}\right)^{1/2}; \; m < n. \tag{10c}$$

To find the maximum rms tilt, the computer program makes a search and prints out the largest value of Y_{mn} for $m < n$. The largest tilt always occurs, as one expects, between the two ends of the cavity. Table I shows the matrix Y for the second post-stripper cavity of the SuperHilac, described in Ref. 1. The assumed rms tolerances for drift tube radius, length and position are 0.1 mm, 0.3 mm, 1 mm. Figure 1 shows $\left(\overline{y(x_m)^2}\right)^{1/2}$ vs. x_m. We also made for each cavity a computer run for each of the tolerances individually to obtain information about their relative importance. Computer time on the CDC 6600 for each run is of the order 1 sec.

Since the values y_m are due to many small errors, the central limit theorem applies. This means specifically that for

$$y_m = y_{mo}: \tag{11a}$$

$$\overline{y}_n = y_{mo} \cdot k_{mn} \cdot (\overline{y_n^2} / \overline{y_m^2})^{1/2}, \tag{11b}$$

$$\overline{(y_n - \overline{y}_n)^2} = \overline{y_n^2} \cdot (1 - k_{nm}^2) \tag{11c}$$

To get a feeling of the shape of y(x) that one can expect, Fig. 2 shows

$$\overline{y_n}/(\overline{y_o^2})^{1/2} \text{ and } \left(\overline{y_n} \pm \left(\overline{(y_n - \overline{y}_n)^2}\right)^{1/2}\right) / \left(\overline{y_o^2}\right)^{1/2} \text{ for } y_{oo} = \left(\overline{y_o^2}\right)^{1/2}.$$

Error Of Resonance Frequency Of Cavity

The relative error of the resonance frequency of the whole cavity due to the

drift tube tolerances is important because all our power amplifiers are driven by the same master oscillator. The relative frequency error of the whole cavity is given by[2]:

$$\Delta\omega/\omega = \tilde{u} = \sum a_n$$

From this follows for the variance of $\Delta\omega/\omega$:

$$\overline{(\Delta\omega/\omega)^2} = \sum_{n,m} \overline{a_n a_m} = \sum_{n,m} A_{nm} \tag{12}$$

Fourier Coefficients Of y(x)

Since $dy(x)/dx=0$ for $x=0$ and $x=L$ (Ref. 2), $y(x)$ can be expanded into a well converging Fourier series:

$$y(x) = \sum f_m \cos(mkx); \; k = \pi/L \tag{13}$$

From Eq. 1 one obtains for f_m:

$$f_m = -\frac{2}{m^2}\frac{K^2}{L\,k^2}\int_0^L \cos(mkx)\cdot u(x)dx$$

Using Eq.'s (4) and introducing the column vector f with components f_m, one can write

$$f = F\,a \tag{14a}$$

$$F_{mn} = -\frac{2K^2}{k^2}\cdot\frac{\cos(mkc_n)}{m^2}\cdot\frac{\sin(mkL_n/2)}{mkL_n/2}. \tag{14b}$$

As was done above, we now form $\overline{ff^T}$ and obtain

$$\overline{ff^T} = F\,A\,F^T \tag{15}$$

Table II represents the results of this computation for the cavity discussed above. As in Table I, the matrix $\overline{ff^T}$ is not printed directly, but printed are the square roots of the diagonal elements of $\overline{ff^T}$, and the correlation coefficients. Only the first few coefficients are of practical significance since the diagonal elements of the printed matrix behave very nearly like m^{-2}. The correlation coefficients are fairly small, so that they can be neglected for all practical purposes. Knowing this from the computer results, it seems quite possible to obtain results that are sufficiently accurate for practical purposes by calculating $\overline{f_m^2}$ with analytical approximations.

Acknowledgement

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2. L. Smith, Encyclopedia of Physics, Vol. 44, (1959); p. 380.
3. E. L. Hubbard, Adv. in Electronics and Electron Physics, Vol. 25 (1968); p. 65.
4. A. M. Winslow, J. Comp. Phys., Vol. 1 (1967); p. 149.
5. J. C. Slater; Microwave Electronics; Van Nostrand 1950; p. 81.

- Table I Y - matrix for Alvarez cavity -

	0	1	2	3	4	5	6	7	8	9	10	11
0	1.8E-04	1.5E-05	4.5E-05	8.1E-05	1.2E-04	1.6E-04	2.1E-04	2.5E-04	2.9E-04	3.2E-04	3.4E-04	3.5E-04
1	1.0E+00	1.7E-04	3.2E-05	7.0E-05	1.1E-04	1.5E-04	2.0E-04	2.4E-04	2.8E-04	3.1E-04	3.3E-04	3.4E-04
2	9.8E-01	9.9E-01	1.5E-04	4.0E-05	8.4E-05	1.3E-04	1.8E-04	2.2E-04	2.6E-04	2.9E-04	3.2E-04	3.2E-04
3	9.3E-01	9.5E-01	9.8E-01	1.2E-04	4.6E-05	9.4E-05	1.4E-04	1.9E-04	2.3E-04	2.6E-04	2.9E-04	3.0E-04
4	8.1E-01	8.3E-01	8.8E-01	9.5E-01	8.7E-05	5.0E-05	1.0E-04	1.5E-04	1.9E-04	2.3E-04	2.5E-04	2.6E-04
5	4.3E-01	4.6E-01	5.3E-01	6.5E-01	8.4E-01	5.5E-05	5.2E-05	1.0E-04	1.5E-04	1.9E-04	2.1E-04	2.3E-04
6	-4.9E-01	-4.8E-01	-4.3E-01	-3.1E-01	-6.2E-02	4.7E-01	4.6E-05	5.3E-05	1.0E-04	1.4E-04	1.7E-04	1.8E-04
7	-9.1E-01	-9.2E-01	-9.1E-01	-8.7E-01	-7.4E-01	-2.9E-01	6.9E-01	7.3E-05	5.1E-05	9.5E-05	1.3E-04	1.4E-04
8	-9.5E-01	-9.6E-01	-9.8E-01	-9.8E-01	-9.1E-01	-6.0E-01	3.8E-01	9.2E-01	1.1E-04	4.6E-05	8.1E-05	9.5E-05
9	-9.3E-01	-9.4E-01	-9.7E-01	-9.8E-01	-9.6E-01	-7.2E-01	2.0E-01	8.1E-01	9.7E-01	1.4E-04	3.7E-05	5.4E-05
10	-8.9E-01	-9.1E-01	-9.4E-01	-9.6E-01	-9.5E-01	-7.6E-01	1.0E-01	7.4E-01	9.2E-01	9.9E-01	1.7E-04	1.8E-05
11	-8.8E-01	-8.9E-01	-9.2E-01	-9.5E-01	-9.4E-01	-7.7E-01	7.4E-02	7.1E-01	9.0E-01	9.7E-01	1.0E+00	1.8E-04

Table II rms Fourier Coefficients and Their Correlation Coefficients

	1	2	3	4
1.	1.7E-4			
2.	-1.5E-2	4.3E-5		
3.	9.4E-3	-9.6E-3	1.9E-5	
4.	6.7E-3	8.3E-3	-4.6E-3	1.1E-5

Figure Captions

Fig. 1 rms field errors vs. x/L

Fig. 2 $\overline{y_n}/y_{oo}$ and $\left(\overline{y_n} \pm \left(\overline{(y_n - \overline{y_n})^2}\right)^{1/2}\right) / y_{oo}$

vs. x/L for $y_{oo} = (\overline{y_o^2})^{1/2}$

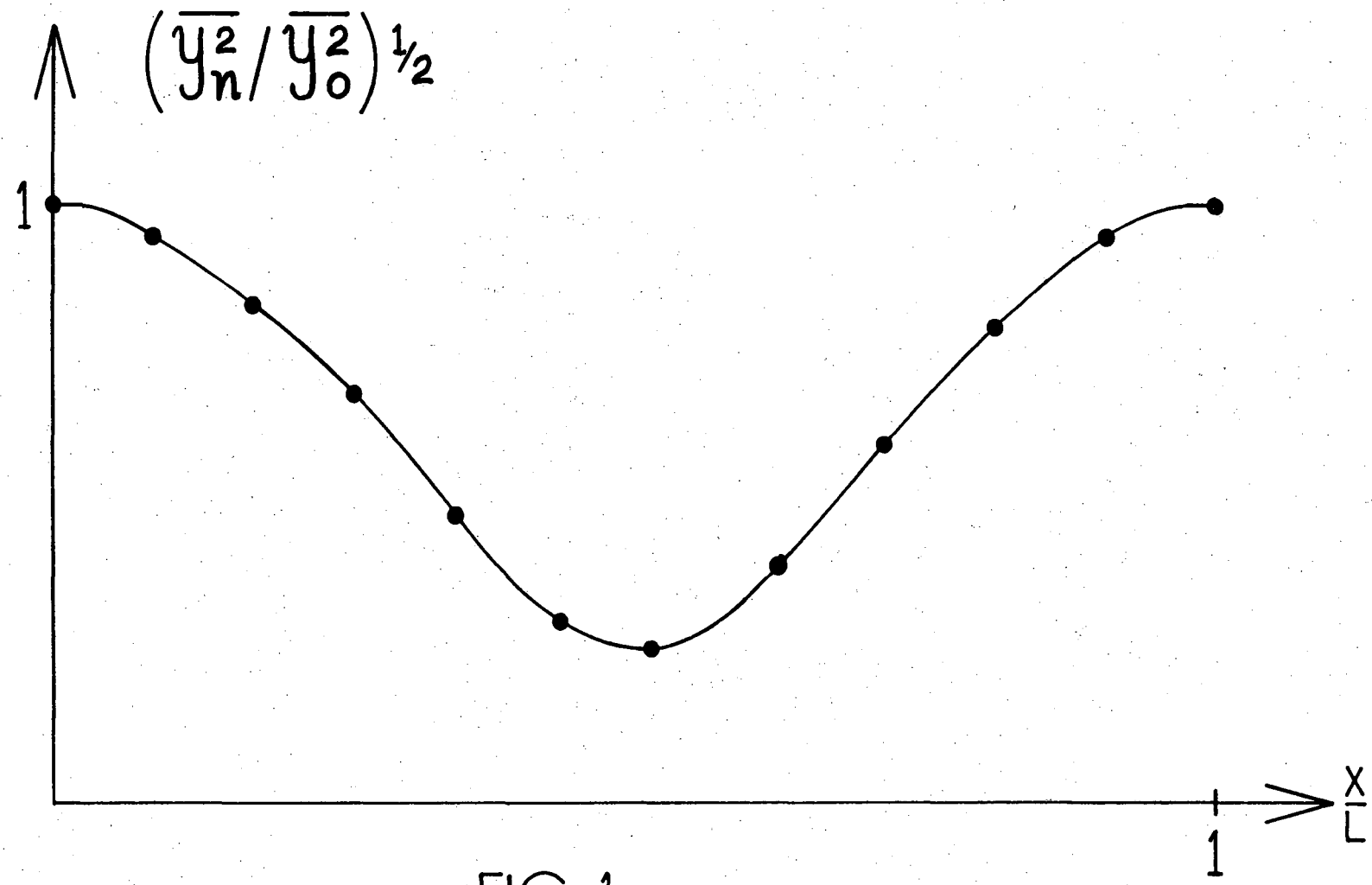

FIG.1

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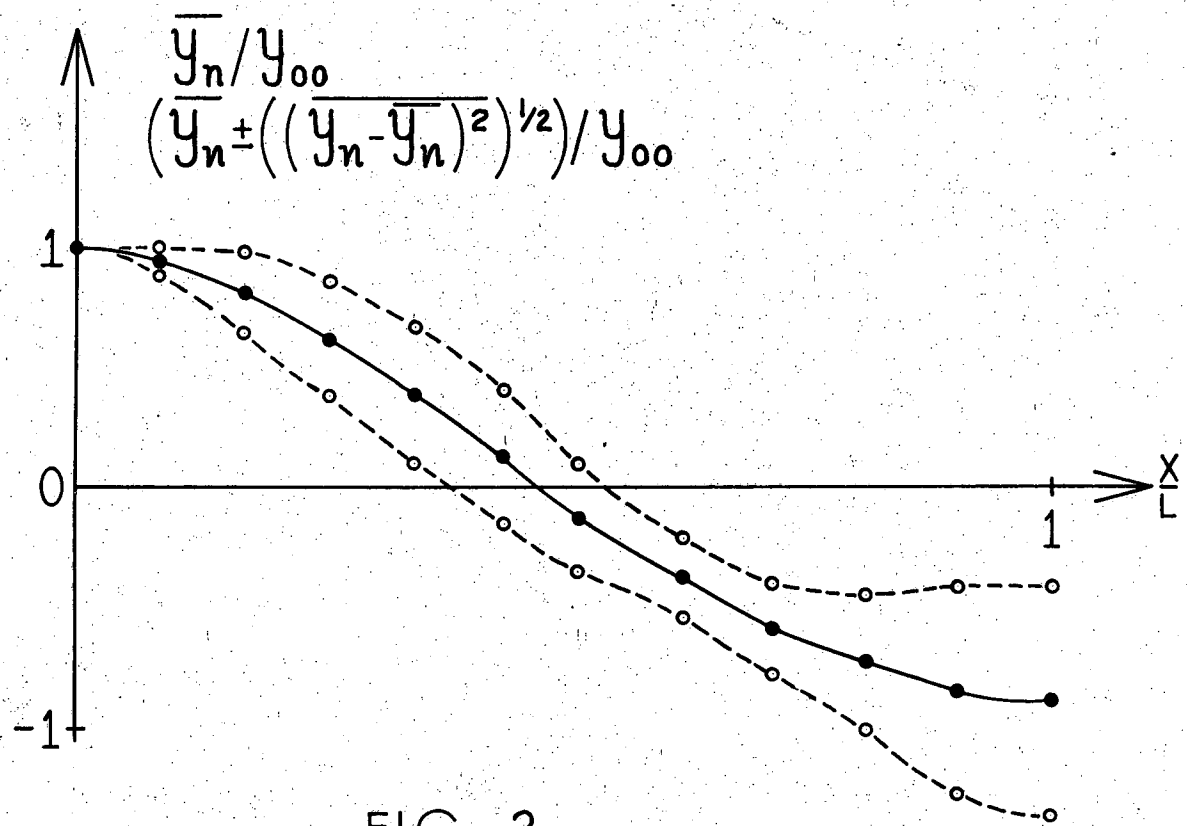

FIG. 2

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